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Applying Fuzzy Inference System by Using Conventional Technique to Solve MADM Problems

Saeed Bahrami[1*], Razali Yaakob[1], Azreen Azman[1], Rodziah Atan[1]
[1]*Faculty of Computer Science and Information Technology, Universiti Putra Malaysia*

Abstract

Fuzzy inference system as a powerful tool to deal with uncertainty is applied in solving multi-attribute decision-making (MADM) problems. So far, the various methods have been applied in order to increase the accuracy of decision making in solving these types of problems with at least computational process. The main objective in this research is that with developing a fuzzy inference system based on a conventional technique as TOPSIS accomplished more accurate ranking. To do so, researcher with modifying the expert's opinion aggregation process type in solving MADM problem will improve final score of alternatives. Indeed, in order to finalize the aggregation of expert's results and makes a unique judgment is utilized TOPSIS technique. As a case-based problem, in this research is examined data regarding a supplier selection problem (SSP) that has been extracted from a validate method. As well, the verifying and validity of the proposed approach is demonstrated with the data of a numerical example of a research and comparative analysis for the problem at hand.

Keywords: Fuzzy inference system, TOPSIS, MADM, Group decision making, Linguistic variables.

1. Introduction

One of the most important tasks that has received a lot of attention in many domains is make-decision such as in engineering, management, mathematics, agriculture and related problems in other fields (Zavadskas et al., 2016; Salih et al., 2019). However, decision making process in the field of engineering often takes place in a complex environment and involves multiple conflicting criteria. Multi criteria decision-making (MCDM) techniques can be broadly categorized into two main groups,

*Corresponding Author
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multiple objective decision-making (MODM) problems and multi-attribute decision-making (MADM) dealing with complex engineering problems (Tavana & Hatami-marbini, 2011).

The opinions of experts, either individually or in groups, are employed to solve MADM problems. Undoubtedly, utilizing the perspectives of experts in a group setting is superior to using the information of a single decision maker (Tavana & Hatami-marbini, 2011) since a single decision maker is limited in their ability to examine all components of the problem, such as knowledge and experience. Group decision-making (GDM) (Tavana & Hatami-marbini, 2011; Zhou & Li, 2014) is essential to guaranteeing the precision and logicalness of decisions.

In real world situations, experts express their opinions by using linguistic variables about alternatives (Zadeh, 1975; Chen, 2000). Naturally, performed linguistic variables are considered imprecise, vague, uncertain or incomplete data. Fuzzy set theory introduced by Zadeh (zadeh, 1965), provides a powerful tool for dealing with ambiguous data. Also, this theory is used to model uncertainty in decision making models happening owing to lack of perfect information. Over the years, there have been successful applications and implementations of fuzzy set theory in MADM. To do so, a wide range of studies have combined MADM techniques with fuzzy set theory called fuzzy MADM (FMADM) (Mardani et al., 2015). Some of them are fuzzy TOPSIS (Su et al., 2013; Ashtiani et al., 2009; Liao & Kao, 2011), fuzzy AHP (Kahraman & Cebeci, 2003; Chen & Yang, 2011; Chen, 2015), fuzzy VIKOR (Ã & Wang, 2009; Shemshadi et al., 2011), fuzzy ELECTREE (Taylor & Sevkli, 2010; Rougendegh & Erkan, 2012), fuzzy ANP (Vinodh et al., 2011; Amy & Yang, 2012; Pang & Bai, 2013) and fuzzy PROMETHEE (Chen et al., 2011; Gupta et al., 2012).

One of the most key challenges in decision making process is to estimate utility function of decision maker. It is usual and convenient for decision maker to present his/her knowledge, utility and preferences as linguistic terms in form of IF-THEN rules. Fuzzy rule-based systems based on IF-THEN rules are widely implemented to improve intelligence system for modeling experts' knowledge and utilities. Several studies have been established to solve MADM problems by means of fuzzy rule-based systems. (Carrera & Mayorga, 2008) demonstrated a modular fuzzy logic system to select an appropriate supplier. In their proposed approach, one expert evaluated the alternatives using crisp values. (Amindoust et al., 2012) presented a fuzzy system for sustainable supplier selection. They collected experts' opinions in form of linguistic variables instead of crisp variables. (Junior et al., 2013) developed a fuzzy system by considering an additional stage including qualification stages with a non-compensatory rule. In addition, they used DOE (design of experiment) to test the rules and extract weights of criteria from the rules. Moreover, (Kumar et al., 2013) proposed a fuzzy inference system similar to (Carrera & Mayorga, 2008) by considering a mechanism for rule designing. (Rezaei & ortt, 2013) proposed an approach based on a fuzzy rule-based system for the segmentation of suppliers. They considered one expert and crisp value for rating. (Mahmoudi et al., 2016) propounded a hybrid approach which employed both fuzzy rule based system and PROMETHEE conventional technique to rank alternatives. In this study has been used linguistic variables and also, this method enable solve MADM problems by means of experts' opinion in form of grouping. In the mentioned studies, most of them were made in group form for rationality to rule and as well, all experts' opinion aggregate to each other at first, then have been used for next steps.

