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Interval network data envelopment analysis model for classification of investment companies in the presence of uncertain data

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Abstract
This paper proposes the interval network data envelopment analysis (INDEA) approach under constant return to scale (CRS) and variable return to scale (VRS) assumptions which can assess the performance of investment companies (ICs) by considering uncertainty and internal structure. The presented approach of the paper is capable to model two-stage efficiency with intermediate measures in a single implementation. Finally, a real-life case study from Tehran stock exchange (TSE) is implemented to demonstrate applicability and exhibit the efficiency and effectiveness of the presented INDEA approach for performance measurement, ranking and classification of ICs in the presence of uncertain data.
Keywords: Investment Company, Uncertainty, Interval Data, Network Data Envelopment Analysis, Interval Data Envelopment Analysis.

1- Introduction

An investment company (IC) invests the money received from investors on a specific investment plan, and each investor will be shared in the investment incomes and risks in proportion to his/her interest in the ICs (Zohdi et al. (2012). So, performance measurement of investment companies such as mutual fund is one of the most important problems in financial markets. Data envelopment analysis (DEA) is one of the non-parametric performance measurement techniques in order to benchmarking, efficiency measurement and ranking of homogeneous decision making units.

This methodology was proposed by Charnes et al. (1978) for the first time and it is based on Farrell's (1957) idea. Charnes et al. (1978) proposed the first DEA model that was based on the constant returns to scale (CRS) assumption and it is called the CCR model. Then, Banker et al. (1984) developed CCR model based on the variable returns to scale (VRS) assumption and they called it the BCC model.

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It should be noted that in measuring the efficiency of an investment company using the data envelopment analysis, uncertainty should be considered. Because, in financial market, we are faced with uncertain data and one of the most important features of financial markets is their uncertainty. Also, one of the most important assumptions in DEA is that the measured data are certain and conclusive. But, a little bias or deviation in data's values can cause significant differences in final results. In worst case, we are faced with unfeasible solutions. Therefore, ranking the results can be invalid, especially when efficiency of a unit is close to another one (Sadjadi & Omrani, 2008).

Another point that should be considered in the proposed model for performance measurement of investment companies is their internal structure. In order to considering internal structure, the network data envelopment analysis must be used. Premachandra et al. (2012), Galagedera et al. (2016) and Premachandra et al. (2016) introduced two stage DEA model for measuring the performance of mutual fund.

Thus, this study proposed interval network data envelopment analysis (INDEA) model for classification and ranking of investment companies (ICs) by considering internal structure and uncertainty. With respect to the uncertain data only known to lie within the upper and lower bounds represented by the intervals and for dealing with uncertainty, the idea of Despotis and Smirlis (2002) will be used.

The rest of this paper is organized as follows. The suggested structure for investment companies will be explained in section 2. The network data envelopment analysis for performance measurement of investment companies will be proposed in section 3. Then, the interval network DEA for evaluating performance of ICs for dealing with uncertainty will be proposed in section 4.The proposed INDEA models in this study are implemented for a case study of Tehran stock exchange (TSE) and the results will be evaluated in section 5. Finally, the conclusions of study and some directions for future researches are given in section 6.

2- Investment companies structure

In this paper, by assuming that the activities of investment companies can be viewed as a two stage process, the network data envelopment analysis (NDEA) model for measuring the performance of an IC will be proposed. As can be seen in figure (1), the overall efficiency of the investment company is decomposed into two stages that the first stage represents the operational management process and second stage represents the portfolio management process.

