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Economic-statistical designs of integrated qss-rs and qss-rgs plans

Samrad Jafarian-Namin[1*] , Parviz Fattahi[1]
[1]*Department of Industrial Engineering, Faculty of Engineering, Alzahra University , Tehran, Iran*

Abstract

This study presents two innovative quick switching sampling (QSS) plans by separately employing repetitive group sampling (RGS) and resubmitted sampling (RS) for Normal inspection, alongside utilizing single sampling (SS) for Tightened inspection, tailored for normally distributed quality characteristics. A Markovian model is employed to calculate acceptance probabilities. Furthermore, we develop the first loss-based model for presenting economic-statistical designs (ESDs) of QSS plans, along with an alternative model based on the average sample number (ASN) objective function. Addressing the limitations of traditional grid search approaches in existing literature, we introduce a Particle Swarm Optimization (PSO)-based solution, enabling the optimal determination of QSS plan decision variables. Through numerical examples, a comprehensive case study, and sensitivity analysis, we demonstrate that the QSS-RS-SS plan, when coupled with the loss-based model, significantly reduces total risks and costs.

Keywords: Normal quality characteristic; quick switching sampling; Resubmitted sampling; Repetitive group sampling; Economic-statistical design; Loss function

1. Introduction

Acceptance sampling plans (ASPs) are critical tools in quality control, employed to assess whether a product lot conforms to specified quality standards. These plans involve inspecting a sample from the lot and making a decision to either accept or reject the entire lot. Over time, various types of ASPs have been developed, each offering distinct advantages depending on the quality characteristic (QC) under inspection. These include single sampling (SS), repetitive group sampling (RGS), multiple dependent state repetitive (MDSR), resubmitted sampling (RS), and quick switching sampling (QSS) plans, among others [1]. QSS plans offer a balanced approach by dynamically adjusting inspection levels based on the quality of sampled items. This adaptability not only leads to cost savings but also ensures effective risk management. Consequently, QSS plans have been widely applied across various contexts, with attribute QSS (AQSS) and Variable QSS (VQSS) plans specifically designed for attribute and variable QCs, respectively.

The introduction of the economic-statistical design (ESD) approach has enabled the optimization of a plan's objective function (OBF) while meeting favorable statistical levels on the operating characteristic (OC) curve. By including constraints that ensure producer and consumer satisfaction, models of ESD with the average sample number (ASN) OBFs were developed to optimize QSS plans. In this context, Balamurali and Usha [2] optimized a VQSS plan for normally distributed QCs with double specification limits. Utilizing the loss-based process capability index, Balamurali and Usha [3] developed VQSS-SS plans based on C_{pmk}. Assuming only one specification limit, Wu et al. [4] presented an ESD model for designing a C_{pk}-based VQSS-SS plan. In general, the policy of changing acceptance criterion (PolAC) facilitates stringent yet simpler inspections, while the policy of changing sample size (PolSS) provides more detailed sample information but at a higher cost. Wu et al. [5] developed two C_{pk}-based VQSS-SS plans and compared their effectiveness. Their findings suggest that both PolAC and PolSS have distinct advantages, and the choice between them should be guided by practical conditions. Wang and Wu [6] and Wang et al. [7] preferred PolSS in designing C_{pm}-based and C_{pmk}-based VQSS-SS plans, respectively, using the ASN as the OBF. Banihashemi et al. [8] proposed two VQSS-

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RS plans based on the process yield index S_{pk}. Their results indicate that PolSS is preferable to PolAC under specific conditions. To leverage the strengths of both PolAC and PolSS, Shu et al. [9] introduced a generalized C_{pm}-based VQSS-SS plan. Recently, Rahimi Yeganeh and Ayoubi [10] developed QSS plans for linear profiles. They utilized PolAC and employed binomial attribute sampling for Normal inspection, while using SS and MDSR for Tightened inspection to provide distinct mixed attribute-variable QSS plans.

Although ESDs have been developed using ASN as OBFs, their optimal results may not always guarantee the lowest cost. Evaluating a lot with a QSS plan can be expensive due to the expected outcomes of inspections. Thus, ESDs have been developed by incorporating inspection-related costs as OBFs. For Weibull distributed lifetime QCs, Balamurali et al. [11] proposed a VQSS-SS plan using PolAC. They demonstrated the plan's minimum total cost, including the cost of inspection per item, internal failure cost and the cost of outgoing defective item, through an ESD model. Considering similar model, Mahalingam and Balamurali [12] optimized an AQSS-RS plan to inspect Binomial and Poisson QCs under PolAC. Moreover, Usha and Balamurali [13] optimized QSS-RS plan under Gamma-Poisson distribution.

To the best of our knowledge, this study is the first to develop QSS-RGS-SS and QSS-RS-SS plans incorporating Normal and Tightened inspection levels for normally distributed QCs. The proposed plans leverage the strengths of both PolAC and PolSS. A Markovian model is employed to capture the system's states and transition probabilities. Furthermore, we introduce the first loss-based ESD model for QSS plans, along with an alternative ASN-based ESD model that meets statistical criteria. Previous research has relied exclusively on grid search (GS) as the solution approach. However, studies such as [14] and [15] have demonstrated the superior performance of Particle Swarm Optimization (PSO) over GS and genetic algorithms in solving Mixed Integer Nonlinear Programming (MINLP) problems. Building on these findings, we propose a PSO-based solution approach to optimally determine the decision variables of QSS plans.

