Journal of Industrial and Systems Engineering
Vol. 9, special issue on supply chain, pp 73 – 87
Winter (January) 2016

Cooperative network flow problem with pricing decisions and allocation of benefits: A game theory approach

Fateme Naseri[1], Ashkan Hafezalkotob[1*]

[1] *Industrial Engineering College, Islamic Azad University, South Tehran Branch, Tehran, Iran.*

chemicaleng.naseri@gmail.com, a_hafez@azad.ac.ir

Abstract

Several real problems in telecommunication, transportation, and distribution industries can be analyzed by network flow models. In revenue management, pricing plays a primary role which increases the profit generated from a limited supply of assets. Pricing decision directly affects the amount of service or product demand. Hence, in traditional maximum flow problem, we assume that the demand of sink nodes depends on price of services or products of that nodes. We first develop a mathematical programming model for decision making of pricing by multiple owners in the maximum flow problem. Afterwards, coalitions between owners will be analyzed via different methods of cooperative game theory. A numerical example is given in order to show how these methods suggest appropriate assignments of extra revenue obtained from the cooperation among the owners.

Keywords:
Network flow; pricing decision; flow game; cooperative game theory; coalition.

1- Introduction

There are many practical problems in which different industries deal with network flow problems, therefore; the literature on this context is extremely comprehensive (Schrijver, 2002). Traffic movement, hydraulic systems, water pipelines and specially, oil, gas and petrochemical pipelines, distribution networks of products, and data communication networks are remarkable examples of these industries. Moreover, there are several examples of systems with their essential purpose for being transportation of goods from one source to an indicated place of use. Since the demand and transshipment increase with progress of these industries, infrastructure development concentrates on developing capacity of the networks (Scotti, 2012). Because of the complex structure of a real network, changing it is not much easy, thus improving and optimized exploitation of the current network would be highly important (Szoplik, 2012).

A network is a directed graph with two sets of symbols: nodes and arcs. In the network flow problems, each arc has a capacity which restricts the flow from passing through it. Moreover, the amount of flow that is going to a node must be equal to the amount of flow going out of the node. The nodes can be interpreted as locations, while the arcs may be roads, pipelines, airlines, etc. which link the location (Ahuja R. K., 1993). Maximum flow problem is one of the most important issues in network flow issues, and in most of models, the arcs are limited in capacity (Winston, 2003). It was first formulated in 1954 by Harris and Ross as a simplified model of Soviet railway traffic flow (Ford and Fulkerson, 1965; Harris and Ross, 1995).

---

*Corresponding author.

The main objective of the maximum-flow problem is transmitting the maximum amount of flow from initial nodes (called the network's source nodes) to terminal nodes (called the network's sink nodes) regarding capacities of the arcs.

Flow games are the games that are associated with flow situations (Saad, Han, Debbah, Hjørungnes, and Basar, 2009). Each player owns one part of the network, and the flow game deals with cooperation of owners to effectively transmit products to the sink nodes. In the network of petroleum products, companies (or even countries) can be assumed as independent owners (i.e. players) while the refineries are the source and consumers are the sinks of this network. As an example, the Russian Federation is the largest exporter of natural gas to European Union. There is a gas transmission network that some countries own and control a part of that. Due to benefits of natural gas against fossil fuel, the demand for gas in Europe is increasing, but the variety of national policies and situations of large exporters for larger dependents of Russian gas has become an eminent problem in European politics into Russia (Pirani and Yafimova, 2009; Hafezalkotob and Makui, 2015). This real example confirms the importance of cooperation among players in a multiple owner network.

Other examples are electricity transmission and wireless or communication networks. In recent years, game theory is used to analyze the communication networks for developing independent, and flexible mobile networks and also to mitigate complexity of distributed algorithms that can show and compare competitive and collaboration scenarios (Reyes, 2005; Schintler et al. 2005). In the framework of wireless networking, the players can be the users or network operators controlling user's devices. According to the practice of game theory, it has assumed that the players are rational, therefore they try to maximize their profits (F´elegyh´azi and Hubaux, 2006; Saad, 2010). The recent surveys in the cooperative paradigm of wireless networks, point out that the assumption of coalitional games-based approaches is quite natural. In this field, the coalitional games demonstrate a very powerful tool for designing fair, practical, and efficient cooperation strategies in the communication networks. Meanwhile, electricity transmission network in Latin America is a real example of multi-owner network where private financier, government or integration of private financier and government own the electricity transmission companies (Hammons, 2001).

In the logistic as well as telecommunication networks, pricing is significant factor which influences the amount of demand in the source nodes. Hence, beyond transshipment decisions, pricing is an important lever that increases profits of the network owners. The companies often build extra capacity during the boom-in-demand periods and shut down some capacity during the slowdown. However, ideas from revenue management propose that a company should first adopt pricing to obtain some balance between demand and supply (Altman and Wynter, 2004). We incorporate pricing and transshipment decisions of the players in the flow game problem. Afterwards, we adopt cooperative game theory methods to address this research question "how different network owners can cooperatively determine transshipment and pricing decisions to achieve the maximum profit?"

This paper is organized in six sections. The related literature is reviewed in Section 2. The mathematical model and assumptions are presented in Section 3. In Section 4, the cooperative game theory methods are discussed briefly. A numerical example is provided in Section 5. The paper is concluded in Section 6 with some suggestions for further research.

2- Literature Review

There are different factors such as pricing decisions which affect the demand of commodities in the logistic networks. Sharkey (2011) considered a class of network flow problems in which the demand levels of the nodes are determined through pricing decisions. He showed that this class of problems with a single pricing decision throughout the network can be solved in a polynomial time under both continuous pricing restrictions and integer pricing restrictions (Sharkey, 2011).

