Journal of Industrial and Systems Engineering
Vol. 15, No. 2, pp. 1-19
Spring (April) 2023

Inpatient bed management considering collaboration strategy to enhance hospital resilience

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Abstract

Hospitals are critical facilities which have a great role to affects the number of mass casualties after disasters. Hence, it is necessary to adopt strategies to increase hospitals preparedness and to improve their resilience. The present paper tries to propose a strategy to cope with surge of demands under disruptions in a hospital. An optimization model for bed management considering collaboration between hospital wards in order to minimize the waiting times of the patients provided in this research and the objective function under the proposed strategy and without the proposed strategy were compared. The results show that the proposed strategy can reduce the patients waiting time under disruptions. Due to the complexity of the proposed model, a Lagrangian relaxation-based heuristic is developed to solve the model. Computational results show that the proposed algorithm is able to reach desirable gap in a reasonable time.

Keywords: Bed management, operations research in health care, patient waiting time, hospital performance

1-Introduction

In recent years, number of disasters both man–made and natural has found on increasing trend. In addition, urbanization and environment degradation have had a great impact on the increase of disasters (Cimellaro et al., 2018).

A disaster could lead to heavy demands for hospitals. It is expected that after a disaster, a huge surge of damaged population requests health services. Natural events such as hurricanes and earthquakes as well as man-made like such as terrorist attacks or mass shootings need efficient hospital responses in order to diminish the number of casualties. Moreover, these events could even lead to performance loss at the hospitals; for example, hurricanes could reduce the number of hospital workforces or cause damage to some infrastructural networks like electrical network. So it is necessary to use the resources efficiently and enable the hospitals to deal with increase in demand when their performance decrease (Shahverdi et al., 2019). Hospitals have to be enabled to manage the surge of demands to prevent system failure (Cimellaro et al., 2018).

Hospitals have a significant role in determining the number of causalities after disasters (Achour and Price, 2010), and their importance has grown up over the last 20 years due to the rise in the number of large scale disasters (Sauer et al., 2009) . Therefore, it is critical to ensure the continuation of the operational state of the hospitals after disasters and avoid over-crowding at hospital wards.

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ISSN: 1735-8272,

Overcrowding may cause social and economic losses. Performance reduction of hospital could be translated into increase in patients' waiting time (Cimellaro et al., 2011) and available beds reduction (Jacques et al., 2014).

Hospital resilience is defined as the ability of the hospital to prepare, plan for, absorb and recover from undesired events, and ensures the continuation of its operation after disasters (Zhong et al., 2014a, Zhong et al., 2014b). Resilience is the capacity of the system to adapt, and the flexibility of the system, which enables it to recover the performance to the normal state (Paturas et al., 2010, Braun et al., 2006). From disaster management perspective, resilience is a capability of the medical facilities to maintain their operational level when facing with an undesirable event and respond to the urge of demand (Bruneau et al., 2003, Cimellaro et al., 2010).

Collaboration between partners in a firm can improve resiliency. Members of a firm should work together in order to deal with challenges in the real world (Banomyong, 2018). Collaboration in the field of supply chain has been investigated by many researchers; they have mentioned the benefits of the collaboration such as lower inventory levels and reduction in the number of warehouses and distribution centers (Horvath, 2001, Min et al., 2005, Sahay, 2003, Zacharia et al., 2009, Lehoux et al., 2014). Collaboration enables the firms to improve their business performance and customer satisfaction and helps them to gain more knowledge (Kahn et al., 2006). Collaboration is defined in the literature as the ability or a culture toward a common goal to create value or bring mutual benefits to the partners (Min et al., 2005, Fawcett et al., 2008). The present research aimed at propose a mathematical model for bed management in hospitals in order to minimize the waiting time of patients. Under disruptions, change of the objective caused severe overcrowding in the hospital and can lead to the cessation of patient care services. Therefore, a collaboration strategy is proposed in the mathematical model to improve resiliency of the hospital against surge of demands. The contributions of the present paper are as follows:

- Modeling the bed management problem to allocate patients to the ward based on their attributes such as gender and requested services from resilience perspective
- Considering bed sharing as a collaboration strategy to enhance the hospital resilience
- Developing a Lagrangian based heuristic to solve the proposed model
- Establishing collaboration among the hospital wards to improve performance level and resiliency

This paper is organized as follows: Section 2 provides a review of bed management models in the literature. Section 3 describes the problem and proposes the optimization model. Section 4 provides the numerical result and compares the results under proposed strategies. Section 5 introduces a novel Lagrangian-based heuristic to solve the model. Section 5 provides computational results of the proposed solution method, and Section 6 presents conclusion and future directions.

2-Literature review

Much of bed allocation and bed management literature is on the application of queuing theory and simulation. Preater (2002) reviewed more than 150 papers on the application of queuing theory in healthcare and categorized them based on the areas of Appointments, Departments, Ambulances, Compartmental Modeling, and Miscellaneous. Dangerfield (1999) surveyed the studies about the application of dynamic flow methods.

There are many papers available on the application of queuing theory in healthcare. Pouraliakbari et al. (2017) tried to propose a model for locating healthcare facilities in the competitive location environments which incorporates the theories of customer choice behavior to patronize the facility. They considered multiple type of facilities and used queuing theory for calculating the traveling time. Cochran and Bharti (2006) proposed a stochastic model for bed planning using queuing theory and discrete event simulation (DES). They sought maximizing bed utilization by balancing the demands. Their case study was an obstetric hospital. Gorunescu et al. (2002a) presented a model using queuing theory to determine optimal bed numbers. They also developed another model using queuing theory and investigated the bed occupancy in case of changing the input parameters of the model (Gorunescu et al., 2002b).

There are papers that have used simulation models. Holm et al. (2013) proposed a DES to calculate the optimal beds in different wards. Since the tradition in under study hospital was to place patients in the corridors when the hospital is overbooked, they minimized the number of patients placed in the crowding beds. Moengin et al. (2014) used DES to evaluate and optimize the number of beds with the objective of balancing the utilization of patients with a view to reduce their waiting time. Harper and Shahani (2002) simulated the flow of patients through hospital wards considering elective and emergency patients in order to find the most suitable beds for such patients. Khasha et al. (2018) used DES for improving patients flow in surgical suits. They stated that the proposed scenarios result 22.15% improvement in patient length of stay.

Optimization models have also been used for resource allocation. Yazdanparast et al. (2018) presented an integrated algorithm for optimizing resource allocation in emergency department. Bachouch et al. (2012) proposed a tool based on integer programming to plan hospital beds efficiently. They provided a schedule that enables nurses to allocate patients based on their attributes. Guido et al. (2018) proposed an optimization model for patients' bed admission scheduling, and solved the problem using metaheuristic algorithm. Li et al. (2009) presented a multi-objective model for allocation of beds in hospitals. They were motivated by a real world problem which was the difference between the occupancy rates of a hospital wards in China. They used queuing theory results as an input to the goal programming approach to solve the problem. Blake and Carter (2002) used goal programming to allocate resources in a hospitals. Wu et al. (2019) investigated blockage in health services, and they provided an optimization model using tandem queuing theory.

