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Truncated life testing under resubmitted sampling plans for Weibull distribution

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Abstract

In a truncated life testing, the test of each item is terminated at a predetermined time which is usually a coefficient of mean, median or other percentiles of lifetime. Life testing and acceptance sampling plans are two major fields of reliability theory and statistical quality control. In a reliability acceptance sampling (RAS) plan the quality characteristic of interest is lifetime. Thus, in designing RAS plans, two subjects of life testing and acceptance sampling plans should be taken into consideration. In this paper, one type of sampling plans, which is known as resubmitted sampling (RS) plans, is proposed for truncated life testing. The items are considered Weibull distributed with a known shape parameter. To obtain the operating characteristic (OC) curve of the RS plan, an equation is derived and to optimize the value of average sample number (ASN), three models are proposed: (I) minimizing ASN in acceptable quality level (AQL), (II) minimizing ASN in limiting quality level (LQL) and (III) minimizing ASN based on the both AQL and LQL. In optimizing the models, the constraints related to the consumer's and producer's risks are taken into consideration. Finally, numerical examples and sensitivity analyses are conducted. According to the results of comparison of RS and single sampling (SS) plans, it cannot be concluded that one scheme monotonically outperforms the other. Moreover, from the aspect of OC curve, the acceptance probability of a given lot under the RS plan is slightly larger than the corresponding value in the SS plans.

Keywords: Life testing, lifetime, reliability, Weibull distribution, acceptance sampling plan

1- Introduction

As a statistical method, acceptance sampling plans play a crucial role in the control and assurance of quality. These plans can be applied in different stages of a manufacturing system, from the start of the process, where the raw materials or purchased items are supplied, to the final stages where the produced items are inspected and sent to the consumers. Traditionally, acceptance sampling plans are classified into attribute and variable plans (Montgomery, 2020). In variable plans, the quality characteristic of interest can be represented as a continuous random variable, while in an attribute plan, the quality characteristic is usually considered and inspected as a discrete random variable. Also, in some studies, combination of variables and attribute sampling plans is proposed (Aslam et al., 2013).

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Different types of sampling plans have been presented including, but not limited to, single sampling, double sampling, sequential sampling, repetitive group sampling, switch sampling, resubmitted sampling and chain sampling plans. An important aspect of a sampling plan is the operating characteristic (OC) curve. An OC curve displays the discriminatory power of a sampling plan. Also, the OC curve is applied in designing sampling plans. One of the common approaches in designing a sampling plan is two points approach. Specifically, a lot at acceptable quality level (AQL) is accepted with at least $(1-\alpha)$ probability and a lot at limiting quality level (LQL) is accepted with at most β probability. The values of α and β are usually referred as the producer's and consumer's risks. Thus, two points, i.e., $(AQL, 1-\alpha)$ and (LQL, β), are specified on the OC curve, and the sampling plan is designed so that passes through these two points. Different sampling plans provide different OC curves for the same values of the two mentioned points.

Generally, a sampling plan can be presented as a hypothesis testing with a null and alternative hypotheses. It is desirable to determine whether the quality characteristic of interest satisfies the condition of the null hypothesis or not, while a certain values of type I and II errors should be satisfied. The OC curve of a sampling plan and the power function of the corresponding hypothesis testing are closely related to each other. Another important aspect of a sampling plan is the average sample number (ASN) to reach a conclusion regarding the submitted lot. Obviously, in designing a sampling plan, minimizing the value of ASN is desirable. In the situations that the sampling and inspecting the items are destructive, e.g., most of the life testing, minimizing the value of ASN is become more important.

One major type of sampling plans that arises in the reliability theory is called reliability acceptance sampling (RAS) plans. In a RAS plan, the quality characteristic of interest is the lifetime of items. Thus, in a RAS plan, two major subjects are addressed: sampling plans and life testing. Life testing has been widely investigated by the researchers of reliability theory and different types of life testing are proposed. Accelerated life testing, truncated life testing, time censoring, failure censoring and hybrid censoring are among the others. In a truncated life testing, the life testing for each item is terminated at a predetermined time which is usually a coefficient of the mean lifetime of the items. For example, if we want to test whether the mean lifetime of the items of a lot is 2000 hours or not, to save the time of the inspection, a random sample can be selected from the lot and the test of each item can be terminated after passing 1000 hours.

In some life testing schemes, the experimenter cannot wait until observing the failure of all items. To cope with this challenge, the concept of censoring has been introduced. In life testing, censoring occurs when the real observation about the failure time of some items is not available. In time censoring life testing, *n* items are randomly selected and placed on the test simultaneously. The test continues until a predetermined time. On the other hand, in failure censoring, *n* items are placed on the test and the test terminates once observing r'th failure ($r \leq n$). In a Hybrid censoring, the life test terminates after passing a predetermined time from the start of the test or after observing r'th failure, which one occurs first. The aim of an accelerated life testing is obtaining the reliability information quickly. To this end, the test units, e.g., material, component, subsystem or entire system, are subjected to one or more accelerated variables that are more than usual operating conditions. Then, the results of the accelerated test are employed to estimate the reliability of the unit during the usual operating conditions.

Although lifetime is a continuous random variable, both attribute and variable sampling plans are proposed in the context of RAS. In designing an attribute RAS plan, during the life testing, the number of failure items is taken into consideration, while in a variable RAS plan, according to the failure time of each item, a suitable statistic is obtained and accordingly lot sentencing is declared.

