

Repairable spare part supply chain: A hybrid priority-based particle swarm approach

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Abstract

The industry life highly depends on spare parts since it is vital to perform maintenance operations, especially in strategic industries. The expensive and low-demand spare parts are a must for the continuation of the production; therefore, they are held in warehouses to meet unexpected demand. These spare parts cause high inventory costs also they require human resources, energy, and budget for the repair operations. It is important to point out that separate optimization of decisions in spare part supply chain leads to sub-optimality so, an integrated mathematical model can outperform a routine model. In this paper, we present a network design and planning model that is integrated with the METRIC model (Multi-Echelon Technique for Recoverable Item Control) that formulates inventory management decisions of the repairable spare parts. This model covers different decisions such as supplier order assignment, stock level in warehouses, flows among the facilities, and location of facilities. Due to the np-hardness of the problem, a hybrid approach is presented that incorporates heuristic and meta-heuristic methods. This approach is used to solve the proposed model that has been never applied in previous researches for such a model.

Keywords: Supply chain, spare part, meta-heuristic, PSO, inventory management

1-Introduction

Spare parts are the important resources used in maintenance and repair operations in every industry that may have high or low value either high or low demand. Maintenance operation plays an important role in logistics and constitutes about 30 percent of costs (Hora, 1987). Several resources are used in maintenance operations including material, tools, energy, and human resources. Materials are the significant resources on which the repair operations highly depend so that the lack of these resources can lead to production disruption. A proper network design and planning help not only reduce the probability of spare part shortage but also it can optimize total costs and prevent unexpected failures. Therefore, proper planning of the assets can prevent unexpected shortages. Spare parts are from these

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types of assets that constitute a prominent part of the assets. Spare parts mainly are of two types: 1) low-demand and high-value, 2) high-demand and low-value. We focus on the repairable spare parts that fall into the first type. The repairable spare parts include about 80% of the overall spare parts' values (Driessen, 2018). A repairable spare part supply chain involves two forward and reverse sections. The closed-loop supply chain integrates the forward and reverse flows. These supply chains can be ranked similarly to forward supply chains to measure performance (Carrasco-Gallego et al., 2012; Paydar et al., 2017; Alborzi, 2019). The spare part shortages are the reason for more than 80% of the downtimes (Kosanoglu et al., 2018), so spare parts should be held to meet demand. It is also vital to consider the stock level and the associated costs (González-Varona et al., 2020). There several issues to be addressed such as shortages, lead-time, and low-quality spare parts (Frandsen et al., 2020). The literature review of the spare part supply chains (SPSC) shows that previous researches focused on the routine SPSCs that do not consider decisions related to order allocation, and inventory management simultaneously. Also, two types of suppliers such as manufacturer and vendors are considered involving the local and foreign supplies. In this paper, we present an integrated mathematical model that considers spare part supply chain decisions into account. The decisions are of two types: strategical and tactical. Strategical decisions are around the issues such as the location of repair and inspection centers and the general structure of the network while tactical decisions involve the inventory management such as stock level, and order level, quantity, and order assignment. Due to complexity of the model, a hybrid approach is presented that incorporates a heuristic and meta-heuristic approach. A priority-based heuristic algorithm is used to encode and decode the solutions. Finally, Particle Swarm Algorithm (PSO) is implemented to optimize the overall procedure. The rest of the paper includes the following sections: First, recent researches are reviewed in section 2. Then, the problem definition is described in section 3. The model formulation is presented in section 4. The solution approach is expressed in section 5. Finally, computational results are discussed in section 6. Section 7 outlines the conclusion and future research opportunities.

2-Literature review

We present the literature review regarding the problem in this paper. The essence of spare parts requires a minimum stock level in warehouses, but it can impose the inventory holding costs and significant room to hold the inventory. So, inventory management can help optimize costs significantly (Wilson, 2020). Integrating all the decisions regarding the spare part supply chain not only help us achieve this goal, but also optimize network design and planning decisions that are under the focus of researchers such as Finkbeiner, M. (2011); Babaveisi et al., 2018; Fathollahi-Fard, A. M., 2018, and Tosarkani & Amin (2019) and Hora (1987).

