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A supply chain coordination model using buyback contract considering effort dependent demand

Mohammad Bagher Fakhrzad[1*], Abolfazl Dehghan[1], Abbasali Jafari-Nodoushan[2]

[1]*Department of Industrial Engineering, Faculty of Engineering, Yazd University, Yazd, Iran*
[2]*Department of Industrial Engineering, Faculty of Engineering, Meybod University, Meybod, Iran*

mfakhrzad@yazd.ac.ir, abolfazldehghan@stu.yazd.ac.ir, a.jafari@meybod.ac.ir

Abstract

This paper deals with the coordination of a two-stage supply chain, including a supplier and a retailer. The final demand is sensitive to the sales promotion and the quality improvement done by the retailer and the supplier, respectively. In the standard newsvendor setting, a buyback contract integrates the decentralized system where both members try to optimize their own profit. We showed that a buyback contract could not thoroughly coordinate the supply chain even though it enhances the whole supply chain profit. Therefore, in this research, we extended a new contract based on a buyback contract with which both members share the costs of efforts. The results showed that this contract can coordinate the channel and provide a win-win condition for supply chain components. The numerical example is used to indicate the results and obtain more insights. The optimal sales and quality efforts and the optimal order level are also determined, resulting in the optimal supply chain profit. Sensitivity analyses are performed in order to investigate the effects of different parameters on decision variables and profit. The results showed that the supply chain performance decreases by incrementing the cost coefficients of sales effort and quality efforts.

Keywords: Buyback contract, supply chain coordination, sales effort, quality improvement effort

1-Introduction

The coordination of the supply chain by considering contracts is one of the subjects that have attracted the attention of many researchers. Contracts in the supply chain are useful tools for establishing and coordinating suppliers and retailers in a decentralized set (Ebrahimzadeh-Afruzi and Aliahmadi, 2020). A contract is determined between two sections, members, or parts based on a treaty. The researchers addressed the various contracts in the literature for supply chain coordination, such as revenue-sharing, quantity-discount, whole-sale price, buyback (BB), sales-rebate, and quantity flexibility contract (Cachon, 2003). In this research, the focus is on the buyback (BB) contract. A BB contract is a deal agreed between the supplier and the retailer where this opportunity is given to the retailer to return its leftover inventory to the supplier for a lower price than the whole-sale price. An important note that is considered in many cases in the real world is that in the supply chains where sales efforts are also considered in order to sell more, the sales efforts costs are mostly made by an entity of the supply chain that directly faces the demand (Ebrahimi et al., 2019). Hence, most of the time, the retailer is supposed to undertake the costs of sales efforts, while

*Corresponding author
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advertising generally leads to increased sales and, as a result, increased profit of the supplier. Accordingly, it is possible to consider the contract for coordination of the supply chain in this case.

Researchers paid considerable attention to the supply chain contracts regarding sales effort in the literature recently (Duc et al., 2018). Cao et al. (2009) developed a model based on a quantity flexibility contract for perishable products in which the costs of sales effort are shared. They showed that the supply chain coordination will be performed, and the profit will be increased by considering the cost sharing of sales effort in the traditional quantity flexibility contract. Jiang and Liu (2014) extended a supply chain coordination model with two-echelon, a supplier and a retailer, where the retailer encounters an uncertain demand and decides to make the sales effort or not. They showed that using a buyback contract cannot attain supply chain coordination when the retailer is responsible for all or part of the sales effort. Also, they proved to improve the supply chain's profits when the supplier does not present sale motivation, and the shortage is allowed. On the other hand, if the retailer is responsible for all sales effort and the supplier presents the sales motivation to the retailer, the buyback contract can gain supply chain coordination. Hosseini-Motlagh et al. (2020) addressed the coordination in a closed-loop supply chain based on energy-saving effort and cost-tarrif contact. The results demonstrated the importance of remanufacturing and energy-saving efforts in the collection process. Wang and Liu (2018) investigated supply chain coordination with two-echelons, a retailer, and a product. They examined the effect of quantity discount and trade credit on coordination of supply chain where market demand is supposed stochastic. Taylor (2002) showed target rebate and returns contract can create coordination if demand is considered based on the retailer's sales effort. However, returns, targets, or linear rebates cannot gain applicable coordination. Cachon and Lariviere (2005) presented a simple quantity discount contract to create supply chain coordination. They also proved that a supply chain with retailers competing in quantities is coordinated based on the revenue sharing contract. Jian et al. (2021) considered the coordination of green closed-loop supply chain consisting of a manufacture and a retailer where a profit-sharing contract is employed. The results showed the profit-sharing contract could improve the relationship between members of the supply chain to achieve sustainable economic and environmental development. Li and Liu (2015) addressed coordination under price and sales effort dependent demand. They investigate three contracts and used the game theory to analyze. The results showed two-part tariff contract create coordination, but the other two contracts cannot coordinate the supply chain. Tian et al., (2020) studied a multi-channel supply chain by sales effort. The results indicate that a great adjustment of supply order quantity indirect channels, retailer's order quantity, consumers' channel preference, and sales effort will cause the system to lose stability and trap into complexity. The revenue sharing contract is one of the contracts that cannot create coordination in the supply chain (Pang et al., 2014). The vital mechanism to create coordination by this contract is sharing the cost of promotion. In some researches, composite contacts are also addressed. Wang et al. (2019) considered the coordination problem with a composite contract where the demand is assumed fuzzy. This contract involves a buyback and a promotion cost-sharing contract. The results showed that the production cost and demand have negative and positive effects on the profits, respectively. Hosseini-Motlagh et al. (2018) studied the coordination in a manufacturer-retailer supply chain where the manufacturer is socially responsible and the retailer invests in promotional efforts and uses a periodic review order-up-to policy for replenishing items.

In many supply chains, the retailer is also responsible for defective items in order to raise the final benefit and the customer satisfaction in the whole supply chain by improving the received goods according to lead time and quality, leading to more supply chain accountability. In this case, the primary point is that you can develop a contract on the same basis in which retailers can receive a higher quality product in return for the cost of advertising.

