Journal of Industrial and Systems Engineering
Vol. 15, No. 2, pp. 20-34
Spring (April) 2023

Truck and drone routing problem with soft time windows

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Abstract

Due to narrow streets, dense traffic, and transportation restrictions, city logistics operations are under increasing pressure and need innovative approaches. Recently coordination of trucks and drones has been used as a new solution, which can improve the efficiency of city logistics operations. This paper also focuses on a truck and drone delivery system. As the major contributions, this paper develops a new mix integer programming model to formulate the hybrid truck and drone routing problem with soft time windows and proposes an effective two-phased metaheuristic algorithm. To evaluate the performance of the proposed metaheuristic, we carried out numerous computational experiments, where the results show the efficiency of the proposed metaheuristic. Finally; a detailed sensitivity analysis is performed.

Keywords: City logistics, last-mile delivery, truck-drone routing problem, metaheuristics, time window

1- Introduction

In recent years, with the increase in E-commerce popularity and customers' expectations for door-to-door and fast delivery, transportation in urban areas has increased which is responsible for economic, environmental, and social destructive effects. As a solution, researchers proposed to use the unmanned aerial vehicle (UAV) for delivery tasks. They travel at direct distances and dense traffic is not reducing their performance. They provide fast deliveries and produce less air pollution. Although this mode of transportation provides several benefits in comparison to ground vehicles, they have some restrictions. UAVs also called drones have limited battery capacities and payloads, and it is not economical to use them independently. For that matter, various studies suggested hybrid truck and drone delivery systems, which take advantage of trucks' large capacity and drones' high speed (Freitas et al. 2019).

Because of urban restrictions and regulations, there are some difficulties for hybrid truck and drone delivery systems. Security and safety are the biggest issues in drone delivery systems (Chen, et al. 2021), which require the drone to fly within the sight radius of its operator. Drones are generally operated by truck drivers, and it is hard to drive and operate the drone at the same time, so the truck should be parked while the drone is flying. It is also contrary to urban logistic ideals to park a trailer or van at a customer location to service another customer. Taking the mentioned issues into account, we worked on the hybrid truck and drone routing problem with time windows. See Fig. 1. In this study, the truck departs from the depot and transports to some parking stations which have enough space for truck parking and drone operations. We also call these parking stations as rendezvous locations. Then the drone takes off and delivers parcels to the customers.