Maintenance and repair operations (MRO) require several resources, such as human resources, material, budget, and time. Spare parts are the essential materials used in MRO, which absorb major capital. As a tangible example, spare part inventory management costs include salaries, orders, and fixed costs (buildings and utilities), which add up to about %20 per year to purchase cost as inventory management costs can be generalized to other spare parts. Jayaraman et al. (1999) presented a programming model for reverse logistics that involves remanufacturing, distribution centers, and collection centers. The proposed model aims to minimize total costs.

Aras et al., (2008) investigated a mathematical model for the facility location problem in a reverse supply chain. The objective function of the model aims to maximize total profit. (Jain & Raghavan, 2009) published a paper regarding the queuing model for inventory planning in a multi-tier supply chain. In this network, manufacturers, warehouses, and vendors are considered. $M/M/\infty$ queuing model is used for a logistics hub and $M/M/1$ for manufacturing centers that aim to minimize total costs.

Sasikumar et al. (2010) considered a tire recycling supply chain in which used products are collected from the customers and first move to the local collection centers and then to the central collection centers. The authors developed a nonlinear model that aims to maximize total profit. (Fonseca et al., 2010) considered an uncertain bi-objective mathematical model for a reverse supply chain involving collection and recycling centers. The proposed model formulates a multi-product and multi-tier network that covers strategic and tactical decisions.

Karamouzian et al., (2011) presented a model for controlling the acceptance of the returned items based on the queuing model. The rate of returned items follows the Poisson process which can be

remanufactured or disposed. The revenue of the remanufacturing is much higher than disposal, but its capacity is limited which causes queue. Due to the difference in costs and revenues of each one, the mixed nonlinear model is formulated to maximize total profit. Zhao et al., (2012) examined order and delivery planning in a distribution center to optimize the operational costs by using the ABC analysis method.

Vahdani et al., (2012) presented a model for minimizing total cost in a supply chain including, remanufacturing, recycling, distribution, collection, and processing centers. M/M/c queuing model is considered in processing centers and the queue length is computed by a chance constraint. The robust optimization with box uncertainty is used to solve the model. (He & Hu, 2014) investigated the emergency humanitarian supply chain and presented a mathematical model. In this supply chain, depot, distribution, and rescue centers are considered. A queuing model is formulated for minimizing response time. M/M/1 queuing model is considered for each node. To solve the model, a genetic algorithm is used.

Sarrafha et al. (2015) presented a bi-objective mathematical model for an integrated production-distribution supply chain. The multi-period and multi-product model with the objective function of minimizing total cost and response time is solved using the Particle Swarm Algorithm. (Hatefi et al., 2015) presented an uncertain model for the forward and reverse logistics network considering facility disruption. The products are collected from the customer and moved to the inspection centers then they are divided into two types of recyclable and non-recyclable. The model optimizes the total costs.

Ahmadi Kurd et al. (2017) investigated an optimization model for a wastewater collection network including treatment plants, storage ponds. This model analyzes the establishment of the canals, and optimal flows to agricultural fields to optimize total costs. (Kim et al., 2018) discuss the customers' demand uncertainty and reverse logistics issues and how it affects the production planning. The authors developed a robust model to maximize total profit in comparison to the routine model.

Sadeghi et al. (2020) presented a model for the automotive spare parts supply chain. The multi-period, multi-product model minimizes the costs. The decision regarding this model includes the routing and facility location decisions. (Fathollahi-Fard et al., 2020) considered a water supply and wastewater collection model under uncertainty. They developed a stochastic model that includes three objective functions that respectively aim to minimize total costs, environmental impacts, and maximize social benefits. (Topan & van der Heijden, 2020) proposed a model for planning the two-echelon, multi-item distribution network. In this network, different decisions including lateral transshipment and emergency shipment are discussed to minimize down-time and total costs.

Rabbani et al., (2020) presented a multi-period and multi-objective model for a sustainable location-allocation problem. In this study, the effect of the level of technologies for the fleet on the cost and emissions of carbon dioxide has been investigated. Also, the shortage of products and its effect on different groups of customers have been examined. In the network, manufacturing centers, warehouses, collection, and recycling centers have been considered and the location of facilities is specified. Objective functions include minimization of costs, environmental impacts, and customer dissatisfaction. The case study is solved with the epsilon-constraint method.

Qin et al., (2021) studied repairable spare part provisioning to maximize the service profit of aging equipment across multiregional markets. A two-echelon repairable spare parts service network is considered including warehouses and the repair centers. The model is solved by a greedy algorithm based on marginal analysis. The model validation is shown using a real case problem. The analyses illustrate the effectiveness of the greedy algorithm.