To the best of our knowledge, the literature about bed planning from optimization perspective is scarce, and a few of them were inspired by a real world problem. In this paper, we propose a model to allocate beds to patients in a hospital using a Lagrangian relaxation based heuristic. It is assumed that each ward of a hospital consists of some rooms, which have finite beds. Based on the domestic regulations, patients who are hospitalized at a room must have same gender. Computational results show that the proposed algorithm can provide efficient lower and upper bounds.

3-Problem description

As mentioned above, the model developed in this paper addresses the allocation of the patients to the beds in hospital wards. We were motivated by the length of waiting list in a hospital in Iran. The process of admitting patients is started by visiting the hospital clinics. When the patients visit the clinic, the physicians examine them and put them in the hospitalization queue, if necessary. It is to be noted that each ward of the hospital has different queues and different averages of length of stay. Then the patients have to wait until the hospital calls them for hospitalization. Figure 1 shows a hospital with two wards; each ward has its own queue for hospitalization, and below of each ward, number of patients in queue is illustrated. It is assumed that there is a rule that the patients hospitalized in a room must have same gender. Based on the care which each patient needs, they can be hospitalized at a different ward out of the requested ward. Figure 1 shows that the patients who are in red have requested to be hospitalized at ward 1 but due to the lack of vacant beds, they were admitted at ward 1 but were hospitalized at ward 2.

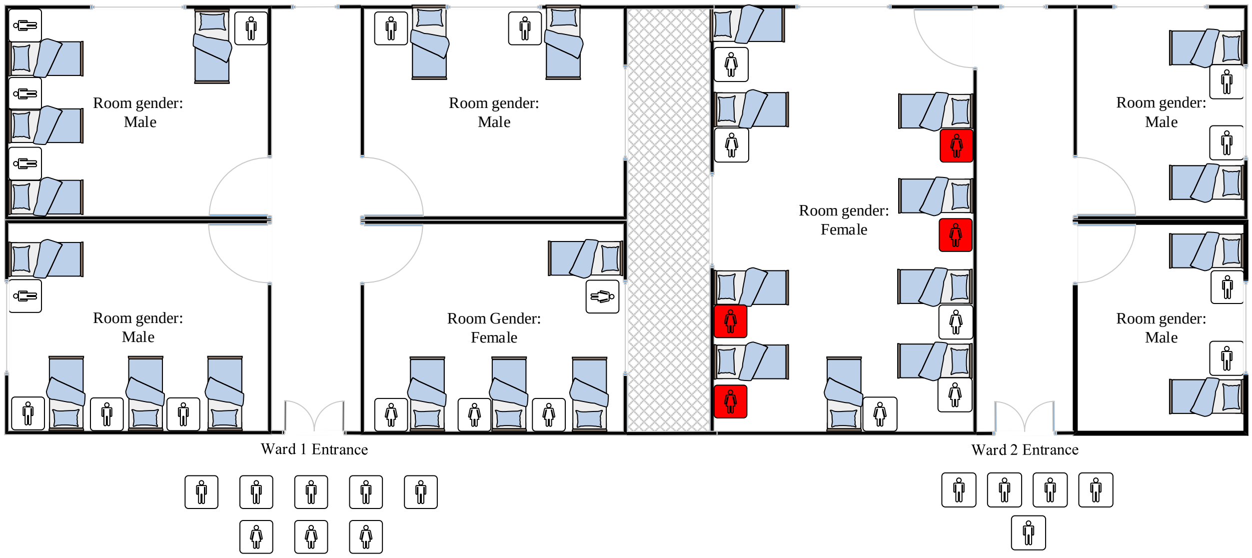

Fig 1. An example of allocating beds to patients

It is assumed that there are two kinds of alliance between the wards: high priority and low priority. The wards can share their beds with other requested wards. The objective function of the model is minimizing the number of patients in the queue for hospitalization. Each ward has a finite number of beds, which are distributed among the finite rooms. The average length of stay of patients at ward and their gender have been considered in the present research,

Sets

I	Set of hospital wards, indexed by i
T	Set of planning horizon, indexed by t
S	Set of gender, if the gender of patient is male then S=1; otherwise, S=2, indexed by s
R	Set of rooms, indexed by r

Parameters

bed_{ri}	Number of beds in room r at ward i
LOS_i	Length of stay of patients at ward i
pat_{sit}	Number of patients with gender s at period t that visit the clinic, and the physician puts them on the queue of ward i
HPS_{ij}	A binary matrix showing the feasibility of high priority sharing between the wards. If ward i can share a bed to ward j and the priority of the sharing is high, the corresponding element would be equal to 1; otherwise, 0.
LPS_{ij}	A binary matrix showing the feasibility of low priority sharing between the wards. If ward i can share a bed to ward j and the priority of sharing is low, the corresponding element would be equal to 1; otherwise, 0.
NR_i	Total number of rooms at ward i
M	A big value

Decision variables

$q_{si\acute{t}t}$	Number of patients with gender *s* at period t who are in queue for hospitalization at ward i and have requested at period $\acute{t}$
x_{rsit}	Number of patients with gender s admitted and hospitalized at room i at ward i at period t
xh_{rsijt}	Number of patients with gender s admitted at ward i and hospitalized at ward j at room *r* at period t using the high priority relationship between the mentioned wards.
xl_{rsijt}	Number of patients with gender s admitted at ward i and hospitalized at ward j at room *r* at period t using the low priority relationship between the mentioned wards.
$numer_{si\acute{\acute{t}}t}$	A binary variable that refers to the state of patients who requested service at period $\acute{\acute{t}}$ at ward *i* at period *t*: it gets the value of 1 if the patients could be serviced, otherwise it gets the value of 0.
H_{it}	A binary variable that refers to the usage of high priority relationship between the wards. If ward i shares at least one bed with other wards the value would be equal to 1; otherwise, 0.
L_{it}	A binary variable that refers to the usage of low priority relationship between wards. If ward i shares at least one bed among other wards the value would be equal to 1; otherwise, 0.
HH_{ijt}	A binary variable that refers to the high priority bed sharing between ward i and j
LL_{ijt}	A binary variable that refers to the low priority bed sharing between ward i and j
$xsum_{sit}$	Number of patients with gender s that admitted at ward i at period t
$tempx_{si\acute{\acute{t}}t}$	An auxiliary free variable which was defined in order to calculate $numer_{si\acute{\acute{t}}t}$.
$rgen_{rsit}$	A binary variable that shows the gender of patients in each room. If the gender of patients in room *r* at ward i at period t is s, it would be equal to 1; otherwise, 0.
avb_{rit}	Number of available beds at room *r* at ward i at period t
$disp_{rijt}$	Number of discharged patients at period t admitted at ward i and hospitalized at room r of ward j.
$nump_{rit}$	Number of patients who are hospitalized at room r of ward i at period t
$numb_{rit}$	A binary variable that refers to the number of patients at each room. If there is at least one patient hospitalized at room r of ward i at period t, it would be 1; otherwise, 0.