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ISSN: 1735-8272,

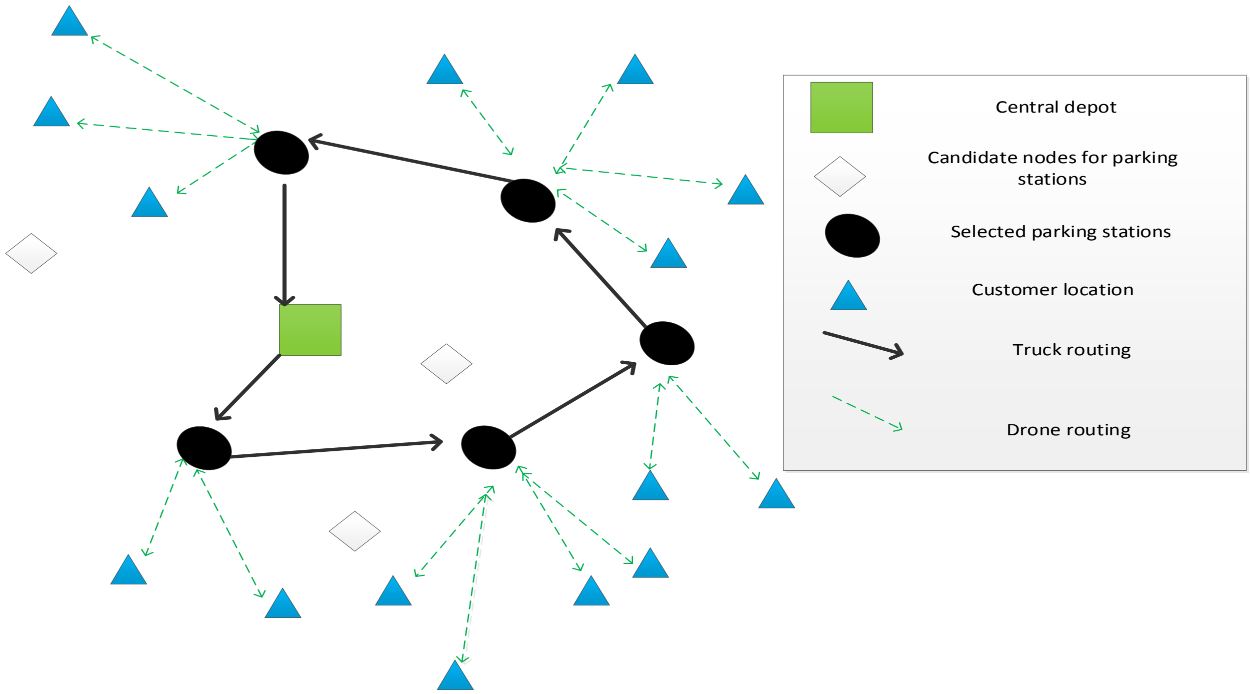

Fig 1. Hybrid truck-drone routing problem

The main contributions of this paper can be summarized as follows:

- This is the first study to model the two-echelon truck-drone routing problem with rendezvous locations and time windows.
- In this paper, we have proposed an effective two-phased metaheuristic algorithm.

The remainder of this paper is organized as follows.

Section 2 represents an overview of related literature. Section 3 represents a mathematical model for the problem. We describe our proposed metaheuristic in section 4. The computational results are provided in section 5, and section 6 outlines future research directions and concludes our study.

2- Literature review

Vehicle routing problem (VRP) is a well-known problem that aims to find vehicle routes to serve customers with minimum costs or minimum delivery time. In recent years, several variations of VRP have been studied in the literature. For instance, Hafezalkotob et al. (2017) considered competition between two distributors and proposed a vehicle routing problem in a competitive environment. Shojaie et al. (2016) considered stochastic demands and time windows and proposed a competitive VRP. Rabani et al. (2021) focused on the transportation of medicines and their wastes and proposed a bi-objective pickup and delivery mathematical model.

A hybrid truck-drone routing delivery system is also a new variation of the *VRP*. Murray and Chu (2015) were the first who coordinate both a delivery truck and a drone for package delivery and proposed the flying sidekick traveling salesman problem (*FSTSP*). In this problem, the drone has maximum traveling distance constraints and it can pick up/deliver only one parcel per flight. There are so many papers worked on this problem (Luo et al., 2021); some extended the problem formulation (Jeong et al., 2019) and some presented a new solution (Freitas et al., 2019).

One extension to FTSP is the traveling salesman problem with drones (*TSP-D*), where the drone is not allowed to start a flight directly from/ to the depot (Bouman et al. 2018, Poikonen et al. 2019, and Wang et al. 2019). Tu et al. (2018) extended this problem to the traveling salesman problem with multiple drones (TSP-mD) in which, the truck travels with m drones instead of one drone.

Murray and Raj (2020) coordinated a truck and a fleet of heterogeneous UAVs and presented the multiple flying sidekicks traveling salesman problem (mFSTSP). Raj and Murray (2020) assumed variable speeds for UAVs and proposed the multiple flying sidekicks traveling salesman problem with variable drone speeds (mFSTSP-VDS).

Another extension of the coordination of trucks and drones is the vehicle routing problem with drones. In this problem, multiple trucks are deployed (Schermer et al. 2019, Poikonen et al. 2017, Chen et al. 2021) which could be equipped with one or more drones.

Although the concept of multi-visits per drone trip is primarily used in surveillance applications (Luo et al., 2021) in city logistics also; it can also reduce costs by eliminating unnecessary truck transportation. We refer to: (Chen et al. 2021, Luo et al. 2021, Kitjacharoenchai et al. 2020)

Most of the related literature focused on the problem in which truck plays the mobile depot role. Considering another application of drones, Pina-Pardo et al. (2021) proposed to use them for truck resupplying and introduced the traveling salesman problem with release dates and drone resupply.

Lu et al. (2017) proposed a two-echelon ground vehicle and a UAV routing problem, in which the ground vehicle can stop recycling and launching the UAV only in rendezvous nodes. Similarly; Karak and Abdelghany (2019) mentioned some real-world city logistics restrictions for the coordination of trucks and drones and proposed the hybrid vehicle-drone routing problem. They adopted the problem to city logistics concept in a way that truck parking stations were different from customers' locations. In this kind of formulation, truck stops were assumed variable, and candidate locations for a parking station could be selected among customers or other feasible locations, so it covers more comprehensive applications of truck and drone coordination. Teimoury and Rashid (2023) also considered the hybrid truck and drone routing problem with non-customer truck stop locations and proposed a large neighborhood search algorithm to solve the problem.