Ghosh et al. (2021) used a game-theoretic approach for supply chain coordination. They provided a model that optimizes the retailer's sales effort, the whole-sale price demanded by the manufacturer, the green level of the product, and the selling price affected by the retailers. They also performed sensitivity analyses of the model by varying market demand, price elasticity coefficient and sales effort. Ma et al. (2013) extended a model based on the two-part tariff contract for a two-stage supply chain where the supplier tries to provide a higher quality product for the retailer and instead the retailer makes the sales effort. They also showed that this contract is not suitable for coordinating the supply chain in this case.

Therefore, they extended a new contract with sales and quality effort that provides enough incentives for each supply chain member in order to enhance collaboration and coordination levels and thus achieve a supply chain with a high level of accountability. Thus, considering both sales and quality improvement efforts establish the coordination. Zhang et al. (2021) studied a dual-channel supply chain with a manufacturer and a retailer. They considered different contracts and proved that advertising costs and revenue sharing contracts make coordination. Gurnani and Erkoc (2008) examined three types of contracts in decentralized distribution channels to consider demand according to manufacturer and retailer. Nikkhoo and Bozorgi-Amiri (2018) studied the coordination model in the humanitarian supply chain using the information-sharing mechanism. They considered three models, including decentralized, centralized and coordinated modes, where the objective is to minimize the total cost. The results showed that the chain cost is high in the decentralized mode, and total costs were reduced in the coordinated mode. Heydari et al. (2021) addressed the green channel coordination using a hybrid contract where the manufacturer can invest and raise the quality of the products. They showed this contract is able to create supply chain coordination. Ranjan and Jha (2019) conducted a partially similar study in supply chain model. Setak et al. (2017) studied a decentralized supply chain in which a manufacturer sells a common generic product through two traditional and online retailers under a free-riding market. The results showed that the information sharing between the manufacturer and traditional retailer is always beneficial for all the supply chain members. Coordination were considered between three decision variables: sales effort, green quality, and the sales price. They showed that in the coordination model, green quality is higher than the decentralized

Table 1 summarizes the reviewed works on supply chain coordination problems and research gaps are shown. The most important difference between the literature mentioned above and our research is that we develop a BB contract providing that demand depends on both the quality-improvement and sales efforts that have not yet been addressed in the literature. The objective is to maximize the expected profit in the supply chain. Table 1 shows the position of the current study in the literature and highlights its advantages.

The main contributions of the study are as follow:

1. It will be proved that a BB contract is not able to create coordination completely even if it improves the supply chain performance.
2. In this study, the new BB contract will be provided in which both members act based on a new form of BB.
3. In this sense, we considered three decentralized, centralized, and coordinated scenarios for each prescribed contribution to show the performance of developed models.

The rest of the paper is organized as follows. The problem description is presented in section 2. Section 3 establishes the centralized supply chain model, and section 4 addresses the decentralized system environment. The new BB contract with parties share and costs of efforts for coordination is presented in section 5. The numerical experiment and the sensitivity analysis are performed in section 6 in order to obtain the optimal levels of variables. Section 7 will present the conclusions and future research.

Table 1. Comparison of literature of supply chain coordination with the current study

Author	demand pattern		contract
	sales effort	quality effort	
Taylor, 2002	✓		target rebate and returns contract
Wang et al., 2018	✓		composite contract
Cao et al., 2009	✓		quantity flexibility
Jiang and Liu, 2014	✓		Buyback
Tian et al., 2020	✓		revenue sharing
Wang and Liu, 2018	✓		trade credit
Pang et al., 2014	✓		revenue sharing
Ghosh et al. 2021	✓		revenue sharing
Cachon and Lariviere, 2005	✓		quantity discount
Jian et al. 2021	✓		profit-sharing contract
Li and Liu, 2015	✓		two-part tariff
Ma et al., 2013	✓	✓	two-part tariff
Hosseini-Motlagh et al., 2020	✓		cost-tarrif contact
Gurnani and Erkoc, 2008	✓	✓	price-only fixed-free general franchise
Heydari et al., 2020		✓	revenue sharing contract
Ranjan and Jha, 2019	✓	✓	profit-sharing contract
Zhang et al. 2021	✓		revenue sharing
Heydari et al., 2021		✓	hybrid contract
Current Study	✓	✓	Buyback

2-Problem description

The system regarded in this research involves two echelons, namely one supplier and one retailer, contracting on replenishment decisions. The retailer orders up to q at the beginning of a sale season for the price of w from the suppliers. All of the rest units are returned from the retailer to them at a price less than w (b) at the end of the season. It is assumed that the demand depends on the quality improvement which is put into by the supplier and sales efforts made by the retailer. Accordingly, e and θ are defined as the sales effort level by the retailer and quality effort level by the supplier, respectively. Hence, the cost of sales and quality effort is $\frac{\eta e^2}{2}$ and $\frac{\xi \theta^2}{2}$ where η and ξ are the unit marketing cost and the quality cost, respectively (Ma et al., 2013). In our model, both members make their decisions simultaneously and reach Nash equilibrium instead of the usual Stackelberg solution. By definition, γ and λ as factors that show the effect of sales and quality effort on demand D, it is obtained according to $D = a + \gamma e + \lambda \theta$ (Gurnani and Erkoc, 2008). The value of a is derived from a uniform distribution $(\mu - \Psi, \mu + \Psi)$ (Cao et al., 2009). The following notations will be used throughout this paper

Parameters

p	*retail price*
g_r	*retailer's goodwill penalty cost*
w	*wholesale price*
c	*supplier's marginal cost per unit*
g_s	*supplier's goodwill penalty cost*
D	*demand* which follows a uniform distribution with the mean μ
γ	*the influence of the marketing effort on demand*
λ	*the impact of the quality effort on demand*
$s(q)$	*the expected sales*
δ	*parameter of buyback contract*
g	*the total goodwill penalty cost*

Decision variables

q	$the\ number\ of\ units\ purchased\ by\ retailer$
e	$sales\ effort\ level\ by\ retailer$
θ	$quality\ effort\ level\ by\ supplier$

3-Model formulation

This section presents the mathematical models according to centralized, decentralized and buyback modes with an objective function and related variables. The objective is to maximize the expected profit and the variables are obtained based on the partial derivative of the objective function with respect to the variables.