$a1_{si\acute{t}t}$	An auxiliary variable to calculate $q_{si\acute{t}t}$
$a2_{si\acute{t}t}$	An auxiliary variable to calculate $q_{si\acute{t}t}$
$a3_{si\acute{t}t}$	An auxiliary variable to calculate $q_{si\acute{t}t}$
$a4_{si\acute{t}t}$	An auxiliary variable to calculate $q_{si\acute{t}t}$

$$\min \sum_{s\in S}\sum_{i\in I}\sum_{\acute{t}=1}^{t}\sum_{t\in T} q_{si\acute{t}t}$$

$$nump_{rit} + avb_{rit} \leq bed_{ri} \qquad \forall\, r \in R, i \in I, t \in T \qquad (1)$$

$$xsum_{sit} = \sum_{r\in R} x_{rsit} + \sum_{r\in R}\sum_{j\in I} xh_{rsijt} + \sum_{r\in R}\sum_{j\in I} xl_{rsijt} \qquad \forall\, s \in S, i \in I, t \in T \qquad (2)$$

$$x_{rsit} + \sum_{j\in I} xh_{rsjit} + \sum_{j\in I} xl_{rsjit} \leq M \times rgen_{rsit} \qquad \forall\, r \in R, i \in I, t \in T, s \in S \qquad (3)$$

$$nump_{rit} = \sum_{s\in S}\sum_{\acute{t}=1}^{t-1} x_{rsit\acute{t}} + \sum_{j\in I}\sum_{s\in S}\sum_{\acute{t}=1}^{t-1} xh_{rsji\acute{t}} + \sum_{j\in I}\sum_{s\in S}\sum_{\acute{t}=1}^{t-1} xl_{rsji\acute{t}} - \sum_{j\in I} disp_{rjit} - \sum_{s\in S}\sum_{\substack{\acute{t}=1\\ \acute{t}>LOS_i}}^{t} x_{rsit(\acute{t}-LOS_i)} \qquad \forall\, r \in R, i \in I, t \in T \qquad (4)$$

$$disp_{rjit} = \sum_{s\in S}\sum_{\substack{\acute{t}=1\\ \acute{t}>LOS_j}}^{t} xh_{rsji(\acute{t}-LOS_j)} + \sum_{s\in S}\sum_{\substack{\acute{t}=1\\ \acute{t}>LOS_j}}^{t} xl_{rsji(\acute{t}-LOS_j)} \qquad \forall\, r \in R, i,j \in I, t \in T \qquad (5)$$

$$numb_{rit} \times M \geq nump_{rit} \qquad \forall\, r \in R, i \in I, t \in T \qquad (6)$$

$$numb_{rit} \leq nump_{rit} \qquad \forall\, r \in R, i \in I, t \in T \qquad (7)$$

$$rgen_{rsit} \leq rgen_{rsit-1} + numb_{rit} - 1 + (1 - numb_{rit}) \times M \qquad \forall\, s \in S, r \in R, i \in I, t \in T - \{1\} \qquad (8)$$

$$rgen_{rsit} \geq rgen_{rsit-1} + numb_{rit} - 1 - (1 - numb_{rit}) \times M \qquad \forall\, s \in S, r \in R, i \in I, t \in T - \{1\} \qquad (9)$$

$$\sum_{s\in S} rgen_{rsit} \leq 1 \qquad \forall\, r \in R, i \in I, t \in T \qquad (10)$$

$$\sum_{s\in S}\sum_{r\in R} rgen_{rsit} \leq NR_i \qquad \forall\, i \in I, t \in T \qquad (11)$$

$$\sum_{r\in R}\sum_{s\in S} xh_{rsijt} \leq M \times HH_{ijt} \qquad \forall\, i,j \in I, t \in T \qquad (12)$$

$$\sum_{r\in R}\sum_{s\in S} xl_{rsijt} \leq M \times LL_{ijt} \qquad \forall\, i,j \in I, t \in T \qquad (13)$$

$$\sum_{r\in R}\sum_{s\in S} xh_{rsijt} \geq HH_{ijt} \qquad \forall\, i,j \in I, t \in T \qquad (14)$$

$$\sum_{r\in R}\sum_{s\in S} xl_{rsijt} \geq LL_{ijt} \qquad \forall\, i,j \in I, t \in T \qquad (15)$$

$$HH_{ijt} + LL_{jit} \leq 1 \qquad \forall\, i,j \in I, t \in T \qquad (16)$$

$$LL_{ijt} + LL_{jit} \leq 1 \qquad \forall\, i,j \in I, t \in T \qquad (17)$$

$$HH_{ijt} + LL_{jit} \leq 1 \qquad \forall\, i,j \in I, t \in T \tag{18}$$

$$\sum_{s\in S} x_{rsit} + \sum_{j\in I}\sum_{s\in S} xh_{rsjit} + \sum_{j\in I}\sum_{s\in S} xl_{rsjit} \leq avb_{rit} \qquad \forall\, r \in R, i \in I, t \in T \tag{19}$$

$$tempx_{si\acute{t}t} = xsum_{sit} - \sum_{k=1}^{\acute{t}} q_{sikt-1} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{20}$$

$$tempx_{si\acute{t}t} \leq M \times numer_{si\acute{t}t} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{21}$$

$$M \times (numer_{si\acute{t}t} - 1) \leq tempx_{si\acute{t}t} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{22}$$

$$M \times numer_{si\acute{t}t} \geq tempx_{si\acute{t}t} + 1 \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{23}$$

$$a1_{si\acute{t}t} \leq numer_{si(\acute{t}-1)t} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{24}$$

$$a1_{si\acute{t}t} \leq numer_{si\acute{t}t} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{25}$$

$$a1_{si\acute{t}t} \geq numer_{si(\acute{t}-1)t} + numer_{si\acute{t}t} - 1 \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{26}$$

$$a2_{si\acute{t}t} \leq numer_{si(\acute{t}-1)t} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{27}$$

$$a2_{si\acute{t}t} \leq 1 - numer_{si\acute{t}t} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{28}$$

$$a2_{si\acute{t}t} \geq numer_{si(\acute{t}-1)t} - numer_{si\acute{t}t} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{29}$$

$$a3_{si\acute{t}t} \leq M \times a2_{si\acute{t}t} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{30}$$

$$a3_{si\acute{t}t} \geq \sum_{k=1}^{\acute{t}} q_{sik(t-1)} + M \times \left(a2_{si\acute{t}t} - 1\right) \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{31}$$

$$a3_{si\acute{t}t} \leq \sum_{k=1}^{\acute{t}} q_{sik(t-1)} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{32}$$

$$a4_{si\acute{t}t} \leq M \times a2_{si\acute{t}t} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{33}$$

$$a4_{si\acute{t}t} \geq xsum_{sit} + M \times \left(a2_{si\acute{t}t} - 1\right) \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{34}$$

$$a4_{si\acute{t}t} \leq xsum_{sit} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{35}$$

$$q_{sikt} = pat_{sit} \times \left(1 - numer_{si(\acute{t}-1)t} - numer_{si\acute{t}t} + a1_{si\acute{t}t}\right) + a3_{si\acute{t}t} - a4_{si\acute{t}t} \qquad \forall\, s \in S, i \in I, \acute{t} \in \{1, \dots, t\}, t \in T \tag{36}$$