From other side, TOPSIS is a suitable method in confronting with MCDM issues which is introduced by Hwang in 1981 (Hwang, 1981). This technique chooses the best alternative in a problem base on finding the shortest distance from the ideal solution of positive and the farthest distance from the ideal solution of negative. It aids decision maker solve the problem through analysis, comparisons and rating of the alternatives. TOPSIS deemed to be one of the major decision making techniques. In recent decades, TOPSIS has been used effectively in many areas. Furthermore, by means of TOPSIS, the researchers able to solve MADM and MODM problems by form individual and group decision making. This technique has a good flexibility to incorporate further propagation to make better choices in various situations. Also, TOPSIS is a technique which works value-based method and by directly compares each alternative according to the information into the evaluation matrices and weights (Awasthi et al., 2011).Therefore, TOPSIS is selected as the major body of extension in this research.

From one side, literature review on studies of fuzzy inference system related to solve MADM problems showed that combination of it and TOPSIS is not been considered and it reveal a gap research. Therefore, this study aims to cover this gap and hybridize TOPSIS technique with fuzzy inference system outputs to rank alternatives under a group decision making problem. On the other side, in the papers mentioned based on fuzzy rule base system in group form, all experts' opinions have been aggregated together at first and then researchers are faced only with one opinion. But, the purpose of this study is that to obtain the final score of each expert by means of fuzzy inference system separately and then obtain the final ranking by using TOPSIS method among experts' opinions**.** This is a main contribution in this research which will obtain compound of uzzy rule base system by means of TOPSIS. The main motivation to establish this research has been derived from two issues. The first one is that, the type of aggregation of experts' opinions effects on the final decision (Wan et al., 2016) and, the second one is that, in carrying out the process of aggregation is faced with losing data challenge (Salih et al., 2018) therefor the results may not be accurate.

The rest of the paper is organized as follows: the proposed method is described in section 2. In section 3, the applicability and validity of the method is demonstrated with an illustrative example and is presented how this method can solve a MADM problem and at the end of this section is showed the validity of the method and finally, conclusion of this paper is presented in section 4.

2. Proposed the Method

In this section an approach by integrating fuzzy inference system method is demonstrated base on a conventional method as TOPSIS. As it clear, in performing any aggregation process in decision-making are faced with losing data challenge (Salih et al., 2018). In the researches mentioned in literature review based on group decision making as Mahmoudi et al., (2016), all experts' opinion is aggregated together in the beginning, and then will be faced only one opinion in the form of a decision matrix. The next computation process to reach the final decision-making is established on the same matrix. This process is shown in Figure 1.

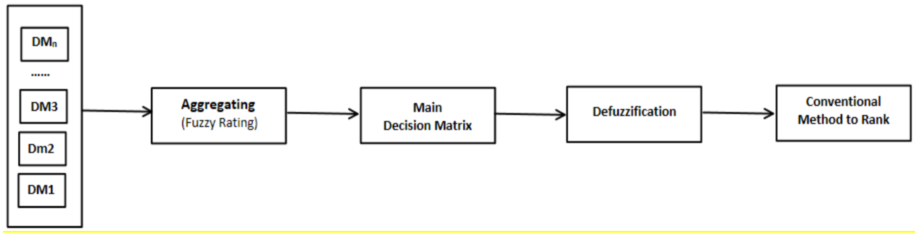

Figure 1: Aggregation type in other method

Certainly in the process of solving MADM problems, the type of aggregation process of experts' opinions affects the final decisions (Wan et al., 2016). Therefore, the purpose is to modify it to improve the solving group MADM problems. In means that, the goal is to develop a new approach based on a fuzzy rule-based method by applying the TOPSIS technique to solve a group MADM problem.

