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Two-stage stochastic programming model for capacitated complete star p-hub network with different fare classes of customers

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Abstract

In this paper, a stochastic programming approach is applied to the airline network revenue management problem. The airline network with the arc capacitated single hub location problem based on complete–star p-hub network is considered. We try to maximize the profit of the transportation company by choosing the best hub locations and network topology, applying revenue management techniques to allocate limited perishable capacity and provide booking limits for all itineraries and fare classes. In order to characterize the uncertainty of demand in the airline market, we introduce stochastic variations caused by seasonally passengers' demands through a number of scenarios. The proposed model deals with finding the location of hub facilities, the assignment of demand nodes to these located hub facilities and allocating the limited capacity of aircraft seats on each rout to different customer classes in order to maximize the profit. Due to the computational complexity of the resulted model, a hybrid algorithm improved by a caching technique based on standard genetic operators is used to find a near optimal solution of the problem. Numerical experiments are carried out on the Turkish network data set. The performance of the solutions obtained by the proposed algorithm is compared with the pure GA and Imperialist Competitive Algorithm in terms of the computational time requirements and solution quality.

Keywords: Revenue management, scenario generation methods, stochastic programming, hub location, seat inventory control, evolutionary algorithms.

1-Introduction

Revenue management strategies arose out of the deregulation in the airline industry in the late 1970s. Mentioned science has helped to improve the profitability of airlines by some instruments include seat inventory control, pricing, forecasting and etc. As a consequence of the deregulation, it is possible for airlines to sell the seats of an aircraft to different customer segments at different prices during the booking period. Airlines have defined some conditions and restrictions as fences for limit switching of the customers among various segments of the market. In the seat inventory control, they manage the capacity with

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deciding about allocating finite flight seat capacity of an aircraft for each customer segment. This instrument is constructed on the principle of offering multiple services with different prices at the same time to various customer classes. In particular, airline companies try to sell high-fare ticket prices as many seats as possible and attempt to minimize the potential loss that happens as an effect of unsold seats. In fact, they reject an early customer with a lower-fare ticket so as to save the seat for the customer with higher-fare ticket prices, but at the same time they confront with the risk of flying with some empty seats. On the other hand, accepting early requests occupy the flight seats, but the company face to the risk of rejecting a latest high-fare request due to the capacity constraints. Under this circumstance, the airline company has fixed and perishable set of assets that should be sold for a price-sensitive population of customers so as to maximize the total expected revenue over the selling horizon (Aslani et al. 2014). Most airlines use hub-and-spoke type of network so as to serve more origin–destination (OD) pairs with fewer flights. This causes the network version problem of revenue management to receive much interest as the fight legs are now shared by multiple origin-destination itineraries. Most of the time, using hub and spoke networks results in lower cost carriers as compared to those of direct flights. Applying optimal inventory control strategies together with choosing the best network topology help the airlines to improve the total expected revenue. This topology uses switching, transshipment and flow consolidation facilities named hubs which remarkably decrease the links required to connect all origins and destinations in transportation networks in purpose of reducing the total transportation cost (Alumur et al., 2012). In other words, consolidating traffic flow in inter-hub transportation and on the spokes, causes to reduce the operational costs in comparison with direct links between all pairs and helps airlines to maximize the revenue.

This paper proposes an extended model that incorporates the revenue management technique into the hub location problem in an environment with the stochastic demand and allocate the fixed amount of capacity to the right customers at the right time. We considered a hub location problem that arises in the design of a complete–star network. In a complete–star p-hub network, there are several nodes in the network that p of them are chosen to be hubs. Each node is assigned to exactly one hub and all of the hubs are connected to each other. To capture the uncertainty of demand, a two-stage stochastic integer programming model with the maximized expected incurred by transfer activities is formulated. The first stage aims to find the optimal locations of p hubs, the allocation of non-hub nodes to the p located hubs and control of protection levels within the network revenue management context, and the second stage provides the decisions on the sold tickets which are influenced by uncertain demands. Due to the NP-hardness of the problem, to solve the proposed model, we are proposing an algorithm based upon the evolutionary genetic algorithm and exact solution method. The rest of the paper is organized as follows:

Section 2 discusses the relevant literature briefly. The integrated revenue management and hub location model is developed in Section 3. In Section 4, we provided a hybrid algorithm improved by a caching technique based on standard genetic operators to solve the problem. Computational results are presented in Section 5. We also investigated the revenue enhancement by using presented integrated model. Moreover, two well-known ratios for stochastic optimization, the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS) are calculated to evaluate the developed model. Finally, in Section 6, general conclusions and some suggestions for future research are given.

2- Literature review

Revenue management began in the airline industry. The methods and techniques in this science attempt to maximize the revenue through demand forecasting and the mathematical optimization of pricing and seat inventory control (McGill and Van Ryzin 1999). Littlewood (1972) is known as a pioneer of revenue management. He introduced a model to manage the capacity of single flight leg for two fare classes.

There is a considerable amount of literature regarding revenue management in the airline industry. Belobaba (1989) applied Littlewood's (1972) rule and introduced effective heuristic known as EMSR (expected marginal seat revenue), for single leg problems with multi independent demand for pairs of fare classes. He also provided a modified EMSR method, called EMSR-b (Belobaba and Weatherford (1996)). EMSR-b differs in aggregating the demand of the products for reducing the problem with p fare classes to two-class problems based on a weighted combination in which seat protection levels near to optimal values better than previous EMSR. The problem of seat allocation in the form of multiple fare classes has been studied by many researchers. Curry (1990), Wollmer (1992), Brumelle and McGill (1993) provided several static models with different distribution of the demand that determine the optimal booking limits for different fare classes in a single-leg problem. A two-class dynamic seat allocation model with passenger diversion is considered by Zhao and Zheng (2001). Their studies have two particular characteristics, in proposed model discount fare cannot be reopened when it closed, the other feature is that they considered some flexible customers, these customers would buy discount fare tickets if available but willing to pay the full fare. Pfeifer (1989) studied a two-fare airline seat allocation problem in a single-period. Williamson (1992) developed a revenue management model that can maximize the total revenue through maximizing the revenue of each single flight leg, separately. He declared that there are many potential itineraries across a hub and spoke airline network. Zhang and Adelman (2009) proposed a network revenue management problem in the case that customers choose among open fare products under some pre-specified choice model. Zhang and Cooper (2005,2009) considered seat allocation problem and pricing issues that customers can choose products from different inventory resources for multiple flights on the same origin and destination. Chen et al. (2010) analyzed the optimal policy for the two-flight and optimal booking problem. Talluri (2001) utilized revenue management approach and proposed a model which incorporates passenger routing along with the seat inventory control. Nechval, et al. (2013) considered the allocation of the finite seat inventory to the uncertain airline customer demand that occurs during the time before the flight is scheduled to depart. The purpose of proposed model is to maximize the revenue by finding the right combination of customers of various fare classes. Feng and Xiao (2006) extended a model to maximize the expected revenue under time and capacity constraints which integrates pricing and inventory control for perishable products for multiple customer classes. Cizaire and Belobaba (2013) proposed a joint approach to optimize fares and booking limits so as to maximize the total revenues generated by the two fare products over the two time periods. Mou and Wang (2014) presented an uncertain programming for network revenue management in which the fares and the demands are considered as uncertain variables.

Many studies involve revenue management techniques in air cargo operations. Revenue management, in air cargo transportation is applied to manage the capacity of dedicated freighters and the passengers of aircraft, simultaneously. As mentioned earlier, at first in overbooking situations the available capacity is estimated, afterwards airlines maximize their expected revenue by accepting or rejecting a booking request. In this regard, Amaruchkul et al. (2007) presented a decision model, which helps airlines to decide about acceptance or rejection of a request from the freight forwarder. In another study, Han et al. (2010) studied the capacity allocation problem in a single-leg air cargo transportation problem. They considered a profit rate for each type of cargo and a random weight for each cargo booking request. Both air cargo RM and air passenger RM problems mainly focus on the issues of overbooking and the allocation of capacity. Nevertheless, the features of air cargo RM differ from air passenger RM. These differences are based on distinct characteristics of cargo types from several aspects such as available capacity estimation, network capacity allocation, and capacity booking behavior. The capacity of a passenger aircraft is fixed by its number of seats, cargo capacity depends on the container types used, which are further specified by multiple dimensions, such as pivot weight, pivot volume, and type, and center of gravity Therefore, air cargo capacity management is much more complicated. A comprehensive review of air cargo operations can be found in (Feng et al., 2015).