As another aspect of truck and drone coordination; some studies considered the dispatch-wait-collect tactic, in which the drone returns to the same locations at which it is dispatched (Li et al. 2020, Chen et al. 2021), and some assumed a drone can visit the truck in a different location instead of its departure location (Wang et al. 2019, Sacramento et al. 2019, Schermer et al. 2019, Luo et al. 2021).

2-1-Research gap

Only a few studies have assumed rendezvous locations in the literature. Only a limited number of studies have addressed the possibility of multiple drone departures per truck stop. In addition, most studies have focused on either completion time or total delivery time dimensions, while few have taken into account operational costs. Although a few studies have addressed customers' hard time windows, there has been no research on truck and drone routing problems with customers' soft time windows. Thus, we proposed a mathematical model for the truck and drone routing problem with soft time windows. This study considers rendezvous locations and multiple drone departures per truck stop and presents a linear formulation and a metaheuristic algorithm.

3- Formal problem description and formulations

This section defines the truck and drone routing problem with rendezvous locations. In this delivery system, parcels are loaded on the truck, and it transfers them to rendezvous locations in the customers' vicinity. Then parcels are loaded on the drone and it delivers them to the customers.

- The drone visits each customer only once.
- There is a sight radius for drone operation.

- The truck only stops at rendezvous locations. Each rendezvous point represents a parking location with sufficient space for truck parking and drone operation.
- The truck has enough capacity to serve all the demands.
- Each customer has a soft time window; if the delivery system services the customer after or before the time window, a penalty cost must be paid.

We defined the problem on *G*= (*N*, *A*) as a directed graph, where A is the set of arcs and N is the set of vertices partitioned into N =$\{0, s+1\} \cup S \cup C$, in which $0 \; and \; s+1$ represent the central depot, S is the set of rendezvous nodes, and C is the set of customer nodes.

3-1- Mathematical model formulation

In this section, we proposed a MIP model for the problem. The objective function calculates the operational costs.

Sets:

N	Set of all nodes, where $N = \mathrm{S} \cup C$,
S	Set of all rendezvous locations, where *0* and *s+1* demonstrate the depot. $S = \{0,1, \dots, s, s+1\}$. *0* demonstrates the depot index where transportation is started and $s+1$ demonstrates the depot index where transportation is finished,
S_0	Set of rendezvous nodes, $S_0 = \{1, \dots, s\}$,
S_1	Set of rendezvous nodes and *0*, $S_1 = \{0,1, \dots, s\}$,
S_2	Set of rendezvous nodes and $S+1$, $S_2 = \{1, \dots, s, s+1\}$,
C	Set of customers, $C = \{1, \dots, c\}$,
R	Set of all possible drone routes for each selected rendezvous location, where R=$\{1, \dots, \lvert r \rvert\}$. $\lvert r \rvert$ is the maximum number of drone routes per rendezvous location.

Parameters:

TCO_{kf}	Truck transportation costs between node *k* and node *f*,
DCO_{if}	Transportation costs for the drone to service customer *i* from rendezvous location *f*,
$d_{i,k}$	Distance between customer (*i*) and rendezvous location (*k*),
ρ	The maximum flying time that a drone can travel for each departure,
SR	Sight radius for operating the drone,
ds	Drone speed,
tt_{fk}	Truck travel time between node *f* and node *k,*
LTW_j	Starting time of time window for customer *j,*
UTW_j	Ending time of time window for customer *j,*
$TWCO$	Time window violation penalty,
M	Big positive number.

Indexes:

i, j	Customer nodes,
k, f	Rendezvous nodes and depot,
r	Route index for each rendezvous location.

Variables:

Z_k	A binary variable, which gets the value of *1* if rendezvous node *k* is selected; otherwise, it gets *0*,
h_{ik}	A binary variable, which gets the value of *1* in case of assigning customer *i* to rendezvous node *k*; otherwise, it gets *0*,
$U_{k,r}^{i}$	A binary variable, which gets the value of *1* in case of assigning customer *i* to a rendezvous node *k* and route *u*; otherwise, it gets *0*,

T_{fk}	A binary variable, which gets the value of *1* if the truck transports from rendezvous node *f* to *k*; otherwise, it gets *0*,
DA_j	Drone arrival time in node *j*,
TA_k	Truck arrival time at rendezvous location *k*,
TD_k	Truck departure time from rendezvous node *k*,
up_k	Order number of rendezvous nodes,
TWV_i	Time window violation for customer *i*
UV_i	Time window violation if customer *i* is serviced after UTW_j,
LV_i	Time window violation if customer *i* is serviced before LTW_j