Fixed price policy of a firm in the supply chains can be often determined by analyzing a supply chain planning problem in which some decisions should be made to meet the demand at the minimum cost. Ahuja el al. (1993) modeled this problem as the minimum cost network flow problem in appropriately defined network. Glover and Klingman (1979) studied on classes of problems including the network problems underlying shipping and distribution decisions. Roson and Hubert (2014) condisered expected costs and benefits of cooperations in distribution networks.

There have been many supply chain planning problems that incorporate pricing decisions into them. For instance, Florian and Klein (1971) and Kunreuther and Schrage (1971) set the foundation for inventory planning problems with price-driven demand production. Besides, Deng and Yano (2006) focused on production and pricing decisions in the multi-period framework of a single product over a limited horizon for a capacity-constrained producer facing price-sensitive demands. Gilbert (1999) considered the problem of jointly determining a single price and production scheduling for a product with the seasonal demand. He assumed that the demands in different periods do not depend on the price. Moreover, Geunes et al. (2006) and Geunes et al. (2008) investigated a single-stage planning implying pricing decision and the demand levels to maximize the profit. On the other hand, Van den Heuvel and Wagelmans (2006) examined inventory planning problems with deterministic price-driven demand.

Thomas (1970) focused on simultaneous making price and production decisions for a single product with a known deterministic demand function while the back orders were not allowed. Kunreuther and Schrage (1973) concentrated on the interrelationship between optimal pricing and inventory decisions for a retailer with an outer distributor and for non-seasonal items. They also developed an algorithm for order and pricing decisions of a manufacturer with single commodity under deterministic demand situation.

Nowadays, cooperative pricing is one of the marketing approaches that focuses on coordination of the prices. When a firm keeps the price steady but its rivals reduces the prices, the firm will probably lose the market to the rivals. However if they all agree upon a reasonable price and keep the price in that level, all the participants will get appropriate and reliable profit. The objective in this paper is to show that the profits of logistic firms rise when the companies coordinately make the pricing decisions.

On the other hand, when the logistic companies collect their transportation requirements, the cost will decrease as well. Lozano et al. (2013) investigated the horizontal cooperation among shippers in order to achieve logistic cost saving. They studied the cost saving that companies can obtain if they merge their requirements, and the most profitable coalition was identified by testing all the possible coalitions. Reyes (2005) used the cooperative game theory to maintain the stable conditions in the logistic network. Hafezalkotob and Makui (2015) investigated cooperative game theory in multiple owner networks under the capacity uncertainty of the networks. They addressed the question that how the independent owners of a network should collaborate to achieve a reliable maximum flow.

Juan et al. 2014 analyzed the cooperative and non-cooperative scenarios about the amount of saving in routing and emission costs and compared them with each other. Sepehri (2011) formulated an integrated multi-period and multi-product supply flow network and discussed about cost and inventory benefits of cooperation in these problems (Lin and Hsieh, 2012).

Transmitting the goods, services and data with high efficiency is an important issue for companies; hence, they have high incentive to cooperate in order to improve the network efficiency. In the collaborative agreements, all aspects of this problem such as deciding about quantity of flow, price of the goods and allocation of the cooperation benefit should be well designed. To the best of our knowledge, there exists no extensive research that considers the cooperative pricing in the network flow problems. Consequently, there are two main contributions in this research. First, we present a mathematical model in which the participants (i.e., logistic companies) of a network determine the price and flow quantity to obtain the maximum profit. Second, when the optimal flow and pricing decisions are calculated for all possible coalitions of the network participants, we propose a wide range of game theory methods to allocate extra benefits of cooperation to the participants.

3- Prerequisites and assumptions

The following assumptions are made to specify the scope of this work for further formulation of the model:

Assumption 1. There is just one source, multi players (owners) and a set of final nodes as the consumers. Each player controls a part of the network and determines the price of the product by itself.

Assumption 2. Each player or coalition of them sets a single price through the network. It means that a single price is specified at all the sink nodes (i.e. customer locations) that have been managed by a player or a coalition. The demand for the costumer nodes depends on the pricing decisions which can be expressed by non-increasing convex functions.
Assumption 3. There is no loss in any node. That is, except for the source and sink nodes, the amount of incoming flow in a node is equal to the amount of outgoing flow. This assumption is common in the flow game models which are considered by the conservation-of-flow constraints (Rockafellar, 1984; Bertsekas, 1998; Lozano et al. 2013).
Assumption 4. There is no uncertain parameter and all the parameters are deterministic.
Assumption 5. The network owners are profit-seeking players (firms). Therefore, the utility of each coalition can be estimated by the generated profit of the corresponding network. According to the main assumption of the cooperative game theory (or transferable utility (TU) games), the utility is transferable. The transferable utility implies that the players can freely transfer the utility (i.e. profit) among themselves. This simplifying assumption is often adopted in the cooperative game theory (Fathabadi and Ghiyasvand, 2007).

4- Model formulation

We consider cooperation in a multi-owner network in which some logistic companies (i.e., players) manage and control specific parts of this network. They independently decide about the amount of flow and the price of goods in order to maximize their own profits. Thus, these companies can individually deliver their product to the consumers, or they may form coalitions and make cooperative decisions about the amount of flow and also the price.
In this section, a network flow problem with pricing decisions for the multiple owner-network is first formulated. Afterwards, we develop a mathematical model for cooperative situations i.e., a coalition of owners of the network. In the cooperative model, the coalition of the owners cooperatively determines the flow in their sub-networks as well as the price of its products.
A directed graph is a set of objects (called vertices or nodes) that are connected together, where all the edges are directed from one vertex to another that can be denoted by $G=(A,V)$, where V is a set of vertices and A is a set of nodes. The directed graph is sometimes called a digraph or a directed network. Each arc $(i,j)\in A_{C_m}$ as a capacity is denoted by u_{ij} and a cost per unit flow is denoted by c_{ij}. It is assumed that the costs are non-negative, i.e., $c_{ij}\geq 0$, for all $(i,j)\in A_{C_m}$ Each node $i\in N$ may be regarded as a supply node, transshipment node, or demand node in the problem. It is assumed that there is a single supply node, $C_m\in N$, in the network. If there are multiple supply nodes, one can add an artificial supply node *s* and an arc from *s* to each of the essential supply nodes to convert the problem to an equivalent single supply node problem. Let *D* be the set of demand nodes in the network. A demand function, $d_i(p_i)$, is associated with each demand node $i\in D$ which assigns the amount of demand at node i as a function of price. The decision variables in our problem are the prices at the demand nodes, p_i for $i\in D$ and the amount of flow placed on each of the arcs, x_{ij} for $(i,j)\in A$. The network flow problem with the pricing decisions for one player can be formulated

$$\text{maximize} \sum_{i\in D} p_i d_i(p_i) - \sum_{(i,j)\in A} c_{ij}x_{ij} \tag{1}$$