Single sampling plans are the most popular plan in practice due to simplicity in administration. In this paper, resubmitted sampling (RS) plan is applied in the context of life testing. General procedure of a resubmitted sampling (RS) plan can be expressed as follows: a sample is taken from the lot according to a desired plan. Based on the information of this sample, it is decided to accept the lot or take another sample. Resubmission from the lot can be applied at most (*m-1*) times. If the lot is not accepted in the *(m-1)'th* resubmission, then the lot is rejected. Even though, it makes sense to combine the results of the previous samples, like the double or sequential sampling, in the RS plan, the results of the previous samples are discarded and the decision is made according to the result of the current sample. As stated by Wu et al. (2012), in some cases, the producers may dispute the result of the first sample. In this state, according to the contract between the producer and consumer, taking another sample may be permitted. In most practical situations, the value of m is considered 2 which means that resampling from the lot is only conducted one time. As an example, for the value of $m=2$ the RS plan is conducted as follows: a random sample is selected from the lot according to the desired plan, then it is decided to accept the lot or take another sample. In the next sampling, if it happens, the final decision regarding the lot is declared.

Truncated life testing has been widely studied by the researchers of reliability theory and quality engineering. Generally, the research conducted regarding the truncated life testing can be classified according to the distribution of the lifetime, sampling plans which is applied, and the parameter of the lifetime population applied to terminate the test, i.e., mean, median or other percentiles. Accordingly, table 1 classifies some research which is closely related to the subject of the current paper.

In this paper, the subject of RAS plans is investigated while the lifetime of items fallows a Weibull distribution with a known shape parameter. Truncated life testing is applied to save the time of inspection so that the life testing of each item is terminated at a predetermined time. Sampling is conducted according to an RS plan. The main novelty of the paper is the application of resubmitted sampling plans in the truncated life testing. To this end, three models are proposed: (I) minimizing ASN in acceptable quality level (AQL), (II) minimizing ASN in limiting quality level (LQL) and (III) minimizing ASN based on the both AQL and LQL. In optimizing the model, the constraints related to the consumer's and producer's risks are taken into consideration. In addition, an explicit equation is derived to compute the OC curve of the proposed plan.

As the best of author's knowledge, this issue has not been investigated so far. The rest of the paper is organized as follows: Section 2 presents some description regarding the truncated life testing for Weibull distribution. In section 3, an equation is derived to obtain the operating characteristic curve of the RS plan. Section 4 presents three mathematical models to optimally determine the RS plans parameters. Section 5 presents an example to clarify the procedure of the RS plans. Section 6 provides some sensitivity analyses. Finally, section 7 concludes the paper.

Table 1. Classification of some studies regarding reliability acceptance sampling plans

Reference	Life testing	Distribution	Sampling plan	Mean/median/other percentiles
(Aslam et al. 2009a)	Truncated life testing	Weibull	Group acceptance sampling	Mean
(Aslam et al. 2010)	Truncated life testing	Log-logistic distribution	Double-acceptance sampling	Median
(Gui and Aslam, 2017)	Truncated life testing	Weighted exponential distribution	Single sampling	Mean
(Aslam et al. 2009b)	Truncated life testing	half exponential power distribution	Double-acceptance sampling	Mean
(Aslam et al. 2016)	Truncated life testing	Burr Distribution	Multiple dependent state repetitive group sampling plan	100qth percentile lifetime of the product
(Tsai et al., 2006)	Truncated life testing	Rayleigh distribution	Single Sampling	Mean
(Aslam, 2019)	Truncated life testing	Weibull distribution	multiple dependent state sampling	Mean
(Rasay et al., 2018)	Truncated life testing	Weibull distribution	Sequential Sampling	Mean
(Wu et al., 2018)	Failure censoring life testing	Weibull/Exponential distribution	Single sampling plan	Order statistic
(Balamurali et al., 2018)	Truncated life testing	Weibull distribution	Quick switching Sampling plan	Mean
(Jun et al., 2010)	Failure censoring life testing	Weibull distribution	Single sampling plan	Order statistic
(Rasay et al., 2020)	Failure censoring life testing	Weibull distribution	Repetitive group sampling plan	Lifetime performance index
(Rasay, and Naderkhani, 2020, June)	Failure censoring life testing	Weibull distribution	Quick switching Sampling plan	Mean
(Divecha, and Raykundaliya, 2020)	Type-I censoring	Generalized exponential	Single sampling plan with EWMA statistic	Order statistic
(Al-Omari et al., 2019)	Truncated life test	Rama distribution	Single sampling plan	Mean
(Yan, A., & Liu, S. (2016)	-	Weibull distribution	Repetitive group sampling plan	Process capability index
Current Study	**Truncated life testing**	**Weibull distribution**	**Resubmitted sampling plan**	**Mean**