Wang & Lin, (2021) proposed an optimization method for spare part supply chain model based on a novel scale-free network. In this study, Q-learning is used that enables dynamic decision-making. The results show that the replenishment time is reduced significantly by using Q-learning.

Karim & Nakade, (2021) developed an inventory-location model for the resilient spare part supply chain network to assess the relation of facility disruption and emission. First, they used queuing theory for formulating inventory model, then a location-inventory model is developed for facility disruption risk with CO₂ emission restriction. The model is solved using MATLAB.

According to the literature review provided in this section, shown in table 1, it is specified that the integrated network design and planning is not accompanied by inventory management decisions that is one of the research gaps in this paper. Additionally, we present a hybrid approach that utilizes the

priority-based method to solve the model using a particle swarm algorithm. All the research gaps are presented in the following:

- Multiple suppliers (vendors and manufacturers) are discussed in our repairable spare part supply chain that is not considered in other researches;
- Integrated decisions regarding strategic and tactical planning such as stock level, flow, and location of facilities, reorder level, and order assignment;
- Stochastic demand is handled by METRIC that outperforms the deterministic models;
- METRIC model utilizes queuing theory enabling decision-makers to assess the performance of the warehouses simultaneously with network design and planning decisions;
- The priority-based procedure is integrated with the particle swarm algorithm is used for the first time in such a model.
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Table 1. Present work vs. reviewed researches

Author(s)	Year	Decision level	Objective function(s)	Uncertainty	Solution method
Hora	1987	TA	MC	×	-
Jayaraman et al.	1999	ST	MC	×	Exact
Aras et al.	2008	ST	MP	×	Heuristic
Jain & Raghavan	2009	TA	MC	×	Heuristic
Sasikumar et al.	2010	ST	MP	×	Exact
Fonseca et al.	2010	ST - TA	MC-MS	✓	Exact
Karamouzian et al.	2011	ST - TA	MP	×	Meta-heuristic
Zhao et al.	2012	TA	MC	×	-
Vahdani et al.	2012	ST	MC	✓	Exact
He & Hu	2014	ST	MR	✓	Meta-heuristic
Sarrafha et al.	2015	ST	MC- MR	×	Meta-heuristic
Hatefi et al.	2015	ST	MC	✓	Exact
Ahmadi Kurd et al.	2017	ST	MC	✓	Exact
Kim et al.	2018	ST - TA	MP	✓	Exact
Sadeghi et al.	2020	ST	MC	×	Exact
Fathollahi-Fard et al	2020	ST	MC-MS	×	Exact
Topan & van der Heijden	2020	ST	MC-MR	×	Exact
Rabbani et al.	2020	ST-TA	MC-MS-MSA	×	Exact
Qin et al.	2021	TA	MC	×	Heuristic
Wang & Lin	2021	TA	MC	×	Meta-heuristic
Karim & Nakade	2021	ST-TA	MC	×	Heuristic
Present paper	2021	ST-TA	MC	✓	Hybrid

MC: Minimizing costs
 MP: Maximizing profit
 MS: Maximizing sustainability
 MR: Minimizing response time
 MSA: Maximizing customer satisfaction
 ST: Strategical
 TA: Tactical

3-Problem definition

3-1-Repairable SPSC

The network flow in the supply chain is illustrated in figure 1. Repair operation is an important part of the logistics that help the industries take advantage of reusing the machines and equipment instead of repairing them to reduce costs. Here, we discuss the corrective maintenance that restores the repaired equipment to the normal operation. The flow in the network begins with a report from the installation bases that leads to the replacement of the defective equipment with an operational one. The defective equipment proceeds to inspection centers to be examined by the repair specialist. The repairability is specified by analyzing the repair cost, repair time, repair capacity and capability, and other technical issues which lead to repair or disposal decision. The unrepairable equipment heads to the disassembly center in where usable spare parts move to repair centers and the rest are disposed. Repairable equipment is repaired in repair centers that require material, tools, and resources (including time, cost, and energy); we discuss time and material in this study. Spare parts are of materials that are supplied by the central warehouses and disassembly centers, used to repair the equipment and disassembly centers. In this paper, we call each equipment as LRU² and spare part as SRU³ i.e. each SRU is a component of LRU.