Based on this method, the opinions of each expert as a set of data that are linguistics converted to a fuzzy number and are placed in a decision matrix. Thereafter, the fuzzy values of each matrix are converted to crisp values (defuzzify) to perform the necessary computational operation. Then, the final score of each alternative with the aggregating operation for each expert's opinion individually is obtained by implementing an FRBS. Then in the final stage, by applying a conventional technique such as TOPSIS, the experts' opinions for each alternative are aggregated and are finally ranked. Because of aggregating each expert's opinion individually instead of aggregating all experts' opinions together as a technique, it needs to be investigated to the extent this action affects the accuracy of decision making. This process is shown in Figure 2.

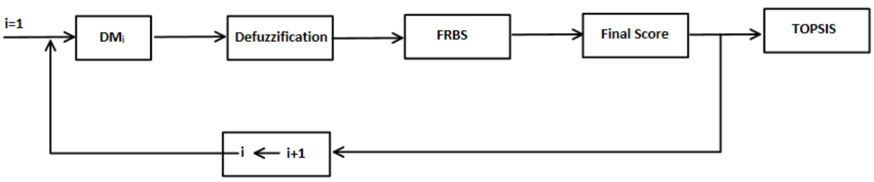

Figure 2: Aggregation type in the proposed method

It is necessary to explain that there are many different methods to defuzzification available such as BOA, COA, COG, and so on. For example, Amindoust (2012) applied some defuzzification methods to show validity of a model through using COA, BOA, MOM, SOM, and LOM , and showed that the obtained ranking results for all alternatives were almost the same. Besides, in another study, Junior et al., (2013), three different defuzification operators including COA, MOM, and FOM were used for ranking suppliers. The output was the same for all three operators. Actually, the COA method is very popular to defuzzification compared to other methods, so to do this action in this method, COA method is used. The output of each method for defuzzifications is very similar and near to each other. The most frequently utilized defuzzification technique is the center of gravity (COG). COG can be presented as follows (Sadi-Nezhad & Sotoudeh-Anvari, 2015; Wang, Yang, Xu, & Chin, 2006).

To do so, three stages are applied. Firstly, in stage 1, the desired data is being prepared. In stage 2, a method is proposed to apply based on the fuzzy rule-based system to rank alternatives by each expert's opinions. Thereafter in stage 3, as the main contribution of our study, the TOPSIS technique is combined with a fuzzy rule-based system for aggregating all experts' ranking and for the final section process.

To describe the proposed method, consider a multi-attribute decision-making problem with *m* alternatives (*i=1, 2,…, m*), *n* criteria (*j=1,2,…,n*) and *K* experts (*k=1, 2, …, K*). Step-by-step, explanations of the proposed hybrid method are presented as follows and in order to understand the graphical scheme of the proposed approach is depicted in Figure 3.