Objective function:

$$Min\ Obj = \sum_{k,f \in S_0} T_{fk} TCO_{kf} + \sum_{k \in S_0, i \in C} h_{ik} DCO_{i,k} + TWCO \sum_{i \in C} TWV_i \tag{1}$$

Subject to:

$$\sum_{k \in S_0} h_{ik} = 1 \qquad \forall i \in C \tag{2}$$

$$h_{ik} \le Z_k \qquad \forall i \in C, k \in S_0 \tag{3}$$

$$\sum_{r \in R} U^i_{k,r} = h_{ik} \qquad \forall i \in C, k \in S_0 \tag{4}$$

$$Z_k = 1, \qquad \forall\ k \in \{0, s+1\} \tag{5}$$

$$\sum_{f \in S_2} h_{ik}\, d_{ik} \le SR \qquad \forall i \in C, k \in S_0 \tag{6}$$

$$\frac{\sum_{k \in S_0} 2 h_{ik} d_{ik}}{ds} \le \rho \qquad \forall i \in C, k \in S_0 \tag{7}$$

$$\sum_{f \in S_2 \neq k} T_{kf} = Z_k \qquad \forall k \in S_1 \tag{8}$$

$$\sum_{f \in S_1 \neq k} T_{fk} = Z_k \qquad \forall k \in S_2 \tag{9}$$

$$T_{fk} \le \frac{Z_f + Z_k}{2} \qquad \forall f, k \in S \tag{10}$$

$$up_k - up_f + M(T_{kf}) \le M - 1 \qquad \forall k, f \in S \tag{11}$$

$$TA_k - (TD_f + tt_{fk}) \ge (T_{fk} - 1).M \qquad \forall f \in S_1, \forall k \in S_2 \tag{12}$$

$$DA_j \ge DA_i + \left(\frac{U^i_{k,r-1} d_{ik}}{ds} + \frac{U^j_{k,r} d_{jk}}{ds} \right) - M\left(1 - U^i_{k,r-1}\right) - M\left(1 - U^j_{k,r}\right) \quad \forall i, j \in C, k \in S_0, r \in R \tag{13}$$

$$DA_j \ge TA_k + \frac{U^j_{k1} d_{jk}}{ds} - M\left(1 - U^j_{k1}\right) \qquad \forall j \in C, k \in S_0 \tag{14}$$

$$TD_k \ge DA_j + \frac{U^j_{k,|r|} d_{jk}}{ds} - M\left(1 - U^j_{k,|r|}\right) \qquad \forall j \in C, k \in S_0 \tag{15}$$

$$DA_j - UV_j \le UTW_j \qquad \forall j \in C \tag{16}$$

$$DA_j + LV_j \ge LTW_j \qquad \forall j \in C \tag{17}$$

$$TWV_j \ge UV_j \qquad \forall j \in C \tag{18}$$

$$TWV_j \ge LV_j \qquad \forall j \in C \tag{19}$$

Binary variables: Z_k, h_{ik}, $U^i_{k,r}$

Integer variables: up_k, uc_i

Positive variables: T_{fk}, TA_k, DA_j, TD_k, UV_j, LV_j, TWV_j

The objective function is minimizing the operational costs. Constraints (2 and 3) assign customers to selected rendezvous locations. Constraint 4 assign customers to a drone route of a selected rendezvous location. Constraint 5 selects depot nodes as truck stop locations. Constraint 6 restrict drone traveling distance in the sight radius of rendezvous locations. Constraint 7 restrict drone traveling duration for its battery endurance. Constraints (8, 9, and 10) ensure that the truck travels only between selected rendezvous locations. Also, if a rendezvous location is selected, it will be visited by the truck only once. Constraint 11 is to evade sub-tours. Constraint 12 calculate truck arrival time in rendezvous locations. Constraints (13 and 14) calculate drone arrival time in customer locations. Constraint *15* calculate truck departure time in rendezvous locations. Constraints (16, 17, 18 and 19*)* calculate time window violations) ensure that customers are serviced in their time window durations. To prepare a better description, Fig. 2 is presented. If the drone arrives before the starting time of the time window, the time window violation equals *LV* and will be calculated based on constraints (17 and 19), and if the drone arrives after the finishing time of the time window, the time window violation equals to *UV* and will be calculated based on constraints (16 and 18).