We also refer to study Çetiner (2013), Lapp and Weatherford (2014), Brumelle and McGill (1993), and Chiang, Chen, and Xu (2006) for an overview on the seat allocation problem and the early researches on passenger revenue management problems.

In trying to offer a more comprehensive framework for revenue management, the performance of a given system, significantly depends on designing optimal networks for routing the traffic. To provide organized transportation between different origins and destinations, utilizing set of hubs and using fewer arcs instead of point-to-point network can reduce transportation costs.

Hub location is one of the most attractive fields in facility location which has been appeared in various applications in the real world, including airline systems, postal delivery systems, cargo delivery systems and telecommunication network design. Since the service network design plays an important role in airline operations, many researchers have been addressed this problem. O'Kelly (1987) was the first one who formulated the discrete hub location problem as a quadratic integer program. After that, this field have attracted attentions of researchers. Toh and Higgins (1985) investigated the hubs application in networks of air transportations systems. They showed that airlines which utilize hub networks are more successful in compared with the airlines with more linear routing systems. Campbell (1994) provided integer programming formulations for both single and multiple version of hub location problems. He introduced hub covering and hub center problems. Jaillet et al. (1996) presented flow-based integer linear programming models for designing capacitated networks for airline networks. They have proposed a set of formulations for mentioned problem, each corresponding to a different service policy. They did not consider a priori structures for hub-network. The resulting network may suggest the use of hubs, if cost efficient. The large, and growing researches, solution techniques and applications on hub location problem is summarized in (Alumur and Kara 2008), (Farahani et al. 2013) and (Campbell and O'Kelly 2012).

In a real-world application in the airline market, demand has some uncertainty during the time. The importance of uncertainty has motivated several researchers to study various stochastic parameters in network design problems. Sim et al. (2009) considered chance constraints in the stochastic p-hub center problem to formulate the service-level guarantees. The only source of uncertainty in their study is travel time. Alumur et al. (2012) proposed the hub location problem by considering two sources of uncertainty contains the set-up costs for the hubs and the demands between origin–destination pairs. Contreras et al. (2011) addressed stochastic uncapacitated hub location problems considering uncertainty in demands and transportation costs. Yang (2009) presented a stochastic programming model to determine the air freight hub location and flight routes planning in the stochastic environment. Adibi and Razmi (2015) developed an uncapacitated multiple hub location problem with stochastic demand and transportation cost. They used 10-node air network in Iran to evaluate their proposed model.

To the best of author's knowledge from a review of the literature, there is no existing joint hub location and seat allocation decision model based upon stochastic programming and the scenario generation method. In the current study, we provided a new formulation in order to maximizes airline's profit by designing best capacitated topology network and routing policies by applying optimal hub and spoke, then optimal seat allocation for different customer classes is carry out for each itinerary regarding to routes capacity. In the subsequent section the problem is explained with more details.

3-Problem description

This section presents a modeling framework for the design of integrated hub location and revenue management model. The aim of the presented model is to find the location of hubs, the optimal flight routes and the number of seats to allocate on each rout for different classes to maximize the revenue. We consider a hub location problem that arises in the design of a complete–star network. There are several nodes in the network that p of them are chosen to be hubs. Each node is assigned to exactly one hub and all of the hubs are connected to each other. Our hub and spoke network configuration is an extended case of flow-based models for designing capacitated networks and routing policies which proposed by Jaillet et al (1996). Mentioned network has applications in telecommunications and transportation.

In figure 1, a complete–star network is depicted. Consider that we have to transfer passengers between two nodes i and m, if both nodes are assigned to the same hub like J, the passengers from node i to node m first goes to hub J and then from hub J goes to node m. In a situation that node i and node l are assigned to different hubs the passengers traffic from node i to node l first goes from node i to its hub like J, and then from hub J to the hub K and lastly the traffic goes from hub K to node l. The limited capacity on arc (i, j) is shared between itineraries from i to the other destination nodes and the itineraries between all origin nodes to the node i. Therefore, we have two types of flight legs in the network. The first one is the links that transfers the traffic between non-hub node and a hub node that we name them as link type 1 and the second type is that transfers the traffic between hubs that we name them as links type 2. The classical discount factor α is incorporated to the model by taking $C2 = \alpha\, C1$. It is assumed customers are classified into different segments or classes based on their sensitivity to prices. Considering the locations of hubs and the fixed and limited capacity for serving uncertain demand, the protection levels that specify the capacity to reserve (protect) for a particular class or set of classes of customers are set at the beginning of the horizon. In addition, the sale decisions are made after the arrival of customer orders as well as when more definitive information becomes available.

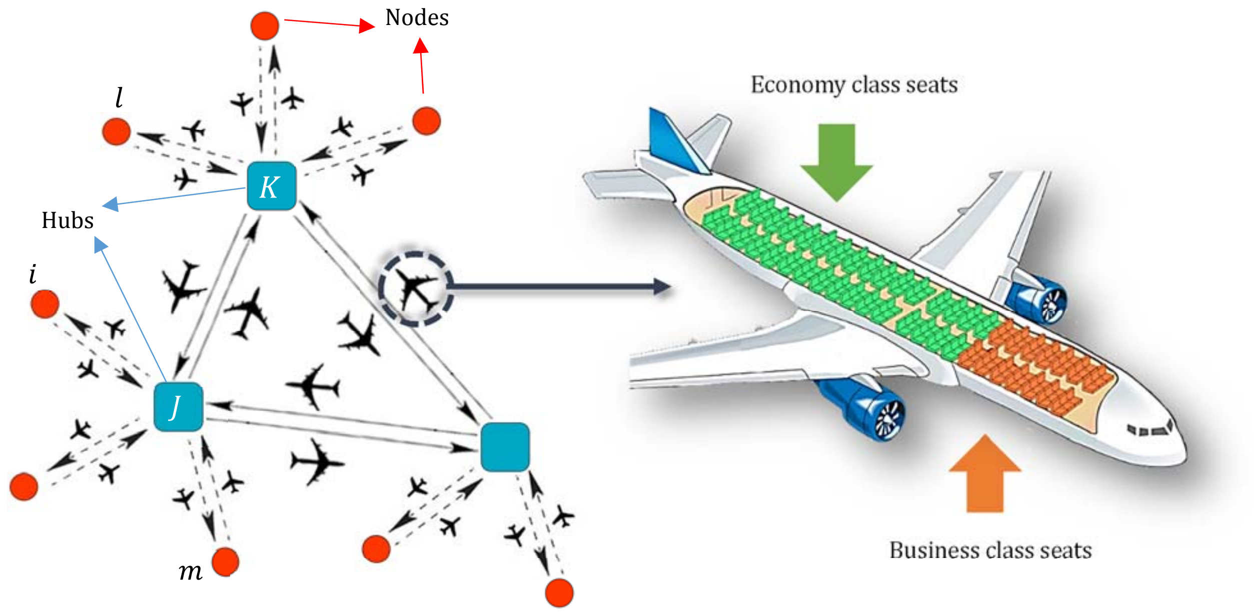


Fig. 1. An example solution of presented hub location and seat allocation problem

The hub location, network design and protection levels that specify the capacity to reserve (protect) for a particular class or set of classes of customers are determined at the beginning of the horizon. In addition, the sale decisions are made after the arrival of customer orders as well as when more definitive information becomes available. On the other hand, the customer classes arrive with stochastic demands. The tickets are sold according to both the protection levels and demands to maximize the total airline's revenue. To employ the stochastic programming (SP) model, consider S scenarios with probabilities pr_{imk}^{s}, $s = 1 \dots S$, of the booking demand. These scenarios may be obtained from stochastic demand models. In this section, first we introduce the non-linear revenue management and capacitated hub location model with different customer classes. In the second part of this section, a linear integer programming is obtained by some additional variables and constraints.