Subject to

$$\sum_{j:(i,j)\in A} x_{sj} - \sum_{j:(j,i)\in A} x_{js} = 0 \tag{2}$$

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = -d_i\left(p_i\right) \text{ for } i \in D \quad (3)$$

$$0 \le x_{ij} \le u_{ij} \text{ for } (i,j) \in A \quad (4)$$

$$c_{ij} \ge 0 \text{ for } (i,j) \in A \quad (5)$$

$$p_i \ge 0 \text{ for } i \in D. \quad (6)$$

The first part of the objective function represents the total revenues received, while the price for each unit of flow is called p_i . The second part of the objective function represents the cost of the flow in the network. c_{ij} stands for the cost of each unit of flow and x_{ij} shows the maximum flow of the arc. The decision variables in this problem are the maximum flow $\left(x_{ij}\right)$ and the price of each unit of flow$\left(p_i\right)$. Constraint (2) emphasizes that there is no loss in each node, whereas Constraint (3) ensures that the demand of each node is met.

We now define the multi-owner network flow problem with the game theory approach. There is a non-empty and finite set of players as $P = \{1, 2, ..., n, .., N\}$. In a network flow with several players (multiple-owner graph) there are N ($N \ge 2$) players (or logistic companies) which own and control different arcs of the network. Let define the set A i.e. $\left\{A_{\{1\}}, A_{\{2\}}, .., A_{\{n\}}, .., A_{\{N\}}\right\}$ such that it represents the arcs of the network graph $G = (A, V)$ owned by N players, that $\bigcup_{n\in N} A_{\{n\}} = A$ and $\bigcap_{n\in N} A_{\{n\}} = \varnothing$. In the flow games, depending on the game type the owners can be players, or they can confirm the coalitions (Reyes, 2005). Let C denote the possible coalition between a subset of the players. The coalition owns all arcs of its members. For example, consider players one and two form a coalition, then the formed coalition owns the arcs of $A_{\{1\}} \cup A_{\{2\}}$.

A simple example is used to explain the model clearly. Assume that there are some goods to be transferred from node so (the source) to node si (the sink) and the maximum possible flow should pass, also the optimum price should determine to reach the maximum benefit in Fig. 1. For this graph $G = (A, V)$, $V = \left\{so, 1, 2, si_1, si_2\right\}$ is the set of nodes and the set of arcs is $A = \left\{(so,1), (so,2), (1, si_1), (1, si_2), (2, si_1), (2, si_2)\right\}$. This graph is a multiple-owner type and belongs to two players; i.e. $P = \left\{1, 2\right\}$. The first player owns the upper arcs i.e. $A_{\{1\}} = \left\{(so,1), (1, si_1), (1, si_2)\right\}$, and the second one owns the lower arcs i.e. $A_{\{2\}} = \left\{(so,2), (2, si_1), (2, si_2)\right\}$. Therefore, the coalition that is formed by the first and second players is $A_{C_m} = A_{\{1,2\}} = A_{\{1\}} \cup A_{\{2\}}$.

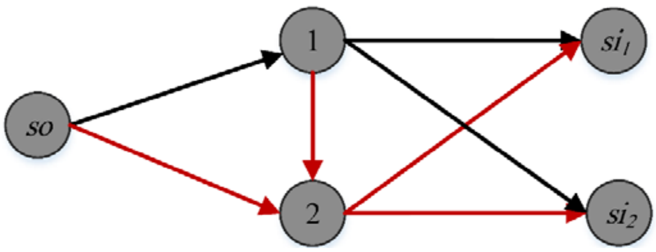

Fig 1. An example of two-owner network with pricing decisions.

The new concept of this paper is firstly, to solve the maximum flow and the pricing model to reach the maximum benefit considering all possible coalitions of the graph owners. According to supper additive property, the benefit of the network flow for any coalition should be greater than the total utilities of members of the coalition. Afterwards, different methods of the cooperative game theory (CGT), namely shapely value, core, τ-value and minmax core are used to allocate the excess benefit to the graph owners. The first of all, indices, input parameters and decision variables are defined for further model formulation.

3-1- Indexes

i, j, k index of nodes;

s index of sink nodes;

m index of coalitions;

E_j initial nodes for node j, i.e. these nodes have arcs with initial node j;

F_j terminal nodes for node j, i.e., these nodes have arcs with terminal node j;

x_{0,C_m} total flow units entering the sink by the arcs owned by coalition C_m.

3-2- Parameters

c_{ij} cost of arc (i, j);

$d_{s,j}$ demand at the node s for coalition j;

u_{ij} capacity of arc (i, j) which limits the flow units that can be transferred through;

π_i profit of player i;

3-3- Decision variables

x_{ij} flow units moving from node i to node j, $(i \neq j)$;

p_i price of each unit of demand at node i.