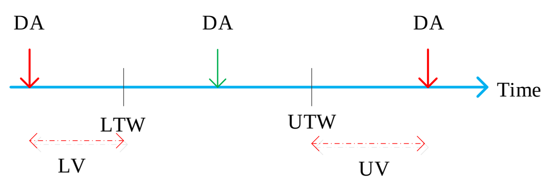

Fig 2. Description of the time window violation

4- Solution method

The hybrid truck-drone routing problem with time windows is a complex optimization problem that requires several considerations: selecting rendezvous locations for truck stop nodes, customer assignment, and scheduling truck and drone routes to optimize operational costs. Due to the computational complexity of this problem, we proposed a two-phased metaheuristic, in which the first phase constructs an initial solution and the second phase improves the solution. Algorithm *1* presents the framework of our two-phased metaheuristic.

Algorithm 1: Hybrid Truck and Drone (HTD) metaheuristic framework
Require: travel time matrix, distance matrix; costs matrix 1-sol ← create an initial solution 2- Sol^* ← solution improvement (Sol) 3- return S^*

4.1- Making an initial solution:

The initial solution phase selects all rendezvous locations, assigns customers to their nearest rendezvous locations, and then randomly schedules customers. After discarding rendezvous locations with no customer, the initial solution phase constructs truck routes. For a better description, the first phase of the proposed algorithm is presented as follows:

Algorithm 2: Create an initial solution

Require: travel time matrix, distance matrix; costs matrix

set_p ← make set of customers assigned for each rendezvous location
    for all $i \in C$, do
        find the nearest rendezvous location (p)
            put i in set_p
    end for
Sol^{d}_{Tsp} ← define customers' sequences
    for all p∈S0, do
        Sol^{dp}_{Tsp} ← randomly schedule customers assigned to rendezvous location p
    end for
discard parking stations with no customer
Sol^{T}_{Tsp} ← Truck scheduling using Regret procedure
return Sol

Where Sol^{d}_{Tsp} defines drone route scheduling, Sol^{dp}_{Tsp} is the drone route scheduling for rendezvous location p, Sol^{T}_{Tsp} is the truck route scheduling for the remaining rendezvous locations and *Sol* defines our initial solution containing selected rendezvous locations, customer assignments, drone routes, and truck routes. In this algorithm, we defined the Regret procedure. The regret process selects the next sequence based on evading the maximum regret. Regret value for scheduling node (j) exactly after node (i) is calculated as follows:

$$regret_{ij} = minimum\left(TCO_{kj}, for\ k \neq selected\ nodes\right) - TCO_{ij} \tag{20}$$

4-2- Improvement phase

This phase of the metaheuristic begins with a solution and generates a neighborhood solution ($\acute{Sol}$) for each iteration. Then *Sol* can be updated based on our Hybrid Simulated Annealing (*HSA*) acceptance criterion. The procedure of the algorithm is continued until in a predetermined number of iterations ($\bar{k}$); the solution is not improved. Algorithm 3 describes the improvement heuristic.

Algorithm 3: Improvement phase

Require: *Sol*, travel time matrix, distance matrix; costs matrix
Ensure: ***Sol***

k←1
    while k≤ $\bar{k}$, do
        $\acute{Sol}$ ←Shake (Sol)
        if $\acute{Sol}$ is not feasible
            k←k+1,
        else
            update the Sol based on *SA*,
            update f_N
            if $f(\acute{Sol}) \leq f(Sol)$
                k←1
            else
                k←k+1
            end if
        end if
    end while

Where *f(sol)* is the objective value, $\acute{sol}$ is the new solution, and f_N is the worst objective value among the last *N* iterations.

For each iteration, the shaking procedure randomly selects an operator among the following operators and constructs a neighborhood solution:

1) Reducing parking stations: This operator randomly selects a rendezvous location, discards it, and assigns its customers to their nearest rendezvous locations. In this operator, the sequence of the remained rendezvous locations will not change.

2) Replacing parking stations: The number of rendezvous locations in this operator will not change, just a selected rendezvous location is replaced with a discarded rendezvous location. For this, a rendezvous location among discarded rendezvous locations in previous iterations is selected, and simultaneously nearest rendezvous locations to that are removed.

3) (1, 1) exchange in the truck route: This operator randomly selects two rendezvous stations and replaces their sequence.

4) Customer insertion: This operator randomly selects a rendezvous location and randomly changes the sequences of its customers.

5) Customer assignment: This operator randomly selects a customer and assigns that to the next nearest rendezvous location. As there may be some rendezvous locations with one customer, this operator also can reduce the number of rendezvous locations.