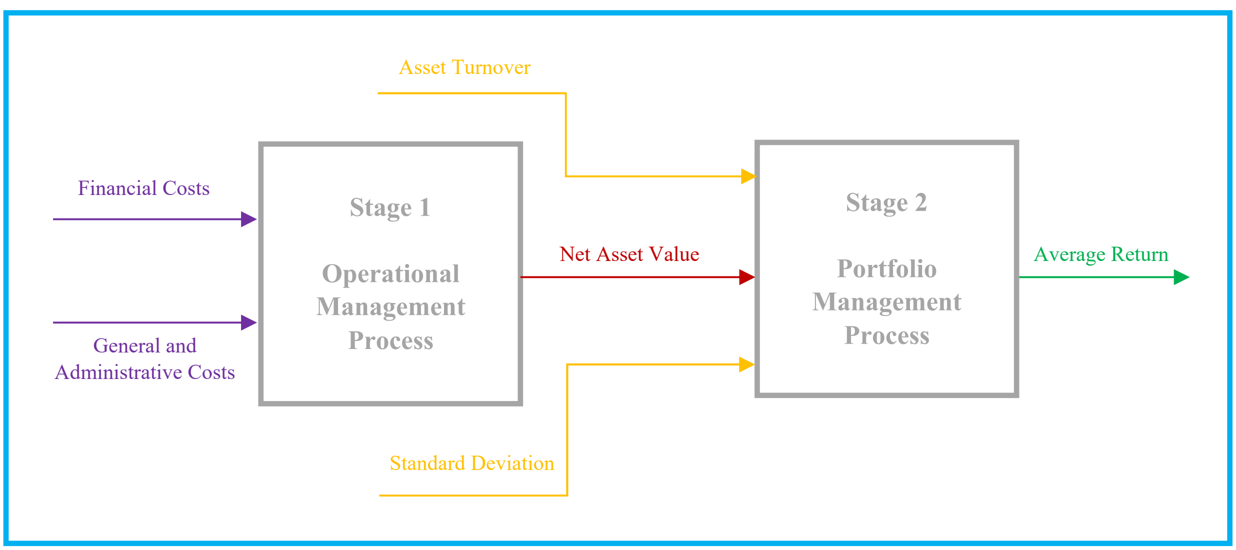

Fig 1. The proposed two-stages structure for evaluating the efficiency of investment companies

It should be noted that financial costs and general and administrative costs are the input variables at first stage and net asset value is the intermediate variable that is output and input variable at first and second stage, respectively. So in first stage, an investment company that can produce the highest net asset value with the least amount of financial costs and general and administrative costs will be more efficient than the other investment companies from operational aspect. Also, asset turnover and standard deviation are another parts of inputs to the second stage and average return is the output variable at second stage. So in second stage, an investment company that can produce the highest average portfolio return with the least amount of net asset value, asset turnover and standard deviation will be more efficient than the other investment companies from portfolio management function.

3- Network DEA for performance measurement of Investment Company

Consider an extended two-stage process with added inputs to the second stage as shown in figure (2), for each set of n homogenous decision making units DMU_j $(j=1,\ldots,n)$ that each DMU) has m inputs x_{ij} $(i=1,\ldots,m)$ in the stage 1, and D outputs z_{dj} $(d=1,\ldots,D)$ less than that stage. These D outputs then become the added inputs to the stage 2, hence behaving as intermediate measures. Another part of inputs that are added to the stage 2 are H inputs γ_{hj} $(h=1,\ldots,H)$ and finally the outputs from the stage 2 are y_{rj} $(r=1,\ldots,s)$.