For a multi-owner network, the class of network flow problems with pricing decisions can be formulated as

$$\text{maximize } v(C_m) = \pi(C_m) = \sum_{i \in A_{C_m}} p_i d_i(p_i) - \sum_{(i,j) \in A_{C_m}} c_{ij} x_{ij} \quad (7)$$

Subject to

$$\sum_{i \in E_j} x_{ij} = \sum_{k \in F_j} x_{jk}, \text{ where } (i, j) \text{ and } (j, k) \in A_{C_m} \quad (8)$$

$$\sum_{i \in E_j} x_{ij} = d_{s,i}(p_i), \text{ where } j \in A_{C_m} \quad (9)$$

$$x_{0,C_m} = \sum_{i \in E_s} x_{i,s}, \text{ where } (i, s) \in A_{C_m}, \quad (10)$$

$$x_{ij} \leq u_{ij}, \quad (11)$$

$$x_{ij} \geq 0 \quad (12)$$

The main objective of the problem is to maximize the profit obtained from total flow through arcs owned by coalition C_m. The first part of the objective function represents the total revenues received by coalition C_m , while the second part represents the cost of the flow in the network. The conservation-of-flow constraint set (8) assures that the flow into each node is equal to the outgoing flow; therefore, no flow unit gets lost while being passed through the nodes. The Constraint sets (9) ensure the amount of flow in network is equal to the demand in the network. The Constraint sets (10) implies that the total units of output flows of the source node is equal to total units of input flows into the sink nodes; hence, no unit gets lost while passing through the network owned by the coalition. The capacity Constraint set (11) guarantees that the flow units passing through the arcs do not exceed their defined capacities.

The owners of the graph have to make collaborative decisions based on their arc capacities. An acceptable solution for real application of the maximum flow problem should remain feasible and almost optimal under all probable scenarios. The maximum flow problem is first presented for the multiple-owner graph under uncertainty, and then the collaboration mechanism between the owners in order to improve the graph efficiency is discussed subsequently.

At first, the model considering the maximum flow for each company have been solved independently. Then, the model is solved again merging the maximum flow for every coalition of two companies, then considering the coalitions of three companies, and so on, until reaching the grand coalition.

The game is supper additive, if the optimal profit for any coalitional scenario is greater than sum of the individual maximum profit of the coalition members i.e.,

$$\pi_{C_m} \geq \sum_{i \in C_m} \pi_i \tag{10}$$

The extra profit of coalition C_m is the difference between coalitional maximum profit and sum of the separate maximum profit, that is

$$EP_{C_m} = \pi_{C_m} - \sum_{i \in C_m} \pi_i \ , \tag{11}$$

These extra profit depends on the synergy between the profits of the collaborating companies. Actually, the extra profit computed with this model are a reliable measure for that synergy.

$$synergy\left(C_m\right) = \frac{EP_{C_m}}{\pi_{C_m}} = 1 - \frac{\sum_{i \in C_m} \pi_i}{\pi_{C_m}} . \tag{12}$$

The criteria of synergy can be used for evaluating effectiveness of cooperation among members of a coalition. A higher synergy criteria implies that the members' cooperation generates a greater extra profit.

4- Cooperative game theory

Game theory can be divided into two sections: non-cooperative and cooperative. The two branches of game theory are different in how they formalize the dependence among the players. In the non-cooperative theory, a game is a model with details of all the moves possible to the players. On the other hand, the cooperative theory normally avoids this level of details, and describes only the consequence that result when the players come together in different coalitions. A game is cooperative, when decision makers work together in a joint activity to reach the common purpose. The game theoretic concepts apply whenever the actions of several agents are interdependent. These agents may be individuals, groups, firms, or any combination of them. The concepts of game theory provide a language to formulate, structure, analyze, and understand the players' behavior.

The cooperative game theory investigates such coalitional games with respect to the relative amounts of power held by various players, or how a successful coalition should divide its proceeds. This is most naturally applied to situations arising in political science or international relations, where the concepts like power are most important. Under cooperative games, players can coordinate their strategies and share the payoff (Barron, 2013).

4-1- Imputation

For a set of players $P = \{1, 2, ..., n, .., N\}$, $v(P)$ defines the available result when all the players cooperate together. A payoff vector $\vec{x} = (x_1, x_2, ..., x_N)$ represents an imputation (or allocation vector) in which x_k is profit allocated to player i. The first desirable property of an imputation is that the total amount received by the players should be $v(P)$. Many imputations of wealth may make up a single solution (Branzei, Dimitrov, and Tijs, 2008) Suppose a real number for each player as x_i, $i = 1, 2, 3, ..., N$, while the set of imputation for a cooperative game is given below:

$$X = \left\{ \vec{x} = (x_1, x_2, ..., x_N) \middle| x_i \geq v(i), \sum_{i=1}^{N} x_i = v(P) \right\}. \quad (13)$$

4-2- Core

Core is the most popular solution concept for the cooperative game. The core of a cooperative game is relevant to the great coalition's stability. According to super additivity, the players has a motive to form the grand coalition thus, the core is the set of allocations as each coalition receives at least the compensations participated with that coalition. One way to describe the fact that one imputation is better than another one is the concept of domination. The core consists of all the stable imputations, that is:

$$core(0) = \left\{ \vec{x} \in X \middle| e(C_m, \vec{x}) = v(C_m) - \sum_{i \in C_m} x_i \leq 0 \right\}. \quad (14)$$

When $e(C_m, \vec{x}) \leq 0$ for all the coalitions, then the players would be happy with the imputation and they no longer wish to switch to another coalition. The core allocations provide the effects with a motive to maintain the grand coalition. It is not guaranteed that the core of games always exists. Actually, in many games, the grand coalition is not stabilized and the core is thus empty. The second issue is that the core would be quite large, so that it can be difficult to select the appropriate core. The last matter is that the allocations which fall in the core can be unfair to some players (Shapley and Shubik, 1972).