3-1- The centralized supply chain

The supply chain is considered as an integrated entity where a single decision-maker decides on the aforementioned issues. The objective function is to maximize the whole expected profit of the supply chain. It is calculated according to equation (1).

$$\pi_T = \pi_s + \pi_r = (p+g).s(q) - cq - \eta\frac{e^2}{2} - \xi\frac{\theta^2}{2} \tag{1}$$

Where

$$s(q) = \min(q, D) = \frac{1}{4\Psi}[2q(\mu + \Psi + \gamma e + \lambda\theta)] - q^2 - (\mu - \Psi + \gamma e + \lambda\theta)^2] \tag{2}$$

In order to obtain the variables q, e, and θ, the partial derivative of equation (1) must be calculated under equation (2) with respect to the above variables and equal to 0 as follows.

$$\frac{\partial\pi_T}{\partial e} = 0 \rightarrow (p+g)\left[\frac{2q\gamma - 2\gamma(\mu - \Psi + \gamma e + \lambda\theta)}{4\Psi}\right] - e\eta = 0 \tag{3}$$

$$\frac{\partial\pi_T}{\partial\theta} = 0 \rightarrow (p+g)\left[\frac{2q\lambda - 2\lambda(\mu - \Psi + \gamma e + \lambda\theta)}{4\Psi}\right] - \theta\xi = 0 \tag{4}$$

$$\frac{\partial\pi_T}{\partial q} = 0 \rightarrow -c + (p+g)\left(\frac{2(\mu - q + \Psi + \gamma e + \lambda\theta)}{4\Psi}\right) = 0 \tag{5}$$

If the second-order derivative of equation (1) by consideration of equation (2) concerning the variables q, e, and θ is calculated, the Hessian matrix of equation (1) will be obtained.

$$H(q,e,\theta) = \begin{bmatrix} \frac{\partial^2\pi_T}{\partial q^2} & \frac{\partial^2\pi_T}{\partial q\partial e} & \frac{\partial^2\pi_T}{\partial q\partial\theta} \\ \frac{\partial^2\pi_T}{\partial e\partial q} & \frac{\partial^2\pi_T}{\partial e^2} & \frac{\partial^2\pi_T}{\partial e\partial\theta} \\ \frac{\partial^2\pi_T}{\partial\theta\partial q} & \frac{\partial^2\pi_T}{\partial\theta\partial e} & \frac{\partial^2\pi_T}{\partial\theta^2} \end{bmatrix} = \begin{bmatrix} -\eta - \frac{\gamma^2(p+g)}{2\Psi} & \frac{\gamma(p+g)}{2\Psi} & -\frac{(g+p)\gamma\lambda}{2\Psi} \\ \frac{\gamma(p+g)}{2\Psi} & \frac{-(p+g)}{2\Psi} & \frac{\lambda(p+g)}{2\Psi} \\ -\frac{(p+g)\gamma\lambda}{2\Psi} & \frac{\lambda(p+g)}{2\Psi} & -\xi - \frac{-(p+g)\lambda^2}{2\Psi} \end{bmatrix}$$

Since the above matrix should be negative according to the variables q, θ, and e, the following relations must be established. These conditions are always true. Hence, the concavity of π_T is proved.

1) $-(\eta + \frac{(p+g)\gamma^2}{2\Psi}) < 0$
2) $\frac{\eta(p+g)}{2\Psi} > 0$
3) $-\frac{\eta(p+g)\Psi}{2\Psi} < 0$

Proposition 1. By solving the equation (3)-(5) simultaneously, the optimal value for the variables q, e, and θ is obtained as order quantity, sales, and quality effort level according to the below results.

$$e_c^* = \frac{\gamma((p+g)-c)}{\eta} \tag{6}$$

$$q_c^* = \frac{(p+g)^2(\eta\lambda^2+\gamma^2\xi) - 2c\eta\xi\Psi + (p+g)[\mu\eta\xi + \eta\xi\Psi - c\eta\lambda^2 - c\gamma^2\xi]}{\eta\xi(p+g)} \tag{7}$$

$$\theta_c^* = \frac{\lambda((p+g)-c)}{\xi} \tag{8}$$

3-2- Decentralized system

The supplier's quality improvement effort is equal to zero in the decentralized system since quality improvement has no benefits for it. In this model, each part works independently and optimizes its profit; therefore, the quality effort is zero. As a result, the retailer wants to maximize its own profit consequently, the value of e and q is obtained according to the objective function of the retailer in which is calculated as follows.

$$\pi_r^d = (p+g_r)s(q) - wq - g_r\mu - \eta\frac{e^2}{2} \tag{9}$$

In order to extract the optimal value of e and q, the partial derivative of equation (9) is taken and set equal to zero. Hence, we will have:

$$\frac{\partial \pi_r^d}{\partial e} = 0 \rightarrow (p+g_r)\left[\frac{\partial s(q)}{\partial e}\right] - \eta e = 0 \tag{10}$$

$$\frac{\partial \pi_r^d}{\partial q} = 0 \rightarrow (p+g_r)\left[\frac{\partial s(q)}{q}\right] - w = 0 \tag{11}$$

If the second-order derivative of equation (9) concerning the variables q, e, and θ is calculated, the Hessian matrix of π_r^d will be obtained.

$$H(\pi_r^d; e, q) = \begin{bmatrix} \frac{\partial^2 \pi_r^d}{\partial e^2} & \frac{\partial^2 \pi_r^d}{\partial e \partial q} \\ \frac{\partial^2 \pi_r^d}{\partial q \partial e} & \frac{\partial^2 \pi_r^d}{\partial q^2} \end{bmatrix} = \begin{bmatrix} -\eta - \frac{\gamma^2(p+g_r)}{2\Psi} & \frac{\gamma(p+g_r)}{2\Psi} \\ \frac{\gamma(p+g_r)}{2\Psi} & \frac{-(p+g_r)}{2\Psi} \end{bmatrix}$$

Since the above matrix should be negative according to the variables q, and e, the following relations must be established where these conditions are always true.