The remainder of this study is structured as follows. Section 2 proposes two QSS plans for normally distributed QCs. Section 3 develops two ESD models for optimization. Section 4 proposes a PSO-based approach for optimizing models. Section 5 evaluates the performance of two models and four plans through simulation studies, followed by demonstrating the application through a real-world example. Then, a sensitivity analysis across various parameter combinations is implemented for thorough evaluation. Finally, Section 7 concludes the study and suggests directions for future research.

2. Proposed QSS plans

This section proposes two QSS plans, one employing RGS and SS inspections and the other using RS and SS inspections.

2.1. VQSS-RGS-SS plan

This proposed plan, utilizing the merits of both PolAC and PolSS, consists of the following steps:

1. RGS plan for Normal inspection. Its decision variables include the sample size (n_N), critical acceptance number (k_{aN}), and rejection number (k_{rN}).
2. SS plan for Tightened inspection. Its decision variables include the sample size (n_T), and critical acceptance number (k_T).

Implementation requires the following steps:

1. Determine decision variables and input parameters.
2. Implement Normal inspection:
    a. Take a random sample of size n_N from the lot, and calculate v_N.
    b. Compare v_N with acceptance and rejection numbers:
        i. If $v_N \geq k_{aN}$, accept the lot, and continue to the next lot under Normal inspection.
        ii. If $v_N < k_{rN}$, reject the lot, and proceed to step 3 for the next lot.
        iii. If the decision is unclear (e.g., $k_{rN} \leq v_N < k_{aN}$), proceed to step 2a.
3. Implement Tightened inspection:
    a. Take a random sample of size n_T ($n_T > n_N$) and calculate v_T.
    b. Compare v_T with the acceptance number:
        i. If $v_T \geq k_T$, accept the lot, and return to step 2 for the next lot.
        ii. If $v_T < k_T$, reject the lot, and proceed to step 3 for the next lot.

Note that the indices in Normal and Tightened inspections are respectively calculated as follows:

$$v_N = \left(\overline{x}_N - LSL\right)/\sigma \quad (1)$$

$$v_T = \left(\overline{x}_T - LSL\right)/\sigma \quad (2)$$

According to the Markovian model, the following transition matrix indicates the probabilities of moving from one state to another (states 1, and 2 respectively indicate Normal, and Tightened inspections):

$$P = \begin{matrix} & 1 & 2 \\ 1 & P_a^N & 1-P_a^N \\ 2 & P_a^T & 1-P_a^T \end{matrix} \tag{3}$$

where the probabilities of acceptance under Normal, and Tightened inspections are obtained as follows, respectively:

$$P_a^N\left(p\right) = \frac{1-\Phi\left(\left(k_{aN}-v_N\right)\sqrt{n_N}\right)}{1-\Phi\left(\left(k_{aN}-v_N\right)\sqrt{n_N}\right)+\Phi\left(\left(k_{rN}-v_N\right)\sqrt{n_N}\right)} \tag{4}$$

$$P_a^T\left(p\right) = P\left(v_T \geq k_T\right) = 1-\Phi\left(\left(k_T-v_T\right)\sqrt{n_T}\right) \tag{5}$$

where p denotes the proportion of nonconforming items and $\Phi(.)$ indicates the cumulative distribution function. Regardless of the system's initial state in a Markov chain, the probability of reaching a specific state in the long-run remains constant. Accordingly, to get the steady-state vector, one can solve these equations:

$$\begin{cases} \left[\pi_N,\pi_T\right] = \left[\pi_N,\pi_T\right]\begin{bmatrix} P_a^N & 1-P_a^N \\ P_a^T & 1-P_a^T \end{bmatrix} \\ \pi_N+\pi_T = 1 \end{cases} \tag{6}$$

where the long-run probabilities of remaining under Normal, and Tightened inspections are respectively calculated as follows:

$$\pi_N = \frac{P_a^T}{1-P_a^N+P_a^T} \tag{7}$$

$$\pi_T = \frac{1-P_a^N}{1-P_a^N+P_a^T} \tag{8}$$

The acceptance probability for this plan is calculated as follows:

$$\pi_a\left(p\right) = \pi_N P_a^N\left(p\right)+\pi_T P_a^T\left(p\right) \tag{9}$$

The OC curve is generated from the points (AQL, 1-α) and (LQL, β). At $p=AQL$, the producer aims to maximize the acceptance probability. This leads to the calculation of the Type I error $\alpha=\pi_a(AQL)$. Conversely, at $p=LQL$, the consumer prefers to minimize the acceptance probability. This leads to the calculation of the Type II error $\beta=\pi_a(LQL)$. The ASN measure in the current plan is defined as follows:

$$ASN = \pi_N ASN_N + \pi_T ASN_T \tag{10}$$

where ASN_N, and ASN_T respectively denote the ASN under Normal, and Tightened inspections as follows:

$$ASN_N = \frac{n_N}{1-\Phi\left(\left(k_{aN}-v_N\right)\sqrt{n_N}\right)+\Phi\left(\left(k_{rN}-v_N\right)\sqrt{n_N}\right)} \tag{11}$$

$$ASN_T = n_T \tag{12}$$

2.2. VQSS-RS-SS plan

This plan consists of the following steps:

1. RS plan for Normal inspection. Its decision variables include the sample size (n_N), critical acceptance number (k_N), and number of resubmissions (r_N).
2. SS plan for Tightened inspection. Its decision variables include the sample size (n_T), and critical acceptance number (k_T).