Notations used for mathematical formulation are as follows:

3-1- Sets and Parameters

N	The number of nodes (spokes) in the hub-spoke network.
P	The number of hubs.
S	The number of scenarios.
K	The number of customer classes.
i, m	Indices for nodes $i, m = 1 \dots N$.
j	Indices for hubs.
s	Indices for scenarios.
k	Indices for customer classes.
f_{im}	Distance from node i to node m.
C_{1k}	Transfer cost (per unit flight and unit distance) between origin and hub node which is defined as leg type 1 for customer class k.

C_{2k} Transfer cost (per unit flight and unit distance) between two hubs which is defined as leg type 2 for customer class k.

d_{imk}^{s} Traffic demand between origin i and destination m for customer class k under scenario s.

pr_{imk}^{s} The probability of traffic demand between origin i and destination m for customer class k under scenario s.

r_{imk} Ticket price of itinerary between origin i and destination m for customer class k.

No_{ij} Number of flights available for itinerary between node i and hub j.

$No'_{jj'}$ Number of flights available for itinerary between hub j and hub j'.

Q_1 Number of seats available in a service cabin at leg type 1.

Q_2 Number of seats available in a service cabin at leg type 2.

F_j Fixed cost for establishing hub j.

3-2- Decision variables

y_{imk}^{s} Number of tickets sold for itinerary between origin i and destination m for customer class k under scenario s.

z_{imk} Protection level for itinerary between origin i and destination m for customer class k.

$x_{ijj'm}$ A binary decision variable, which is 1 if traffic from node i to node m goes through hubs located at node j and j' and 0 otherwise.

w_{ij} A binary decision variable, which is 1 if node i is connected to hub j and 0 otherwise.

With these set of variables, we can obtain a nonlinear formulation as follows

3-3- Non-linear capacitated model of hub location and revenue management

The nonlinear formulation of the proposed hub location and revenue management problem is as follows:

$$Max\ \sum_i \sum_m \sum_k \sum_s pr_{imk}^{s}\, r_{imk}\, y_{imk}^{s} - \sum_i \sum_j \sum_k C_{1k}[\, f_{ij}(\sum_{m\in I/\{i\}} \sum_s pr_{imk}^{s} \frac{y_{imk}^{s}}{Q_1}) + f_{ji}(\sum_{m\in I/\{i\}} \sum_s pr_{mik}^{s} \frac{y_{mik}^{s}}{Q_1})]w_{ij} - \sum_k \sum_j \sum_{j'} \sum_i \sum_{m\in I/\{i\}} \sum_s C_{2k} \left[(f_{jj'} \left(pr_{imk}^{s} \frac{y_{imk}^{s}}{Q_2}\right) + f_{jj'} \left(pr_{mik}^{s} \frac{y_{mik}^{s}}{Q_2}\right)) x_{ijj'm}\right] - \sum_j F_j w_{jj} \tag{1}$$

$$\sum_j \sum_{j'} x_{ijj'm} = 1 \quad \forall i, m \tag{2}$$

$$x_{ijj'm} \le w_{ij} \quad \forall i, j, j', m \tag{3}$$

$$x_{ijj'm} \le w_{mj'} \quad \forall i, j, j', m \tag{4}$$

$$w_{ij} \le w_{jj} \quad \forall i, j \tag{5}$$

$$\sum_j w_{ij} = 1 \quad \forall i \tag{6}$$

$$\sum_j w_{jj} = P \tag{7}$$

$$y_{imk}^{s} \le d_{imk}^{s} \quad \forall i, m, k, s \tag{8}$$

$$y_{imk}^{s} \le Z_{imk} \quad \forall i, m, k, s \tag{9}$$

$$\sum_i \sum_m \sum_k \frac{Z_{imk}}{Q_2} x_{ijj'm} + \frac{Z_{mik}}{Q_2} x_{ijj'm} \leq No'_{jj'} \quad \forall j,j' \tag{10}$$

$$\sum_m \sum_k \frac{Z_{imk}}{Q_1} + \sum_m \sum_k \frac{Z_{mik}}{Q_1} \leq \sum_j No_{ij} * w_{ij} + M\, w_{ii} \quad \forall i \tag{11}$$

$$w_{ij} \in \{0,1\} \quad \forall i,j,j',m \tag{12}$$

$$x_{ijj'm} \in \{0,1\} \quad \forall i,j,j',m \tag{13}$$

$$y_{imk}^{s}, Z_{imk} \in Z_{+} \forall i,m,k,s \tag{14}$$

The objective function (1) maximizes the total profit of the hub network considering the revenue of selling tickets to different classes and itineraries, total transportation cost between all non-hub nodes and the hub nodes, total transportation cost between hubs and total installation hub costs. Constraint (2) ensures that each origin/destination pair (i,m) is allocated to one pair of hub nodes (j,j'). Constraints (3) and (4) guarantee that the demand from origin node i to destination node m cannot be allocated to a hub pair (j,j') unless both nodes (j,j') are defined as hub nodes. Constraint (5) assures that no node is assigned to a location unless a hub is opened at that site. Constraint (6) enforces single allocation for each node. Constraint (7) states that there must be exactly P hubs. Constraint (8) and (9) insure that the number of sold tickets for each origin destination itineraries should be less than the demand and the protection level. Constraint (10) establish the capacity constraints of link type 2 between hub node j and hub node j'. Also, constraint (11) shows the capacity limit of link type 1 between node i and hub node j. Constraint (12-14) define the types of decision variables.

3-4- Linear capacitated model of hub location and revenue management

We add some variables and a set of linear constraints to obtain the linear integer programming. To linearize the proposed non-linear model, following equations are added to the model:

$$V_{ijmk}^{s} \geq (1 - w_{ij})(-M) + y_{imk}^{s} \quad \forall i,m,j,k,s \tag{15}$$

$$V_{mjik}^{s} \geq (1 - w_{ij})(-M) + y_{mik}^{s} \quad \forall i,m,j,k,s \tag{16}$$

$$O_{ijj'mk}^{s} \geq (1 - x_{ijj'm})(-M) + y_{imk}^{s} \; \forall i,m,j,j',k,s \tag{17}$$

$$O_{mjj'ik}^{s} \geq (1 - x_{ijj'm})(-M) + y_{mik}^{s} \; \forall i,m,j,j',k,s \tag{18}$$

$$G_{ijj'mk} \geq (1 - x_{ijj'm})(-M) + Z_{imk} \quad \forall i,m,j,j',k \tag{19}$$

$$G_{mjj'ik} \geq (1 - x_{ijj'm})(-M) + Z_{mik} \quad \forall i,m,j,j',k \tag{20}$$

$$O_{ijj'mk}^{s}, V_{ijmk}^{s}, G_{mjj'ik} \geq 0 \quad \forall i,j,j',m,k,s \tag{21}$$

Using constraint (15), the variable y_{imk}^{s} becomes the lower bound for V_{ijmk}^{s}, if $w_{ij} = 1$, and using constraints (17) and (20), the variable y_{imk}^{s} and $\mathrm{Z_{imk}}$ becomes the lower bound for $O_{ijj'mk}^{s}$ and $G_{ijj'mk}$ respectively, if $x_{ijj'm} = 1$. As we minimize the value of $O_{ijj'mk}^{s}$ and V_{ijmk}^{s}, in the objective function and according to the utility of the lower value of $G_{ijj'mk}$ in the capacity constraint, they will attain the lower bound. In line with additional constraints (15-21), the objective function and constraint (10) are also changed to equations (22) and (23) respectively.

$$Max\ \sum_i \sum_m \sum_k \sum_s pr_{imk}^s\, y_{imk}^s\, r_{imk} - \sum_i \sum_j \sum_k C_{1k}[\, f_{ij}(\sum_{m\in I/\{i\}} \sum_s pr_{imk}^s \frac{V_{ijmk}^s}{Q_1}) + f_{ji}(\sum_{m\in I/\{i\}} \sum_s pr_{mik}^s \frac{V_{mjik}^s}{Q_1})] - \sum_k \sum_j \sum_{j'} \sum_i \sum_{m\in I/\{i\}} C_{2k}\left[f_{jj'}\left(pr_{imk}^s \frac{O_{ijj'mk}^s}{Q_2}\right) + f_{jj'}\left(pr_{mik}^s \frac{O_{mjj'ik}^s}{Q_2}\right)\right] - \sum_j F_j w_{jj} \tag{22}$$

$$\sum_i \sum_{m\in I/\{i\}} \sum_k (\frac{G_{ijj'mk}}{Q2} + \frac{G_{mjj'ik}}{Q2}) \quad \le NO'_{jj'} \qquad \forall j, j' \tag{23}$$

4- Solution method

The model presented in section 3 is a mixed-integer linear programming model and is coded by the GAMS 24.1.3. However, only the small-sized instances can be solved to optimality using the general purpose MIP solvers. Due to this limitation, a hybrid meta-heuristic algorithm combined the genetic algorithm and the exact solution approach is used.