$$\sum_{s\in S} xh_{rsijt} \leq HPS_{ij} \times M \qquad \forall\, i,j \in I, r \in R, t \in T \tag{37}$$

$$\sum_{s\in S} xl_{rsijt} \leq LPS_{ij} \times M \qquad \forall\, i,j \in I, r \in R, t \in T \tag{38}$$

$$\sum_{r\in R}\sum_{j\in I}\sum_{s\in S} xh_{rsijt} \leq M \times H_{it} \qquad \forall\, i \in I, r \in R, t \in T \qquad (39)$$

$$\sum_{r\in R}\sum_{j\in I}\sum_{s\in S} xl_{rsijt} \leq M \times L_{it} \qquad \forall\, i \in I, r \in R, t \in T \qquad (40)$$

$$\sum_{r\in R}\sum_{j\in I}\sum_{s\in S} xh_{rsijt} \geq H_{it} \qquad \forall\, i \in I, r \in R, t \in T \qquad (41)$$

$$\sum_{r\in R}\sum_{j\in I}\sum_{s\in S} xl_{rsijt} \geq L_{it} \qquad \forall\, i \in I, r \in R, t \in T \qquad (42)$$

$$L_{it} \leq H_{it} \qquad \forall\, i \in I, t \in T \qquad (43)$$

$$q_{sitt-1} = pat_{sit} \qquad \forall\, i \in I, s \in S, t \in T \qquad (44)$$

$$numer_{si0t} = 1 \qquad \forall\, i \in I, s \in S, t \in T, r \in R \qquad (45)$$

$$xh_{rsiit} = 0 \qquad \forall\, i \in I, s \in S, t \in T, r \in R \qquad (46)$$

$$xl_{rsiit} = 0 \qquad \forall\, i \in I, s \in S, t \in T, r \in R \qquad (47)$$

$$q_{si\acute{t}t} \geq 0, x_{rsit} \geq 0, xh_{rsijt} \geq 0, xl_{rsijt} \geq 0, xsum_{sit} \geq 0, avb_{rit} \geq 0, disp_{rijt} \geq 0, nump_{rit} \geq 0, a3_{si\acute{t}t} \geq 0, a4_{si\acute{t}t} \geq 0, numer_{si\acute{t}t} \in \{0,1\}, \; H_{it} \in \{0,1\}, L_{it} \in \{0,1\}, HH_{ijt} \in \{0,1\}, LL_{ijt} \in \{0,1\}, rgen_{rsit} \in \{0,1\}, numb_{rit} \in \{0,1\}, a1_{si\acute{t}t} \in \{0,1\}, a2_{si\acute{t}t} \in \{0,1\} \qquad \forall\, s \in S, i, j \in I, \acute{t} \in \{1, \dots, t\}, t \in T, r \in R \qquad (48)$$

Objective function tries to minimize the total number of patients in queue. Constraint 1 ensures that summation of the occupied and unoccupied beds at each room is less than the total number of beds. Constraint 2 calculates the total number of admitted patients at each ward at each period. Constraint 3 ensures that the gender of the patients hospitalized at each room is the same as the gender of the room. Constraint 4 calculates the number of patients hospitalized at each room at each period. Constraint 5 calculates the number of discharged patients whose admitted and hospitalized wards are different at each period. Constraints 6 and 7 transform the number of patients hospitalized at each room to a binary variable. Constraints 8 and 9 determine the gender of the room at each period; if the entire beds of a room are vacant at the beginning of a period, these constraints allow that the gender of the room is chosen without considering the gender of the room at the previous period; otherwise the gender of the room must be the same as the previous period. Constraint 10 ensures that a room has only one gender. Constraint 11 ensures that the number of rooms which can hospitalize patients is less than the total number of hospital rooms. Constraints 12 - 15 transform the variables related to the bed sharing among wards to the corresponding binary variables. Constraints 16 - 18 indicate if a ward shares its beds with other wards whether uses high or low priority relationships, sharing in the reverse order cannot be done. Constraint 19 ensures that the total number of hospitalized patients at each room must be less than the total number of available beds. Constraints 20 - 36 calculates the number of patients who are in queue. Constraints 37 and 38 ensure that the sharing between wards happens if the corresponded elements in relationships matrices are equal to one. Constraints 39 to 42 transform the sharing variables to corresponding binary variables. Constraint 43 indicates that if the model uses high priority sharing then the low priority is possible too, but if the high priority relationship is not possible, the model could not use low priority relationship. Constraint 44 adds the demand of each gender at each ward at each period to the end of the corresponding number of patients in queue at the previous period. Constraint 45 adds one at the beginning of the auxiliary binary variable to calculate the number of patients in queue. Constraints 46 and 47 ensure that sharing cannot be done among the rooms of a same ward. Constraint 48 is the general constraint and indicates the upper and lower bounds of the variables.

4-Numerical results

In order to investigate the efficiency of the proposed strategy to share beds among hospital wards, 21 random problems for an assumed hospital were generated and solved under two different strategies. The first strategy refers to a hospital whose beds can be shared among wards and the other one related to a hospital with fixed number of beds at each ward which can be assigned to the patients whom were hospitalized at that wards. Test problems were generated with different number of wards, rooms and period and input parameter of the problem such as number of patients are generated randomly using uniform distribution. As can be seen in table 1**Table 1**, the problems under sharing strategy have better objective function than the problems which the respected strategy was not used but the running times are greater due to the complexity of the model.

Based on the result, it can be said that by increasing the input patients in the hospital, the number of patients who are in queue increases. Each hospital has a bed management procedure to assign patients to the beds. Based on the reason which cause patients to refer, the hospital determines a ward to hospitalize that patient. But if that ward can't admit more patients, due to the capacity, the admitted patients must remain in the hospital until at least one patient discharge from that ward. By increasing the number of admitted patients, the waiting time becomes longer and may cause overcrowding issues at the hospital. Therefore, a strategy proposed in this research in order to cope with the conditions which the demand for hospitalization is increased. As mentioned, the hospital assign the patients to the wards based on reason for referral. It's proposed that each hospital determines a ward as secondary ward for reason for referral. For example, a patient come to the hospital with heart disease, the ward 1 is primary ward and ward 2 is secondary ward. In other word, the patient is assigned to the ward 1, if the patient can't be admitted at ward 1, the bed manager can admit patient at ward 2 which is the secondary ward for his/her disease. If ward 2 has no capacity, the patient remains in the queue until a patient from ward 1 or ward 2 is discharged. In addition, figure 2 provides a guideline to use the proposed model in a hospital.