The VQSS-RS-SS plan uses the RS plan for Normal inspection and the SS plan for Tightened inspection. Implementation requires the following steps:

1. Determine decision variables and input parameters.
2. Implement Normal inspection:
    a. Take a random sample of size n_N from the lot, and calculate v_N.
    b. Compare v_N with the acceptance number:
        i. If $v_N \geq k_N$, accept the lot, and continue to the next lot under Normal inspection.

ii. If the lot is not accepted after $(r_N\text{-}1)^{st}$ resubmissions, reject it, and proceed to step 3 for the next lot.
iii. Otherwise, resubmit the lot and proceed to step 2a.

3. Implement Tightened inspection:
   a. Take a random sample of size n_T ($n_T>n_N$) and calculate v_T.
   b. Compare v_T with the acceptance number:
      i. If $v_T \geq k_T$, accept the lot, and return to step 2 for the next lot.
      ii. If $v_T<k_T$, reject the lot, and proceed to step 3 for the next lot.

After utilizing the Markovian model, the probability of acceptance under Normal inspection is obtained as follows:

$$\begin{aligned} P_a^N(p) &= P(v_N \geq k_N)\times\left[1+P(v_N<k_N)+\left(P(v_N<k_N)\right)^2+\ldots+\left(P(v_N<k_N)\right)^{r_N-1}\right] \\ &= 1-\left(P(v_N<k_N)\right)^{r_n} = 1-\left(\Phi\left((k_N-v_N)\sqrt{n_N}\right)\right)^{r_n} \end{aligned} \quad (13)$$

The formulas of long-run probabilities, acceptance probability, and ASN measure for this plan are the same as introduce in the previous subsection. The ASN under Normal is calculated as follows:

$$ASN_N = n_N + n_N P(v<k_N) + n_N\left(P(v<k_N)\right)^2+\ldots+n_N\left(P(v<k_N)\right)^{r_n-1} = n_N\frac{1-\left(P(v<k_N)\right)^{r_n}}{1-P(v<k_N)} \quad (14)$$

3. Optimization models

To present the first ESD and obtain decision variables, a model M1 that integrates a non-linear OBF is described as follows:

$$\begin{aligned} &\textit{Minimum} \quad E_L \\ &\textit{Subject to} \quad \pi_a(AQL) = 1-\alpha \geq 1-\alpha^U \\ &\qquad \pi_a(LQL) = \beta \leq \beta^U \\ &\qquad k_T > k_{aN} > k_{rN} > 0 \\ &\qquad n_T > n_N \geq 2 \\ &\qquad r_N \in N^+ \end{aligned} \quad (15)$$

where this optimization model is subject to several constraints. First, it ensures a high probability of accepting conforming items from the producer's perspective. Second, it safeguards the consumer to accept the nonconforming items with a lower chance. Third, the acceptance number for Tightened inspection is required to be greater than that for Normal inspection.

As an alternative, model M2 is proposed to determine the optimal decision variables by considering *ASN* as an OBF:

$$\begin{aligned} &\textit{Minimum} \quad ASN \\ &\textit{Subject to} \quad \pi_a(AQL) = 1-\alpha \geq 1-\alpha^U \\ &\qquad \pi_a(LQL) = \beta \leq \beta^U \\ &\qquad k_T > k_{aN} > k_{rN} > 0 \\ &\qquad n_T > n_N \geq 2 \\ &\qquad r_N \in N^+ \end{aligned} \quad (16)$$

To convert the models into unconstrained forms and explore feasible solutions, it is necessary to penalize the OBFs by accounting for deviations from the three constraints. The following section introduces a PSO-based solution approach designed to optimize the models

4. Solution approach

We aim to identify decision variables by optimizing various models. Model M1, formulated as a MINLP problem, presents challenges due to the nonlinearity of the OBF and constraints, as well as the inclusion of both discrete and continuous decision variables. To address these challenges, a PSO-based approach is proposed for optimizing different models of the RGS plan. The pseudocode for this approach is outlined below:

Step 1. Initialization:

Set bounds for decision variables in the model.

Set PSO input parameters: inertia weight (w), recognition (c_1), and social (c_2) learning factors, population size (N_P), and iteration number (N_I).
Set iteration counter r=1.

Step 2. Generate Initial Population:
For each particle y, denoted as Prt_y^r, from 1 to N_p:
Randomly initialize position $X_y^r=[x_{y1}^r, \ldots, x_{yi}^r, \ldots]$ and velocity $V_y^r=[v_{y1}^r, \ldots, v_{yi}^r, \ldots]$.

Step 3. Evaluation:
For each particle y from 1 to N_p:
Calculate the penalized fitness value $fp(Prt_y^r)$.

Step 4. Find Personal Best:
For each particle y from 1 to N_p:
If $fp(Prt_y^r) \leq fp(pbest_y^{r-1})$:
Update $pbest_y^r = Prt_y^r$.
Else:
Retain $pbest_y^r = pbest_y^{r-1}$.