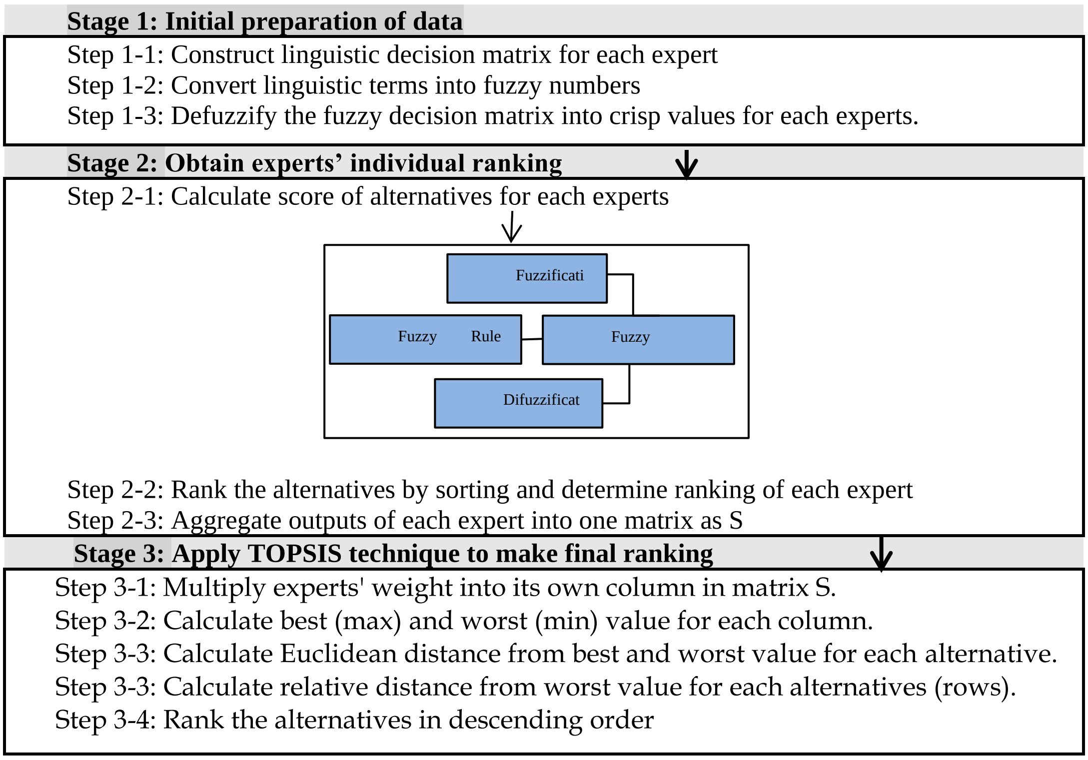

Figure 3: the scheme of the Proposed Method

Now, the following shows in more detail what happens at each stage.

Stage1: *Initial preparation of data*

The combination of steps under this stage is as follows:

Step 1: In this step linguistic decision matrix for each expert is constructed by using linguistic opinion. It means that each expert opinion about the relationship between each alternative with criteria is placed into a decision matrix in form of linguistic. Indeed, in this step, no conversion is done.

Step 2: Convert linguistic terms into fuzzy numbers $\tilde{D}_k$; k=1,2,...,K

Step 3: Defuzzify the fuzzy decision matrix into crisp values for each expert. D_k ; k=1,2,...,K . The meaning of defuzzification is that converting fuzzy numbers into crisp values.

Imagine that there is a fuzzy number with 4 values (by trapezoidal) namely A = (a, b, c, d). Triangular fuzzy number (TFN) is a particular case of trapezoidal fuzzy number with b=c (Sadi-

Nezhad & Sotoudeh-Anvari, 2015). The defuzzified value can be obtained as follow (Sadi-Nezhad & Sotoudeh-Anvari, 2015):

$$\alpha=\frac{1}{3}\left[a+b+c+d-\frac{cd-ab}{(c+d)-(a+b)}\right] \quad (1)$$

When b=c the above formula becomes as follows:

$$\alpha=\frac{a+b+d}{3} \quad (2)$$

Finally, the outputs of this stage are K crisp matrix associated each individual expert as follows:

$$D_k=\left[x_{ijk}\right]_{n\times m};\quad k=1,2,...,K \quad (3)$$

Where D is a decision matrix for each expert (k) and indexes of i, j shows the value of row and column numbers.

Stage2: *Obtain experts' individual ranking*

In this stage, a fuzzy rule base system is applied to rank the alternatives by using each individual expert's opinions and the individual ranking of alternatives for each expert is obtained. To do so, the following steps are performed.

Step 1: Calculate the score of the alternatives for each expert by applying a fuzzy rule base system. To do so, a fuzzy inference system is designed with *m* inputs variables where *m* is several criteria. The proposed fuzzy rule base system for each expert and each alternative is iterated and as the output of the each iteration s_{ik} is obtained as the score of alternative *ith* under expert *kth*.

Step 2: Rank the alternatives by sorting s_{ik} and determine the ranking of each expert as $S_k=[s_{ik}]_{n\times 1}$

Step 3: Aggregate outputs of each expert into one matrix as $S=[s_{ik}]_{n\times k}$

Stage 3: *Apply the TOPSIS technique to make the final ranking*

In this stage by using matrix S and applying the TOPSIS technique final ranking of alternatives is obtained. The following steps are performed in stage 3.

Step 1: Multiply the expert's weight into his column in matrix S and then construct a weighted matrix $V=[v_{ik}]_{n\times k}$.