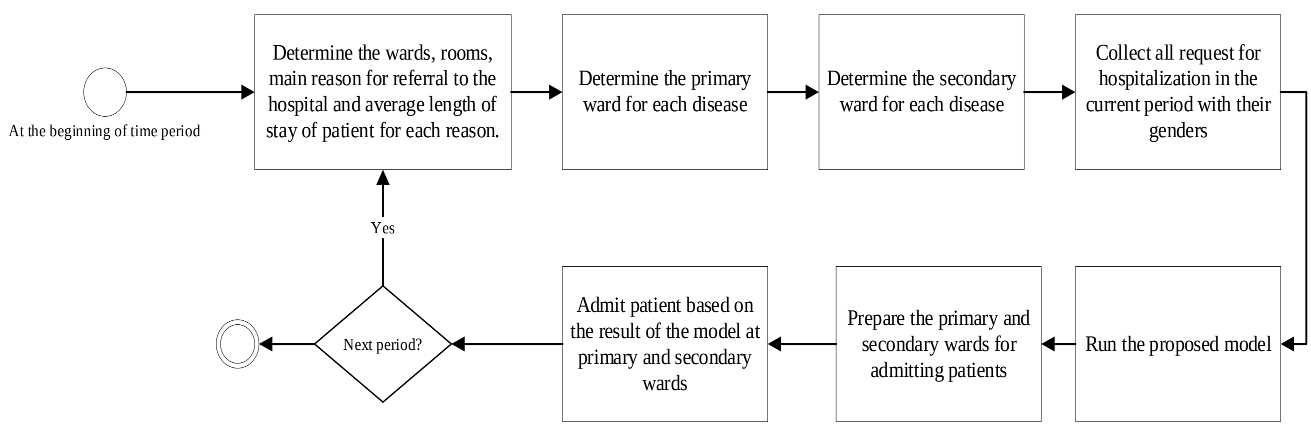

Fig 2. Flow-chart of implementing the proposed model in a hospital

It also was shown in the table 1 that Cplex could not solve the problem under sharing strategy in the 1-hour time limit. Therefore, a solution method based on the Lagrangian relaxation is proposed to solve the larger problem in reasonable time.

Table 1. Configuration of the assumed hospital

Num	Ward	Room	Period	LOS	Patients	Under sharing strategy			Without sharing strategy		
						OFV	Time(Seconds)	Gap%	OFV	Time(Seconds)	Gap%
1	3	3	3	[1,3,2]	(1,8)*	48	0.50	0	52	0.21	0
2	3	3	3	[1,3,2]	(1,10)	71	1.07	0	74	0.14	0
3	3	3	3	[1,3,2]	(1,12)	146	0.84	0	175	0.2	0
4	3	3	4	[1,3,2]	(1,8)	73	2.84	0	151	0.35	0
5	3	3	4	[1,3,2]	(1,10)	75	0.52	0	134	0.28	0
6	3	3	4	[1,3,2]	(1,12)	118	11.73	0	179	0.27	0
7	4	3	3	[1,3,2,3]	(1,8)	71	0.59	0	80	0.22	0
8	4	3	3	[1,3,2,3]	(1,10)	75	0.52	0	160	0.25	0
9	4	3	3	[1,3,2,3]	(1,12)	107	0.62	0	179	0.27	0
10	4	3	4	[1,3,2,3]	(1,8)	96	0.87	0	164	0.44	0
11	4	3	4	[1,3,2,3]	(1,10)	128	2.68	0	212	0.37	0
12	4	3	4	[1,3,2,3]	(1,12)	93	1.22	0	205	0.48	0
13	4	3	5	[1,3,2,3]	(1,8)	333	1h**	3.16	287	1h**	10.88
14	4	3	5	[1,3,2,3]	(1,10)	314	1h**	15.61	342	239.04	0
15	4	3	5	[1,3,2,3]	(1,12)	345	1h**	20.48	399	2.16	0
16	4	4	5	[1,3,2,3]	(1,8)	163	1h**	3.07	168	4.64	0
17	4	4	5	[1,3,2,3]	(1,10)	259	1h**	17.16	281	4.69	0
18	4	4	5	[1,3,2,3]	(1,12)	445	1h**	14.36	448	4.35	0
19	4	3	7	[1,3,2,3]	(1,8)	471	1h**	21.73	487	1h**	12.25
20	4	3	7	[1,3,2,3]	(1,10)	701	1h**	23.00	726	1h**	8.33
21	4	3	7	[1,3,2,3]	(1,12)	761	1h**	22.82	846	1h**	11.12

* Uniform distribution was used to generate input parameter.

** Cplex could not reach the optimal solution in h1- hour time limit.

5-Solution method

5-1-Lagrangian relaxation

Lagrangian relaxation was presented in the 1970s and is a useful approach for solving the complex optimization problems and has been used in health care optimizations (Fisher, 1985). The idea of this technique is to separate the constraints from problem between "easy" and "hard" constraints, and then add the "hard" constraints to the objective function, with each constraint multiplied by a Lagrangian multiplier. The new problem becomes much easier to solve and has some nice properties that help solve the original problem. Two properties that help solve the original problem. Two properties are particularly helpful:

- Lagrangian relaxation provides good – quality upper bounds (in a maximization problem). The bounds from this method better than those resulting from linear relaxation.
- While searching upper bounds, there are several ways to obtain feasible, high quality solutions.

Zhou et al., (2016) and Augusto et al., (2010) are two examples which used Lagrangian relaxation to use hospital resources efficiently. Zhou et al., (2016) used Lagrangian relaxation to optimize the schedule of surgery room. They considered three stages consisting preoperative, operative and post-operative. The model tries to allocate the resources to the patients during the respected stages. Augusto et al., (2010) provided an optimal schedule of operations. They also tried to optimize the allocation of the resources to the patients. Both of the mentioned studies relaxed the capacity constraints to solve the model by Lagrangian relaxation approach.

5-2-Lower bound

Constraints 1, 8 and 9 are relaxed to make the problem simpler, and u_{rit} , $u1_{rsit}$, $u2_{rsit}$ are added as Lagrangian multipliers to these constraints, respectively to ensure that the solutions are not too far from the feasible ones. Therefore, Constraints 1, 8 and 9 will be deleted from the set of constraints and the following expressions will be added to the objective function:

$$\sum_{r\in R}\sum_{i\in I}\sum_{t\in T} u_{rit}(avb_{rit}+nump_{rit}-b_{ri}) \tag{49}$$
$$\sum_{r\in R}\sum_{i\in I}\sum_{t\in T-\{1\}}\sum_{s\in S} u1_{rsit}\,(rgen_{rsit}-rgen_{rsit-1}-numb_{rit}+1-(1-numb_{rit})\times M) \tag{50}$$
$$\sum_{r\in R}\sum_{i\in I}\sum_{t\in T-\{1\}}\sum_{s\in S} u2_{rsit}\,(rgen_{rsit-1}-rgen_{rsit}+numb_{rit}-1-(1-numb_{rit})\times M) \tag{51}$$