New branded equipment and spare parts are supplied by suppliers including vendors and manufacturers. Vendors deal with the purchase and distribution of spare parts while manufacturers use raw materials to produce spare parts and equipment. Both manufacturers and vendors are of two types of local and foreign origins. Price, quality, and response time are the factors that affect the order assignment to the suppliers. Equipment and spare parts are held in central and local warehouses. Central warehouses supply the local warehouses where they supply the installation bases (end users). The orders for the repairable spare parts are released based on demand and base stock replenishment policy (s-1, s) due to the low demand and high value of the spare parts. The proposed model takes the following decisions into account:

- Order assignment to suppliers based on price, response time, and quality;
- Reorder level of the central warehouses;
- Flows among the facilities such as inspection and repair centers, local and central warehouses local warehouses and end-users, and warehouses and repair centers;
- Repair assignment based on repair capacity, response time, and repair capability;
- Stock level in local and central warehouses.

4-Mathematical description

In this section, we present the model details including assumptions, indices, parameters, decisions variables, objective function, and constraints.

4-1-Assumptions

- All the spare parts follow base stock (s-1, s) replenishment policy and Poisson process for demand;
- Central warehouses do not face shortages;
- Lead-time for local warehouses is stochastic, but the travel time between local and central warehouses are constant;
- SRUs are components of LRU and each SRU lays only in one LRU;

² Line-Replaceable Unit

³ Shop-Replaceable Unit

4-2-Indices and sets

$s \in S$	Spare part
$w \in W$	Warehouse
• $w_1 \in W_1 \subseteq W$	• Central warehouse
• $w_2 \in W_2 \subseteq W$	• Local warehouse
$r \in R$	Repair center
$i \in I$	Inspection center
$l = \{w, r, i\}$	All the Facilities
$c \in C$	End-User (Installation base)
$d \in D$	Disassembly center
$m \in M$	Raw Material Manufacturer
$s' \in S'$	Supplier
• $s'_1 \in S'_1$ where $S'_1 \subseteq S'$	• Local vendors
• $s'_2 \in S'_2$ where $S'_2 \subseteq S'$	• Foreign vendors
• $s'_3 \in S'_3$ where $S'_3 \subseteq S'$	• Local manufacturer
• $s'_4 \in S'_4$ where $S'_4 \subseteq S'$	• Foreign manufacturer

4-3-Parameters

$f_{l'}$	Establishment cost of facility $l' = \{r, i\}$
$t_{sl'}$	Transportation cost of spare parts s between facility l and l'
$o_{ss'w_1}$	Ordering cost of spare part s from central warehouse w_1 to supplier s'
d_{sc}	Spare part s demand from end-user c
rw_{sr}	Work (man-hour) for repairing spare part s in repair center r
cap_r	Capacity of repair center r
rs_{rs}	1, if repair center r has the capability of repairing spare part s , 0 otherwise
$sc_{ss'}$	Spare part s supply capacity of supplier s'
v_{sd}	Probability of SRU s usability in disassembly center d
ψ_{si}	Probability of LRU s repairability in inspection center i
$rp_{s_1s_2}$	Probability that spare part $s_1 \in s$ is used for repairing spare part $s_2 \in s$
$pc_{ss'}$	Purchase cost of spare part s for the supplier s'
$g_{s_1s_2}$	1, if SRU $s_1 \in s$ is component of LRU $s_2 \in s$, 0 otherwise
sv_s	Salvage value of each ton of defected spare part s
wt_s	Spare part s weight
h_{sw}	spare part s holding cost in warehouse w
$df_{ss'}$	spare part s defect rate from supplier s'
def_s	maximum acceptable defect rate of spare part s
md_s	minimum acceptable on-time delivery rate for spare part s
$del_{ss'}$	on-time delivery rate for spare part s from supplier s'
I_{sw}^0	Initial inventory of spare part s in warehouse w
$\tau_{sw_1w_2}$	Travel time of spare part s from central warehouse w_1 to local warehouse w_2
bc_{sw}	Spare part s backorder cost in warehouse w
$\mu_{ss'w_1}$	Leadtime of spare part s from supplier s' to central warehouse w_1
τ_{sw_1}	
$= \sum_{s'} \mu_{ss'w_1}$	Leadtime of spare part s supplied by central warehouse w_1
rc_{sr}	Spare part s repair cost in repair center r
dc_{sd}	Spare part s disassembly cost in disassembly center d