4-3- Minimax-Core

Minimax core is a rule that is used to minimize the possible absence for the worst case. A value is attributed to each situation or condition of the game. This value is computed by recourse of a position assessment function and it demonstrates how good it would be for a player to reach that situation. The player then takes the move that maximizes the minimum valence of the situation resulting from the competitors' possible subsequent moves (Palais, 1970). The following linear programming problem acquires the minimax core as follows:

$$Min \varepsilon \quad (15)$$

Subject to:

$$e(C, \vec{y}) = \upsilon(C) - \sum_{i \in C} x_i \leq \varepsilon, \quad for\, all\ C \subset P, C \neq P. \quad (16)$$

4-4- Tau-Value

The τ-value is defined as the efficient allocation τ:

$$\sum_{i \in P} \tau_i = \upsilon(P) \quad (17)$$

Such that:

$$\tau = m + \alpha(M - m) \quad (18)$$

For some values of α, where M and m are the utopia payoffs and the minimum rights vectors, respectively. The τ-value can only be computed if the game is of admissible compromise (Driessen, 1985).
Shapley value is a classical cooperative solution contents, it allocates a proportional benefit of coalition to each player for minimizing the maximum dissatisfaction. It encourages the players to join to a coalition (Geunes et al. 2006). The Shapley value $\phi(\upsilon)$ assigns to each agent i his expected marginal contribution, presuming that each of the n! orders occurs almost likely (Barron, 2013):

$$\phi_i(\upsilon) = \frac{1}{n!} \sum_{\sigma \in \pi_N} m_i^{\sigma}, i \in N. \tag{19}$$

5- Numerical example

In this section, a network flow model is presented consisting of one source (S), three players (A, B, C) (see Fig. 2), in which one commodity is delivered from the source to a client at nodes 6,7,8 and 9, without any loss at the other nodes. The numbers shown on the arcs are, first the cost of carrying each unit of the flow, and second the capacity of the arc.
This model can be considered as a gas pipeline network in which three companies aim to deliver the product from a refinery to their consumers. Furthermore, it may represent a single source model for the mobile networks, each player manages a part of the network, and they want as many active flows as possible over the time. These firms determine the price of their products or services at the demand point as well.

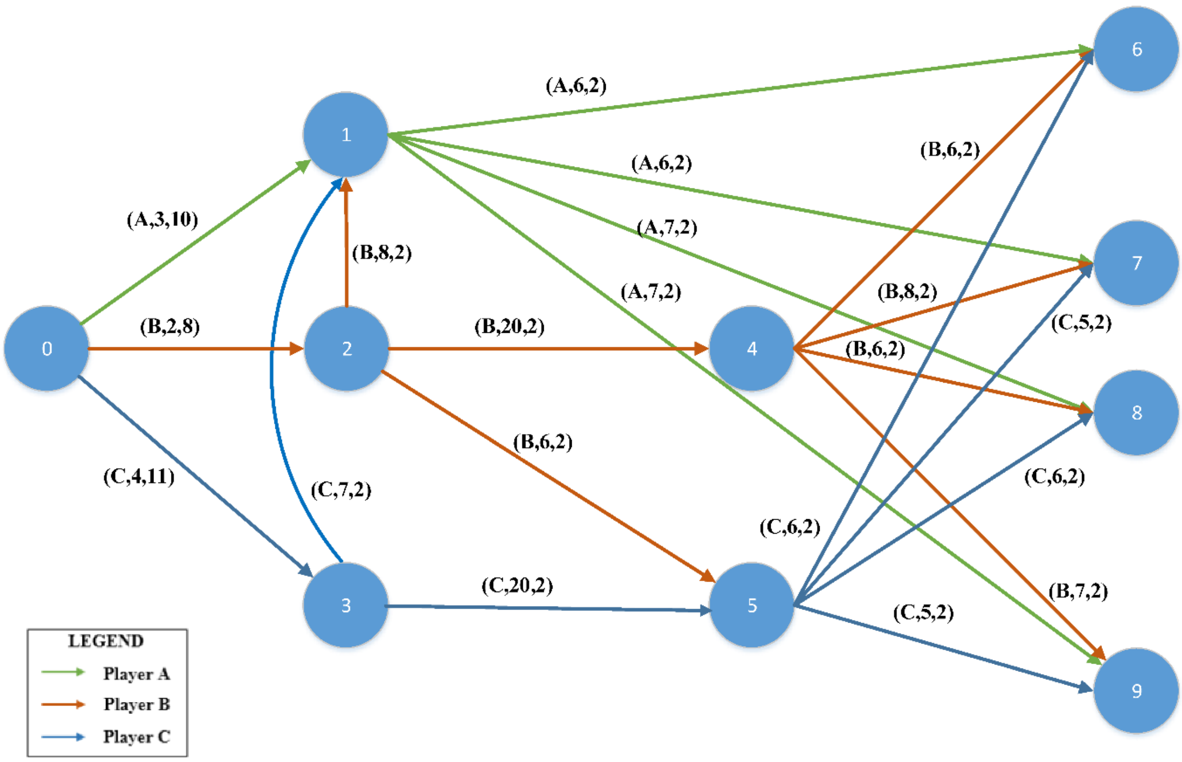

Fig 2. Numerical example network

Model (7) – (13) must be solved seven times, so that for each possible coalition S: {A}, {B}, {C}, {AB}, {AC}, {BC} and {ABC} the maximum profit and corresponding extra profit and synergy can be computed as listed in Table1. As shows in this table, the maximum price belongs to the players {B},{C} and the coalition {B,C}, while the minimum price is related to the coalition {A,B,C}. Due to the principle rules of economy, when the price rises, the demand will goes down, so it is clear that the profit will decline down as well. According to the price of players {B} and {C}, it is not beneficial that players {B} and {C} form a coalition. The sum of profits for the players {B} and {C} is equal to the profit of coalition {B,C}. The maximum synergy goes to the grand coalition {A,B,C}, so this coalition has the largest portion from the extra profit of coalition to the profit of coalition. In other words, the coalition {A,B,C} is the best coalition to form.