Genetic algorithm is a stochastic search approach which imitates the process of evolution and natural selection for finding the near-optimal solution. The primary idea was introduced by Holland (1975). GA works with an initial population of solutions that usually randomly initialized. Each population is represented by chromosomes treat as individuals, whose fitness is calculated by the corresponding objective function value. Individuals of a given population goes through some procedure named evolution consisting of cross over and mutation. Cross over and mutation are used to generate population. In cross over, two chromosomes (parents) are combined (mated) to produce a new chromosome (offspring). The mutation operator makes random changes in individual's genes. This cycle of evaluation-selection-reproduction is repeated until a well-defined stopping criterion is satisfied. Proposed evolutionary algorithm for solving large-scale problem is composed of meta-heuristic and exact solution based on CPLEX and GA. In fact, our problem can be parsed into two sub-problems: hub location problem which is an NP-hard problem (Kara and Tansel 2000) and seat allocation problem that obtains protection levels and sold tickets considering principal of revenue management. The genetic algorithm is utilized to find different structures for the hub location network and then to take advantage of the optimal solutions; the CPLEX solver is used in obtaining the overall revenue and other variables based on the network configuration proposed by GA.

4-1- Modified caching genetic algorithm

Caching GA is a modified version of the genetic algorithm with the purpose of avoiding unnecessary calculation of objective values for repetitive individuals during the GA operations (Kratica et al. 2007). Kratica et al. (2007) applied caching techniques to introduce two efficient version of genetic algorithms for solving NP-hard problem. They concluded that implementing caching technique has a significant enhancement in the GA running time. They utilized simple and useful caching strategy called Least Recently Used (LRU) with pre-specified data storage.

In the caching process, the objective function value of each individual is stored in a cache table which is called hash-row table. The main advantage of this method is that if we meet the same genetic code during the operation of the genetic algorithm, we can use the stored information in cache-table instead of repeating the calculations. We applied mentioned technique in our proposed algorithm to reduce the time-consuming part of the algorithm which calculates the objective value, protection levels and sold tickets values and as a result improves the performance of evolutionary algorithm. Same as referred work, we used LRU caching strategy and store a certain amount of calculated values in a cache-table of the size $Ncache = 3000$. We also considered an additional operator named immigration. In migration operator, some immigrants (random members) entrant to the society in each period which keep the algorithm away from local optima during the execution (Ebrahimi-zade et al. 2015). It is worth mentioning that after obtaining the network structure from the individual's genetic code, by fixing values of w_{ij} and then $x_{ijj'm}$, the initial mixed non-linear integer programming problem (NMIP) is reduced to a linear mixed integer programming (MIP) sub problem that could be solved by CPLEX without the complexity of linearization. According to the mentioned description we used the link between MATLAB software and GAMS. Complete flowchart of the algorithm is provided in figure 2.

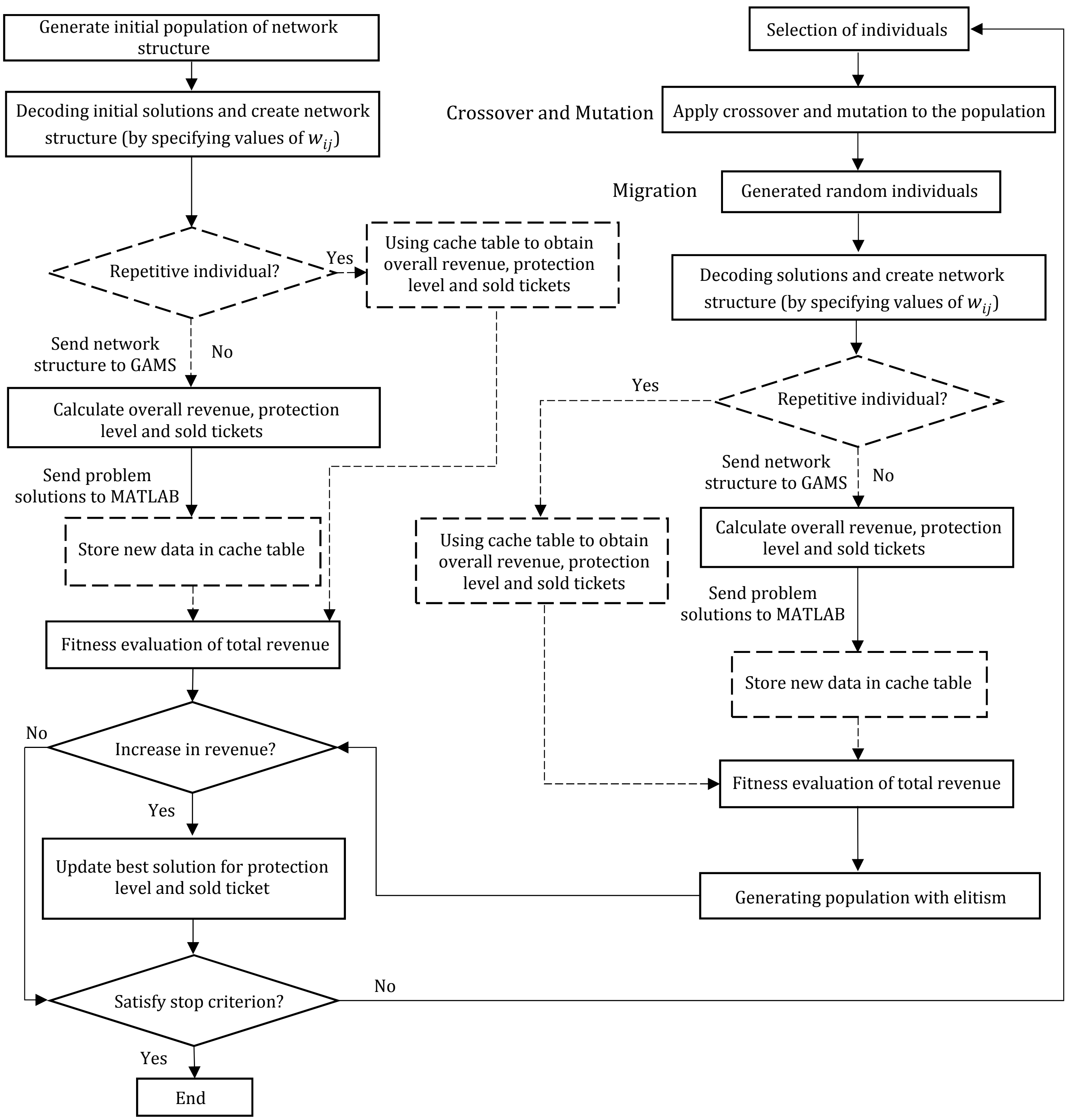


Fig. 2 Flowchart of process of proposed evolutionary algorithm based on genetic algorithm

4-2- Representation of the Solution

The efficiency of genetic algorithm is noticeably affected by chromosome structure. Chromosome structure contains all the information related to solve the problem. The solution of the hub location problem represents the network configuration by demonstrating the location of hubs and allocation of nodes to hubs. We used a n dimension matrix contains numbers between [0,1] to represent the given network as a chromosome structure in which n is the number of nodes. Figure 3 illustrates the example solution of GA which is decoded by decoding process.