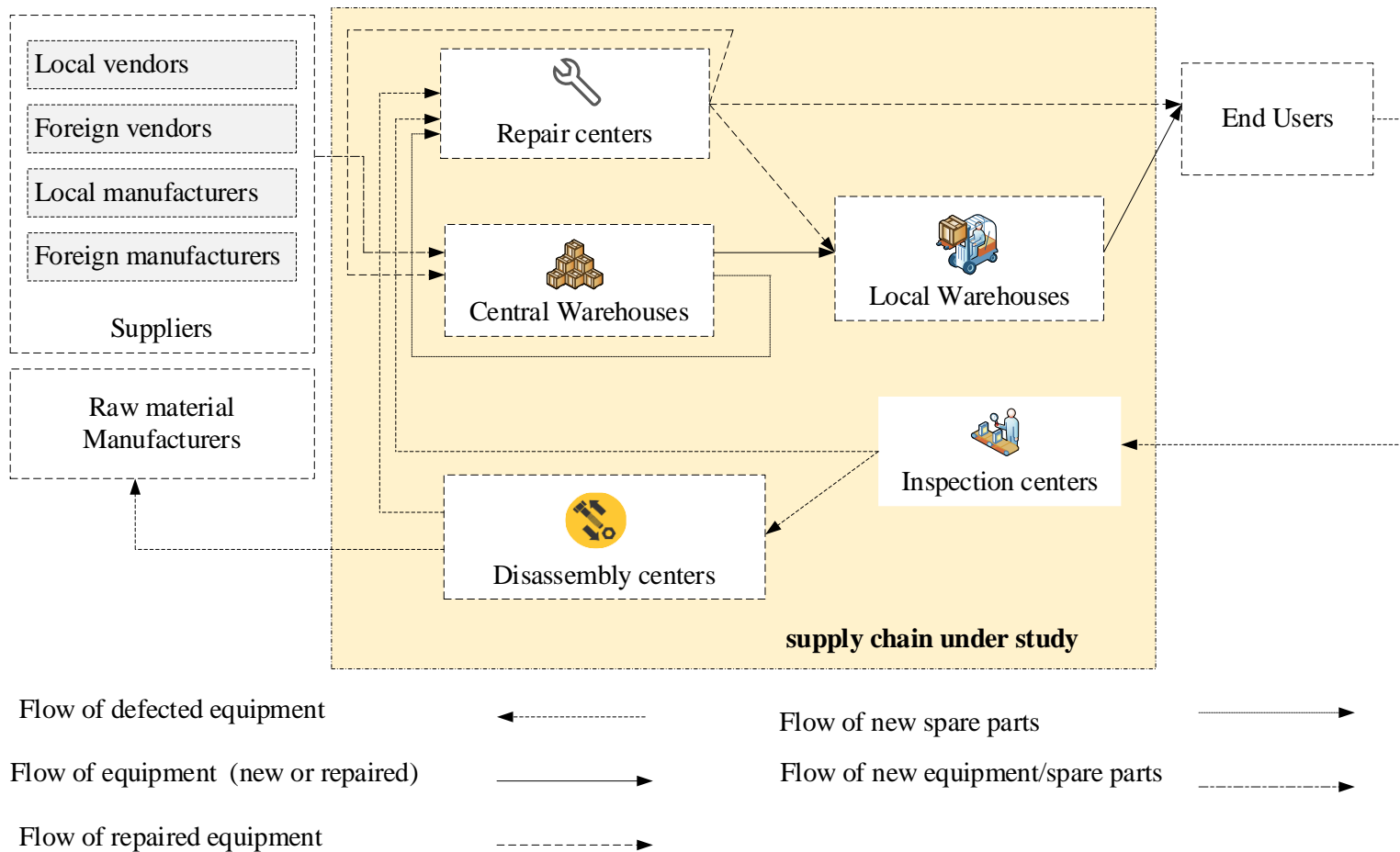


Fig 1. Network flow in the repairable spare part supply chain

4-4-Decision variables

$x_{ss'w_1}$	Amount of spare part s from supplier s' to central warehouse w_1
x_{srw_1}	Amount of spare part s from repair center r to central warehouse w_1
$y_{sw_1w_2}$	Amount of spare part s from the central warehouse w_1 to local warehouse w_2
y_{srw_2}	Amount of spare part s from repair center r to local warehouse w_2
z_{sw_2c}	Amount of spare part s from local warehouse w_2 to end-user c
z_{src}	Amount of spare part s from repair center r to end-user c
x_{sci}	Amount of spare part s from end-user c to inspection center i
y_{sir}	Amount of spare part s from inspection center i to repair center r
z_{sid}	Amount of spare part s from inspection center i to disassembly center d
w_{sdr}	Amount of spare part s from disassembly center d to repair center r
w_{sdm}	Amount of spare part s from disassembly center d to raw material manufacturer m
q_{sw_1r}	Amount of spare part s from central warehouse w_1 to repair center r
$k_{l'}$	1, if facility l' is open, 0 otherwise
I_{sw}^p	The average on-hand inventory of spare part s in warehouse w
I_{sw}^n	The average shortage of spare part s in warehouse w
S_{sw}	Stock position of spare part s in warehouse w
w_{sw_1}	Average waiting time for replenishment spare part s in central warehouse w_1
\bar{t}_{sw_2}	Average replenishment time of spare part s in local warehouse w_2
R_{sw}	Reorder level of spare part s at warehouse w
$Q_{ss'w_1}$	Order quantity of spare part s from the supplier s' for central warehouse w_1
λ_{sw}	Demand of spare part s in warehouse w

4-5-Objective function and constraints