4-2-1-Acceptance procedure

In this paper, *HSA* accepts all neighborhood solutions if $f(\acute{Sol}) \leq f(Sol)$. For the new solution with $f(\acute{Sol}) > f(Sol)$, operators approve the new worse solution with the following probability function.

$$e^{\left(\frac{-(f(\acute{Sol})-f(Sol))}{T}\right)} \tag{19}$$

Where T (temperature) is reduced after each iteration with $T = T.\varepsilon$, and $0 < \varepsilon < 1$. For the first iteration, T is considered equal to the objective value calculated by the initial solution.

To avoid local optimal solutions, the first operator also accepts a solution rejected by the *SA* probability function, if $f(\acute{Sol}) < f_N$.

5- Computational study

Our proposed algorithm was implemented in Python 3.9.12, and the CPLEX is used to solve the proposed MIP model. All computations are performed on a LENOVO Laptop with Intel(R) Core (TM) i7-9750HF CPU @ 2.60 GHz, 2.59 GHz, and 16 GB installed RAM running Windows 10.

5-1- Test instances

To evaluate the performance of *the proposed solution method*, we generated 250 instances. We generated customer locations and rendezvous locations in a 6000*6000 m2 square. Customers must be covered with the rendezvous locations, so we created them in a way that at least one rendezvous node is in sight radius of each second-class or third-class customer. The sight radius is considered to be 500 meters for this data instance. We have assumed that the depot is located at the vertex of the square (0, 0). We assumed 10 meters per second for truck speed and *20* meters per second for drone speed. We assumed the drone travel costs are equal to 0.01 dollars per kilometer. Also, the truck travel costs are 0.5 dollars per kilometer. For small-size instances, we considered random time windows with 900 seconds duration, and for large-size instances, we considered 1-hour duration for time window durations. It is also assumed that the time window violation penalty is 5 dollars per hour. In this way, we created 25 instance types and generated 10 instances for each type.

5-2- Performance of the proposed metaheuristic

In this section, we considered 200 instances with various customer sizes (8, 12, 16, 24, and 32) and different rendezvous location sizes (4, 6, 8, 10, and 12). Karak and Abdelghany (2019) also studied the two-echelon truck and drone routing problem and proposed a MILP and three heuristic algorithms based on the Clarke and Wright algorithm. One of their proposed algorithms was DDH. To analyze the performance of our MILP and metaheuristic, we adopted their formulation with our proposed problem. We compared the performance of Karak and Abdelghany's (2019) MILP with our proposed MILP and HTD in table *1* for small-size and medium-size instances. For this analysis, we limited the computational time to 1800 seconds. As is demonstrated, our MILP outperforms Karak and Abdelghany's (2019) formulation. The maximum CPU time for HTD for the mentioned instances is 0.46 seconds, and the maximum average gap is 0.0264, which is a reasonable performance for a newly implemented metaheuristic.

We compared HTD with DDH in Table *2* for large-size instances. As it is depicted, although DDH solves the instances in less CPU time, HTD outperforms DDH in terms of objective value which highlights the effectiveness of our proposed metaheuristic.

Table 1. Performance of the *MDR* and *AHTD* for small-sized instances

No.	Total number of nodes	Number of customers	Number of rendezvous locations	Number of Instances	CPLEX: Karak and Abdolghany's (2019) MILP		CPLEX: Our MILP		HTD		
					Average cost	CPU time (sec)	Average cost	CPU time (sec)	Average cost	CPU time (sec)	Average gap (%)
1	12	8	4	10	4.33	0.54	4.33	0.08	4.33	0.02	0
2	14	8	6	10	4.18	3.11	4.18	0.27	4.22	0.02	0.95
3	16	12	4	10	4.91	1.24	4.91	0.15	4.91	0.02	0
4	18	12	6	10	5.33	9.06	5.33	0.42	5.38	0.04	0.93
5	20	12	8	10	4.50	92.18	4.50	1.61	4.57	0.05	1.55
6	20	12	10	10	4.66	1620.67	4.66	5.70	4.73	0.06	1.50
7	20	16	4	10	6.14	3.61	6.14	0.82	6.17	0.05	0.48
8	22	16	6	10	5.71	28.42	5.71	2.14	5.75	0.06	0.70
9	24	16	8	10	5.93	366.89	5.93	6.87	6.02	0.07	1.51
10	26	16	10	10	5.42	1800	5.38	47.44	5.49	0.09	2.04
11	28	24	4	10	6.47	11.55	6.47	1.24	6.52	0.07	0.77
12	30	24	6	10	6.19	108.24	6.19	2.99	6.27	0.08	1.29
13	32	24	8	10	6.61	1800	6.57	18.30	6.68	0.11	1.67
14	34	24	10	10	6.20	1800	6.05	202.55	6.21	0.15	2.64
15	36	24	12	10	6.25	1800	6.23	1614.92	6.30	0.22	1.12
16	36	32	4	10	8.09	1614.30	8.09	2.67	8.14	0.09	0.61
17	38	32	6	10	8.53	1800	8.38	9.04	8.51	0.12	1.55
18	40	32	8	10	8.29	1800	8.21	113.56	8.37	0.17	1.94
19	42	32	10	10	-	1800	8.17	1800	8.11	0.24	-0.73
20	44	32	12	10	-	1800	8.08	1800	7.96	0.46	-1.48