Step 5. Find Global Best:
For each particle y from 1 to N_p:
If $fp(pbest_y^r) \leq fp(gbest^{r-1})$:
Update $gbest^r = pbest_y^r$.
Else:
Retain $gbest^r = gbest^{r-1}$.

Step 6. Termination Check:
If $r=N_I$:
Stop and return $gbest^r$ as the optimal solution.
Else:
Proceed to the next step.

Step 7. Update Velocities and Positions:
Increment iteration counter: $r=r+1$.
Generate random numbers r_p and r_g from Uniform(0, 1).
For each particle y from 1 to N_p:
Update velocity.
Update position.
Repeat from Evaluation step.

Assume that z_i indicates the i^{th} decision variable, with its bounds defined by L_i and U_i. In Step 2, the values of x_{yi}^r and v_{yi}^r are sampled from Uniform(L_i, U_i) and Uniform($-|U_i-L_i|$, $|U_i-L_i|$), respectively. For the discrete sample size, we set $n=z_1=\min\{L_1+\lfloor Rv(U_1-L_1+1)\rfloor, U_1\}$, where $\lfloor\,\rfloor$ denotes the floor function and Rv~U(0,1).

5. Performance evaluation and analysis

This part evaluates the performance of two models. Additionally, it compares the QSS-RGS-SS, QSS-RS-SS, RGS, and RS as well as provides extended tables for practical applications. To illustrate the concepts, a numerical example, and a case study are presented and analyzed. Table 1 summarizes all the input parameters used throughout this section.

5.1. Numerical example

This study examined the QSS-RS-SS plan for the optimization and comparison of both models. Tables 2 and 3 present the outcomes of the comparison between model M1 and model M2, evaluating cost, *ASN*, and total risks across various lot sizes. The total risks ($\alpha+\beta$) were calculated using the formula $(1-\pi_a(AQL))+\pi_a(LQL)$. Across all lot sizes and designs, the results reveal the following key findings:

- In general, both *ASN* and cost tend to increase as the lot size increases, while total risks do not exhibit a consistent trend.

Table 1. Nominal values of the input parameters

Numerical example	μ	σ	USL	LSL	Δ	δ	A	K
	20	0.6667	23	17	3	16.33	11.1	100
	C_p	α^U	β^U	C_{ins}	C_{pr}			
	1.50	0.05	0.10	7	25			
	Design	1	2	3	4	5	6	7
	AQL	0.00040	0.00065	0.00100	0.00150	0.00250	0.00400	0.00650
	LQL	0.00200	0.00325	0.00500	0.00750	0.01250	0.02000	0.03250
	Design	8	9	10	11	12		

	AQL	0.01000	0.01500	0.02500	0.04000	0.06500		
	LQL	0.05000	0.07500	0.12500	0.20000	0.32500		
Practical example	μ	σ	*USL*	*LSL*	Δ	δ	*A*	*K*
	0.2025"	0.025"	0.315"	0.09"	0.1125"	0.065"	11.1	0.14
	C_p	α^U	β^U	C_{ins}	C_{pr}	*N*	*AQL*	*LQL*
	1.50	0.05	0.10	7	25	2500	0.01	0.03