2- Truncated life testing for Weibull distribution

Generally speaking, the lifetime of an item is a non-negative random variable. Weibull distribution has been widely applied to represent the random variable associated with the lifetime of products (Fallahnezhad et al., 2020). More specifically, consider the Weibull distributed items with the following cumulative distribution function (c.d.f):

$$F(t;\nu,\lambda) = 1 - \exp[-(\lambda t)^{\nu}] \tag{1}$$

Where λ and ν are the scale and shape parameters of Weibull distribution, respectively. It is assumed that the shape parameter is known. Practitioners would probably have some prior knowledge of the model parameters when they select the acceptance sampling plan. The shape parameter of Weibull distribution can be easily estimated by the data of several failure times (Aslam and Jun, 2009a). Hence, the Weibull distributed item has the following mean:

$$\mu = \frac{\Gamma(\frac{1}{\nu})}{\lambda.\nu} \tag{2}$$

Where $\Gamma(\bullet)$ is the gamma function. It is desirable to design an acceptance sampling plan to assure that the lifetime mean of the product is at least μ_0. The lot is accepted if there is enough evidence that $\mu \geq \mu_0$ at the certain level for the consumer's and producer's risks. More specifically, for each submitted lot, the following hypothesis testing is conducted:

$$\begin{cases} H_0: \mu \geq \mu_0 \\ H_1: \mu < \mu_0 \end{cases} \tag{3}$$

For convenience's sake, it is assumed that the termination time of the test, t_0 , for each item, is a coefficient of the target lifetime mean ,μ_0. Thus $t_0 = g\mu_0$ where g is a coefficient factor. For example, if we want to determine whether the lifetime mean of the product is more than 2000 hours and set $g = 0.5$ then the test of each item is terminated after passing 1000h from the start of the test.
For a specified value of test termination time as t_0, the probability of failure for each item ,*p*, is equal to the c.d.f of Weibull distribution at t_0 . Thus, the value of *p* can be computed as follows:

$$p = p(t < t_0) = 1 - \exp[-(\lambda t)^{\nu}] = 1 - exp\left\{-\left(\frac{\mu}{\mu_0}\right)^{-\nu}\left[gT(1+\frac{1}{\nu}))\right]^{\nu}\right\} \tag{4}$$
$$= 1 - exp\left\{-r^{-\nu}\left[g\Gamma(1+\frac{1}{\nu})\right]^{\nu}\right\}$$

The quality level of each item is denoted by *r* which is expressed based on the ratio of its lifetime mean (μ) to the target value (μ_0). Hence, the quality level of each item is stated as follows: $r = \frac{\mu}{\mu_0}$. The producer's risk is the probability of rejecting a good lot, while the consumer's risk is the probability of accepting a bad lot. The producer's risk and consumer's risk are denoted by α and β, respectively. The producer wants that the rejection probability of the lot at the higher quality level, denoted by p_1, becomes smaller than α. On the other hand, the consumer wants that the probability of accepting a lot at the lower quality level, denoted by p_2, becomes smaller than β. Assume that the ratio of $\frac{\mu}{\mu_0}$ at the higher quality level and lower quality level are r_1 and r_2 , respectively ($r_1 > r_2$). Indeed, the values of r_1 and r_2 correspond to the acceptable quality level (AQL) and limiting quality level (LQL). For the specified values of shape parameter of Weibull distribution (v), test termination coefficient (g) and quality level of each item (r), equation 4 determines the probability of item failure before t_0. Hence, the values of p_1 and p_2 corresponding to r_1 and r_2 can be obtained using equation 4.

3- Computing the Operating Characteristic (OC) curve of the RS plan

The proposed RS plan can be stated as follows. Take a random sample with size *n* from the lot. The truncated life test is conducted for the selected items which means that life testing for each item is terminated at time t_0, then it is determined whether the item is failed or not. If the total number of failure items in the sample is less than a criterion as *c*, then the lot is accepted, otherwise, another sample is taken from the lot. These stages continue at most *(m-1)* times. In each resubmission, it is decided to accept the lot or take another sample. If the lot is not accepted in the *(m-1)'th* resubmission, then the lot is rejected.

Accordingly, each time a sample is taken from the lot, using binomial distribution, the acceptance probability of the lot is computed as follows:

$$P_a(n,c|r) = \sum_{j=1}^{c-1} \binom{n}{j} p^j (1-p)^{n-j} \quad (5)$$

In equation 5, the value of p is the failure probability of each item during the truncated life test which is computed using equation 4. The eventual probability of acceptance of the lot can be computed as follows:

$$\pi_a(m,n,c|r) = \sum_{j=1}^{m} [1 - P_a(n,c|r)]^{j-1} P_a(n,c|r) = 1 - [1 - P_a(n,c|r)]^m \quad (6)$$

Thus, the OC curve of the RS plan can be obtained using equation 6. For a submitted lot that its quality level is r, the OC curve displays the acceptance probability of the lot under the RS plan with parameters m,n and c. The OC curve displays discriminatory power of a sampling plan. In an ideal OC curve, the acceptance probability of a lot with $\mu \geq \mu_0$ is 1 while the acceptance probability of a lot with $\mu < \mu_0$ is zero. Figure 1 displays an ideal OC curve with respect to the mean lifetime. Figure 2 shows a real OC curve with respect to $r = \frac{\mu}{\mu_0}$.