Table 2. Comparing the HTD and DDH for large-size instances

No.	Number of customers	Number of rendezvous locations	Number of Instances	DDH		HTD		
				Obj.	CPU time (s)	Obj.	CPU time(s)	Gap (%)
1	50	15	10	15.87	0.64	16.14	1.15	-1.68
2	75	20	10	16.10	1.18	16.40	2.08	-1.81
3	100	25	10	33.29	2.74	33.99	4.46	-2.08
4	125	30	10	47.08	7.68	48.04	10.10	-2.03
5	150	35	10	50.95	22.43	52.15	33.10	-2.35

We analyzed the sensitivity of operational costs for various time window widths in figure 3. It is illustrated that increasing the time window width reduces truck stop locations, which may result in lower truck transportation costs and fewer time window violations. As it is depicted, for shorter durations of the time window, it has more impact on operational costs. Also, for smaller drone speeds, the operational costs show more sensitivity due to increasing the time window width.

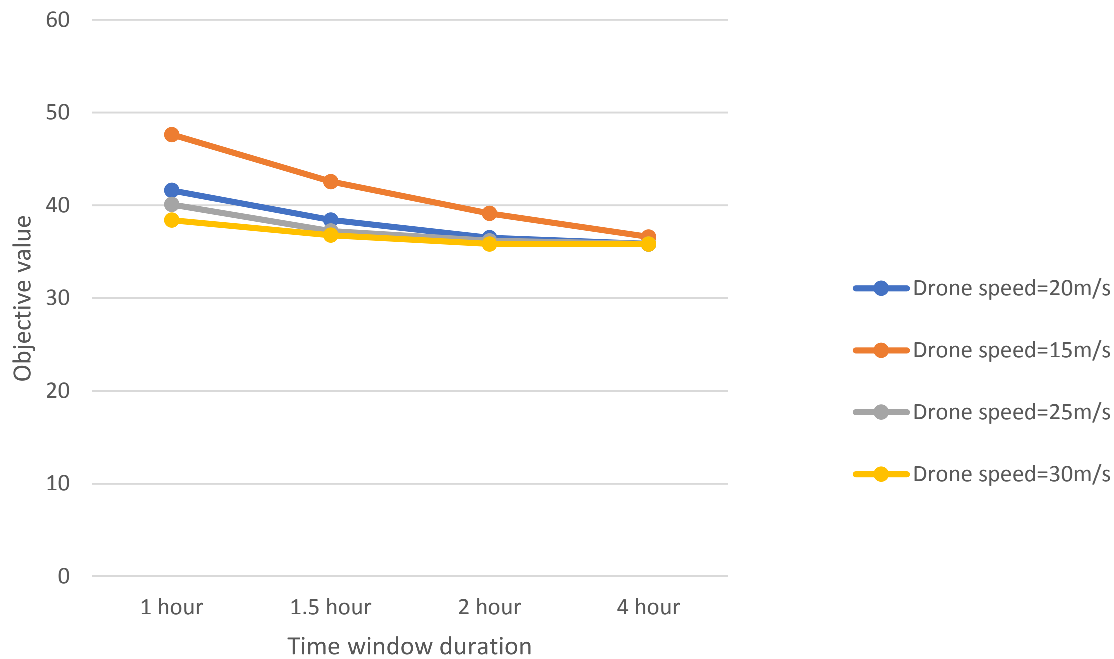

Fig 3. The performance of the delivery system for various values of drone speed and time window widths

Because of tall buildings in urban areas, to ensure the safety of the drone and its parcels, we assumed a sight radius for drone operation (*SR*). Figure 4 and figure 5, depict the overall costs of the system and the number of selected parking stations for various sight radiuses. As it is demonstrated, for small values of drone operational costs the objective value and number of selected rendezvous locations show high sensitivity due to the increase of sight radius.