Step 2: Calculate the best max and min value for each column as v_k^+ and v_k^-

Step 3: Calculate Euclidean distance from maximum and minimum value for each alternative (rows) as:

$$C_i^+=\sqrt{\sum_{k=1}^{K}(v_{ik}-v_k^+)^2}\quad,\quad C_i^-=\sqrt{\sum_{k=1}^{K}(v_{ik}-v_k^-)^2} \quad (4)$$

Step 3: Calculate relative distance from minimum value for each alternatives (rows) as:

$$C_i=\frac{C_i^-}{C_i^-+C_i^+} \quad (5)$$

Step 4: Rank the alternatives in descending order

3. Validation and verifying of the proposed method

In this section, the performance and validity of the proposed method is evaluated with a supplier selection problem (SSP) which has been investigated by Mahmoudi et al., (2016). In that research, to show the validity of their proposed method has been defined a numerical example. Suppose that there are many suppliers to provide the main material for a company and their main purpose is to select a suitable supplier which strongly affects the production process. Certainly, in the next step, initial screening is the best action which must be established by a committee of experts. After this, the committee has chosen 10 experts, four candidate suppliers (a_1, a_2, a_3, and a_4) settle to further valuation. All 10 experts were demanded to prepare a list of criteria that could be used to evaluate suppliers and ultimately four criteria consisting of C1: *quality*, C2: *Price*, C3: *lead time delivery*, C4: *Flexibility* are selected. In this study, all judgments of experts were stated in form of linguistic variables. As well, to the performance ratings of the alternatives were defined seven linguistic variables. These linguistic variables and their fuzzy numbers are shown in Table 1 and Figure 4 derived from (Mahmoudi et al., 2016). Also, the DMs judgments or preferences were described in form of trapezoidal fuzzy numbers (TFN).

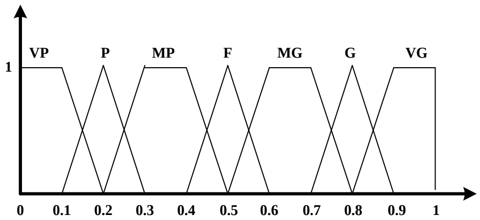

Figure 4: Linguistic variables for the fuzzy rates of Alternatives (Mahmoudi et al., 2016)

Table 1: The linguistic variables for the performance rating and their associated fuzzy numbers (Mahmoudi et al., 2016)

Linguistic Variable	Fuzzy number
Very Poor (VP)	(0.0,0.0,0.1,0.2)
Poor (P)	(0.1,0.2,0.2,0.3)
Moderately Poor (MP)	(0.2,0.3,0.4,0.5)
Faire (F)	(0.4,0.5,0.5,0.6)
Moderately good (MG)	(0.5,0.6,0.7,0.8)
Good (G)	(0.7,0.8,0.8,0.9)
Very good (VG)	(0.8,0.9,1,1)

The performance rating of the alternative concerning the four criteria provided by each individual expert is presented in Table 2.

Table 2: The performance rating of the suppliers with by DMs (Mahmoudi et al., 2016)

Experts	Suppliers	Criteria				Experts	Suppliers	Criteria			
		C1	C2	C3	C4			C1	C2	C3	C4
Exp1	a_1	MG	MG	G	VG	Exp6	a_1	VG	G	MG	G
	a_2	MG	G	G	MG		a_2	G	G	G	VG
	a_3	G	F	MG	VG		a_3	G	MG	MG	G
	a_4	VG	G	MG	G		a_4	MG	MG	G	VG
Exp2	a_1	G	G	G	VG	Exp7	a_1	MG	G	G	MG
	a_2	MG	MG	MG	MG		a_2	VG	G	MG	G
	a_3	VG	F	MG	VG		a_3	MG	MG	MG	MG
	a_4	G	VG	G	G		a_4	MG	MG	G	VG
Exp3	a_1	MG	G	MG	MG	Exp8	a_1	VG	F	MG	VG
	a_2	MG	G	G	G		a_2	MG	G	MG	MG
	a_3	G	MG	MG	G		a_3	G	MG	MG	G
	a_4	G	VG	MG	MG		a_4	G	VG	MG	MG
Exp4	a_1	MG	MG	G	VG	Exp9	a_1	G	MG	MG	G
	a_2	MG	MG	MG	MG		a_2	MG	MG	G	VG
	a_3	G	MG	MG	G		a_3	VG	F	MG	VG
	a_4	VG	G	MG	G		a_4	G	VG	G	G
Exp5	a_1	MG	G	G	MG	Exp10	a_1	G	MG	MG	G
	a_2	VG	F	MG	VG		a_2	VG	F	MG	VG
	a_3	G	VG	MG	MG		a_3	VG	G	MG	G
	a_4	G	G	G	VG		a_4	G	VG	G	VG

This method has been implemented using Matlab software. Data were then imported into the written code. Indeed, the data has been derived from the (Mahmoudi et al., 2016) paper. The proposed method following is established step by step with more details in each stage as followed.