Table 2. Optimal results of M1 with QSS-RS-SS plan and different lot sizes

N	Design	z_1	z_2	z_3	z_4	z_5	E_L	$\pi_a(AQL)$	$\pi_a(LQL)$	*ASN*	$\alpha+\beta$
100	1	2	2.48	4	8	4.00	64.04	1.00	0.10	4.87	0.10
	2	3	1.94	2	6	4.00	62.74	0.99	0.10	4.39	0.11
	3	2	1.39	2	6	4.00	57.88	0.99	0.10	3.82	0.11
	4	2	1.24	2	5	4.00	54.87	1.00	0.10	3.37	0.10
	5	2	1.04	2	4	4.00	51.80	1.00	0.10	2.91	0.10
	6	3	0.14	1	4	4.00	54.25	1.00	0.10	3.45	0.10
	7	2	0.92	3	3	4.00	46.92	1.00	0.10	2.47	0.10
	8	2	0.93	3	2	4.00	45.09	1.00	0.10	2.04	0.10
	9	2	0.82	4	2	4.00	44.65	1.00	0.10	2.05	0.10
	10	2	0.01	2	2	3.57	45.66	1.00	0.10	2.01	0.10
	11	2	0.01	3	2	3.56	46.27	1.00	0.03	2.01	0.03
	12	2	0.01	4	2	3.38	47.81	1.00	0.00	2.02	0.00
200	1	3	2.64	4	7	4.00	94.15	1.00	0.10	5.06	0.10
	2	2	1.98	3	7	4.00	89.87	1.00	0.10	4.32	0.10
	3	4	1.88	2	5	4.00	90.49	1.00	0.10	4.48	0.10
	4	2	1.87	4	5	4.00	81.67	1.00	0.10	3.44	0.10
	5	2	0.78	2	5	4.00	81.63	1.00	0.10	3.36	0.10
	6	2	1.04	3	4	4.00	78.30	1.00	0.10	2.92	0.10
	7	2	0.92	3	3	4.00	76.16	1.00	0.10	2.47	0.10
	8	2	0.54	3	3	4.00	76.27	1.00	0.10	2.46	0.10
	9	2	0.01	2	3	3.68	77.40	1.00	0.10	2.45	0.10
	10	2	0.07	3	2	4.00	75.07	1.00	0.10	2.01	0.10
	11	2	0.01	2	3	2.54	81.03	1.00	0.10	2.47	0.10
	12	2	0.01	3	2	2.70	78.79	1.00	0.04	2.03	0.04
500	1	4	2.44	3	8	4.00	181.02	1.00	0.10	5.92	0.10
	2	4	2.30	3	6	4.00	176.74	1.00	0.10	5.02	0.10
	3	3	1.47	2	7	4.00	174.49	1.00	0.10	4.81	0.10
	4	3	1.97	4	5	4.00	168.59	1.00	0.10	4.01	0.10
	5	2	1.48	4	5	4.00	165.63	1.00	0.10	3.40	0.10
	6	3	0.88	2	4	4.00	168.15	1.00	0.10	3.45	0.10
	7	3	1.36	3	4	3.44	171.60	1.00	0.08	3.53	0.08
	8	2	0.31	2	4	3.51	168.68	1.00	0.10	2.91	0.10
	9	4	0.01	1	4	3.22	178.35	1.00	0.08	4.00	0.08
	10	3	0.60	2	6	2.27	184.15	1.00	0.10	4.39	0.10
	11	2	0.01	3	3	2.90	171.15	1.00	0.10	2.47	0.10
	12	2	0.01	4	2	3.22	170.77	1.00	0.01	2.02	0.01
1000	1	5	2.18	2	9	4.00	324.03	1.00	0.10	6.83	0.10
	2	6	2.08	2	7	4.00	321.76	1.00	0.10	6.48	0.10
	3	2	0.91	2	9	4.00	314.82	1.00	0.10	5.15	0.10
	4	2	1.29	3	7	4.00	310.21	1.00	0.10	4.26	0.10
	5	2	1.02	3	6	4.00	309.49	1.00	0.10	3.81	0.10
	6	2	0.43	2	5	3.94	311.13	1.00	0.10	3.36	0.10
	7	2	0.62	3	4	4.00	309.60	1.00	0.10	2.91	0.10
	8	2	0.51	3	4	3.81	313.65	1.00	0.04	2.96	0.04
	9	3	0.58	3	3	3.74	316.96	1.00	0.10	3.02	0.10
	10	2	0.59	4	3	3.18	318.17	1.00	0.10	2.50	0.10
	11	2	0.01	2	2	2.93	326.89	1.00	0.10	2.02	0.10
	12	2	0.01	4	2	3.15	324.45	1.00	0.01	2.02	0.01

- Model M1 is generally more cost-effective than model M2, with only a few exceptions.
- Model M1 is generally more reliable in terms of total risks, with exceptions to Designs 10 and 11.
- Model M2 consistently requires smaller sample sizes compared to model M1.

Model M2 appears to be more efficient in terms of sample size requirements. However, model M1 proves to be more cost-effective and reliable, despite requiring larger sample sizes. Thus, reducing the *ASN* does not automatically result in lower costs. This insight contradicts traditional beliefs and underscores the necessity of incorporating cost- based objectives when developing sampling strategies. In conclusion, it is recommended to utilize model M1 for its cost-effectiveness, particularly when balancing sample size and financial considerations.

5.2. Case study

A company receives weekly lots of 2500 2-inch plastic pipe segments for an assembly process (see "support.minitab.com"). Pipe wall thickness is evaluated through variable sampling plans. The dataset obtained from a shipment includes the values: 0.17350, 0.18100, 0.25430, 0.18030, 0.20280, 0.20010, 0.19310, 0.15850, 0.19740, 0.19260, 0.20860, 0.25600, 0.18610, 0.22610, 0.18210, 0.21240, 0.24890, 0.16120, 0.21750, 0.20460, 0.23310, 0.21350, 0.18940, 0.21600, 0.21430, 0.15930, 0.15640, 0.18920, 0.19030, 0.23660, 0.18980, 0.20700, 0.21350, 0.21460, 0.23570, 0.18170, 0.17860, 0.21120, 0.19840, 0.15180, 0.20390, 0.20340, 0.18820, 0.20540, 0.19680, 0.19420, 0.22960, 0.16400, 0.21950, 0.21590, 0.20280, 0.23030, 0.19970, 0.16890, 0.20970, 0.17500, 0.18000, 0.16360, 0.22600, 0.21070, 0.17100, 0.22380, 0.21870, 0.16940, 0.18000, 0.23110, 0.20610, 0.18040, 0.21890, 0.14140, 0.20640, 0.24050, 0.19940, 0.17080, 0.18670, 0.15930, 0.20950, 0.19170, 0.18070, 0.17970, 0.24660, 0.22670, 0.20460, 0.24230, 0.15070, 0.21670, 0.20730, 0.20870, 0.19730, 0.22010, 0.16750, 0.23090, 0.22380, 0.18900, 0.18050, 0.16270, 0.21350, 0.20330, 0.17320, 0.20830, 0.20750, 0.22770, 0.14630, and 0.24710. Before inspecting the received lot, the results of various models and sampling plans were compared. The findings are summarized in Table 5. Based on the model comparison:

- M1 is more cost-effective and has lower total risks but requires larger sample sizes.
- M2 is more efficient in terms of sample size requirements but is more expensive and carries higher total risks.

Regarding the plan comparison:

- QSS-RS-SS is the best-performing plan overall, offering the lowest costs, smallest ASN values, and lowest total risks for both models.
- RS is the least efficient method, with the highest costs, largest ASN values, and elevated total risks.