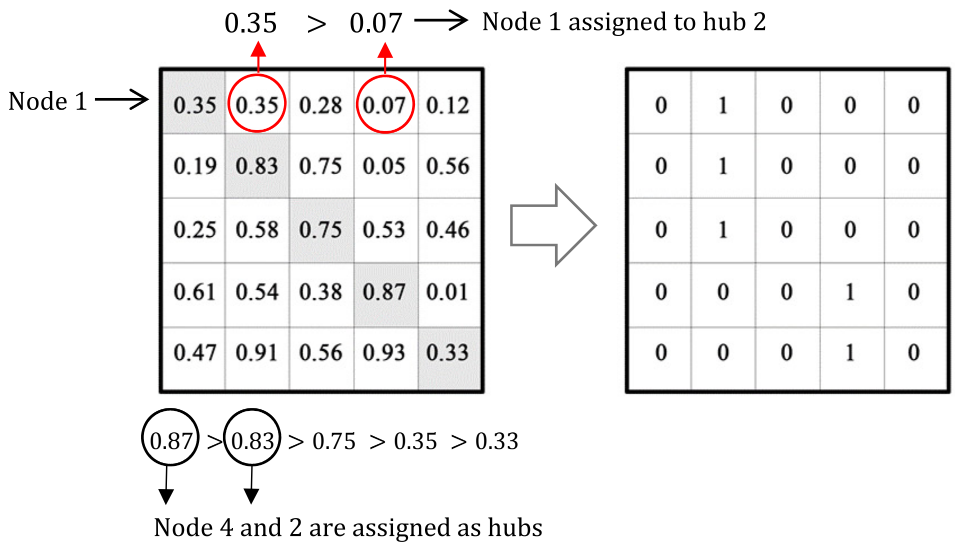


Fig. 3 Chromosome structure and decoding process

In this example, there are five nodes that two of them should be determined as hubs. First, the numbers in the diagonal of matrix are sorted; the larger numbers are assigned as a hub until the number of hubs are completed. For example, node 2 and 4 are specified as hubs. In the following, the non-hubs nodes are assigned to hubs by comparing the values at the intersection of non-hub node's column and the rows which are assigned as hubs, and the highest number determines the hub number which the mentioned node should be assigned to that. This approach ensures that exactly p distinct hub indices are established as hubs and each ordinary node is only allowed to connect into one hub. The values which are equal to one at the main diagonal will be considered as a hub and on the other elements with value 1 show the allocated demand nodes. A sample solution related to figure 3 is depicted in figure 4.

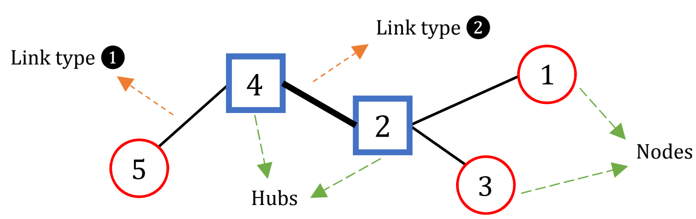


Fig. 4 Sample network solution

In the current study, we applied three genetic operators consist of crossover, mutation and immigration. Thus, the following subsections are dedicated to discuss about these operators.

4-3- Crossover operator

Crossover operator combines two parent solutions of the population, so as to create one or more offspring. There are different ways to combine variable values from the two parents into new variable values in the offspring. We used equations (24) and (25) to obtain new variable values from two parents. So p_1new and p_2new comes from a combination of the two corresponding parent's variable values like as follows (Radcliffe 1991):

$$p_1new = b\,p_{fn} + (1 - b)p_{sn} \tag{24}$$

$$p_2new = (1-b)\,p_{fn} + b\,p_{sn} \tag{25}$$

Where

b = random matrix with size of parents on the interval $[0, 1]$

p_{fn} = n dimension matrix of first parent

p_{sn} = n dimension matrix of second parent

4-4- Mutation operator

The purpose of mutation in GAs is to prevent the problem ending in a local solution. Mutation is an important part of the genetic algorithm which helps to preserve the diversity and keeping the algorithm away from the local optima during the executions. Figure 5 demonstrates the process of two mutation operators used in this study. The algorithm randomly selects one of the mutation operators to change the chromosome structure.

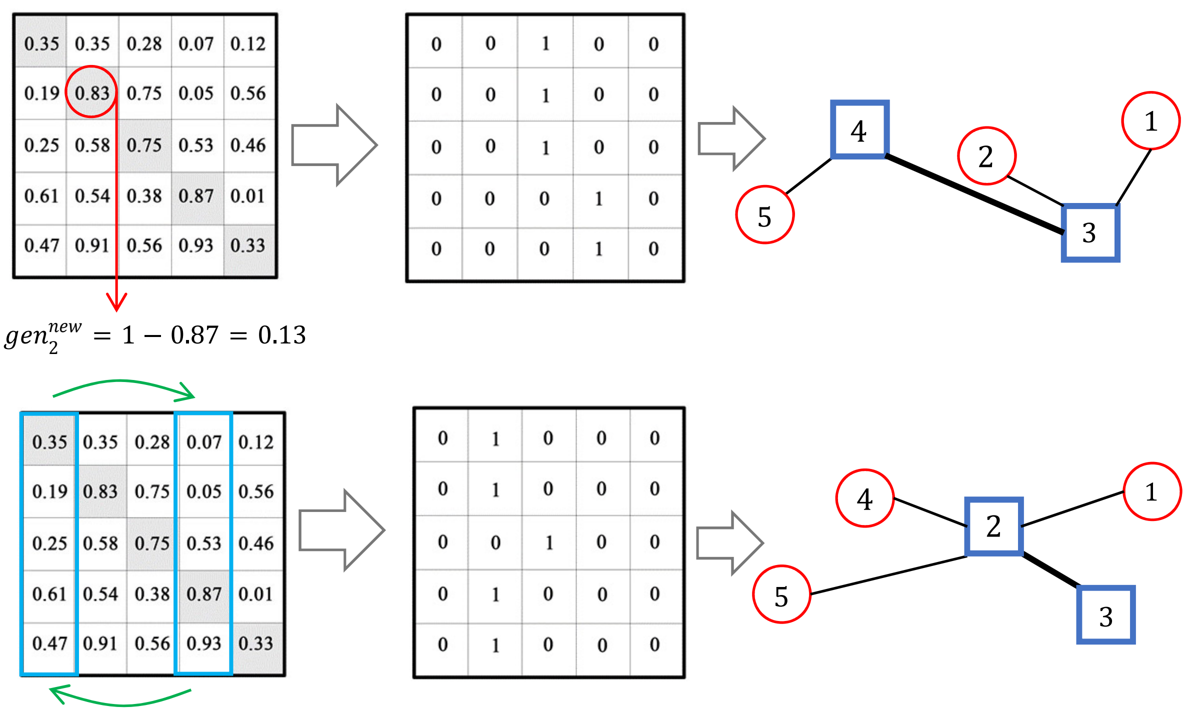


Fig. 5. The effect of different mutations on chromosomes and the resulting network

In this study we considered two mutation operators: exchange mutation in which we select two random columns and then swap the genes on these columns and in second mutation operator we choose a gene in the main diagonal that causes the node to be a hub by chance and then the related value is changed to

$gen_j^{new} = 1 - gen_j^{old}$, as a result the hub node changes to a non-hub node so that the next priority will be selected as a hub.

4-5- Immigration operator

Same as mutation operator, immigration operator plays a significant role in helping the algorithm to avoid from local optima. As what happens in many real-world societies, there is a set of individuals named immigrants which enter to the existing population during the time. These immigrants are created randomly and cause a proper search in the regions of the search space and helps to improve algorithm's capability. Immigration operator is proposed by Michael et al. (1991) in purpose of increasing the exploration while maintaining nearly the same level of exploitation for the given population size. In this study we applied immigration as a GA operator. Accordingly, in each generation of proposed GA, number of offsprings, mutants and immigrants added to the main population, and then after sorting, better individuals move to the next generation.