$$Max z = \left[\sum_s \sum_{s'} \sum_{w_1} t_{ss'w_1} x_{ss'w_1} \right. \quad (1)$$

$$+ \sum_s \sum_r \sum_{w_1} t_{srw_1} x'_{srw_1} + \sum_s \sum_r \sum_{w_2} t_{srw_2} y'_{srw_2} + \sum_s \sum_r \sum_c t_{src} z'_{src} \quad (2)$$

$$+ \sum_s \sum_{w_1} \sum_{w_2} t_{sw_1w_2} y_{sw_1w_2} \quad (3)$$

$$+ \sum_s \sum_{w_2} \sum_c t_{sw_2c} \times z_{sw_2c} \quad (4)$$

$$+ \sum_s \sum_c \sum_i t_{sci} \times x''_{sci} \quad (5)$$

$$+ \sum_s \sum_i \sum_r t_{sir} \times y''_{sir} + \sum_s \sum_i \sum_d t_{sid} \times z''_{sid} \quad (6)$$

$$+ \sum_s \sum_d \sum_r t_{sdr} w'_{sdr} + \sum_s \sum_d \sum_m t_{sdm} \times w''_{sdm} \quad (7)$$

$$+ \sum_s \sum_r \sum_{w_1} t_{srw_1} \times q'_{sw_1r} \quad (8)$$

$$+ \sum_s \sum_{w_1} \sum_{s'} o_{ss'w_1} \times x_{ss'w_1} \quad (9)$$

$$+ \sum_s \sum_{s'} \sum_{w_1} pc_{ss'} \times x_{ss'w_1} \quad (10)$$

$$+ \sum_s \sum_w h_{sw} I_{sw}^p \quad (11)$$

$$+ \sum_s \sum_{w_2} b_{c_{sw_2}} I_{sw_2}^n \quad (12)$$

$$+ \sum_s \sum_{w_2} \sum_{i} r_{c_{sr}} y_{sir}'' \quad (13)$$

$$+ \sum_s \sum_i \sum_d d_{c_{sd}} \times z_{sid}'' \quad (14)$$

$$+ \sum_{l \in \{r,i\}} f_l k_l \quad (15)$$

$$- \sum_s \sum_d \sum_m s v_s \times w_{sdm}'' \quad (16)$$

Equations (1) to (8) calculate transportation costs among facilities. Equations (9) and (10) are respectively the ordering and purchase costs. Equations (11) and (12) present the expected holding cost and shortage costs in warehouses. Equation (13) and (14) express the repair and disassembly costs. Equation (16) is the facility establishment costs for the potential repair and inspection centers. Salvage cost is shown in the last equation.