Table 3. Optimal results of M2 with QSS-RS-SS plan and different lot sizes

N	Design	z_1	z_2	z_3	z_4	z_5	E_L	$\pi_a(AQL)$	$\pi_a(LQL)$	ASN	$\alpha+\beta$
100	1	2	2.51	3	6	4.00	73.33	0.97	0.10	4.02	0.13
	2	2	2.28	3	5	4.00	65.62	0.98	0.10	3.51	0.12
	3	2	2.14	3	4	4.00	63.31	0.98	0.10	3.05	0.12
	4	2	2.08	3	3	4.00	67.20	0.97	0.10	2.62	0.13
	5	2	1.69	3	3	4.00	52.76	0.99	0.10	2.54	0.11
	6	2	1.88	4	2	4.00	53.02	0.99	0.10	2.22	0.11
	7	2	0.95	2	2	4.00	52.86	0.99	0.10	2.02	0.11
	8	2	0.56	2	2	4.00	46.78	1.00	0.10	2.01	0.10
	9	2	0.58	3	2	4.00	44.63	1.00	0.10	2.02	0.10
	10	2	0.01	2	2	4.00	48.28	1.00	0.01	2.00	0.01
	11	2	0.01	3	2	4.00	46.54	1.00	0.00	2.01	0.00
	12	2	0.01	3	2	4.00	65.02	0.98	0.00	2.02	0.02
200	1	2	2.51	3	6	4.00	119.26	0.97	0.10	4.02	0.13
	2	2	1.77	2	6	4.00	101.98	0.99	0.10	3.86	0.11
	3	2	2.35	4	4	4.00	96.19	0.99	0.10	3.14	0.11
	4	2	2.08	3	3	4.00	116.53	0.97	0.10	2.62	0.13
	5	2	1.04	2	4	4.00	83.11	1.00	0.10	2.91	0.10
	6	2	0.94	2	3	4.00	83.18	0.99	0.10	2.47	0.11
	7	2	0.92	3	3	4.00	76.16	1.00	0.10	2.47	0.10
	8	2	0.56	2	2	4.00	79.05	1.00	0.10	2.01	0.10
	9	2	0.17	2	2	4.00	75.34	1.00	0.10	2.01	0.10
	10	2	0.01	2	2	4.00	81.45	1.00	0.01	2.00	0.01
	11	2	0.01	3	2	4.00	77.20	1.00	0.00	2.01	0.00
	12	2	0.01	3	2	4.00	113.34	0.98	0.00	2.02	0.02
500	1	2	2.51	3	6	4.00	257.04	0.97	0.10	4.02	0.13
	2	2	1.95	2	5	4.00	276.55	0.97	0.10	3.45	0.13
	3	2	1.59	2	5	4.00	207.80	0.99	0.10	3.39	0.11
	4	2	2.08	3	3	4.00	264.58	0.97	0.10	2.62	0.13
	5	2	1.34	2	3	4.00	224.33	0.98	0.10	2.49	0.12
	6	2	1.37	3	3	4.00	175.91	1.00	0.07	2.52	0.08
	7	2	1.30	3	2	4.00	181.32	0.99	0.10	2.07	0.11
	8	2	0.54	3	3	3.99	163.96	1.00	0.10	2.46	0.10
	9	2	0.10	2	3	3.59	167.01	1.00	0.10	2.45	0.10
	10	2	0.01	2	2	4.00	180.95	1.00	0.01	2.00	0.01
	11	2	0.01	2	2	3.96	364.52	0.95	0.00	2.01	0.05
	12	2	0.01	2	2	3.32	379.73	0.95	0.00	2.02	0.05
1000	1	2	2.19	2	6	4.00	590.87	0.96	0.10	3.95	0.14
	2	2	2.28	3	5	4.00	438.74	0.98	0.10	3.51	0.12
	3	2	1.81	2	4	4.00	544.96	0.96	0.10	2.98	0.14
	4	2	2.28	4	3	4.00	438.82	0.98	0.10	2.72	0.12
	5	2	0.20	1	4	4.00	395.93	0.99	0.10	2.91	0.11
	6	2	1.36	2	2	4.00	573.92	0.96	0.10	2.05	0.14

N	Design	z_1	z_2	z_3	z_4	z_5	E_L	$\pi_a(AQL)$	$\pi_a(LQL)$	ASN	$\alpha+\beta$
	7	2	0.95	2	2	4.00	400.02	0.99	0.10	2.02	0.11
	8	2	0.56	2	2	4.00	337.27	1.00	0.10	2.01	0.10
	9	2	0.17	2	2	4.00	317.80	1.00	0.10	2.01	0.10
	10	2	0.01	2	2	4.00	346.78	1.00	0.01	2.00	0.01
	11	2	0.01	2	2	3.96	713.98	0.95	0.00	2.01	0.05
	12	2	0.01	2	2	3.32	746.44	0.95	0.00	2.02	0.05

Figure 1 demonstrates that the QSS-RS-SS plan, when used with both M1 and M2, produces the smallest ASN curves. Figure 2 further reveals that the QSS-RS-SS plan with M1 generates the most closely aligned OC plot. Considering the trade-offs between cost, sample size, and risk, this analysis recommends using model M1 with the QSS-RS-SS plan. For the recorded 104 samples from a single received lot, the steps for implementing the QSS-RS-SS plan with model M1 are outlined below:

1. Determine decision variables n_N=4, k_N=0.89, r_N=3, n_T=7, and k_T=3.72. Moreover, set input parameters.
2. Implement Normal inspection:
    a. Take a random sample of size n_N from the lot. The measured values are recoded as (x_1, x_2, x_3, x_4)=(0.2146, 0.2423, 0.1732, 0.2087). The sample mean equals to 0.2097. Thus, v_N is calculated as (0.2097-0.090)/0.025=4.788.
    b. Compare v_N with the acceptance number:
        i. Since $v_N \geq k_N$, accept the lot, and continue to the next lot under Normal inspection.