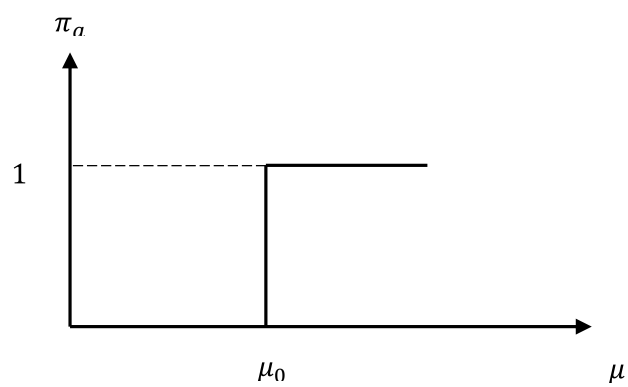

Fig 1. An ideal OC curve

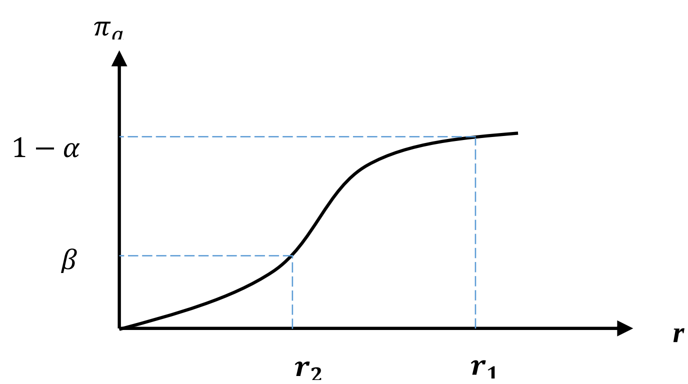

Fig 2. A real OC curve

4- Computing the Value of ASN and optimally design the RS plan

The value of average sample number (ASN) for the proposed RS can be computed as follows:

$$ASN = n + n\big(1 - P_a(n,c|r)\big) + n(1 - P_a(n,c|r))^2 + \cdots + n(1 - P_a(n,c|r))^{m-1} = \frac{n[1-(1-P_a(n,c|r))^m]}{P_a(n,c|r)} \tag{7}$$

Minimizing the value of ASN can be considered as a desired criterion in an acceptance sampling plan. One of the common approaches for designing a sampling plan is based on the specifying two points on the OC curve. More specifically, the sampling plan is designed so that the lot at the acceptable quality level (AQL) or r_1 is accepted with the probability of at least $(1-\alpha)$, and the lot at the limiting quality level (LQL) or r_2 is accepted with the probability of at most β. The sampling plan is designed so that passes through these two points: $(r_1, 1-\alpha)$ and (r_2, β). These two statements can be included as the constraints of the sampling plan.

According to equation 7, the value of ASN depends on the quality level of the lot. Thus, in proposing an optimization model for the RS plan, different objective functions can be considered. Based on Wu et al. (2012) and Wu (2012), three approaches are proposed as follows: (I) minimizing ASN in AQL (II) minimizing ASN in LQL and (III) minimizing the ASN based on the both AQL and LQL.

Accordingly, the following three models can be presented. The first one, corresponding to the first approach, is as follows:

$$Minimize\ ASN(m,n,c) = \frac{n[1-(1-P_a(n,c|r_1))^m]}{P_a(n,c|r_1)} \tag{8}$$
$$1 - [1 - P_a(n,c|r_1)]^m \geq 1 - \alpha$$
$$1 - [1 - P_a(n,c|r_2)]^m \leq \beta$$
$$m,n,c \in integer, n \geq c$$

The second, corresponding to the second approach, i.e., minimizing ASN in LQL, is as follows:

$$Minimize\ ASN(m,n,c) = \frac{n[1-(1-P_a(n,c|r_2)^m]}{P_a(n,c|r_2)} \tag{9}$$
$$1 - [1 - P_a(n,c|r_1)]^m \geq 1 - \alpha$$
$$1 - [1 - P_a(n,c|r_2)]^m \leq \beta$$
$$m,n,c \in integer, n \geq c$$

Finally, the third model is as follows:

$$Minimize\ ASN(m,n,c) = 0.5\left\{\frac{n[1-(1-P_a(n,c|r_1))^m]}{P_a(n,c|r_1)} + \frac{n[1-(1-P_a(n,c|r_2))^m]}{P_a(n,c|r_2)}\right\} \tag{10}$$
$$1 - [1 - P_a(n,c|r_1)]^m \geq 1 - \alpha$$
$$1 - [1 - P_a(n,c|r_2)]^m \leq \beta$$
$$m,n,c \in integer, n \geq c$$

Each model determines the values of m, n and c so that the value of *ASN* can be minimized. The constraints of the model guaranty the producer's and consumer's risks.
Finally, the steps of the proposed RS plan can be stated as follows:
1- Randomly select n items from the lot.
2- Conduct the life testing for each item: the life testing of each item is terminated at $t_0 = g\mu_0$.
3- Enumerate the total number of the failure items during the test. If the total number of failure items is less than$c$, then accept the lot. Otherwise go to the next step.
4- Repeat step 1 to 3 (*m-1*) times. In the case that the lot is not accepted in the (m-1)'th resubmission, reject the entire lot.
Figure 3 also shows these steps. It should be noted that the proposed RS plan reduces to single sampling plan if $m = 1$.