Stage 1: ***Initial preparation of data***

This stage is implemented in three steps. The description of each step is given below.

Step1: in this step, linguistic variables in Table 1 by fuzzy numbers are defined and then, the decision matrix in form of linguistic for each expert from Table 2 is constructed separately. For example, the decision matrix for the first expert is as follows.

$$D_1 = \begin{bmatrix} MG & MG & G & VG \\ MG & G & G & MG \\ G & F & MG & VG \\ VG & G & MG & G \end{bmatrix}$$

Step2: In this step, linguistic terms are converted into fuzzy numbers; a fuzzy decision matrix for all experts is created. For instance, the fuzzy decision matrix (D_1 displayed in step1) is shown for expert 1 in the table (Table 3). This step is formed a matrix with fuzzy values by name $\widetilde{D}_k$ that k=1, 2, …,10

Table 3: Fuzzy decision matrix of expert1 ($\widetilde{\boldsymbol{D}}_{\boldsymbol{1}}$)

Suppliers	**Criteria**			
	C1	**C2**	**C3**	**C4**
a_1	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1.0, 1.0)
a_2	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)
a_3	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)	(0.5, 0.6, 0.7, 0.8)	(0.8, 0.9, 1.0, 1.0)
a_4	(0.8, 0.9, 1.0, 1.0)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)

Step3: The fuzzy decision matrix is defuzzified into crisp values for each expert. For instance, a crisp decision matrix is shown for expert1 in table (Table 3). This step has formed a matrix with crisp values with a name of D_k that k=1, 2, …, 10. Defuzzification includes converting fuzz numbers into crisps values. As mentioned before, various techniques can be used to carry out this conversion. One of the most frequently utilized defuzzification techniques is the center of gravity (COG). COG can be presented as follow (Sadi-Nezhad & Sotoudeh-Anvari, 2015):

$$\alpha = \frac{\int x\mu_{\tilde{B}}(x)dx}{\int \mu_{\tilde{B}}(x)dx} \tag{6}$$

For trapezoidal fuzzy number (TrFN), namely Ã= (a, b, c, d) the defuzzified value can be generated by Eq. (1) and when b=c then is used formula Eq. (2).

For example, by converting the fuzzy values of expert1 based on Equation (6), the crisp decision matrix is obtained through COG method and the output of this calculation is shown in Table 4.

Table 2: Crisp decision matrix of expert1 ($\boldsymbol{D}_{\boldsymbol{1}}$)

Suppliers	**Criteria**			
	C1	**C2**	**C3**	**C4**
a_1	0.65	0.65	0.80	0.92
a_2	0.65	0.80	0.80	0.65
a_3	0.80	0.50	0.65	0.92
a_4	0.92	0.80	0.65	0.80

For instance, suppose that, the fuzzy number for alternative 1 (a_1) and criteria 1 (C1) is equal Ã = (0.5, 0.6, 0.7, 0.8). The question is that how can transfer this fuzzy number to a crisp? To do that is used Eq. (1). $\alpha = \frac{1}{3}\left[0.5 + 0.6 + 0.7 + 0.8 - \frac{0.7\times0.8-0.5\times0.6}{(0.7+0.8)-(0.5+0.6)}\right] = 0.65$

Stage 2: ***Obtain experts' individual ranking***

In this stage, the final score of all alternatives and its associated rank is calculated for each expert. To do so, a fuzzy inference system with four inputs (criteria) and one output (final score of alternatives) was designed and values of crisp decision matrixes (D_k) were involved in fuzzy system separately for each expert and alternative. A Fuzzy Toolbox or a writing code in Matlab software has been used to design a fuzzy inference system. So the steps to perform this stage will be as follows.

Step1: The score of the alternatives for each expert is calculated in the same way as mentioned above. The output values of this step are placed in a matrix-like $D_k = [X_{ijk}]_{n\times m}$. For example, the output for expert 1 was presented in Table 5.