Average shortage and on-hand inventory in warehouses for LRUs and SRUs are presented in equation (18), (19), (20), and (21). According to Little law, the expected waiting time is calculated by equation (20).

$$\lambda_{sw_1} = \sum_{w_2}^{s_{sw_1}} y_{sw_1 w_2} \quad \forall s, w_1 \quad (17)$$

$$I_{sw_1}^p = \sum_{j_s=1}^{s_{sw_1}} j_s \times \frac{e^{-\lambda_{sw_1} \tau_{sw_1}} (\lambda_{sw_1} \tau_{sw_1})^{s_{sw_1} - j_s}}{(s_{sw_1} - j_s)!} \quad \forall s(LRU), w_1 \quad (18)$$

$$I_{sw_1} = I_{sw_1}^p - I_{sw_1}^n, I_{sw_1} = s_{sw_1} - \lambda_{sw_1} \tau_{sw_1} \quad (19)$$

$$I_{sw_1}^n = I_{sw_1}^p - (s_{sw_1} - \lambda_{sw_1} \tau_{sw_1}) \quad (20)$$

$$w_{sw_1 h} = \frac{I_{sw_1 h}^n}{\lambda_{sw_1 h}}, \lambda_{sw_1 h} \neq 0 \quad \forall s, w_1, h \quad (20)$$

$$\bar{\tau}_{sw_2 h} = \sum_{w_1, y_{sw_1 w_2}^{(1)} > 0} (\tau_{sw_1 w_2} + w_{sw_1 h}) \quad \forall s, w_2, h \quad (21)$$

$$I_{sw_2 h}^p = \sum_{j_s=1}^{s_{sw_2 h}} j_s \times \frac{e^{-\lambda_{sw_2 h} \bar{\tau}_{sw_2 h}} (\lambda_{sw_2 h} \bar{\tau}_{sw_2 h})^{s_{sw_2 h} - j_s}}{(s_{sw_2 h} - j_s)!} \quad \forall s, w_2, h \quad (22)$$

$$I_{sw_2 h} = I_{sw_2 h}^p - I_{sw_2 h}^n, I_{sw_2 h} = s_{sw_2 h} - \lambda_{sw_2 h} \bar{\tau}_{sw_2 h} \quad \forall s, w_2, h \quad (23)$$

The constraints are presented in the following that includes the network design, planning, and inventory management equations.

$$\sum_r y_{src}' + \sum_{w_2} z_{sw_2 c} = d_{sc} \quad \forall s, c \quad (24)$$

$$I_{sw_1}^p = \sum_{j_s=1}^{s_{sw_1}} j_s \times \frac{e^{-\lambda_{sw_1} \tau_{sw_1}} (\lambda_{sw_1} \tau_{sw_1})^{s_{sw_1} - j_s}}{(s_{sw_1} - j_s)!} \quad (25)$$

$$I_{sw_1}^n = I_{sw_1}^p - (s_{sw_1} - \lambda_{sw_1} \tau_{sw_1}) \quad \forall s, w_1 \quad (26)$$

$$w_{sw_1} = \frac{I_{sw_1}^n}{\lambda_{sw_1}}, \lambda_{sw_1} \neq 0 \quad (27)$$

$$\bar{\tau}_{sw_2} = \sum_{w_1, y_{sw_1 w_2}^{(1)} > 0} (\tau_{sw_1 w_2} + w_{sw_1}) \quad \forall s, w_2 \quad (28)$$

$$I_{S_{w_2}}^p = \sum_{j_s=1}^{S_{w_2}} j_s \times \frac{e^{-\lambda_{sw_2} \bar{t}_{sw_2}} (\lambda_{sw_2} \bar{t}_{sw_2})^{S_{w_2}-j_s}}{(S_{w_2} - j_s)!} \quad \forall s, w_2 \quad (29)$$

$$I_{S_{w_2}}^n = I_{S_{w_2}}^p - (S_{w_2} - \lambda_{sw_2} \bar{t}_{sw_2}) \quad (30)$$

$$I_{S_{w_1}}^0 + \sum_{s'} x_{ss'w_1} + \sum_r x'_{srw_1} = S_{S_{w_1}} + \sum_r q'_{sw_1r} + \sum_{w_2} y_{sw_1w_2} \quad \forall s, w_1 \quad (31)$$

$$I_{S_{w_2}}^0 + \sum_{w_1} y_{sw_1w_2} + \sum_r y'_{srw_2} = S_{S_{w_2}} + \sum_c z_{sw_2c} \quad \forall s, w_2 \quad (32)$$

$$\sum_d z''_{sid} = \sum_c (1 - \psi_{si}) \times x''_{sci} \quad \forall s, i \quad (33)$$

$$\sum_d y''_{sir} = \sum_c \psi_{si} \times x''_{sci} \quad \forall s, i \quad (34)$$