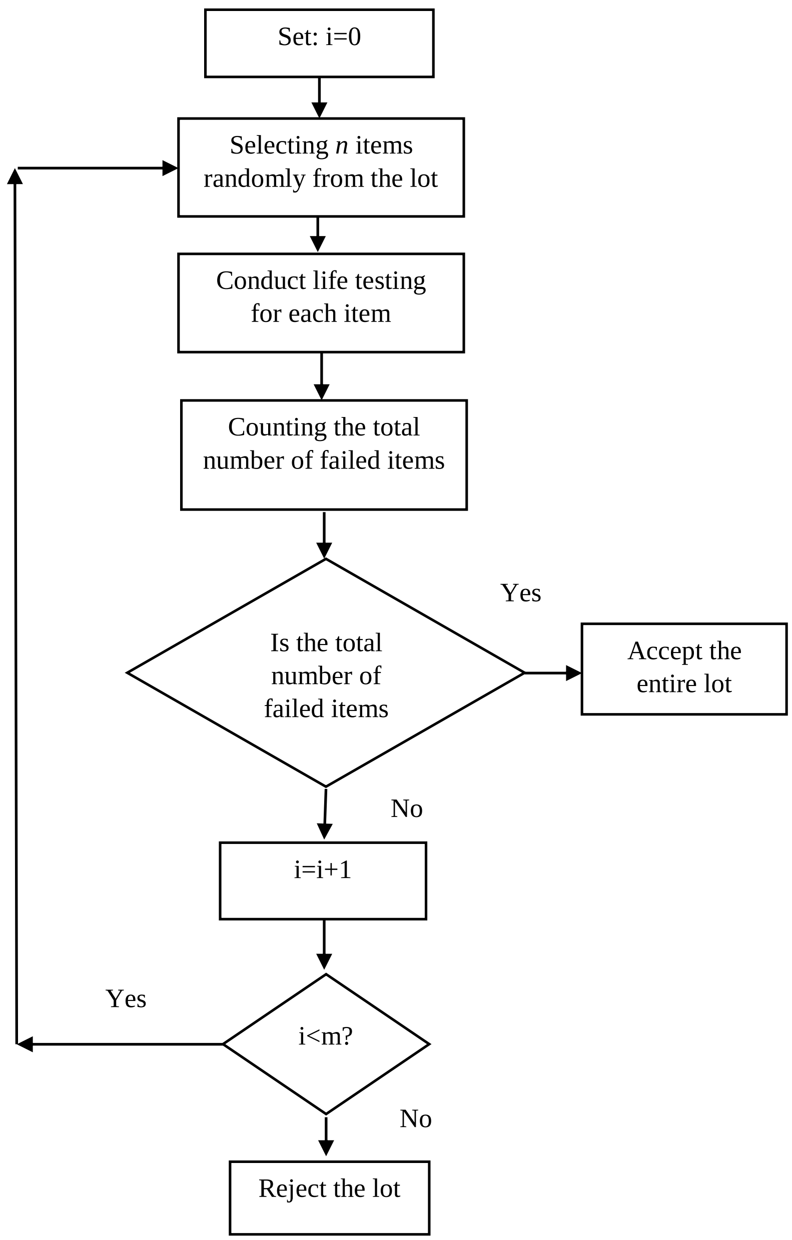

Fig 3. Flowchart of the proposed resubmitted sampling plan

5-Example

Suppose that the lifetime of a product follows a Weibull distribution with the shape parameter $v=2$. It is desirable to design a RS plan to assure that the lifetime mean of this product is at least 1000 hours, while the experiment must be terminated after 800 hours. The consumer wants that the risk of accepting a lot with the mean value of 1000 becomes less than 0.05 and the producer wants the risk of rejecting a lot with the mean value of 4000 becomes less than 0.01. Based on this information, the following data is derived: g=0.8, $r_1=4$, $r_2=1$, v=2, α=0.01, β=0.05 and $\mu_0=1000$.

The results of optimized RS plans and the SS plan are shows in table 2. As the value of $m = 2$ is broadly suggested for designing the RS plan (Wu et al., 2012), we use $m = 2$ in the RS plan. According to the results of the table, the three types of RS plans lead to the same values for the *n* and *c*, but they are different in the values of ASN.

According to the results of table 2, the RS plan is conducted as follows: a sample with size 12 is randomly taken from the lot. The truncated life test is performed and if the failure becomes 0 or 1 (less than 2) then accept the entire lot; otherwise another sample is taken. In the next sampling, if the failures are 0 or 1, accept the lot, otherwise reject it. The results of the SS plan are also shown in the last row of table 2. According to the SS plan, a sample with size 14 is selected from the lot and if the number of failures is less than 3 (0, 1 or 2) then the lot is accepted. If the equality level r_1 is considered in designing the RS plan, i.e., RS1, then the RS plan leads to a smaller ASN in comparison with the SS plan. On the other hand, for the RS2 and RS3, the SS plan leads to a smaller ASN.

Table 2. Optimized sampling plans of the example

Sampling plan	ASN	n	m	c
RS1	12.61	12	2	2
RS2	23.75	12	2	2
RS3	18.18	12	2	2
SS	14	14	1	3

Figure 4 shows the OC curve of the RS and SS plans. As the figure shows, there is a little differences between the OC curves of the RS and SS plans. Approximately, for the values of $1.8 \leq r < 3.8$, the SS plan leads to a smaller acceptance probability. For example, for a lot that its quality level is 2.8, which means that its mean lifetime is 2800 hours, under the SS plan, the acceptance probability of the lot is 0.94 and the RS plan provides a bit greater acceptance probability.