1) $-(\eta + \frac{(p+g_r)\gamma^2}{2\Psi}) < 0$
2) $\frac{\eta(p+g_r)}{2\Psi} > 0$

In exchange for the retailer's effort to raise the demand, the supplier makes an effort to raise the quality, which affects the demand level. The profit function of the supplier is:

$$\pi_s^d = g_s s(q) + wq - cq - \xi\frac{\theta^2}{2} - g_s\mu \tag{12}$$

In order to extract the optimal value of quality effort Θ, the partial derivative of equation (12) is taken and set equal to zero. We will have:

$$\frac{\partial \pi_s^d}{\partial \theta} = 0 \rightarrow g_s\frac{2q\lambda - 2\lambda(\mu - \Psi + \gamma e + \lambda\theta)}{4\Psi} - \xi\theta = 0 \tag{13}$$

The Nash equilibrium is found by solving equations (10), (11), and (13) to extract the optimal values of variables. Hence the following relations are satisfied.

$$e_d^* = \frac{\gamma((p+g_r)-w)}{\eta} \tag{14}$$

$$q_d^* = \frac{(p+g_r)^2\gamma^2\xi + \mu\eta(p+g_r)\xi + \eta(p+g_r)\xi\Psi + g_s\eta(p+g_r)\lambda^2 - 2\eta\xi w\Psi - g_s\eta\lambda^2 w - (p+g_r)\gamma^2\xi w}{(\eta\xi(p+g_r))} \tag{15}$$

$$\theta_d^* = \frac{g_s\lambda((p+g_r) - w)}{(p+g_r)\xi} \tag{16}$$

3-3- Supply chain coordination via buyback contract

The supplier charges w from the retailer for each unit of the product purchased, and it pays the retailer b for each remaining product according to the buyback contract. Therefore, the retailer's profit and the supplier's cost due to return products are calculated based on the following equation so that $(q - s(q))$ and q show the number of return products and the number of units purchased, respectively. Also, we have:

$$p + g_r - b = \delta(p+g) \rightarrow b = p + g_r - \delta(p+g) \tag{17}$$

$$w_b = b + \delta c \tag{18}$$

Considering quality improvement efforts by supplier and sales efforts by the retailer, the profits of the retailer and supplier are as follow:

$$\pi_r^{BB} = b\big(q - s(q)\big) + (p+g_r)s(q) - g_r\mu - \eta\frac{e^2}{2} - wq \tag{19}$$

$$\pi_s^{BB} = (w - c_s)q + g_s s(q) + b(s(q) - q) - g_s\mu \tag{20}$$

Taking the partial derivative and the second-order derivative of equation (19) as the profit of the retailer is taken concerning the variables *q* and *e*, and we will have:

$$\frac{\partial\pi_r^{BB}}{\partial e} = 0 \rightarrow (p+g_r)\left[\frac{\partial s(q)}{\partial e}\right] + b\left[\frac{\big(q - s(q)\big)}{\partial e}\right] - \eta e = 0 \tag{21}$$

$$\frac{\partial\pi_r^{BB}}{\partial q} = 0 \rightarrow (p+g_r)\left[\frac{\partial s(q)}{\partial q}\right] - w + b\left[\frac{\big(q - s(q)\big)}{\partial q}\right] = 0 \tag{22}$$

$$H(\pi_r^{BB}; e, q) = \begin{bmatrix} \frac{\partial^2\pi_r^{co}}{\partial e^2} & \frac{\partial^2\pi_r^{co}}{\partial e\partial q} \\ \frac{\partial^2\pi_r^{co}}{\partial q\partial e} & \frac{\partial^2\pi_r^{co}}{\partial q^2} \end{bmatrix} = \begin{bmatrix} -\eta - \frac{\gamma^2(p+g_r)}{2\Psi} + \frac{\gamma^2 b}{2\Psi} & \frac{\gamma(p+g_r)}{2\Psi} - \frac{b\gamma}{2\Psi} \\ \frac{\gamma(p+g_r)}{2\Psi} - \frac{b\gamma}{2\Psi} & \frac{-(p+g_r)}{2\Psi} + \frac{b}{2\Psi} \end{bmatrix}$$

If the following relations were established, the matrix H would be negative according to the variables *q* and *e*, where these conditions are always true

1) $\frac{b\gamma^2}{2\Psi} < \eta + \frac{(p+g_r)\gamma^2}{2\Psi}$
2) $(p + g_r) > b$

Also, taking the partial derivative of equation (20) as the supplier's profit concerning the variables Θ, the relation (23) is satisfied.

$$\frac{\partial\pi_s^{BB}}{\partial\theta} = 0 \rightarrow g_s\left[\frac{2q\lambda - 2\lambda(\mu - \Psi + \gamma e + \lambda\theta)}{4\Psi}\right] - \xi\theta - b\left(0 - \left(\frac{2q\lambda - 2\lambda(\mu - \Psi + \gamma e + \lambda\theta)}{4\Psi}\right)\right) = 0 \tag{23}$$

It is noteworthy that the equation (20) is concave according to Θ.