The sub – gradient method was used in order to solve the Lagrangian dual problem (Fisher, 1985). The initial values of the multipliers are considered zero and after solving the Lagrangian relaxation problem and using the sub gradient method, the multipliers are updated. Step sizes are used to update the value of the multipliers, and norm is used to calculate the step size. Norms and step sizes in iteration k are, calculated respectively as follows:

$$norm_1^k=\sum_{r\in R}\sum_{i\in I}\sum_{t\in T}(avb_{rit}+nump_{rit}-b_{ri})^2 \tag{52}$$
$$\begin{aligned} norm_2^k = {} & \sum_{r\in R}\sum_{i\in I}\sum_{t\in T-\{1\}}\sum_{s\in S}(rgen_{rsit}-rgen_{rsit-1}-numb_{rit}+1-(1-numb_{rit})\times M)^2 \\ & +\sum_{r\in R}\sum_{i\in I}\sum_{t\in T-\{1\}}\sum_{s\in S}(rgen_{rsit-1}-rgen_{rsit}+numb_{rit}-1-(1-numb_{rit}) \\ & \times M)^2 \end{aligned} \tag{53}$$

Therefore, step sizes and Lagrangian multipliers in iteration k can be, respectively calculated as follows:

$$\mu_1^k=\theta_k\frac{UB^k-LB^k}{norm_1^k} \tag{54}$$
$$\mu_2^k=\theta_k\frac{UB^k-LB^k}{norm_2^k} \tag{55}$$
$$u_{rit}^{k+1}=max\{u_{rit}^k+\mu_1^k(avb_{rit}+nump_{rit}-b_{ri}),0\} \tag{56}$$
$$u1_{rsit}^{k+1}=max\{u1_{rsit}^k+\mu_2^k(rgen_{rsit}-rgen_{rsit-1}-numb_{rit}+1-(1-numb_{rit})\times M),0\} \tag{57}$$
$$u2_{rsit}^{k+1}=max\{u2_{rsit}^k+\mu_2^k(rgen_{rsit-1}-rgen_{rsit}+numb_{rit}-1-(1-numb_{rit})\times M),0\} \tag{58}$$

Where, UB^k and LB^k are the best upper bound found so far and the calculated Lagrangian function, respectively, and $\theta_k$is a constant and its initial value is 1. If the value of the lower bound does not improve for *m* consecutive iterations the value of θ_k will be halved. Figure 3 illustrates the flow chart of the proposed algorithm.

5-3-Upper bound

The other factor affecting the efficiency of the proposed algorithm is upper bound. In the present paper, a heuristic method is used to generate upper bound for the algorithm. It is obvious that the solution of the relaxed problem is not feasible. So, at each iteration, the solution of the relaxed problem is used to generate an upper bound. A relaxed solution may have two possible violations from a feasible solution. The first is that the gender of the room may change even though the whole patients have not been discharged at the end of the previous period. The second possible violation is that the hospitalized patients are more than the total number of beds at each room. Therefore, a three-stage algorithm proposed to generate a feasible solution. The proposed heuristic method checks the gender of the rooms from the first period. At each period, the algorithm checks the gender of the patients hospitalized at the current period and those hospitalized at the previous periods. If the gender of the room is not the same as the previous periods and some patients, hospitalized at the previous periods, have not been discharged so far, the algorithm changes the gender of the room and make all of the patients hospitalized at the current period equal to zero. The second stage of the proposed method checks the number of patients hospitalized at each room. If Constraint 1 is violated, the algorithm will reduce the number of hospitalized patients until it does not violate the related constraint. The final stage of the proposed algorithm checks the unoccupied beds at each room. If unoccupied beds could be found in a room, the algorithm checks the other hospitalized patients; if there is at least one patient hospitalized at the current period, the algorithm increases the number of hospitalized patient to the minimum value of the patients who are in queue and the number of unoccupied beds. Otherwise, it checks the length of the queue for other hospital wards and chooses the longest queue and hospitalize them at the room until it does not violate other constraints.

5-4-Stopping criteria

Three different criteria were used to terminate the algorithm. The first criterion is running 200 iterations, the second criterion refers to when the step size become smaller than ε, and the third one is the gap between the upper bound and the lower bound as follows:

$$Gap = \frac{UB^k - LB^k}{UB^k} < \gamma \tag{59}$$

The algorithm will stop if at least one of the termination criterion is met.

6-Computational results

In order to evaluate the proposed algorithm, 26 random samples were generated. The test problems were solved by the Cplex solver. The Lagrangian relaxation algorithm was coded by the MATLAB 2017 linked with the Cplex Solver. A Core(TM) i7-2600k 3.40 GHz, 8 GB RAM was used for the calculations. The configuration of the assumed hospital consisting of the number of wards, the number of beds in each room and the length of stay at each ward is available in table 2. The results of running the test problems by two mentioned soft wares are shown in table 3. It is worth noting that an hour time limit was set on the running time of solving the main problems by the Cplex. Other parameters consisting of γ, m and ε were considered as 0.05, 5 and 0.1, respectively. Figure 4 and figure 5 show the trend of changes in lower bound, upper bound and step sizes.