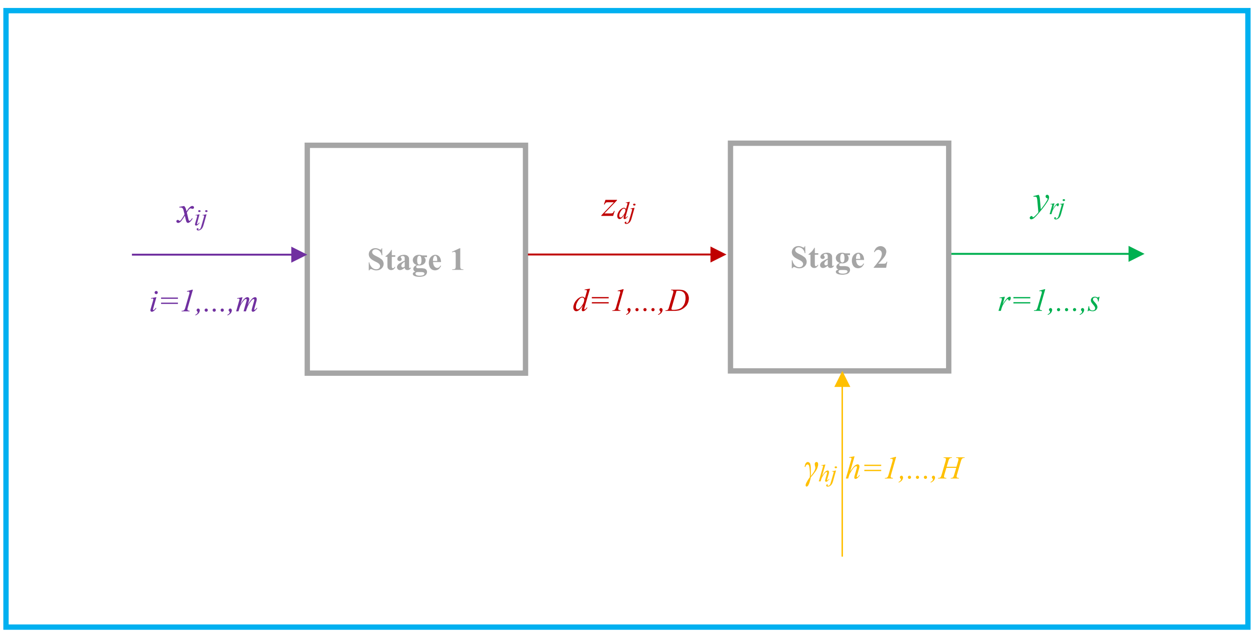

Fig 2. Two-Stage (s) Process with Added Inputs to the Second Stage

Now, by applying the idea of Chen and Zhu (2004) for dealing with two-stage efficiency with intermediate measures in a single implementation, the extended network data envelopment analysis model for a two-stage process with added inputs to the stage 2, are proposed. The NetDEA models based on constant return to scale (CRS) and variable return to scale (VRS) are as models (1) and (2):

<table>
<tr><td colspan="4">Network Data Envelopment Analysis Model</td></tr>
<tr><td colspan="2">Constant Return to Scale (CRS)</td><td colspan="2">Variable Return to Scale (VRS)</td></tr>
<tr><td>$$\begin{aligned} \Psi_p = \quad & \text{Min} \quad \Theta_p - \Phi_p \\ \text{S.t.} \quad & \sum_{j=1}^{n} \lambda_j \, x_{ij} \le \Theta_p x_{ip}, \quad \forall i \\ & \sum_{j=1}^{n} \lambda_j \, \tau_{gj} \ge \tau_{gp}, \quad \forall g \\ & \sum_{j=1}^{n} \lambda_j \, z_{dj} \ge \widehat{z}_{dp}, \quad \forall d \\ & \sum_{j=1}^{n} \mu_j \, z_{dj} \le \widehat{z}_{dp}, \quad \forall d \\ & \sum_{j=1}^{n} \mu_j \, \gamma_{hj} \le \gamma_{hp}, \quad \forall h \\ & \sum_{j=1}^{n} \mu_j \, y_{rj} \ge \Phi_p y_{r0}, \quad \forall r \\ & \lambda_j, \mu_j \ge 0, \quad \forall j \end{aligned}$$</td><td>(1)</td><td>$$\begin{aligned} \Psi_p = \quad & \text{Min} \quad \Theta_p - \Phi_p \\ \text{S.t.} \quad & \sum_{j=1}^{n} \lambda_j \, x_{ij} \le \Theta_p x_{ip}, \quad \forall i \\ & \sum_{j=1}^{n} \lambda_j \, \tau_{gj} \ge \tau_{gp}, \quad \forall g \\ & \sum_{j=1}^{n} \lambda_j \, z_{dj} \ge \widehat{z}_{dp}, \quad \forall d \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \sum_{j=1}^{n} \mu_j \, z_{dj} \le \widehat{z}_{dp}, \quad \forall d \\ & \sum_{j=1}^{n} \mu_j \, \gamma_{hj} \le \gamma_{hp}, \quad \forall h \\ & \sum_{j=1}^{n} \mu_j \, y_{rj} \ge \Phi_p y_{r0}, \quad \forall r \\ & \sum_{j=1}^{n} \mu_j = 1 \\ & \lambda_j, \mu_j \ge 0, \quad \forall j \end{aligned}$$</td><td>(2)</td></tr>
</table>

It should be noted that in models (1) and (2), $\Theta^* \le 1$, $\Phi^* \ge 1$ and as a result $\Psi^* \le 0$. Also, DMU under evaluation will be efficient in overall if it be efficient in stages 1 and 2 simultaneously.

4- Interval network DEA for performance measurement of an IC under uncertainty

In this section, the interval network data envelopment analysis (INDEA) will be proposed for measuring the performance of investment companies under uncertain situation that the data for inputs, intermediate and outputs are not known exactly. It should be noted that the uncertain data only known to lie within the upper and lower bounds represented by the intervals.