Table 3: Final score of suppliers based on Expert1 data (S_1)

Suppliers	Final score
a_1	4.049
a_2	5.106
a_3	4.324
a_4	6.087

Step2: the alternatives are ranked by sorting and determine the ranking of each expert as matrix $S_k = [s_{ik}]_{n\times 1}$.

Step3: Finally, the output of each expert (each expert has the role of a criterion in this table) is aggregated into one matrix as s. The aggregated matrix of scores based on all experts' data was presented in Table 6.

Table 4: Final score of suppliers for all experts (S)

Suppliers	Experts										
	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	Total
a_1	4.049	5.658	5.213	4.049	5.106	6.087	5.106	4.121	5.643	5.643	**50.675**
a_2	5.106	3.890	5.106	3.890	4.121	5.658	6.087	5.213	4.049	4.121	47.241
a_3	4.324	4.121	5.643	5.643	6.717	5.643	3.890	5.643	4.121	4.121	49.866
a_4	6.087	6.627	6.717	6.087	5.658	4.049	4.049	6.717	6.627	6.627	**59.245**

Stage 3: ***Apply TOPSIS technique to make final ranking***

In this stage, the TOPSIS technique is applied to obtain the final ranking of suppliers. In in this method the weight of all experts is considered the same; therefore matrix *V* was equal to matrix *S*.

Step1: Multiply expert's weight into its own column in matrix S. $V = [v_{ik}]_{n\times k}$

Step2: In this step, the values of v_k^+ and v_k^- the best (max) and the worst (min) value of each expert are obtained which is shown in Table 7.

Table 5: the best and worst value of the experts

v_k^+	6.087	6.627	6.717	6.087	6.717	6.087	6.087	6.717	6.627	6.627
v_k^-	4.049	3.890	5.106	3.890	4.121	4.049	3.890	4.121	4.049	4.121

Step3: In this step, Euclidean distance from the best and the worst value for each alternative is calculated, according to the following formulas.

$$C_i^+ = \sqrt{\sum_{k=1}^{k} [v_{ik} - v_k^+]^2} \tag{7}$$

$$C_i^- = \sqrt{\sum_{k=1}^{k} [v_{ik} - v_k^-]^2} \tag{8}$$

$$C_i = \frac{C_i^-}{C_i^- + C_i^+} \tag{9}$$

For this purpose $[v_{ik} - v_k^+]^2$ and $[v_{ik} - v_k^-]^2$ is obtained. The value of each of them is shown in Table 8 and Table 9. After that, C_i^+ and C_i^- are obtained by using (7) and (8) Equations and then C_i is calculated according to the Equation of (9). Then, the final sequences of suppliers are obtained as Table 10.

As mentioned above, to obtain Euclidean distance from best values $(v_{ik} - v_k^+)^2$ is calculated. For instance, to reach 4.153 for alternative a_1, (4.049-6.087) is calculated which is equal to (-2.038). After that, the square of (-2.038) is calculated that equal to 4.153. Other values are obtained in the same way (see Table 8).

Table 6: Euclidean distance from best values $(v_{ik} - v_k^+)^2$

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10
a_1	4.153	0.940	2.262	4.153	2.595	0.000	0.962	6.740	0.969	0.969
a_2	0.962	7.494	2.595	4.830	6.740	0.184	0.000	2.262	6.647	6.280
a_3	3.108	6.280	1.155	0.198	0.000	0.198	4.830	1.155	6.280	6.280
a_4	0.000	0.000	0.000	0.000	1.123	4.154	4.153	0.000	0.000	0.000

Also, to obtain Euclidean distance from the worst values $(v_{ik} - v_k^-)^2$ is calculated. For instance, to reach 4.153 for alternative a_4, (6.087-4.049) is calculated which is equal to (+2.038). After that, the square of (+2.038) is calculated that is equal to 4.153. Other values are obtained in the same way (see Table 9).