The below relations are established by considering and solving equations (21), (22), and 23.

$$e_{BB}^{*} = \frac{\gamma\big((p+g_r) - w\big)}{\eta} \tag{24}$$

$$q_{BB}^{*} = -\frac{\begin{matrix}(p+g_r)^2\gamma^2\xi - b(p+g_r)\gamma^2\xi + b\gamma^2\xi w - (p+g_r)\gamma^2\xi w - b\mu\eta\xi \\ +b\eta\xi y + \mu\eta(p+g_r)\xi + \eta(p+g_r)\xi\Psi - 2\eta\xi w\Psi + g_s\eta(p+g_r)t^2 \\ -b\eta\lambda^2 w - g_s\eta\lambda^2 w + b\eta(p+g_r)\lambda^2 - (p+g_r)\xi\end{matrix}}{(\xi)\big(b\eta - \eta(p+g_r)\big)} \tag{25}$$

$$\Theta_{BB}^{*} = -\frac{b(p+g_r)\lambda + g_s(p+g_r)\lambda - b\lambda w - g_s\lambda w}{(\xi)(b - (p+g_r))} \tag{26}$$

Proposition 2. The profit of the entire supply chain is not equal in two situations based on the conventional buyback contract and under the centralized system.

Proof. The profit of the entire supply chain should be investigated for two situations conventional buyback contract and the centralized system. It is necessary to compare the optimal values obtained for the decision variables in two systems. After the comparison, it is obvious that $e_c{}^{*} > e_{BB}{}^{*}$, $\Theta_c{}^{*} > \Theta_{BB}{}^{*}$ and $q_{BB}^{*} < q_c^{*}$ and consequently $\pi_T^{c}(e_c{}^{*}, q_{co}^{*}, \Theta_c{}^{*}) > \pi_T^{BB}(e_{BB}{}^{*}, q_{BB}^{*}, \Theta_{BB}{}^{*})$. As a result, the profit in the system based on the conventional buyback contract is less than the centralized system.

Thus, we propose a new buyback contract so that the profit will be equal in both systems. In the new buyback contract, both members divide the costs of sales and quality efforts between themselves. Since the conventional buyback contract cannot coordinate the supply chain, the new contract as cost sharing (CS) contract is suggested where the shared marketing cost by retailer and supplier is equal to β_1 and $1\text{-}\beta_1$, respectively. Likewise, the amount paid by the retailer for quality improvement is β_2, and the amount paid by the supplier for quality improvement is $1\text{-}\beta_2$. In this new model, the benefits and costs of the supply chain are shared simultaneously and fairly.

Accordingly, the profit for the retailer and supplier is calculated based on the below equations.

$$\pi_r^{cs} = (p+g_r)s(q) + b\big(q - s(q)\big) - \beta_1\eta\frac{e^2}{2} - \beta_2\xi\frac{\theta^2}{2} - g_r\mu - wq \tag{27}$$

$$\pi_s^{cs} = g_s s(q) + (w-c)q - (1-\beta_1)\eta\frac{e^2}{2} - (1-\beta_2)\xi\frac{\theta^2}{2} - g_s\mu - b(q - s(q)) \tag{28}$$

To extract the optimal value for variables e, Θ, and q, will have:

$$\frac{\partial\pi_r^{cs}}{\partial e} = 0 \rightarrow (p+g_r)\left[\frac{2q\gamma - 2\gamma(\mu - \Psi + \gamma e + \lambda\theta)}{4\Psi}\right] - \beta_1 e\eta + b\left(0 - \left(\frac{2q\gamma - 2\gamma(\mu - \Psi + \gamma e + \lambda\theta)}{4\Psi}\right)\right) = 0 \tag{29}$$

$$\frac{\partial\pi_r^{cs}}{\partial q} = 0 \rightarrow -w + (p+g_r)\left(\frac{2(\mu - q + \Psi + \gamma e + \lambda\theta)}{4\Psi}\right) + b\left(1 - \left(\frac{2(\mu - q + \Psi + \gamma e + \lambda\theta)}{4\Psi}\right)\right) = 0 \tag{30}$$

$$\frac{\partial\pi_s^{cs}}{\partial\theta} = 0 \rightarrow g_s\left[\frac{2q\lambda - 2\lambda(\mu - \Psi + \gamma e + \lambda\theta)}{4\Psi}\right] - \beta_2\xi\theta - b\left(0 - \left(\frac{2q\lambda - 2\lambda(\mu - \Psi + \gamma e + \lambda\theta)}{4\Psi}\right)\right) = 0 \tag{31}$$

Hessian matrix of π_r^{CS} is calculated based on the second-order derivatives of equation (27) concerning the variables q and e.

$$H(\pi_r^{cs}; e, q) = \begin{bmatrix} \frac{\partial^2 \pi_T}{\partial q^2} & \frac{\partial^2 \pi_T}{\partial q \partial e} \\ \frac{\partial^2 \pi_T}{\partial e \partial q} & \frac{\partial^2 \pi_T}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -\beta_1 \eta - \frac{\gamma^2 (p + g_r)}{2\Psi} + \frac{b\gamma^2}{2\Psi} & \frac{\gamma (p + g_r)}{2\Psi} - \frac{b\gamma}{2\Psi} \\ \frac{\gamma (p + g_r)}{2\Psi} - \frac{b\gamma}{2\Psi} & \frac{-(p + g_r)}{2\Psi} + \frac{b}{2\Psi} \end{bmatrix} \tag{32}$$

The above matrix is negative definite for q and e if the two following relations are satisfied. Notably, both of them are always true.

1) $\frac{b\gamma^2}{2\Psi} < \beta_1 \eta + \frac{(p+g_r)\gamma^2}{2\Psi}$
2) $(p + g_r) > b$