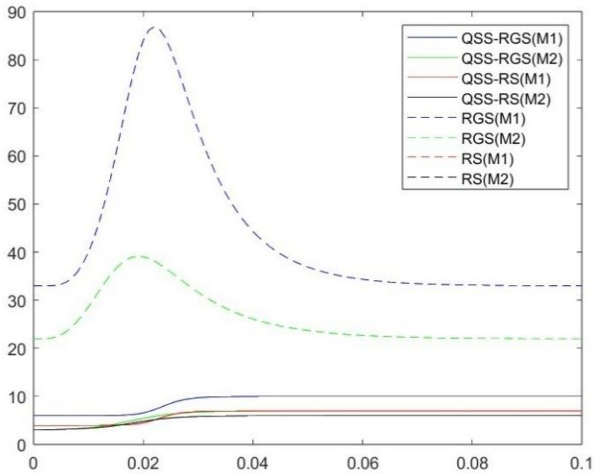

Figure 1. ASN curves of various models and plans in the case study (horizontal and vertical axes indicate p and ASN, respectively)

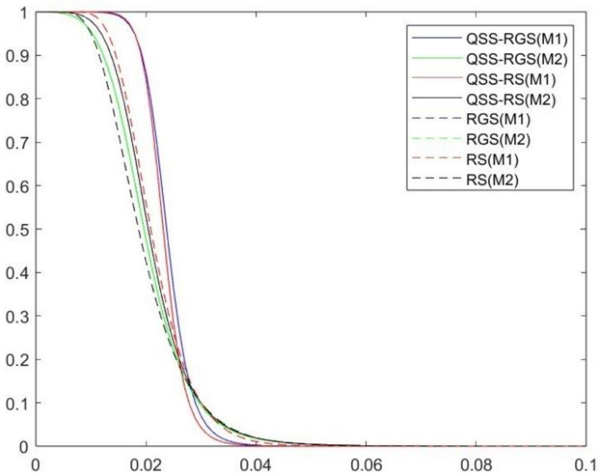

Figure 2. OC curves of various models and plans in the case study (horizontal and vertical axes indicate p and $\pi_a(p)$, respectively)

Table 4. Comparing the optimal results of different models and plans in the case study

Model	Method	z_1	z_2	z_3	z_4	z_5	E_L	$\pi_a(AQL)$	$\pi_a(LQL)$	ASN	$\alpha+\beta$
M1	QSS-RGS	6	1.02	0.22	10	3.35	17643.84	1.00	0.07	7.87	0.07
	QSS-RS	4	0.89	3	7	3.72	17624.62	1.00	0.04	5.44	0.04
	RGS	33	2.17	1.86			18034.10	1.00	0.10	52.72	0.10

	RS	49	2.11	2			18185.51	1.00	0.10	73.92	0.10
M2	QSS-RGS	3	1.59	0.70	7	2.92	18402.59	0.96	0.09	5.11	0.14
	QSS-RS	3	1.60	3	6	2.99	17978.79	0.98	0.10	4.61	0.12
	RGS	22	2.20	1.95			18666.93	0.95	0.10	30.16	0.15
	RS	29	2.18	2			18755.39	0.95	0.10	45.97	0.15

6. Sensitivity analysis

Taguchi's orthogonal-array design is utilized to investigate the influence of input parameters on the solutions of M1 using the QSS-RS-SS plan. Seven independent variables are examined, with E_L serving as the response variable. A three-level design is implemented for the factors, defined as follows: α^U=A=(0.01,0.05,0.10), β^U=B=(0.01,0.05,0.10), K=C=(1,10,100), N=D=(500,1000,2500), AQL=E=(0.010,0.015,0.025), C_{ins}=F=(3,7,15), C_{pr}=G=(12,25,50). The levels of these independent variables are assigned to the L_{27} design, and the optimization outcomes are presented on the left and right sides of Table 5. Table 6 displays the delta values, which indicate the range between the highest and lowest E_L values. The ranking row reveals that variations in K levels exert the most substantial influence on E_L. The optimal input parameter levels correspond to the lowest EL values.

Subsequently, Minitab's output for identifying significant factors is provided in Table 7. ANOVA, conducted at a 5% significance level, identifies the parameters significantly affecting E_L. Based on Tables 6 and 7, E_L decreases as K, N, and AQL parameters are reduced. Conversely, a decrease in α^U leads to an increase in E_L. Selecting a β^U value between its upper and lower levels results in a reduction in E_L. The regression model's adequacy is confirmed, with R-Sq=99.71% and R-Sq(adj)=99.37%. Additionally, the normality of residuals is validated using the Anderson–Darling test, with AD=0.36 and a P-value 0.422 (> 0.05).