Table 1. Optimal profit and synergy for each of the possible coalitions

Coalition	Sales	Price	Profit	Extra profit	Synergy
{A}	13	10.5	81.67	0	0
{B}	12	12	60	0	0
{C}	13	12	60	0	0
{A,B}	38	10.5	149.5	7.8	0.05
{A,C}	50	9.5	163.33	21.7	0.13
{B,C}	25	12	120	0	0
{A,B,C}	54	11	274.5	72.8	0.26

In Fig. 3 the amount of sales is compared between the sum of independent sales of players and the sales after forming a coalition. As shown on this chart, the amount of sales have been increased when the companies forms a coalition. For example the total sales of players A and B, when they sell their products separately, is 25 units, but, when they form the coalition, their sales rise to 38 units together.

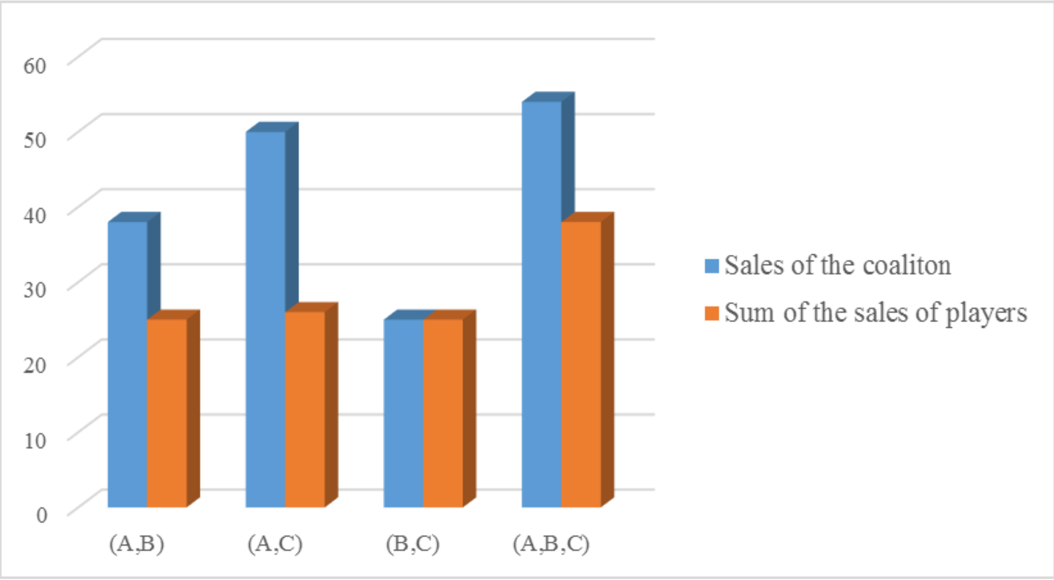

Fig 3. Comparison column chart between total sales of players and sales of coalitions

In some cooperative games, a tension occurs when the participants decide about sharing the obtained extra profit and when in the player's opinion the extra profit was not shared in a fairy manner. Therefore, if they thought in this way, they will not be willing to join a coalition. Therefore, allocating the benefit of cooperation would be a high-priority problem for the corporations. The achieved extra profit can be fairly allocated CGT methods. Table 2 summarizes the allocations found, in this instance, by different CGT methods, namely the Shapley value, the τ -value and the core-center. All but the last method have been computed using TUGlab (Mirás Calvo, 2006). The least (Minimax) core problem is solved by an optimization software to achieve the results of the last column (Soyster, 1973).

Table 2. Allocation of the extra profit $v(P)$=({A, B, C}) =72.8 according to CGT methods

Company	Shapley	τ –value	Core center	Minimax core
{A}	29.1	28.1	26.2	26.3
{B}	18.4	19.7	21.2	21
{C}	25.3	25	25.4	25.5

The satisfaction of a coalition C_m can be defined as the excess sum of their allocated extra profit shares imputation if they form the grand coalition over the total profit if the coalition C_m , i.e.:

$$F_C(C_m, \vec{x}) = \sum_{i \in C_m} x_i - v(C_m), \tag{20}$$

Table 3 shows the related satisfaction values for each coalition in absolute terms $F_C(C_m, \vec{x})$ and in relative terms $F_C(C_m, \vec{x})/v(C_m)$ i.e., as a percentage of the corresponding coalition profit $v(C_m)$. Actually, adding a new partner to the coalition sometimes degrades the satisfaction because the obtained benefit is not sufficient to expiate complexity of the increased collaboration. To avoid this, the cooperative game theory was adopted to minimize the maximum dissatisfaction (Lozano et al. 2013). Coalition {A, C} has the minimum satisfaction among all the coalitions. As shown in Table 3, the largest minimum satisfaction in the absolute terms, belongs to the core center. Furthermore, there is no difference among Shapley, τ –value and core center in the largest maximum relative satisfaction but Core center and Minimax core are the largest absolute maximum satisfaction.
According to the definition, the methods with the largest minimum relative satisfaction are the Minimax Core, Core center and τ –value (32%). These methods work to impose fairness by maximization of the minimum satisfaction.

Table 3. Coalition satisfactions for Shapley value, τ -value and core-center.

Coalition	Shapley value	τ –value	Core center	Minimax core
{A}	29.1	28.1	26.2	26.3
	36%	0.34%	0.32%	0.32%
{B}	18.4	19.7	21.2	21.0
	31%	0.33%	0.35%	35%
{C}	25.3	25	25.4	25.5
	42%	0.42%	0.42%	42%
{A,B}	39.7	40	39.6	39.5
	32%	32%	32%	32%
{A,C}	32.7	31.4	29.9	30.1
	33%	32%	32%	32%
{B,C}	43.7	43.7	46.6	46.6
	36%	37%	39%	39%
Min $F_s(EP, y)$	18.4	19.7	21.2	21.0
Min $F_s(EP, y)/\pi_{c_m}$	31%	32%	32%	32%
Max $F_s(EP, y)$	43.7	43.7	46.6	46.6
Max $F_s(EP, y)/\pi_{c_m}$	42%	42%	42%	42%
Sum $F_s(EP, y)$	188.9	187.9	188.9	189
Sum $F_s(EP, y)/\pi_{c_m}$	192%	192%	191%	191%

Figure 4 illustrates the relative locations of the solutions to these 3-player games in barycenteric coordinates. The shaded area is related to the core (EP), which is not empty in this case, though it may happen sometimes if all of solutions lie into the core.