Thereafter, according to Eq. (7) C_i^+ is obtained. For instance, to reach 4.873, at first the sum of all

Table 7: Euclidean distance from worst values $(v_{ik} - v_k^-)^2$

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10
a_1	0.000	3.126	0.011	0.025	0.971	4.153	1.481	0.000	2.539	2.315
a_2	1.118	0.000	0.000	0.000	0.000	2.588	4.830	1.193	0.000	0.000
a_3	0.076	0.054	0.287	3.073	6.740	2.539	0.000	2.315	0.005	0.000
a_4	4.153	7.494	2.595	4.830	2.361	0.000	0.025	6.740	6.647	6.280

values of the row of alternative a_1 include (4.153+0.940+2.262+4.153+2.595+0.000 +0.962+6.740 +0.969 +0.969) is calculated which is equal to (+23.745). After that, the squared of (+23.745) is calculated that is equal to 4.873. Other alternative values as well other values for C_i^- and C_i are obtained based on Eq. (8) and Eq. (9) in the same way (see Table 10).

Table 8: values of C_i^+, C_i^- and C_i

	C_i^+	C_i^-	C_i	Result of the Proposed method (sequence of alternatives)
a_1	4.873	14.623	0.7500	2
a_2	6.164	9.728	0.6121	4
a_3	5.430	15.089	0.7354	3
a_4	3.071	41.127	0.9305	1
				$a_4 > a_1 > a_3 > a_2$

As stated in Table 10, results of the sequence of alternatives based on our proposed method indicate that supplier a_4 is the best among all alternatives. Therefore, the ranking of each alternative is demonstrated in the last row of this table.

4. Comparison Result of the proposed Method

In this section, the proposed method is compared with Mahmoudi's et.al (2016) method and it is shown that the propose method is highly capable of solving group MADM problems. The reasons for choosing Mahmoudi's method as follow. First, the opinions of experts in this method are both in group form and in linguistic form. Second, in this method has been used fuzzy inference system to development, and also the body of the ranking operation therein is dependent on one of the conventional techniques. Third, this method has been strongly evaluated. According to the compatibility of the conditions and features of this method with the proposed method, eventually, Mahmoudi's method (Mahmoudi et al., 2016) was chosen for this comparison.

For this comparison, Mahmoudi's method data (refer to table 4 in (Mahmoudi et al., 2016)) was imported in the proposed method. The comparison output of the proposed method with Mahmoudi's method briefly is shown in Table 11. The main purpose in this portion is that proper analysis shows that ranking with the proposed method is more reliable and more accurate than Mahmoudi's method.

Table 11: Comparison of the two methods

Alternatives	Proposed method	(Mahmoudi et al., 2016) method
a_1	2	1
a_2	**4**	**4**
a_3	**3**	**3**
a_4	1	2
Sequences	$a_4 > a_1 > \boldsymbol{a_3} > \boldsymbol{a_2}$	$a_1 > a_4 > \boldsymbol{a_3} > \boldsymbol{a_2}$

As shown in Table 11, the ranking results based on the proposed method indicate that supplier a_4 is the best among all alternatives. Also from this table is observed that in both methods of Mahmoudi's and the proposed method Suppliers a_2 and a_3 are as third and fourth choices respectively, which shows that the proposed approach is valid. But the ratings of a_1 and a_4 alternatives are different. To illustrate the correctness and the accuracy of the proposed approach, according to Table 6, the outputs of a_1 and a_4 are compared one by one, respectively with each other for 10 experts. In this table, the total value of the score concerning all experts for a_1 and a_4 are 50.67 and 59.24 respectively (refer to Table 6). Therefore, it is verifiable that a_4 take precedence over a_1. The inconsistency in examples can be resultant from the fact that in Mahmoudi's method the expert's opinion is aggregated at the begging of the algorithm but in the proposed method the ranking alternatives for each expert are established separately and at the end, the task has aggregated the outputs. Finally, the results show that in the new approach due to losing fewer data more accurate results are obtained.

5. Conclusion

In this study, it is focused to solve an MADM issue by means of fuzzy uncertainty under group decision making situation to rank of alternatives. In this research, fuzzy inference system and TOPSIS conventional technique as a main contribution was hybridized to solve group MADM problems with aggregating the opinions of each expert individually. Although based on mathematical formulations the conventional decision making techniques were constructed but in some situation they cannot provide decision maker utility and preferences. Therefore, it is applied fuzzy inference system to modeling decision maker preferences and utilities. In order to increase accuracy, a classical technique namely TOPSIS was combined with our FRBS method. Numerical analysis showed that our proposed method had suitable efficiency and led to more accurate results for solving group MADM problems. To combine FRBS with ANP method to solve issues with network relationship and cope with the interdependence among different criteria is recommended as a future research. Also, considering proposed method with Fuzzy Type 2 concept can be a proper choice to provide in the future researches.

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