Solving equations 29-31, we get the optimal value of e, q, and Θ. Hence, we will have:

$$e_{cs}^* = \frac{\gamma\big((p + g_r) - w\big)}{\eta \beta_1} \tag{33}$$

$$q_{cs}^* = -\frac{\begin{array}{c}(p + g_r)^2 \gamma^2 \xi - b(p + g_r)\gamma^2 \xi + b\gamma^2 \xi w - (p + g_r)\gamma^2 \xi w - \beta_2 (p + g_r)^2 \gamma^2 \xi - b\beta_1 \mu \eta \xi \\ + b\beta_1 \eta \xi y + \beta_1 \mu \eta (p + g_r)\xi + \beta_1 \eta (p + g_r)\xi \Psi - 2\beta_1 \eta \xi w \Psi + b\beta_1 \eta (p + g_r)\lambda^2 \\ + g_s \beta_1 \eta (p + g_r) t^2 - b\beta_1 \eta \lambda^2 w + b\beta_2 (p + g_r)\gamma^2 \xi - g_s \beta_1 \eta \lambda^2 w - b\beta_2 \gamma^2 \xi w \\ + \beta_2 (p + g_r)\gamma^2 \xi w - i1\beta_2 \eta (p + g_r)\xi \Psi + 2\beta_1 \beta_2 \eta \xi w \Psi + b\beta_1 \beta_2 \mu \eta \xi - b\beta_1 \beta_2 \eta \xi \Psi - \beta_1 \beta_2 \mu \eta (p + g_r)\xi\end{array}}{(\xi - \beta_2 \xi)(b\beta_1 \eta - \beta_1 \eta (p + g_r))} \tag{34}$$

$$\Theta_{cs}^* = -\frac{b(p + g_r)\lambda + g_s (p + g_r)\lambda - b\lambda w - g_s \lambda w}{(\xi - \beta_2 \xi)(b - (p + g_r))} \tag{35}$$

Proposition 3. The defined cost sharing contract strongly coordinates the supply chain. The shared profits earned by retailer and supplier $\pi_r^{cs} = \delta \Pi + \mu(\delta g - g_r)$ and $\pi_s^{cs} = (1 - \delta)\Pi - \mu(\delta g_s - (1 - \delta) g_r)$ respectively, where $\beta_2 = \beta_1 = \delta$ and $\beta_1, \beta_2, \delta \in [0,1]$.

Proof. If the total profit in this system is assumed as Π, which is equal to the total profit in the centralized system, the following relation is valid.

$$\Pi = \pi_T^c (e_c^*, q_c^*, \Theta_c^*) \tag{36}$$

The profit for the retailer and supplier is determined as follows.

$$\pi_r^{cs} = \delta \Pi + \mu(\delta g - g_r) \tag{37}$$

$$\pi_s^{cs} = (1 - \delta)\Pi - \mu(\delta g_s - (1 - \delta) g_r) \tag{38}$$

According to the above equations, for the higher value of δ, the profit of retailer and supplier is higher and lower, respectively. In other words, all the profits in the supply chain are for the retailer ($\Pi(e_c^*, q_c^*, \Theta_c^*) = \pi_r^{cs}(e_c^*, q_c^*, \Theta_c^*)$) if we have:

$$\delta = \frac{\Pi(e_c^*, q_c^*, \Theta_c^*) + \mu g_r}{\Pi(e_c^*, q_c^*, \Theta_c^*) + \mu g} \leq 1 \tag{39}$$

And in return, the supplier gets all the profits of the supply chain if the below condition is satisfied.

$$0 \leq \delta = \frac{\mu g_r}{\Pi(e_c^*, q_c^*, \Theta_c^*) + \mu g} \tag{40}$$

Table 2 summarizes the optimal values for the variables e and Θ by contracts.

Table 2. The optimal value for effort levels in the various contract

Optimal values	Centralized	Decentralized	buyback (BB)	Cost Sharing (CS)
e	$e_c^* = \frac{\gamma((p+g)-c)}{\eta}$	$e_{d2}^* = \frac{\gamma((p+g_r)-w)}{\eta}$	$e_{co}^* = \frac{\gamma(p-w)}{\eta}$	$e_{CS}^* = \frac{\gamma(p-w)}{\beta_1\eta}$
Θ	$\theta_c^* = \frac{\lambda((p+g)-c)}{\xi}$	$\theta_{d2}^* = \frac{g_s\lambda((p+g_r)-w)}{(p+g_r)\xi}$	$\theta_{co}^* = -\frac{b\lambda(p-w)}{\xi(b-p)}$	$\theta_{CS}^* = -\frac{b\lambda(p-w)}{\xi(1-\beta_2)(b-p)}$

4-Computational experiments

In this section, the numerical experiments are performed in two stages. At first, we study the efficiency of the contracts and obtain the optimal variables and profits based on three examples presented in Table 3. Also, sensitivity analysis and impact of the variables on models' performance are performed and their profit is investigated in the second stage.

Table 3. The data of numerical examples

Problem	μ	Ψ	η	ξ	Γ	Λ	δ	P	c	w	g_r	g_s	g	b
1	1200	500	60	60	25	25	0.5	40	10	25	5	5	10	20
2	1000	300	55	65	20	30	0.4	44	12	31.2	4	6	10	26.4
3	800	200	60	50	30	20	0.7	36	8	15.4	6	4	10	9.8

The results of the first stage are provided in table 4. The results show some variables have similar values in the cost sharing and centralized contract. Also, the supply chain has the best performance in cost sharing contract, equal to that of a centralized system. The supply chain performance is improved in the Buyback scenario than the decentralized system, but the buyback cannot gain the same total profit as the centralized and cost sharing system. Accordingly, the buyback system cannot coordinate the supply chain completely in such conditions. However, we can look at it as an improving mechanism where each member can promote its profit. Consequently, the total profit will be promoted that it is the main contribution of the paper. The results show that the decentralized system has the lowest profit, therefore both parts in the supply chain had better work out a mechanism like a buyback or a cost sharing contract to obtain a higher profit. The results showed the proposed contract could improve the relationship between members of the supply chain to achieve sustainable economic that which can help the managers in decision making.