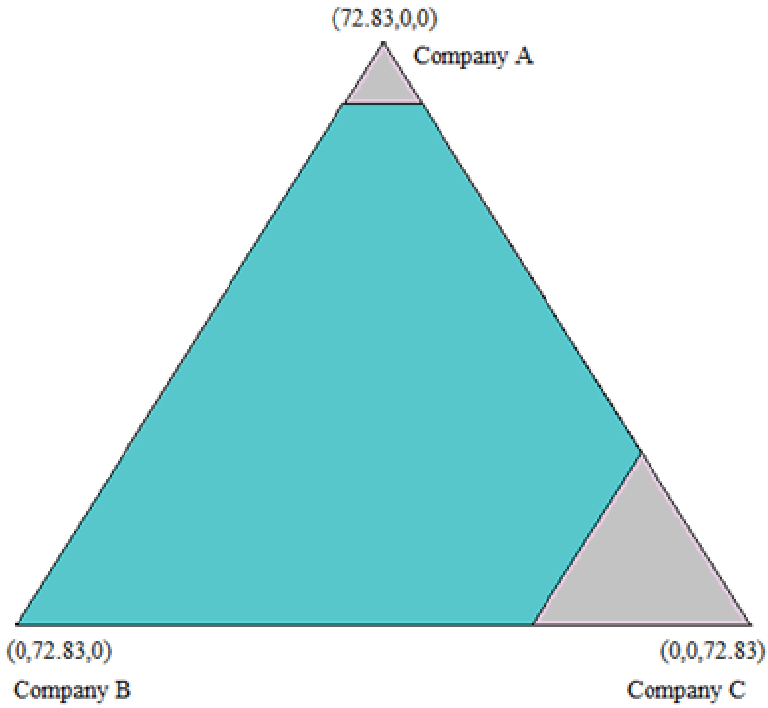

Fig 4. Core and EP allocations for different 3-player games

From the numeric results the following managerial insights were derived:

- Cooperative pricing will hold the market in a stable condition. The price determined by coalition of logistic companies has an important role in the business. As shown in Table 1 the price of each unit in the coalition is lower than the average price of players. It is also demonstrated, that as the price is decreasing, the profit is increasing.
- The comparison between the mount of sales obtained from separate sale and coalitional sales, as shown in Fig. 2, shows when the players form a coalition, the sales is greater than when they sell their products separately. As explained in the last notation, the coalitional price is lower than the average price in the separately sales, and due to the well-known principle of economics, when the price goes down, the demand goes up.
- The efficiency of the network increases due to cooperation of logistic companies.
- Cooperative game theory approach introduces some tools to allocate the extra profit and to choose the best allocation system to maximize the player's satisfaction. The fair allocation of extra profit encourages the players to continue their participants.
- The results of Table 3 shows that each concepts of CGT (core, shapley-value, τ-value, minimax-core) represents a different consequence and the logistic companies can choose their favorite method in the contract to maximize their satisfaction.

6- Conclusion and Future Research

In this paper a mathematical programming model is proposed to determine the price and the maximum flow simultaneously. We used this model for each player and then for each possible coalition to find the largest benefit. It is shown that when two or more logistic companies colligate, the price will decrease and the sale amount will increase. The minimum price belongs to the grand coalition and this coalition has the maximum extra profit and synergy. Thus the efficiency of logistic network increases due to cooperation of companies. For allocating the extra profit of cooperation, fairness is the most important problem. We proposed the methods of cooperative game theory such as Shapley value, τ –value, core center and minimax-Core.

There are various issues and suggestions to be discussed in the future researches. First, in this paper, it was assumed that the demand of products is deterministic, but in reality the demands can be non-deterministic. Secondly, the players were assumed to have similar information about each other, but the model can considered under asymmetric information condition. Third, a single price has been set in this paper for each unit of products for all the consumers, However, in the real-world problems, each consumer has a certain value for the companies, so that the companies will set a specific price for each customer, and applying this issue will make the problem interesting.

References

Ahuja, R. K. (1993). Network Flows: Theory, Algorithms and Applications. *Prentice Hall*.

Ahuja, R., Magnanti, T., & Orlin, J. (1993). Network Flows: Theory, Algorithms, and Applications. *Prentice-Hall*.

Ahuja, Ravindra K.Magnanti; Thomas L.; Orlin, James B. (1993). Network Flows: Theory, Algorithms and Applications. *Prentice Hall*.

Altman, E., & Wynter, L. (2004). Equilibrium, games, and pricing in transportation and telecommunication networks. *Networks and Spatial Economics, 4*(1), 7-21.

Barron, E. N. (2013). *Game Theory: An Introduction* (2nd edition ed.). New York: John Wiley & Sons.

Bertsekas, D. (1998). Network Optimization - Continuous and Discrete Models. *Athena Scientific*.

Branzei, R., Dimitrov, D., & Tijs, S. (2008). *Models in cooperative game theory* (Vol. 556). Springer Science & Business Media.

Deng, S., & Yano, C. (2006). Joint production and pricing decisions with setup costs and capacity constraints. *Manage. Sci., 52*, 741-756.

Driessen, T. (1985). Contributions to the Theory of Cooperative Games: The τ-value and k-Convex Games. *Ph.D. Thesis, U. of Nijmejen*.

F´elegyh´azi, M., & Hubaux, J.-P. (2006). Game theory in wireless networks: tutorial. *Technical Report LCA-REPORT*.