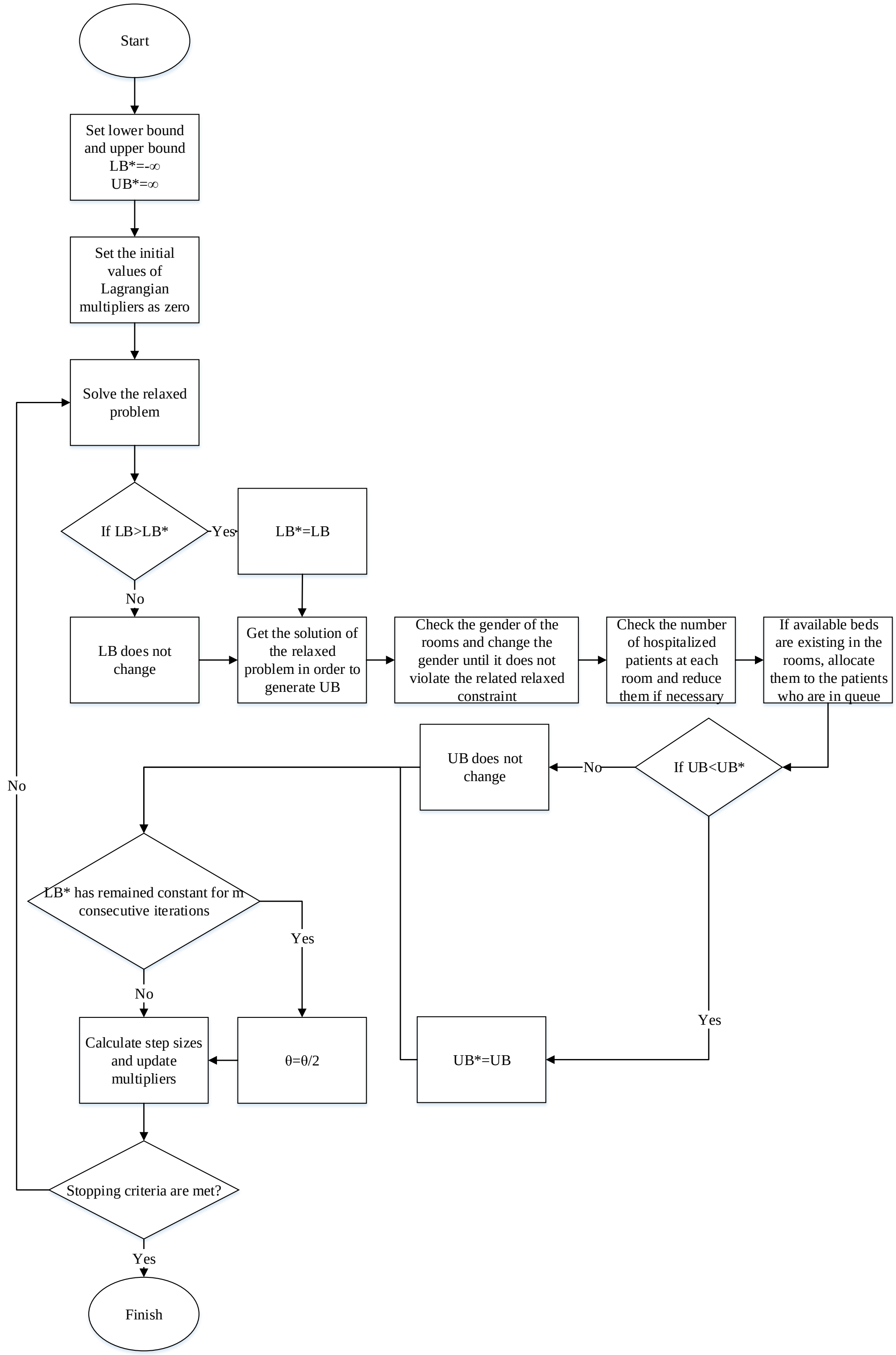

Fig 3. Flow-chart of the proposed Lagrangian based heuristic algorithm

Table 2. Configuration of the assumed hospital

Number of wards	Number of rooms	Length of stay LOS_i	Number of beds bed_{ri}
3	2	[1,3,2]	$\begin{matrix} 3 & 2 & 3 \\ 4 & 3 & 1 \end{matrix}$
3	3	[1,3,2]	$\begin{matrix} 4 & 4 & 2 \\ 4 & 3 & 3 \\ 1 & 1 & 4 \end{matrix}$
4	2	[1,3,2,2]	$\begin{matrix} 4 & 4 & 2 & 2 \\ 4 & 3 & 3 & 3 \end{matrix}$
4	3	[1,3,2,2]	$\begin{matrix} 4 & 4 & 2 & 2 \\ 4 & 3 & 3 & 3 \\ 1 & 1 & 4 & 4 \end{matrix}$

Table 3. Results of random samples

Sample Num.	Ward	Room	Period	Number of patients	Cplex		Lagrangian relaxation			
					OFV	Time(Seconds)	UB	LB	Time(seconds)	Gap%
1	3	2	3	(1,8)*	100	1.16	107	98.17	193.487	8.25%
2	3	2	3	(1,10)	112	0.61	113	111.03	144.62	1.74%
3	3	2	3	(1,12)	164	0.63	174	163.4	239.8	6.09%
4	3	2	3	(1,14)	184	0.67	198	182.69	311.45	7.73%
5	3	3	3	(1,8)	60	2.03	71	58.111	224.32	18.15%
6	3	3	3	(1,10)	93	0.72	98	90.95	45.89	7.19%
7	3	3	3	(1,12)	110	1.02	120	108.2343	97.004	9.80%
8	3	3	3	(1,14)	312	25.94	328	309.61	942.72	5.61%
9	3	2	4	(1,8)	124	14.08	135	122.36	727.07	9.36%
10	3	2	4	(1,10)	205	7.28	223	200.81	832.7	9.95%
11	3	2	4	(1,12)	271	7.63	288	267.31	752.25	7.18%
12	3	2	4	(1,14)	312	4.13	335	308.52	831.05	7.90%
13	3	2	5	(1,8)	157	470.9	181	155.33	1778.04	14.18%
14	3	2	5	(1,10)	282	102.25	316	280.63	910.95	11.19%
15	3	2	5	(1,12)	369	113.76	404	365.19	1120.82	9.61%
16	3	2	5	(1,14)	420	250.5	461	418.56	1158.49	9.21%
17	3	3	5	(1,8)	215	1h**	249	213.1	1123.24	14.42%
18	3	3	5	(1,10)	134	1h	152	130.78	1662.02	13.96%
19	3	3	5	(1,12)	333	2619.4	382	323.75	1244.14	15.25%
20	3	3	5	(1,14)	343	1772.8	381	342.03	1431.11	10.23%
21	4	2	5	(1,8)	220	1h	236	217.78	2019.44	7.72%
22	4	2	5	(1,10)	369	1h	383	364.51	1212.23	4.83%
23	4	2	5	(1,12)	400	1h	438	398.33	1681.37	9.06%
24	4	2	5	(1,14)	407	1h	449	401.67	1743.65	10.54%
25	4	3	5	(1,8)	175	1h	195	173.85	2346.14	10.85%

Table 3. Continued

Sample Num.	Ward	Room	Period	Number of patients	Cplex		Lagrangian relaxation			
					OFV	Time(Seconds)	UB	LB	Time(seconds)	Gap%
26	4	3	5	(1,10)	301	1h	330	295.75	1984.79	10.38%
27	4	3	5	(1,12)	420	1h	466	414.5	2519.7	11.05%
28	4	3	5	(1,14)	575	1h	621	567.73	1795.48	8.58%

* Numbers in the parentheses show the lower bound and upper bound for uniform distribution