Table 4. The optimal value of the variables and profits

Contract	Variable	Problem 1	Problem 2	Problem 3
Decentralized	E	8.33	6.1091	10.5231
	Q	1375.9	1061.3	1148.6
	Θ	0.9259	0.9692	0.7896
	π_r	14991	11039	15314
	π_s	20998	20494	14006
	π_T	35989	31533	29320
Centralized	E	16.67	15.2727	17.5385
	Q	2333.3	2053.7	1733
	Θ	16.67	19.3846	13.8182
	π_r	17937	15375	18982
	π_s	30730	32452	17345
	π_T	48667	47827	36327
Buyback	E	8.33	6.1091	12.2769
	Q	1916.7	1637.8	1381.7
	Θ	8.33	11.6308	4.1455
	π_r	22480	18568	22659
	π_s	22480	24995	10195
	π_T	44496	43563	32836
Cost Sharing	E	16.67	15.2727	17.5385
	Q	2333.3	2053.7	1733
	Θ	16.67	19.3846	13.8182
	π_r	24334	19131	26229
	π_s	24334	28696	10098
	π_T	48667	47827	36327

In the following, the sensitivity analysis of the problem is performed. At first, we study the effect of η on e and q. According to Table 2, the value of η does not have any impact on Θ because Θ is independent of η. Therefore, we make this analysis for e and q. For numerical study, we assume$\mu = 1200,\ \Psi = 500,\ \lambda = 25,\ \gamma = 25,\ \xi = 60,\ p = 40,\ c = 10,\ \delta = 0.5,\ \beta_1 = \beta_2 = 0.5,\ w = 25$, and $b = 20$. Figure 1 displays the impact of η on the e and q based on three scenarios, i.e., decentralized (dec), BB, and CS model. Figure 1 shows that the sales effort and quantity for retailers are the highest in the CS model. However, the value of e in BB and decentralized model is equal. Based on figure 1, it is conclusive that the values of e and q decrease as η increases. In addition, the less value of η, the less value of e, and for the values of η near to 0, the value of e proliferates. Accordingly, the diagram of e in proportion to η is a vertical asymptote.

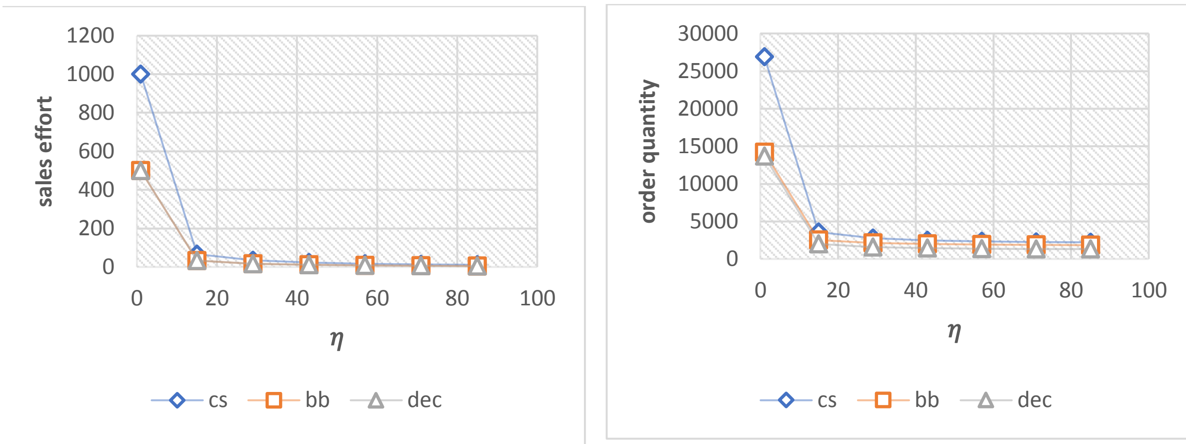


Fig 1. The effect of η on e and q

In order to examine the effect of the ξ on q and quality effort, the same parameters are selected $\mu = 1200, \Psi = 500, \lambda = 25, \gamma = 25,\ p = 40, c = 10, \delta = 0.5,\ \beta_1 = \beta_2 = 0.5, w = 25\ and\ b = 20$, let $\eta = 60$. As mentioned before, the optimal decision for quality effort in the decentralized model is zero because the quality effort is not beneficial for the supplier. However, in BB and CS models, the value of quality effort is decreasing in ξ. Also, figure 2 shows that the supplier's quality effort and q will be the highest in CS. Based on the above figures, it is conclusive that the values of Θ and q decrease as ξ increases. In addition, the less value of ξ, the less value of e, and for the values of ξ near to 0, the value of Θ increases. Accordingly, the diagram of Θ in proportion to ξ is a vertical asymptote.

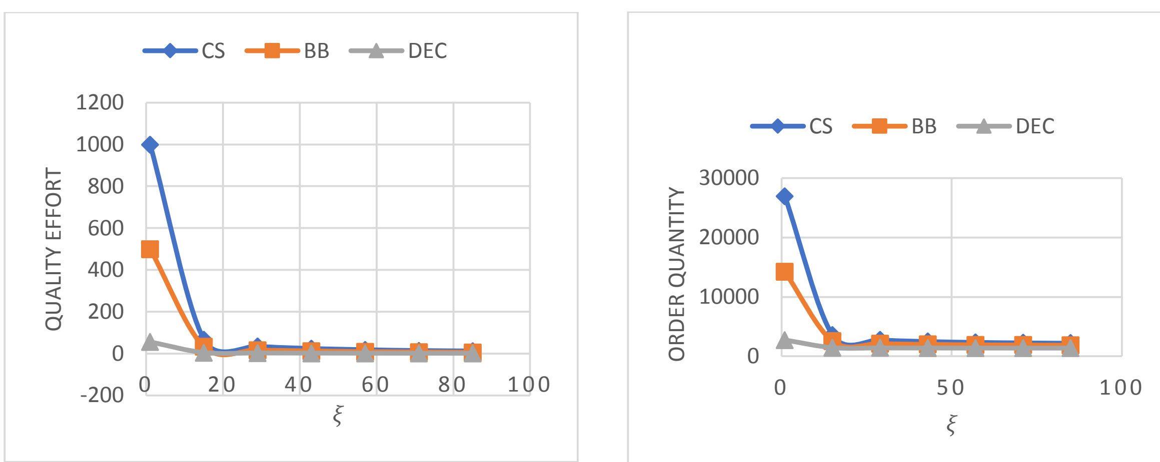
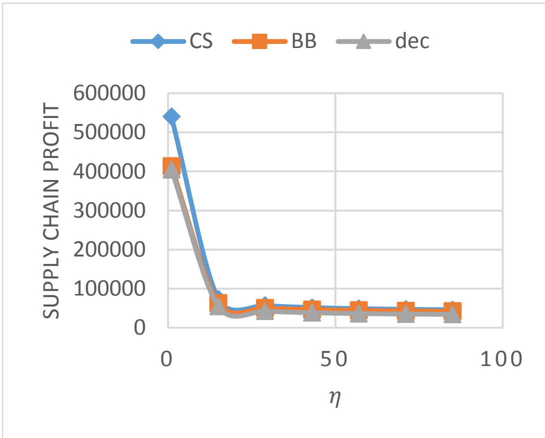