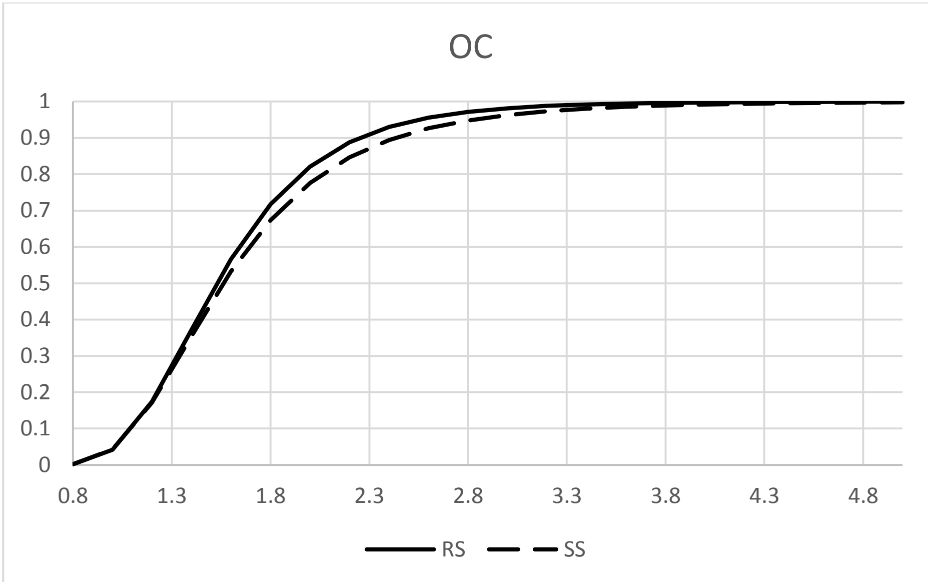

Fig 4. The OC curve of the SS and RS plans

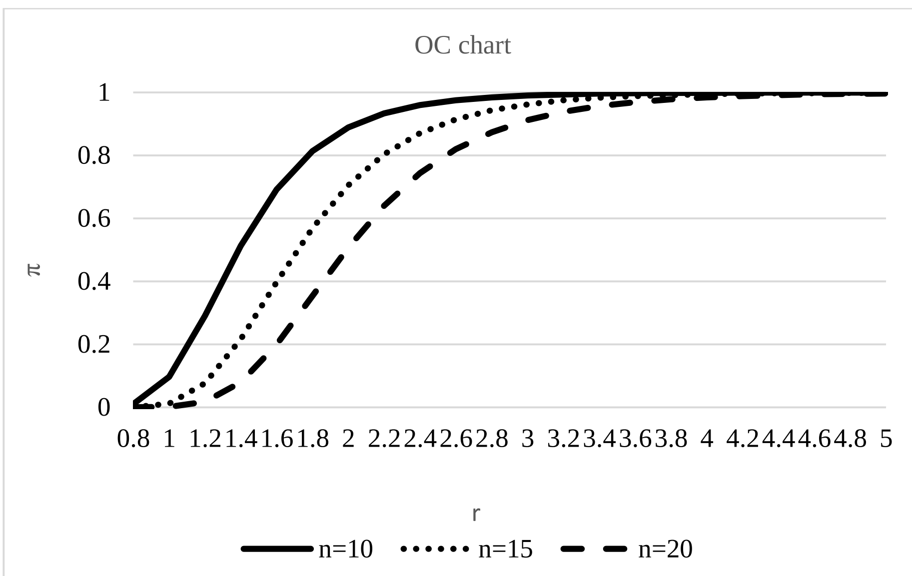

Fig 5. OC curves for different values of sample size; g=0.8; v=2; m=2; c=2

Figure 5 displays OC curves for different values of sample size. As the figure shows, for the fixed value of *c*, increasing the sample size decreases the probability of lot acceptance. Figure 6 displays the OC curve for different values of acceptance criterion. As the figure shows, for the larger values of *c*, the OC curve leads to a bigger probability of acceptance. Figure 7 compares the OC curves of the SS and RS plans. According to this figure, the RS plans lead to a larger probability of acceptance. Finally, figure 8 illustrates the OC curves for different values of the shape parameters of Weibull distribution. As the figure shows increasing the values of shape parameter increases the probability of acceptance of the lot.

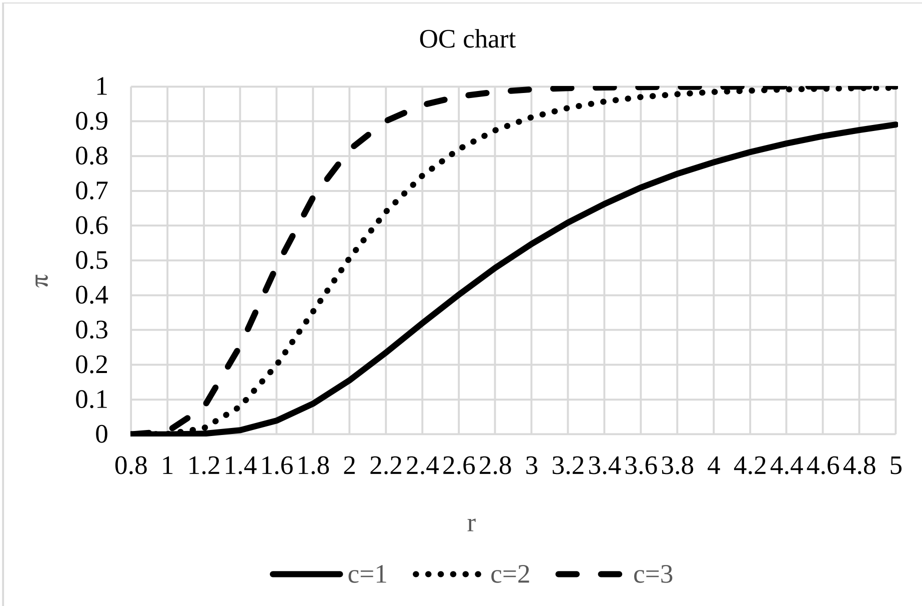

Fig 6. OC curves for different values of acceptance criterion; g=0.8; v=2; m=2; n=20