By considering the $x_{ij} \in \left[x_{ij}^L, x_{ij}^U \right], z_{dj} \in \left[z_{dj}^L, z_{dj}^U \right], \tau_{gj} \in \left[\tau_{gj}^L, \tau_{gj}^U \right]$ and $y_{rj} \in \left[y_{rj}^L, y_{rj}^U \right]$ with upper and lower bounds of the intervals given as constants and assumed strictly positive, the efficiency for each DMU can be an interval. In order to calculating the lower and upper bound of efficiency, the decision maker (DM) must run two separate models. The lower bound of interval efficiency is obtained from the pessimistic viewpoint that is proposed in models (3) and (4) for CRS and VRS assumptions, respectively:

Interval Network Data Envelopment Analysis (INDEA)Model- (Lower Bound)			
Constant Return to Scale (CRS)		*Variable Return to Scale (VRS)*	
$\Psi_p^L = \text{ Min } \Theta_p - \Phi_p$ S.t. $\sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j x_{ij}^L + \lambda_p x_{ip}^U \leq \Theta_p x_{ip}^U, \quad \forall i$ $\sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j \tau_{gj}^U + \lambda_p \tau_{gj}^L \geq \tau_{gp}^L, \quad \forall g$ $\sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j z_{dj}^U + \lambda_p z_{dj}^L \geq \widehat{z}_{dp}, \quad \forall d$ $\sum_{\substack{j=1\\j\neq p}}^{n} \mu_j z_{dj}^L + \mu_p z_{dp}^U \leq \widehat{z}_{dp}, \quad \forall d$ $\sum_{\substack{j=1\\j\neq p}}^{n} \mu_j \gamma_{hj}^L + \mu_p \gamma_{hp}^U \leq \gamma_{hp}^U, \quad \forall h$ $\sum_{\substack{j=1\\j\neq p}}^{n} \mu_j y_{rj}^U + \mu_p y_{rp}^L \geq \Phi_p y_{rp}^L, \quad \forall r$ $\lambda_j, \mu_j \geq 0, \quad \forall j$	(3)	$\Psi_p^L = \text{ Min } \Theta_p - \Phi_p$ S.t. $\sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j x_{ij}^L + \lambda_p x_{ip}^U \leq \Theta_p x_{ip}^U, \quad \forall i$ $\sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j \tau_{gj}^U + \lambda_p \tau_{gj}^L \geq \tau_{gp}^L, \quad \forall g$ $\sum_{\substack{j=1\\j\neq p}}^{n} \lambda_j z_{dj}^U + \lambda_p z_{dj}^L \geq \widehat{z}_{dp}, \quad \forall d$ $\sum_{j=1}^{n} \lambda_j = 1$ $\sum_{\substack{j=1\\j\neq p}}^{n} \mu_j z_{dj}^L + \mu_p z_{dp}^U \leq \widehat{z}_{dp}, \quad \forall d$ $\sum_{\substack{j=1\\j\neq p}}^{n} \mu_j \gamma_{hj}^L + \mu_p \gamma_{hp}^U \leq \gamma_{hp}^U, \quad \forall h$ $\sum_{\substack{j=1\\j\neq p}}^{n} \mu_j y_{rj}^U + \mu_p y_{rp}^L \geq \Phi_p y_{rp}^L, \quad \forall r$ $\sum_{j=1}^{n} \mu_j = 1$ $\lambda_j, \mu_j \geq 0, \quad \forall j$	(4)

The upper bound of interval efficiency is obtained from the optimistic viewpoint that is proposed in models (5) and (6) for CRS and VRS assumptions, respectively:

Interval Network Data Envelopment Analysis (INDEA)Model - (Upper Bound)			
Constant Return to Scale (CRS)		*Variable Return to Scale (VRS)*	
$\Psi_p^U = \quad \text{Min} \quad \Theta_p - \Phi_p$ S.t. $\sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j x_{ij}^U + \lambda_p x_{ip}^L \le \Theta_p x_{ip}^L , \quad \forall i$ $\sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j \tau_{gj}^L + \lambda_p \tau_{gj}^U \ge \tau_{gp}^U , \quad \forall g$ $\sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j z_{dj}^L + \lambda_p z_{dj}^U \ge \hat{z}_{dp} , \quad \forall d$ $\sum_{\substack{j=1 \\ j \neq p}}^{n} \mu_j z_{dj}^U + \mu_p z_{dp}^L \le \hat{z}_{dp} , \quad \forall d$ $\sum_{\substack{j=1 \\ j \neq p}}^{n} \mu_j \gamma_{hj}^U + \mu_p \gamma_{hp}^L \le \gamma_{hp}^L , \quad \forall h$ $\sum_{\substack{j=1 \\ j \neq p}}^{n} \mu_j y_{rj}^L + \mu_p y_{rp}^U \ge \Phi_p y_{rp}^U , \quad \forall r$ $\lambda_j , \mu_j \ge 0 , \quad \forall j$	(5)	$\Psi_p^U = \quad \text{Min} \quad \Theta_p - \Phi_p$ S.t. $\sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j x_{ij}^U + \lambda_p x_{ip}^L \le \Theta_p x_{ip}^L , \quad \forall i$ $\sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j \tau_{gj}^L + \lambda_p \tau_{gj}^U \ge \tau_{gp}^U , \quad \forall g$ $\sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j z_{dj}^L + \lambda_p z_{dj}^U \ge \hat{z}_{dp} , \quad \forall d$ $\sum_{j=1}^{n} \lambda_j = 1$ $\sum_{\substack{j=1 \\ j \neq p}}^{n} \mu_j z_{dj}^U + \mu_p z_{dp}^L \le \hat{z}_{dp} , \quad \forall d$ $\sum_{\substack{j=1 \\ j \neq p}}^{n} \mu_j \gamma_{hj}^U + \mu_p \gamma_{hp}^L \le \gamma_{hp}^L , \quad \forall h$ $\sum_{\substack{j=1 \\ j \neq p}}^{n} \mu_j y_{rj}^L + \mu_p y_{rp}^U \ge \Phi_p y_{rp}^U , \quad \forall r$ $\sum_{j=1}^{n} \mu_j = 1$ $\lambda_j , \mu_j \ge 0 , \quad \forall j$	(6)

It should be noted that models (3) and (5) for CRS assumption and models (4) and (6) for VRS assumption provide each DMU with a bounded interval in which its possible efficiency scores lie from the worst to the best. With respect to interval efficiency scores of stage 1, stage 2 and overall, decision making units can be classified in three subsets as equations (7) to (9), respectively:

$$E^{++}=\left\{j\in J\mid\Theta_j^L=1\right\}$$ (7)

$$E^{+}=\left\{j\in J\mid\Theta_j^L<1 \quad and \quad \Theta_j^U=1\right\}$$

$$E^{-}=\left\{j\in J\mid\Theta_j^U<1\right\}$$

$$E^{++}=\left\{j\in J\mid\Phi_j^L=1\right\}$$ (8)

$$E^{+}=\left\{j\in J\mid\Phi_j^L>1 \quad and \quad \Phi_j^U=1\right\}$$

$$E^{-}=\left\{j\in J\mid\Phi_j^U>1\right\}$$

$$E^{++}=\left\{j\in J\mid\Psi_j^L=0\right\}$$ (9)

$$E^{+}=\left\{j\in J\mid\Psi_j^L<0 \quad and \quad \Psi_j^U=0\right\}$$

$$E^{-}=\left\{j\in J\mid\Psi_j^U<0\right\}$$

It should be noted that by inversing the optimal value of variable Φ in models (3) to (6), the efficiency score of second stage will be calculated.

5- Case study and numerical results

In this section, the implementation of the proposed interval network data envelopment analysis models based on the CRS and VRS assumptions for performance measurement of investment companies will be presented for a real world case study from Tehran Stock Exchange (TSE). So, financial data for 10 investment companies in TSE are extracted from March 2016 to March 2017. After collecting data, interval network data envelopment analysis models will be run. The results and ranking and classification of investment companies based on INDEA model with CRS assumption are introduced in tables 1 and 2, respectively:

Table 1. The results of INDEA model with CRS assumption for investment companies

IC	Stage 1		Stage 2		Overall	
	Θ^L	Θ^U	Φ^L	Φ^U	Ψ^L	Ψ^U
IC 01	0.158496	0.353673	1.678250	**1**	-1.519755	-0.646327
IC 02	0.375896	0.838850	5.312105	2.443028	-4.936209	-1.604178
IC 03	0.517713	**1**	10.582360	4.748422	-10.064640	-3.748422
IC 04	**1**	**1**	12.555540	5.626305	-11.555540	-4.626305
IC 05	0.103532	0.231009	**1**	**1**	-0.896468	-0.768991
IC 06	0.079260	0.176870	**1**	**1**	-0.920740	-0.823130
IC 07	0.190748	0.425661	4.056551	1.817812	-3.865803	-1.392151
IC 08	**1**	**1**	28.743620	12.718900	-27.743620	-11.718900
IC 09	0.717899	**1**	2.122912	**1**	-1.405014	**0**
IC 10	0.711790	**1**	2.504609	1.122093	-1.792820	-0.122093

Table 2. The ranking and classification of investment companies using INDEA model with CRS assumption

IC	Stage 1		Stage 2		Overall	
	Rank	Classification	Rank	Classification	Rank	Classification
IC 01	8	E^-	3	E^+	5	E^-
IC 02	6	E^-	7	E^-	7	E^-
IC 03	5	E^+	8	E^-	8	E^-
IC 04	1	E^{++}	9	E^-	9	E^-
IC 05	9	E^-	1	E^{++}	2	E^-
IC 06	10	E^-	1	E^{++}	3	E^-
IC 07	7	E^-	6	E^-	6	E^-
IC 08	1	E^{++}	10	E^-	10	E^-
IC 09	3	E^+	4	E^+	1	E^+
IC 10	4	E^+	5	E^-	4	E^-

Also, the results and ranking and classification of investment companies based on INDEA model with VRS assumption are introduced in tables3 and 4, respectively:

Table 3. The results of INDEA model with VRS assumption for investment companies

IC	Stage 1		Stage 2		Overall	
	Θ^L	Θ^U	Φ^L	Φ^U	Ψ^L	Ψ^U
IC 01	0.336321	0.607595	1	1	-0.663679	-0.392405
IC 02	0.731707	1	1	1	-0.268293	0
IC 03	0.534493	1	4.472305	2.993570	-3.937812	-1.993570
IC 04	1	1	10.794940	5.218697	-9.794941	-4.218697
IC 05	1	1	1	1	0	0
IC 06	0.159375	0.290595	1	1	-0.840625	-0.709406
IC 07	0.245415	0.488680	3.588589	1.668920	-3.343174	-1.180240
IC 08	1	1	22.675520	12.437730	-21.675520	-11.437730
IC 09	0.730450	1	1.878983	1	-1.148533	0
IC 10	0.866406	1	2.486714	1	-1.620308	0

Table 4. The ranking and classification of investment companies using INDEA model with VRS assumption

IC	Stage 1		Stage 2		Overall	
	Rank	Classification	Rank	Classification	Rank	Classification
IC 01	8	E^-	1	E^{++}	3	E^-
IC 02	5	E^+	1	E^{++}	2	E^+
IC 03	7	E^+	8	E^-	8	E^-
IC 04	1	E^{++}	9	E^-	9	E^-
IC 05	2	E^{++}	1	E^{++}	1	E^{++}
IC 06	10	E^-	1	E^{++}	5	E^-
IC 07	9	E^-	7	E^-	7	E^-
IC 08	3	E^{++}	10	E^-	10	E^-
IC 09	6	E^+	5	E^+	4	E^+
IC 10	4	E^+	6	E^+	6	E^+

As it can be seen in the results of interval network DEA models that are presented in tables 1 and 3, the results of stage 1, stage 2 and overall in CRS model worse than VRS model. Also, number of efficient DMUs in VRS model is more than CRS model.

6- Conclusions and future directions

In this study, by using the idea of Chen and Zhu (2004), two-stage DEA model for evaluating performance of investment companies was extended. Then, the interval network DEA model for dealing with uncertain data was proposed. It should be noted that NDEA and INDEA models were presented in CRS and VRS assumptions. Finally, for solving and showing validation of the proposed models in this study, the INDEA models are implemented for a real case study of 10 investment companies of Tehran stock exchange (TSE) in order to ranking and classification these ICs. For the future studies, the network DEA models for dealing with uncertainty could be proposed based on other approaches such as fuzzy mathematical programming and chance constrained programming.

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