5-Computational results

In this section, we reported the results of computational experiments by implementing the model on four subsets of Turkish network data set presented by Tan and Kara (2007). These subsets are selected from the first 5, 6, 10, 15 and 20 elements of the aforementioned dataset. We also evaluated the performance of implementation of caching technique on modified GA in comparison with the exact solution obtained by GAMS SOLVER 24.1.3 and the performance of the solutions found by the proposed improved genetic algorithm is compared with pure GA and Imperialist Competitive Algorithm (ICA). The meta-heuristic algorithms are implemented in the MATLAB program and linked to the GAMS with ILOG CPLEX 12.5 64-Bit optimization routines. All programs are run on an Intel core i5-3337U (1.8 GHz) with 6 GB of RAM. To model the uncertainty of data, we applied stochastic programming by using a finite number of scenarios. These scenarios and their relevant probabilities represent an approximation of the probability distribution given by the random data. This method helps to avoid the difficulty of continuous distributions. In the current study, we utilized a scenario tree-based stochastic programming approach to produce demand scenarios. Other input parameters are generated according to table 1.

Table 1. Generated input parameters

Number of flights between hubs	$DU(3{,}6)$
Number of flights between nodes and hubs	$DU(1{,}3)$
The setup costs for a hub	$U(1200000{,}2400000)$
The unit transportation costs (C)	$U\ [7, 14]$
Demand scenarios (high/nominal /low)	125%/100%/75%
Demand classes (economic / business)	80%/20%
Scenario Probabilities	0.19/0.59/0.22
Ticket price for each itinerary for economic class	$U(600{,}3600)$
Ticket price for each itinerary for business class	$1.5 \times$ ticket price for corresponding economic class

In table 1, $U[a, b]$ denotes a continuous uniform distribution function with upper bound a and lower bound b, and $DU(a, b)$ denotes discrete uniform distribution with integer parameters between a and b. Parameter C refers to the transfer cost per unit flight and unit distance between origin and hub for business class seats. This is calculated by equation $C_{11} = C$, and for economic seats $C_{12} = \beta C$ where $\beta \sim U\ [0.5{,}1]$. We sampled the nominal demand from Turkish network data set. Then, we considered deviations from the nominal scenario as 125%, and 75% in response to the seasonal demand variation. The capacity of aircrafts in leg between hub and nodes have 100 seats capacity while 200 capacity of seats is considered for aircrafts that transfer passengers between hubs. Since parameter calibration has a significant effect on the efficiency of meta-heuristic algorithms, in the next section we applied Taguchi design method for setting the parameters.

5-1- Parameters setting

Taguchi method provides a fractional factorial experiment instead of full factorial experiments in order to reduce the number of experiments. It divides the factors into signal and noise factors. This method strives to minimize the effect of noise for determining the optimal level of the signal factors. For a comprehensive study on the Taguchi method, we refer the readers to study Roy (2001).

In current problem, the L9 design of the Taguchi method is applied for genetic algorithm by using the Minitab 16.2 software. The relative percentage deviation (RPD) formulation is used to change objective function values to non-scale data so a lower response level is more desirable. Since quality characteristic is considered as relative percentage deviation, we selected "Smaller is better" type.
Accordingly, the RPD is calculated as follows:

$$RPD = \frac{|Alg_{sol} - Best_{sol}|}{|Best_{sol}|} \times 100 \tag{26}$$

Where, Alg_{sol} is the objective function value obtained by the meta-heuristic algorithms and $Best_{sol}$ is the optimal solution obtained for each instance by the GAMS software.
In order to study the behavior of different parameters of the proposed evolutionary algorithm, we considered four factors involved in our genetic algorithm, namely, the crossover percentage, mutation percentage, immigration percentage, and the population size and for ICA we considered four factors consist of number of imperialists, number of colonies, revolution rate and assimilation rate. The considered levels of the parameters are shown in table 2.

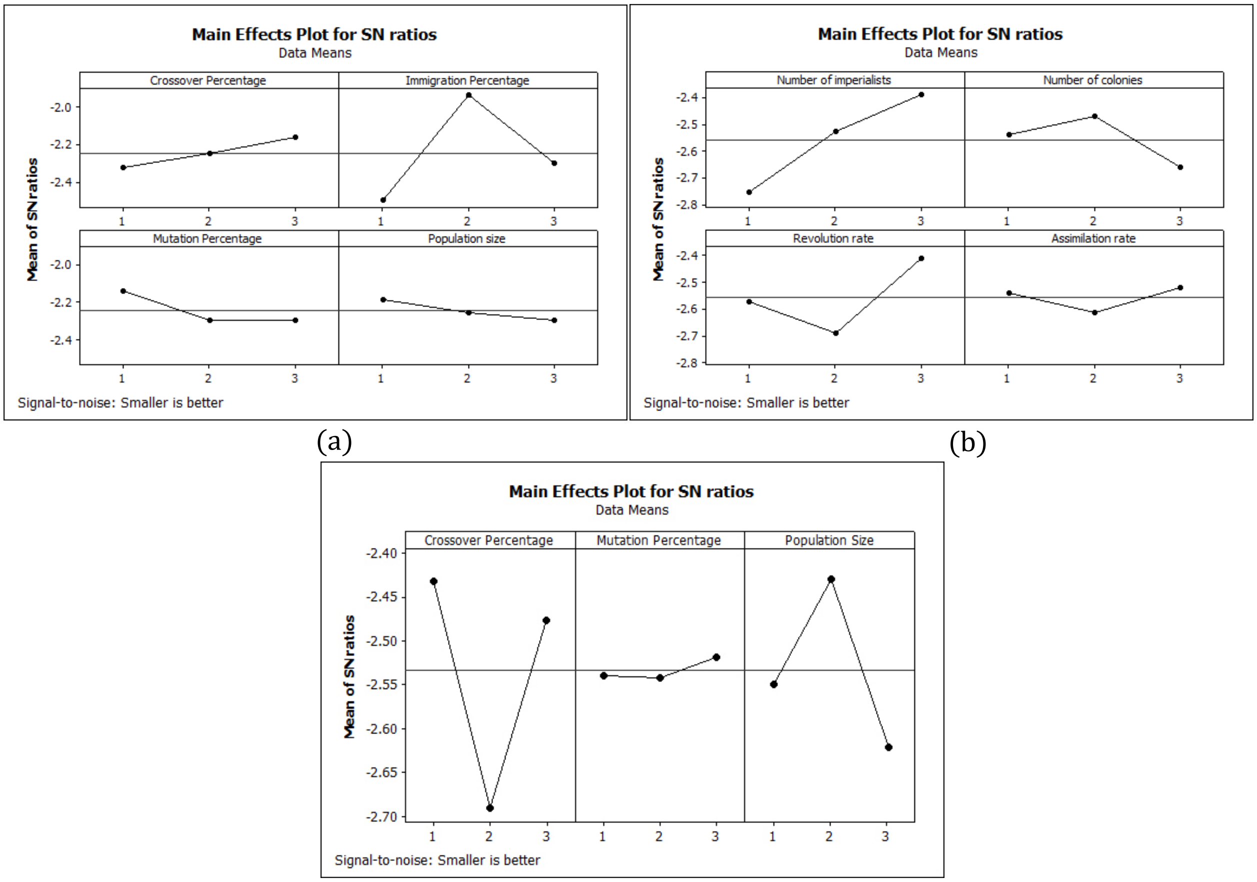
Table 2. Levels of parameters of meta-heuristic algorithms

	Parameter	Level 1	Level 2	Level 3
Modified GA	Crossover percentage	0.60	0.70	0.80
	Mutation percentage	0.10	0.15	0.20
	Immigration percentage	0.05	0.10	0.15
	Population size	50	70	90
GA	Crossover percentage	0.60	0.70	0.80
	Mutation percentage	0.10	0.15	0.20
	Population size	50	70	90
ICA	Number of imperialists	5	10	15
	Number of colonies	50	70	90
	Revolution rate	0.10	0.15	0.20
	Assimilation rate	0.60	0.70	0.80

Relative percentage deviation (RPD) for the objective function value is utilized to compare the levels of parameters. The results of implementing Taguchi design method in MINITAB 16.2 is shown in figure 6. Largest value of the S/N ratio in the graphs for each parameter shows a higher performance level. Desired parameters for each of the algorithms are shown in table 3.