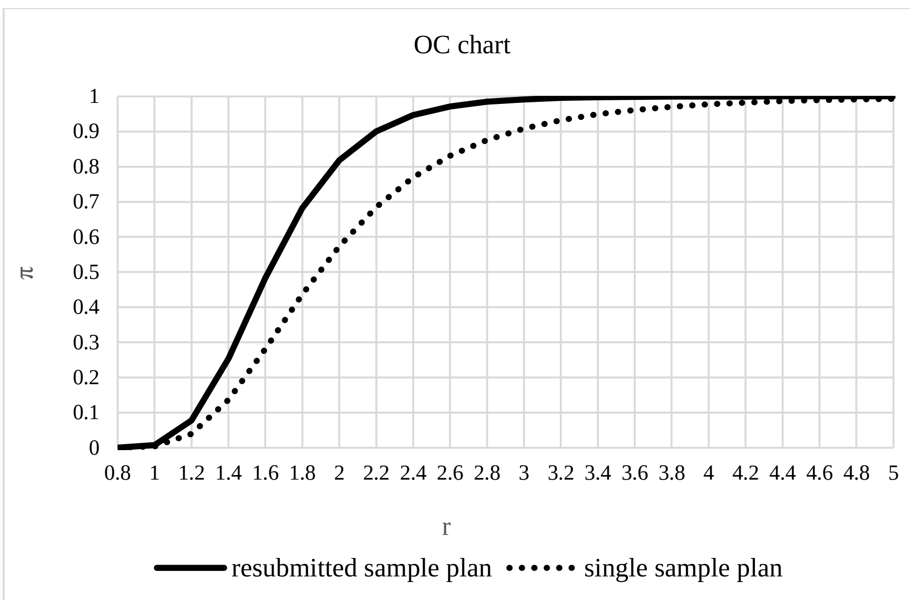

Fig 7. Comparison of the OC curves for SS and RS plans. g=0.8; v=2; n=20; c=3; m=1,2

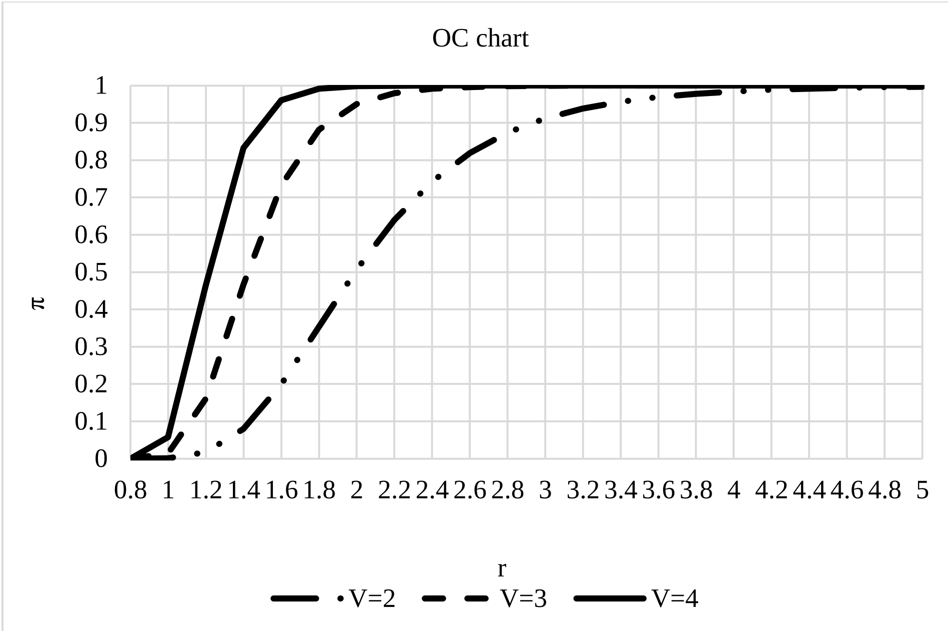

Fig 8. OC curves for different values of shape parameter; g=0.8; n=20 ;c=2; m=2

6- Sensitivity analysis

In this section, to gain some insight regarding the proposed RS plan, some sensitivity analyses are conducted. The example of the previous section is considered and the parameters are changed. The results of the analyses are shown in table 3. In the first row of the table, the results of the base example are presented. In the second row, the effect of changes on the termination coefficient of the test, g, is presented. As the results indicate, increasing the value of g from 0.8 to 1 leads to a significant reduction in the value of *ASN* and *n*. In the third row of this table, the value of producer's risk is increased from 0.01 to 0.05. This change leads to a decrease in the value of ASN. Also, as a result of this change, the acceptance criterion changes from 2 to 1. Decreasing the value of the consumer's risk, as the third row shows, increases the *ASN* and *n*. Decrease the value of *r* corresponding to the AQL , i.e., r_1, increases the *ASN*, *n* and *c*. On the other hand, decreasing the value of *r* corresponding to the LQL leads to a reduction in the value of *ASN*, *n* and *c*. Finally, the last row of the table presents the effect of change in the shape parameter of Weibull distribution. According to this result, increasing the value of the shape parameter from 2 to 3 increases the value of *ASN*.

Different optimized RS plans are displayed in tables 4 and 5. In table 4 the value of *m* is considered 2 while in table 5, it is 3. It should be noted that the sensitivity analyses and the results of table 4 and 5 are obtained considering the third model of RS plan, i.e., minimizing the value of ASN in AQL and LQL. For example, according to the results of table 4, for a sampling plan with $\alpha = 0.01, \beta = 0.05, r_2 = 1, r_1 = 3, g = 0.8$, the parameters of the optimized RS plan, which is obtained from the optimization of the third model, are as follows:$n = 16, c = 3$ and $ASN = 24.2611$. On the other hand, if the resubmission from the lot is permitted 2 times, i.e., *m=3*, according to the results of table 5, for the same values of sampling plan, the parameters of the optimized RS plan are as follows: $n = 13, c = 2$ and $ASN = 26.89$.