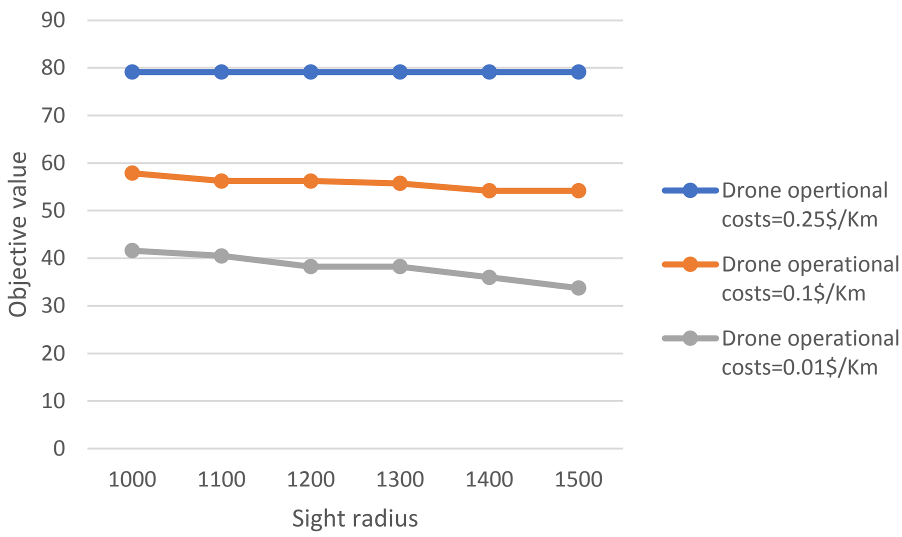

Fig 4. Sensitivity of the system's objective value for various values of drone operational costs and sight radiuses

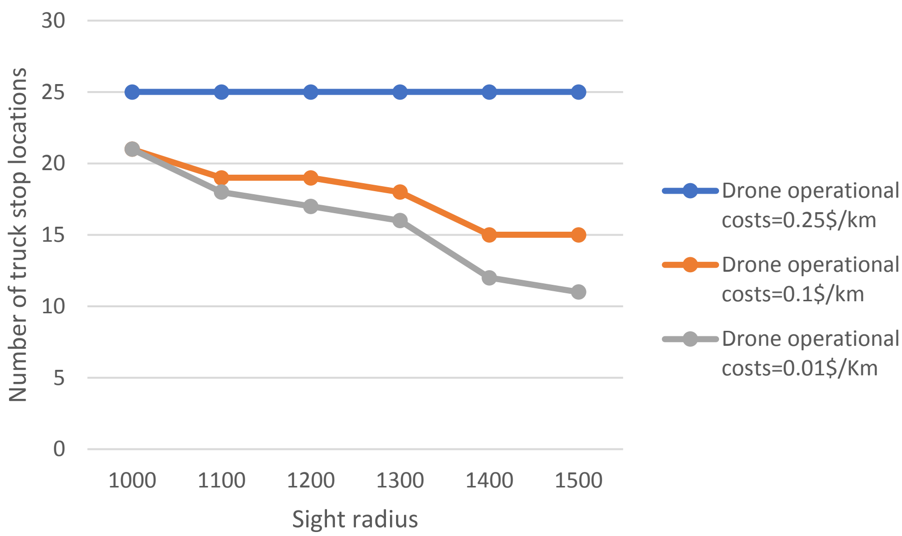

Fig 5. Sensitivity of the number of selected rendezvous locations for various values of drone operational costs and sight radiuses

6- Conclusion

In this paper, we considered the hybrid truck and drone routing problem with soft time windows and proposed a new effective *MILP* model for the problem. The hybrid truck-drone routing problem is known as an *NP-hard* problem, so we developed a two-phased metaheuristic, which provides high-quality solutions with reasonable runtimes for practical and large-size instances.

Applying the proposed heuristic, we have investigated some aspects of the problem and obtained some insights. The overall cost of the delivery system is largely determined by the width of the time window and drone speed. According to the computational results, increasing the sight radius improves the overall costs, which shows that truck and drone delivery systems could be more effective in urban areas with shorter buildings and wider view space.

It would be more practical to consider a delivery system with multiple trucks and multiple drones. The model also could be extended by considering an uncertain environment. Finally, there are also opportunities to present more efficient heuristics.

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