Table 3. Tuned parameters for the proposed GA, pure GA and ICA

Parameter (GA)	Modified GA	GA	Parameter	ICA
Crossover percentage	0.80	0.60	Number of imperialists	15
Mutation percentage	0.10	0.20	Number of colonies	70
Immigration percentage	0.10	-	Revolution rate	0.20
Population size	50	70	Assimilation rate	0.80



(a) (b)

(c)

Fig. 6 Mean S/N ratios for the proposed GA (a), ICA (b), and pure GA (c)

5- 2- Experimental Results

In this section, we reported the results of computational experiments to evaluate the effectiveness of modified GA using MATLAB software in comparison with the exact solution obtained using GAMS SOLVER 24.1.3. Table 4 provides the results of proposed algorithm and exact solution for different instances with discount factor $\alpha = 0.2, 0.4$. In the first and second columns, instance's dimensions n and p are given. The optimum solutions are presented in column three if existed otherwise it is marked with dash. The next column shows computational time in GAMS. We run GA algorithms 10 times and then best obtained solution of the executions is presented in the fifth column. The solution quality is evaluated by $agap = \frac{1}{10}\sum_{i=1}^{10} gap_i$ where $gap_i = 100 \times \frac{Best.sol - sol_i}{Best.sol.sol}$, sol_i indicates the best solution which found in the ith execution, and $Best.sol$ is the optimum solution if it is found and otherwise it is the best solution obtained from all GA runs. In the next column standard deviation of average gap $\sigma = \sqrt{\frac{1}{10}\sum_{i=1}^{10}(gap_i - agap)^2}$ is presented. In table 4 $eval$ represents the average number of fitness function evaluations while *cache* shows the average percentage of using cache table to obtain fitness function instead of evaluating it. In fact, it could be considered as a measure of time saving during the executions.

We set the total running time 1 hour as a criterion and proposed GA algorithm terminates if the best solution is not changed during 10 numbers of iterations.

Achieved results obviously demonstrate that the proposed algorithm is an effective solution approach for solving the problem. As can be seen from table 4, the proposed GA reached the optimum solution in small-sized instances. Therefore, for larger instances where exact methods cannot provide optimum solutions in a reasonable amount of time, proposed algorithm can be used instead. Also, a little gap in achieving the near-optimal solution by the proposed algorithm confirms that presented solution algorithm is efficient in finding the problem solutions.

Table 4. Results of optimum solution and proposed GA for different instances

n	p	Opt.sol	CPU(s)	Modified GA	CPU(s)	eval	cache	agap%	σ%
		α=0.2							
5	2	659158.29	7.65	opt	21.41	77	0.94	0	0
6	2	1787108.03	30.90	opt	53.96	176	0.90	0	0
10	3	12363273.13	1621.58	opt	331.16	951	0.54	0.48	0.70
	5	-	-	21266242.43	362.78	864	0.57	0.81	0.91
	7	-	-	32628615.77	231.72	541	0.64	0.58	0.67
15	3	-	-	17196953.80	549.97	1279	0.45	0.79	1.37
	5	-	-	26359882.08	579.48	1393	0.52	1.01	0.74
	7	-	-	39264897.70	650.27	1548	0.47	0.87	1.38
20	3	-	-	21514879.42	1228.01	1641	0.42	1.93	2.81
	5	-	-	31827540.37	1271.86	1742	0.48	2.01	2.77
	7	-	-	46208622.17	1054.92	1482	0.41	1.96	1.90
		α=0.4							
5	2	632572.01	7.65	opt	21.59	78	0.94	0	0
6	2	1587746.27	30.90	opt	50.37	162	0.91	0	0
10	3	12128211.78	1659.22	opt	325.87	922	0.56	0.47	0.75
	5	-	-	20901188.92	376.81	881	0.52	0.96	1.14
	7	-	-	31919150.74	240.19	550	0.61	0.62	0.73
15	3	-	-	16485618.15	537.13	1249	0.48	0.90	0.77
	5	-	-	25855391.72	577.18	1389	0.53	1.04	1.01
	7	-	-	38899114.85	656.01	1552	0.45	0.91	1.33
20	3	-	-	20845895.96	1225.36	1636	0.43	1.84	2.61
	5	-	-	30993845.54	1266.47	1733	0.48	2.42	2.54
	7	-	-	45253468.55	1098.10	1496	0.39	1.84	2.01

As can be seen from table 4, in the experiments with 5 and 6 nodes, exact solution performs faster than the modified hybrid GA. Nevertheless, the computational time in exact method has an exponential growth by increasing the problem size. According to table 4, changes in discount factor α does not have a meaningful effect on the computational time, however the objective total revenue decreases with increasing the discount factor in most cases.

In order to evaluate the performance of the proposed modified genetic algorithm, we also compared the quality of its solutions with those of solutions obtained by a standard GA algorithm and imperialist competitive algorithm. The results of the comparison for various size instances of the problem are presented in table 5. We run ICA and GA ten times to solve it. The total CPU-time and objective function value of the best obtained solution and GAP and standard deviation of ten obtained solutions by ICA and modified GA and pure GA are presented.

Table 5. Computational results for the proposed GA, pure GA and ICA

n	p	Modified GA				Pure GA				ICA			
		Obj.value	CPU(s)	agap%	σ%	Obj.value	CPU(s)	agap%	σ%	Obj.value	CPU(s)	agap%	σ%
α=0.2													
5	2	659158.29	21.41	0	0	659158.29	172.32	0	0	659158.29	170.10	0	0
6	2	1787108.03	53.96	0	0	1787108.03	187.17	0	0	1787108.03	176.55	0	0
10	3	12363273.13	331.16	0.48	0.70	12363273.13	624.17	1.34	0.84	12363273.13	622.74	1.06	0.73
	5	21266242.43	362.78	0.81	0.91	21266242.43	538.69	1.20	1.43	21266242.43	518.39	1.34	1.16
	7	32628615.77	231.72	0.58	0.67	32628615.77	444.75	1.94	2.33	32628615.77	416.04	2.19	1.96
15	3	17196953.80	549.97	0.79	1.37	17196953.80	1074.45	2.44	2.30	17196953.80	996.03	2.70	2.14
	5	26359882.08	579.48	1.01	0.74	26359882.08	886. 15	2.56	2.84	26359882.08	880.62	2.50	2.64
	7	39264897.70	650.27	0.87	1.38	39264897.70	1001.48	3.22	3.78	39264897.70	984.48	2.71	3.01
20	3	21514879.42	1228.01	1.93	2.81	21514879.42	1310.01	3.87	3.16	21514879.42	1388.74	4.01	3.33
	5	31827540.37	1271.86	2.01	2.77	31827540.37	1604.03	3.79	3.62	31827540.37	1573.98	3.80	3.52
	7	46208622.17	1054.92	1.96	1.90	46208622.17	2054.33	4.10	4.42	46208622.17	2014.16	4.12	3.65
α=0.4													
5	2	632572.01	21.59	0	0	632572.01	168.16	0	0	632572.01	171.1	0	0
6	2	1587746.27	50.37	0	0	1587746.27	196.47	0	0	1587746.27	184.15	0	0
10	3	12128211.78	325.87	0.47	0.75	12128211.78	602.69	1.16	0.92	12128211.78	584.14	1.21	1.02
	5	20901188.92	376.81	0.96	1.14	20901188.92	516.10	1.83	1.08	20901188.92	505.10	1.93	1.29
	7	31919150.74	240.19	0.62	0.73	31919150.74	504.00	2.09	2.16	31919150.74	467.50	2.44	1.98
15	3	16485618.15	537.13	0.90	0.77	16485618.15	1001.11	2.18	1.46	16485618.15	1019.48	2.01	1.66
	5	25855391.72	577.18	1.04	1.01	25855391.72	915.76	3.03	3.11	25855391.72	875.41	2.82	3.06
	7	38899114.85	656.01	0.91	1.33	38899114.85	1011.19	2.79	3.01	38899114.85	1018.20	2.09	2.69
20	3	20845895.96	1225.36	1.84	2.61	20845895.96	1347.22	4.15	3.28	20845895.96	1329.02	3.60	3.09
	5	30993845.54	1266.47	2.42	2.54	30993845.54	1456.25	4.09	3.87	30993845.54	1516.25	3.66	3.13
	7	45253468.55	1098.10	1.84	2.01	45253468.55	2025.17	3.95	4.02	45253468.55	2044.07	4.70	3.97
Total average :			577.75	0.97	1.18		892.80	2.26	2.16		884.37	2.22	2.00

The results indicate the superiority of proposed algorithm in compared with pure GA and ICA in all problem instances regarding to the quality of the solutions and total computational time.