Table 3. Results of sensitivity analyses

Sampling plan	**ASN**	**n**	**m**	**c**
Base example	18.18	12	2	2
g=1	12.13	8	2	2
$\alpha = 0.05$	12.82	8	2	1
β=0.01	24.66	16	2	2
$r_1 = 3$	24.26	16	2	3
$r_2 = 0.5$	3.04	2	2	1
$v = 3$	21.8	11	2	1

Table 4. Different optimized RS plans

m=2			g=0.8						g=1					
V=2			$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.01$			$\boldsymbol{\alpha = 0.05}$		
β	r_2	r_1	n	c	ASN	n	c	ASN	n	c	ASN	n	c	ASN
0.05	0.5	2	4	2	6.1249	2	1	3.2043	3	2	4.6179	3	2	4.6179
		3	4	2	6.0156	2	1	3.0878	3	2	4.5215	2	1	3.1583
		4	2	1	3.0430	2	1	3.0430	2	1	3.0916	2	1	3.0916
	1	2	26	6	39.7033	20	4	31.8667	18	6	27.5842	13	4	20.5822
		3	16	3	24.2611	12	2	18.6873	11	3	16.7283	8	2	12.4794
		4	12	2	18.1810	8	1	12.8172	8	2	12.1332	5	1	7.9948
0.1	0.5	2	3	2	4.4843	2	1	3.2043	3	2	4.6179	1	1	1.5675
		3	3	2	4.4393	2	1	3.0878	1	1	1.5202	1	1	1.5202
		4	2	1	3.0430	2	1	3.0430	1	1	1.5023	1	1	1.5023
	1	2	21	5	32.0507	17	4	26.1958	14	5	21.2856	12	4	18.6727
		3	10	2	15.2497	10	2	15.2497	10	3	15.0745	7	2	10.7530
		4	10	2	14.9354	6	1	9.3684	7	2	10.5095	4	1	6.2701

Table 5. Different optimized RS plans

m=3			g=0.8						g=1					
V=2			$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.01$			$\boldsymbol{\alpha = 0.05}$		
β	r_2	r_1	n	c	ASN	n	c	ASN	n	c	ASN	n	c	ASN
0.05	0.5	2	4	2	8.1006	3	1	6.6083	3	2	6.1122	2	1	4.4246
		3	3	1	6.2563	3	1	6.2563	2	1	4.1802	2	1	4.1802
		4	3	1	6.1363	3	1	6.1363	2	1	4.0967	2	1	4.0967
	1	2	24	5	49.4578	17	3	37.3160	16	5	32.8883	11	3	23.9722
		3	13	2	26.8954	13	2	26.8954	9	2	18.7598	9	2	18.7598
		4	13	2	26.1423	9	1	19.2353	9	2	18.1822	6	1	12.8800
0.1	0.5	2	4	2	8.1006	2	1	4.2181	3	2	6.1122	2	1	4.4246
		3	2	1	4.0634	2	1	4.0634	2	1	4.1802	2	1	4.1802
		4	2	1	4.0111	2	1	4.0111	2	1	4.0967	2	1	4.0967
	1	2	19	4	39.2909	15	3	31.7688	13	4	27.0244	10	3	21.1773
		3	11	2	22.1899	7	1	15.1909	8	2	16.4013	5	1	11.0498
		4	7	1	14.5192	7	1	14.5192	8	2	15.9884	5	1	10.5157

7- Conclusion

In a resubmitted sampling (RS) plan, a sample is taken from the lot according to a desired plan. Based on the information of this sample, it is decided to accept the lot or take another sample. Resubmission from the lot can be applied at most (*m-1*) times. If the lot is not accepted in the *(m-1)'th* resubmission, then the lot is rejected. Unlike double or sequential sampling, in the RS plan, the results of the previous samples are discarded and the decision is made according to the result of the current sample. In this paper, RS plans are proposed in the context of life testing. The lifetime of items is considered as a Weibull distribution with known shape parameter. First, an equation is derived to compute and display the OC curve. Then, mathematical models are proposed to optimally determine the RS plan parameters. Minimizing the average sample number to reach a decision regarding the submitted lot is considered as the objective function of the mathematical models. The models are designed so that satisfy the constraints related to the consumer's and producer's risks. Numerical example and sensitivity analyses are conducted regarding the key parameters of the RS plans. In addition, the performances of the RS plan and single sampling plans are compared. The main novelty of the paper is proposing RS plans in the context of truncated life testing. The results of sensitivity analyses show that producer's and consumer's risks have significant effects on the parameters of the RS and SS plans so that by decreasing the values of these parameters, ASN and sample size increase noticeably and the criterion employed to decide about the lot becomes tighter. Given specified parameters, increasing the sample size decreases the probability of lot acceptance. Also, increasing the shape parameter of Weibull distribution increases the value of ASN. The comparative studies show that from the aspect of OC curve, the acceptance probability of a given lot under the RS plan is slightly larger than the corresponding value in the SS plans. The current study can be extended in several directions: employing the proposed RS plan for other distributions e.g., log-normal and gamma distributions, comparing the results of the proposed RS plans with RGS and sequential sampling are among others.

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