** The best solution that found by the Cplex under 60 minutes.

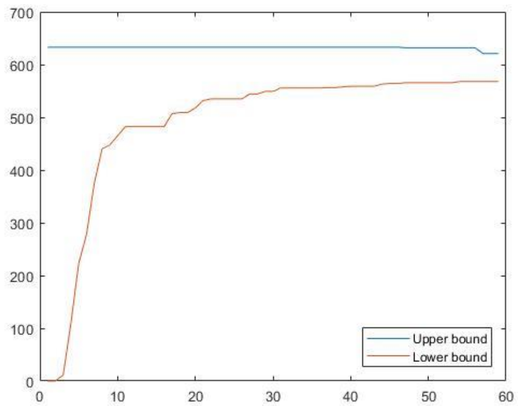

Fig 4. Lower bound and upper bound trends for sample number 28

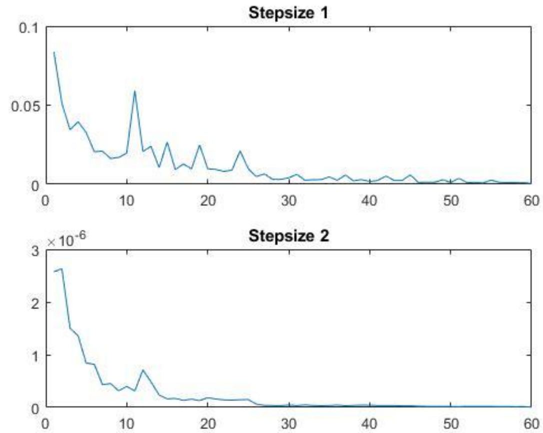

Fig 5. Step sizes trends for sample number 6

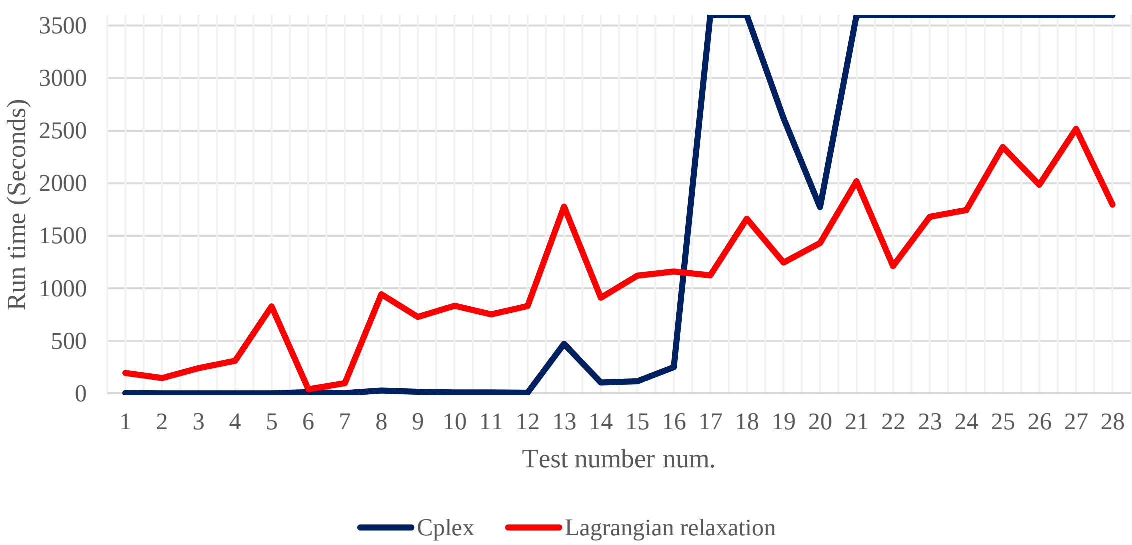

Fig 6. Comparing the run time needed by the Cplex and Lagrangian relaxation for solving the test problems

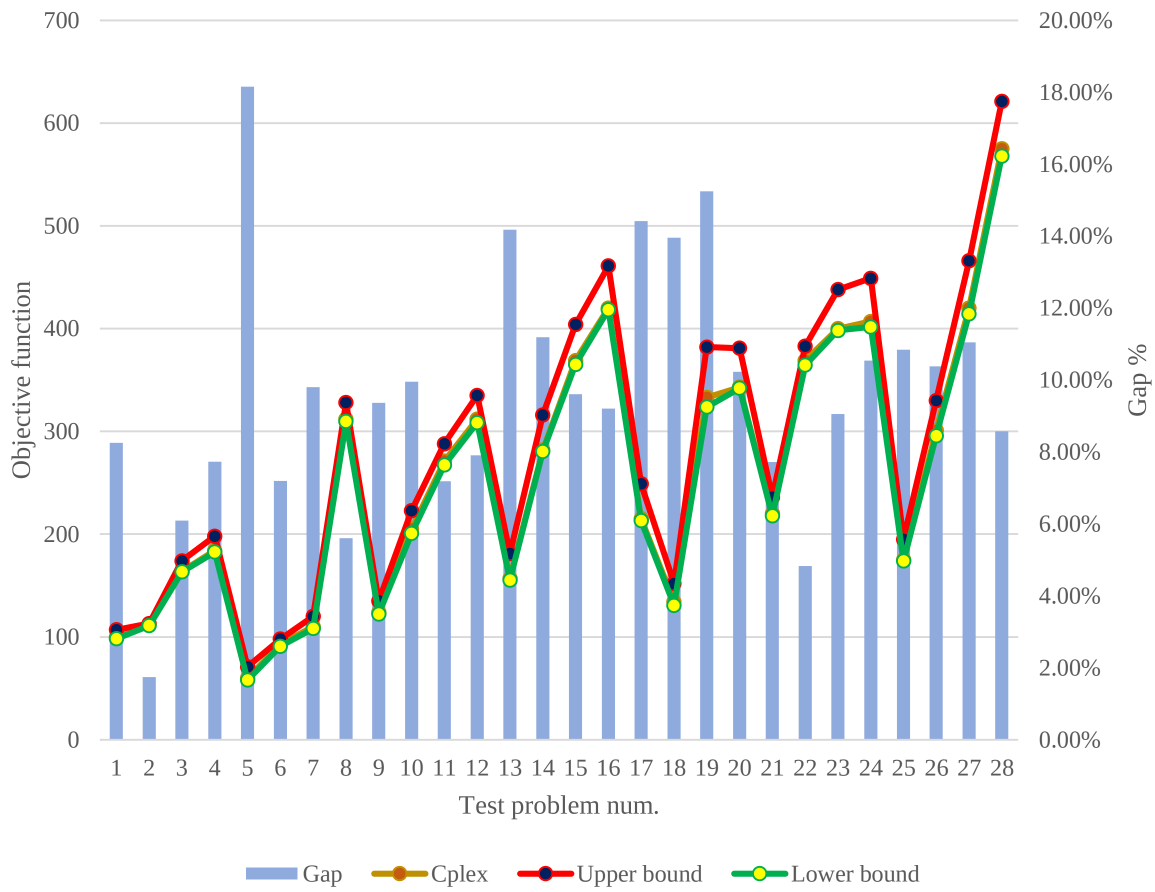

Fig 7. Comparing the objective function obtained by the Cplex and with lower and upper bounds given by Lagrangian relaxation algorithm

7-Conclusion and directions for further researches

In this paper, collaboration between hospital wards as a strategy for improving the hospital resilience was investigated. The model allocates patients to the hospital wards considering the patients' gender, destination ward, length of stay and the room of the wards. The model aims at minimizing the total number patients in a queue. Under the disruptions, the demand for receiving healthcare at hospitals increases. Therefore a strategy is needed for improve resiliency of the hospital against this issue. The collaboration between hospital wards, considered as strategy to cope with the surge of demands. The results show that the proposed strategy can improve the waiting time of the patients at the hospital compare with the situation which the hospital don't share beds between wards.

Since Lagrangian relaxation approaches have been successfully used to solve different problems, a Lagrangian-based heuristic was developed to solve the proposed model. Two groups of constraints consisting of gender of room constraints and capacity of room were relaxed in order to make the problem simpler. A lower bound was obtained by solving the relaxed model. Due to the infeasibility of the solution of relaxed model, a heuristic method was used to make the solution feasible and also consider it as an upper bound for the main problem. If the gap between the mentioned bounds is less than the tolerance or the size of the norm becomes smaller than the pre specified limit, the algorithm will stop; otherwise the Lagrange multipliers are updated until at least one of the stopping criteria is met. The computational results show that the proposed algorithm is able to reach a near optimal solution to the problem.

Future works can add the operating rooms to the proposed model and develop an integrated model. It is an interesting subject to propose an integrated model that be able to consider the operating room together with the hospital rooms.

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