Fathabadi, H., & Ghiyasvand, M. (2007). A new algorithm for solving the feasibility problem of a network flow. *Appl. Math. Comput., 192*, 429-438.

Florian, M., & Klein, M. (1971). Deterministic production planning with concave costs and capacity constraints. *Manage. Sci., 18*, 12-20.

Ford, L. R., & Fulkerson, D. R. (1965). Maximal flow through a network. *Canadian Journal of Mathematics* , 399.

Geunes, J., Merzifonluo˘glu, Y., & Romeijn, H. (2008). Capacitated procurement planning with price-sensitive demand and general concave revenue functions. *Eur. J. Oper. Res., 194*(2), 390-405.

Geunes, J., Romeijn, H., & Taaffe, K. (2006). Requirements planning with dynamic pricing and order selection flexibility. *Oper. Res., 54*(2), 394-401.

Gilbert, S. (1999). Coordination of pricing and multi-period production for constant price goods. *Eur. J. Oper. Res., 114*(2), 330-337.

Glover, F., & Klingman, D. (1979). Network applications in industry and government. *IIE Trans.*, 363-376.
Hafezalkotob, A., & Makui, A. (2015). Cooperative Maximum-Flow Problem under Uncertainty in Logistic Networks. *Applied Mathematics and Computation*(250), 5930604.

Hammons, T. (2001). Electricity Restructuring in Latin America Systems with Significant Hydro Generation. *Rev. Energ. Ren: Power Engineering*, 39-48.

Harris, T. E., & Ross, F. S. (1995). Fundamentals of a Method for Evaluating Rail Net Capacities. *Research Memorandum*.

Juan, A.; Faulin, J;, Pérez-Bernabeu, E.; Jozefowiez, N. (2014). Horizontal cooperation in vehicle routing problems with backhauling and environmental criteria. *Procedia-Social and Behavioral Sciences, 111*, 1133-1141.

Kunreuther, H., & Schrage, L. (1971). Optimal pricing and inventory decisions for non-seasonal items. *Econometrica, 2*, 193-205.

Kunreuther, H., & Schrage, L. (1973). Joint pricing and inventory decisions for constant priced items. *Manage.Sci., 19*, 732-738.

Lin, C.-C., & Hsieh, C.-C. (2012). A cooperative coalitional game in duopolistic supply-chain competition. *Networks and Spatial Economics, 12*(1), 129-146.

Lozano, A., Moreno, P., Adenso-Díaz, B., & Algaba, E. (2013). Cooperative game theory approach to allocating benefits of horizontal cooperation. *Eur. J. Oper. Res.*, *229*, 444-452.

Lozano, S., Moreno, P., Adenso-Díaz, B., & Algaba, E. (2013). Cooperative game theory approach to allocating benefits of horizontal cooperation. *Eur. J. Oper. Res., 229*(2), 444-452.

Mirás Calvo, M. S. (2006). *TUGlab: A Cooperative Game Theory Toolbox.* Retrieved (accessed 08.01.13, from http://webs.uvigo.es/mmiras/TUGlab/TUGl abICM06.pdf

Palais, R. (1970). Critical point theory and the minimax principle. *Proc. Sympos. Pure Math., 15*, 185-212.

Pirani, S., & Yafimova, K. (2009). The Russo-Ukrainian gas dispute of January 2009: a comprehensive assessment. *Oxford Institute for Energy Studies*, 59.

Reyes, P. M. (2005, September). Logistics networks: A game theory application for solving the transshipment problem. *Applied Mathematics and Computation, 168*(2), 1419-1431.

Rockafellar, R. (1984). Network Flows and Monotropic Programming. *John Wiley & Sons*.

Roson, R., & Hubert, F. (2014). Bargaining Power and Value Sharing in Distribution Networks: A Cooperative Game Theory Approach. *Networks and Spatial Economics,* 1-17.

Saad, W. (2010). "Coalitional Game Theory for Distributed Cooperation in Next Generation Wireless Networks", Ph.D. dissertation. *University of Oslo*.

Saad, W., Han, Z., Debbah, M., Hjørungnes, A., & Basar, T. (2009). Coalitional game theory for communication networks. *IEEE Signal Processing Mag., 26*, 77-97.

Schintler, Laurie A;Gorman, Sean P;Reggiani, Aura;Patuelli, Roberto;Gillespie, Andy;Nijkamp, Peter;Rutherford, Jonathan. (2005). Complex network phenomena in telecommunication systems. *Networks and Spatial Economics, 5*(4), 351-370.

Schrijver, A. (2002). On the history of the transportation and maximum flow problem. *Math Program*(91), 437-445.

Scotti, M. (2012). Networks in Social Policy Problems. *Cambridge University Press*.

Sepehri, M. (2011). Cost and inventory benefits of cooperation in multi-period and multi-product supply. *Scientica Iranica, 18*(3), 731-741.

Shapley, L., & Shubik, M. (1972). The Assignment Game I: The Core. *International Journal of Game Theory, 1*, 111-130.

Sharkey, T. C. (2011). Network flow problems with pricing decisions. *Optimization Letters, 5*(1), 71-83.

Soyster, A. (1973). Convex programming with set-inclusive constraints and applications to inexact linear programming. *Oper. Res., 21*, 1154-1157.

Szoplik, J. (2012). The Gas Transportation in a Pipeline Network, Advances in Natural Gas Technology. *Dr. Hamid Al-Megren (Ed.)*.

Thomas, J. (1970). Price-production decisions with deterministic demand. *Manage. Sci., 16*(11), 747–750.

Van den Heuvel, W., & Wagelmans, A. (2006). A polynomial time algorithm for a deterministic joint pricing and inventory model. *Eur. J. Oper. Res., 170*(2), 463-480.

Winston, W. L. (2003). Operations Research: Applications and Algorithms, fourth ed. *Cengage Learning*.