Table 5. Optimal results for the generated trials through the Taguchi L_{27} design

L_{27} Design								Optimal Results				
Trial	A	B	C	D	E	F	G	E_L	$\pi_a(AQL)$	$\pi_a(LQL)$	ASN	$\alpha+\beta$
1	1	1	1	1	1	1	1	2516.26	1.00	0.01	4.99	0.01
2	1	1	1	1	2	2	2	2680.71	1.00	0.01	4.04	0.01
3	1	1	1	1	3	3	3	2936.15	1.00	0.01	4.00	0.01
4	1	2	2	2	1	1	1	50027.45	1.00	0.05	4.38	0.05
5	1	2	2	2	2	2	2	53019.10	1.00	0.05	4.51	0.05
6	1	2	2	2	3	3	3	57433.65	1.00	0.05	4.39	0.05
7	1	3	3	3	1	1	1	250100.18	1.00	0.10	5.90	0.10
8	1	3	3	3	2	2	2	264990.29	1.00	0.10	4.45	0.10
9	1	3	3	3	3	3	3	286988.63	1.00	0.09	10.92	0.09
10	2	1	2	3	1	2	3	125082.26	1.00	0.01	5.01	0.01
11	2	1	2	3	2	3	1	132560.54	1.00	0.01	5.50	0.01
12	2	1	2	3	3	1	2	143443.63	1.00	0.01	6.48	0.01
13	2	2	3	1	1	2	3	50039.14	1.00	0.05	3.96	0.05
14	2	2	3	1	2	3	1	53034.90	1.00	0.05	3.51	0.05
15	2	2	3	1	3	1	2	55599.77	1.00	0.05	511.75	0.05
16	2	3	1	2	1	2	3	5031.75	1.00	0.10	3.93	0.10
17	2	3	1	2	2	3	1	5382.43	1.00	0.10	4.70	0.10
18	2	3	1	2	3	1	2	5766.28	1.00	0.10	7.61	0.10
19	3	1	3	2	1	3	2	100104.34	1.00	0.01	4.97	0.01
20	3	1	3	2	2	1	3	105427.10	1.00	0.00	1000.50	0.00
21	3	1	3	2	3	2	1	114755.58	1.00	0.01	4.48	0.01
22	3	2	1	3	1	3	2	12587.87	1.00	0.05	5.39	0.05
23	3	2	1	3	2	1	3	13274.76	1.00	0.05	6.80	0.05
24	3	2	1	3	3	2	1	14384.35	1.00	0.05	5.45	0.05
25	3	3	2	1	1	3	2	25064.65	1.00	0.10	3.83	0.10
26	3	3	2	1	2	1	3	26504.52	1.00	0.10	4.38	0.10
27	3	3	2	1	3	2	1	28700.25	1.00	0.10	3.01	0.10

Table 6. Effects of independent parameters on E_L

Factor	A	B	C	D	E	F	G
Level 1	107855	81056	7173	27453	68950	72518	72385
Level 2	63993	39933	71315	55216	72986	73187	73695
Level 3	48978	99837	142338	138157	78890	75121	74746
Delta	58877	59903	135164	110704	9939	2604	2362
Rank	4	3	1	2	5	6	7

Table 7. ANOVA

Source	D.F.	Adj. S.S.	Adj. M.S.	F	P-value
A	2	16847161883	8423580942	196.90	0.000
B	2	16896507548	8448253774	197.48	0.000
C	2	82283358522	41141679261	961.70	0.000
D	2	59715967641	29857983820	697.94	0.000

E	2	449795621	224897810	5.26	0.023
F	2	32907281	16453641	0.38	0.689
G	2	25201892	12600946	0.29	0.750
Residual	12	513362378	42780198		
Total	26				

7. Conclusions

This research pioneers the development of loss-based economic-statistical designs of integrated VQSS plans for inspection of normally distributed quality characteristics, a previously unexplored area. Two plans were proposed: a QSS-RS-SS and a QSS-RGS-SS. A Markovian model was employed to represent the system's states and transition probabilities. Optimal decision variables for both plans were derived through the solution of ESD models, with the *ASN* and Loss-based terms as the objective functions. To thoroughly evaluate the proposed plans and models, simulation studies were performed across a wide range of parameter combinations, followed by a real-world implementation. Furthermore, a sensitivity analysis was conducted to assess the impact of some input parameters on the optimal results. The findings consistently highlighted the superior performance of the QSS-RS-SS plan with Loss-based model, characterized by its ability to deliver both higher acceptance probabilities and reduced costs.

The present study addresses significant gaps in prior research by offering a detailed evaluation of various models and plans, proposing a computationally efficient optimization method based on PSO, and demonstrating the effectiveness of the proposed plan through a practical case study. The integration of the QSS-RS-SS strategy with the Loss-based model, optimized via the PSO method, presents a robust solution for achieving a balance between economic and statistical objectives in quality control. These results underscore the importance of adopting advanced sampling techniques to minimize quality-related losses and optimize resource utilization. Industries seeking to enhance quality assurance while managing operational expenses should explore the implementation of QSS-RS-SS designs, especially when dealing with large lot sizes or high-cost products. Future research can extend this study in developing new models, proposing efficient solution procedures, and evaluating novel QSS plans.

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