5-3- Revenue Improvement by Using Proposed Integrated Model

In order to evaluate the revenue improvement by considering both hub location and revenue management problem in an integrated model, consider that airline company sells tickets to a single class using sum of demands for two classes and averaged prices and determines the network configuration. Then, the obtained network structure is considered as an input structure for solving the revenue management sub-problem. The resulting solution obtained from this method in which we used the predetermined network configuration (method 1 in table 6) is compared with the revenue deriving from the proposed model (method 2 in table 6). The obtained revenues from these two methods for some different instances are as follows:

Table 6. Comparing the revenue with predetermined network model and proposed integrated model ($\alpha = 0.2$)

Node	P	Method 1	Method 2	Improvement
10	3	10425247.51	12363273.13	1938025.62
10	5	17882238.79	21266242.43	3384003.64
15	3	11410541.03	17196953.80	5786412.77
15	5	18545929.32	26359882.09	7813952.77

As can be seen from table 6, in all studied instances, proposed model helps to improve the revenue by applying optimal hub and spoke network. The results clearly show the benefits of using an integrated model to gain more profit.

5-4- Evolution of diversity in algorithms

Diversity of generations in evolutionary algorithms has an important influence on the efficiency of the algorithms. This means that fast decreases in the diversity of population leads to premature convergences. So, as to evaluate the diversity, we utilized a measure which presented by Topcuoglu et al. (2005).

In this method, the hamming distance on assigned matrices is used for measuring the diversity. For example, for assigned matrices M1 and M2, the hamming distance between them equals to the amount of changes in the assignments which converts M1 into M2.

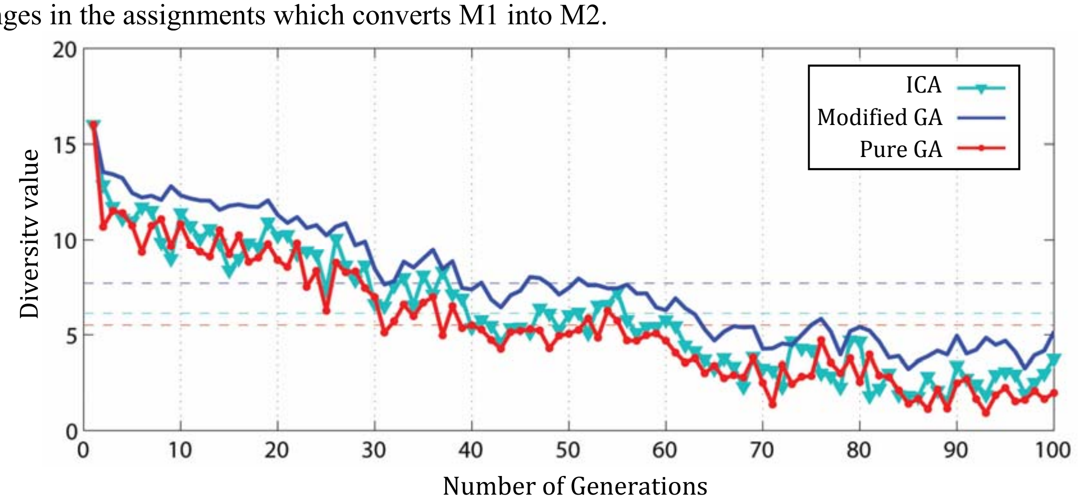


Fig. 7. Diversity values for the algorithms with problem size 20

The average hamming distance between each individual with the best solution and all other individuals in the population considered as a diversity of each population. Figure 7 shows diversity of the population for the problem size 20. We generated initial population once and utilized it for all algorithms. As shown by figure 7, the problem size 20 has an initial population with diversity value of 16. This value signifies that the algorithm generates completely distinct initial populations so as to prevent premature convergence. Furthermore, the diversity values decrease when the number of generations increase. As we can see from figure 7, modified GA performs better at diversifying the search in compared with pure GA and ICA.

5- 5- Value of stochastic programming

Since, it is computationally difficult to solve the stochastic models, for real-world problems, researchers tend to solve simpler versions. For example, they may solve the deterministic program by replacing all random variables with their expected values, or they may combine solution of deterministic programs which correspond to one particular scenario. In order to evaluate the accuracy, we utilized two concepts, the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS) (Birge and Francois 2011).

Expected Value of Perfect Information which estimates the maximum amount a decision maker would be ready to pay in return for complete information in future. Consider Q^* is the optimal value of the stochastic programming and ξ is a random variable with realizations corresponding to the different scenarios, and $\bar{Q}(\xi)$ is the optimal value of the deterministic problem corresponding to each scenario. The wait-and-see value (WS), corresponds to the expected value of the optimal objective for each scenario $WS = E_{\xi}(\bar{Q}(\xi))$. Therefor, the expected value of perfect information (EVPI) is calculated by $EVPI \ = WS - Q^*$.

The other concept is VSS that can be used to find out whether putting extra effort into modeling and solving stochastic programming is beneficial. $\bar{Z}(\bar{\xi})$ is the optimal decision of the first stage in the deterministic problem where all random variables are replaced by their expected values. The VSS is obtained by $VSS \ = Q^* \ - EEV$, where $EEV = E_{\xi}(\bar{Q}(\bar{\xi}), \xi)$. In fact, a higher VSS points out the advantageous of using stochastic programming approach.

Table 7 shows the results of WS, EVPI and VSS for four problem instances, and the best obtained objective value for the two-stage stochastic programming problem, which optimized with three scenarios.

Table 7. Computational results for wait-and-see and EVPI of two-stage stochastic programming ($\alpha = 0.2$)

n	p	Q_0^*	WS	EVPI	EEV	VSS
10	5	21266242.43	22907499.84	1641257.41	20327507.41	938735.02
15	3	17196953.80	18852431.95	1655478.15	16646567.11	550386.69
15	5	26359882.08	28364017.64	2004135.56	25031937.31	1327944.77
20	5	31827540.37	33671927.85	1844387.48	29796591.72	2030948.65

The results of table 7 show the advantage of the stochastic programming solution over deterministic approaches. In particular, the large values for VSS indicate the beneficial use of more complicated modeling techniques. Also, high values of EVPI in studied instances imply that perfect information about future would be helpful to significantly improve the objective function.

6- Conclusion

In this paper, we have formulated and solved a stochastic programming model for airline network revenue management and air freight hub location problem in order to maximize the airline profit by classifying customers and determining the protection levels in a stochastic environment using complete-star network. The problem is NP-hard and it is impossible to solve large scale problems of this type in reasonable amount of time. Therefore, we applied the evolutionary algorithm that includes standard genetic operators and exact solution by linking MATLAB software to GAMS.

The proposed model strives to find the optimal locations of p hubs, the allocations of non-hub nodes to the p located hubs and control of protection levels within the network revenue management context. Computational study with different number of hubs and nodes was carried out based on Turkish network data set. The results clearly demonstrate that integrated model for hub location and revenue management problem would help to improve the total revenue of airline companies.

In the proposed algorithm, an immigration operator is utilized for a better search in the solution space, also computational performances of the algorithm were improved by caching technique. The performance of the modified GA is compared with the pure GA and imperialist competitive algorithm (ICA). The results corroborated capability of the proposed algorithm in achieving high-quality solutions in a reasonable time. Moreover, a considerable percentage of run-time savings is obtained by using caching technique. Additionally, the solution quality of proposed genetic search approach is quite satisfactory, and it prevents the population from premature convergence by preserving the diversity.

For future works, addressing fuzzy demands for the problem, analyzing the problem with robust optimization and presenting a bi-objective model for maximizing reliability of transportation network as the second objective could be valuable research subjects.

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