Fig 2. The effect ot ξ on q and Θ

We now examine how the parameters η and ξ affect the profit of the supply chain under different supply chain situations where the same parameters are used. As it is expected, according to figure 3 and figure 4, the supply chain performance degenerates as the value of η and ξ increase. Also, these figures show that the whole supply chain profit will be the highest in cost sharing contract. However, the results show that the BB contract improves the whole supply chain profit than the decentralized system even though it cannot reach the centralized system profit.



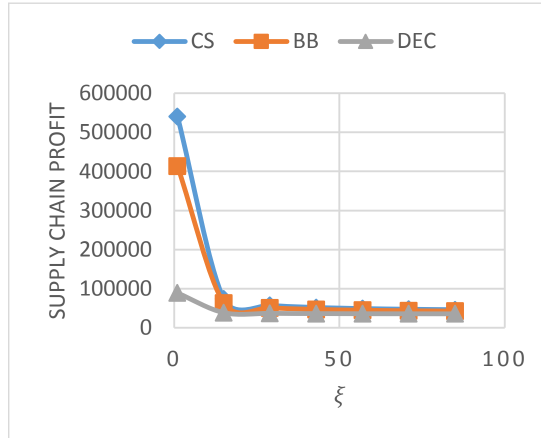


Fig 3. The impact of η on the profit of supply chain **Fig 4.** The impact of ξ on the profit of supply chain

In this section, we examine how η and ξ affect the profit of the supply chain simultaneously. For numerical study, we let $\mu = 1200, \Psi = 500,\ \lambda = 25, \gamma = 25, p = 40, c = 10, \delta = 0.5, \beta_1 = \beta_2 = 0.5, w = 25\ and\ b = 20$, and $\eta \epsilon [5,85], \xi \epsilon [5,85]$. When parameters η and ξ have a large value, supply chain profits are stable and relatively low. The reason is when the value of sales and quality-improvement efforts coefficients are large, the retailer does not make the sales effort. Likewise, the supplier ignores the quality improvement, leading to low demand and low profits.

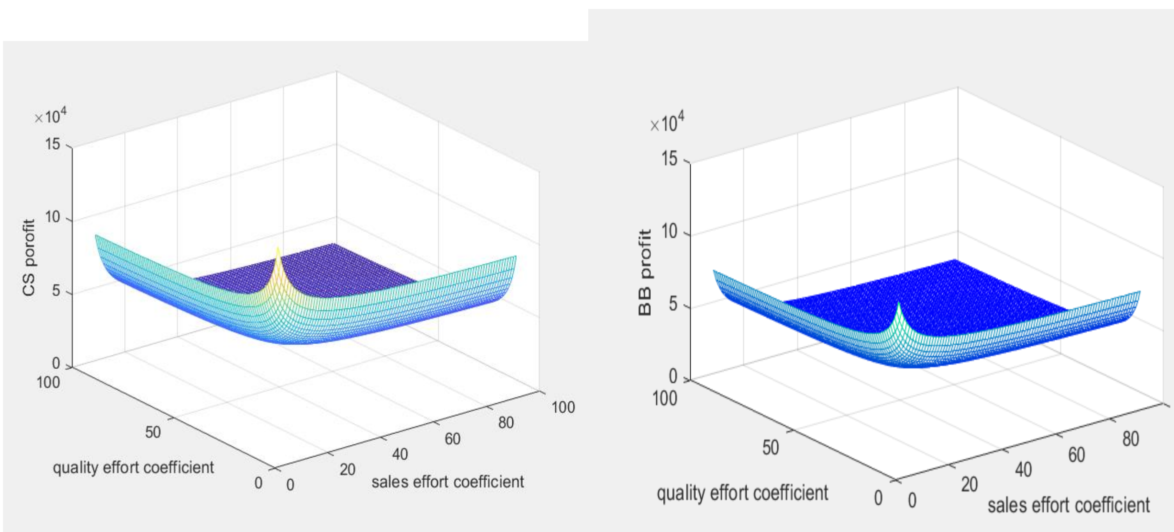


Fig 5. The joint sales and quality efforts cost coefficient's impacts on the profit of the supply chain

5-Conclusions

In this paper, supply chain coordination is considered where one supplier and one retailer act under an effort-dependent demand. It is assumed that the demand is affected by two promoting efforts, i.e., quality improvement executed by the supplier and sales effort, which is put into action by the retailer. Four decision-making structures are analyzed to obtain the supply chain coordination. At first, the centralized contract and the decentralized model are studied. The results indicated that they cannot coordinate the supply chain, therefore two other contracts were extended. The first one is a buyback contract suggested to coordinate the replenishment, quality improvement, and sales efforts. The results showed that a buyback contract promotes each member's profit under such a situation, but it cannot effectively coordinate the supply chain. Consequently, a new contract is proposed in which both parts share the costs of their promoting efforts. The results demonstrated that the supply chain could get coordinated effectively by means of this contract. Also, sensitivity analysis is performed to study the supply chain performance changes when sales effort and quality cost coefficients vary. The results showed that the supply chain performance decreases by incrementing the cost coefficients of sales effort and quality efforts. The limitations of the research are the constant demand and parameters γ and λ; therefore, considering the demand dependent on some other affecting factors such as price, the inventory level of the retailer, and the season is a topic for future research. The other subject to future research is uncertainty in the parameters γ and λ, which measures the influence of marketing and quality improvement efforts. Also, a competitive supply chain under quality and marketing effort can be considered.

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