$$\sum_i y''_{sir} = \sum_{w_1} x'_{srw_1} + \sum_{w_2} y'_{srw_2} + \sum_c z'_{src} \quad \forall s, r \quad (35)$$

$$\sum_{w_1} w_{s_1w_1r} + \sum_d w'_{s_1dr} = rp_{s_1s_2} \sum_i y''_{s_2ir} \quad \forall s_1, s_2, r \quad (36)$$

$$\sum_r w'_{s_2dr} = \sum_i v_{s_1d} g_{s_1s_2} z''_{s_2id} \quad \forall s_1, s_2, d \quad (37)$$

$$\sum_m w''_{s_2dm} = \sum_i (1 - v_{s_1d}) g_{s_1s_2} z''_{s_2id} \quad \forall s_1, s_2, d \quad (38)$$

$$\sum_{w_1} x_{ss'w_1} \leq sc_{ss'} \quad \forall s, s' \quad (39)$$

$$\sum_{w_1} y''_{sir} \leq M \times rs_{rs} \quad \forall s, r \quad (40)$$

$$\sum_s \sum_i r w_{sr} \times y''_{sir} \leq cap_r \quad \forall r \quad (41)$$

$$\sum_i x''_{sci} = d_{sc} \quad \forall s, c \quad (42)$$

$$\frac{\sum_{s'} \sum_{w_1} df_{ss'} x_{ss'w_1}}{\sum_{w_1} \sum_{s'} x_{ss'w_1}} \leq def_s \quad \forall s \quad (43)$$

$$\frac{\sum_{s'} \sum_{w_1} md_{ss'} x_{ss'w_1}}{\sum_{w_1} \sum_{s'} x_{ss'w_1}} \geq md_s \quad \forall s \quad (44)$$

$$\begin{aligned} k_l &\in \{0,1\} && \forall l \\ x''_{sci} &\geq 0, int && \forall s, c, i \\ y''_{sir} &\geq 0, int && \forall s, i, r \\ z''_{sid} &\geq 0, int && \forall s, i, d \forall s, i, d \\ w''_{sdr} &\geq 0, int && \forall s, d, r \\ w''_{sdm} &\geq 0, int && \forall s, d, m \\ x_{ss'w_1} &\geq 0, int && \forall s, s', w_1 \\ x'_{srw_1} &\geq 0, int && \forall s, r, w_1 \\ y_{sw_1w_2} &\geq 0, int && \forall s, w_1, w_2 \\ y'_{srw_2} &\geq 0, int && \forall s, r, w_2 \\ z_{sw_2c} &\geq 0, int && \forall s, w_2, c \\ z'_{src} &\geq 0, int && \forall s, r, c \\ S_{sw} &\geq 0, int && \forall s, w \\ I_{sw}^p, I_{sw}^n &\geq 0 && \forall s, w \end{aligned} \quad (45)$$

Equation (24) shows the balance of input to installation bases and demand. Equations (25) through (30) show the inventory management model formulation associating the METRIC model. Equations (31) and (32) calculates the balance of flows in warehouses. Equations (33) and (34) present the flow to inspection and disassembly centers considering the probability of repairability. Equation (35). is the balance of flow in the repair center. Each equipment requires a certain amount of material used in the

repair operation shown in equation (36). The output of the disassembly centers is calculated using Equations (37) and (38). Equation (39) is the capacity constraint of the suppliers that shows the maximum available supply of the spare parts. Equation (40) ensures that assignments to repair centers are performed considering the capability of the repair centers for the repair operations. Equation (41) is the maximum repair capacity of the repair centers and equation (42) expresses the balance between the return flow to inspection centers and the demand of installation bases. Equation (43) and (44) ensures the maximum defect rate and the minimum on-time delivery rate that is acceptable for each spare part from each supplier. The domain of variables is shown in equation (45).

5-Solution method

In this paper, we follow the priority-based encoding and decoding procedures. The network is composed of nodes that are connected with the edges. In the priority-based representation, the nodes have a specific priority value and the nodes with higher priorities are selected first. Indeed, each particle is a form of permutation that encodes the connection among the nodes. PSO iterates over the priority-based procedure to optimize the final solution.

5-1-Particle swarm optimization (PSO)

Particle swarm optimization (PSO) works with some so-called particles originally taken from the movements of the fish and bird (Eberhart & Kennedy, 1995). PSO algorithm is a population-based metaheuristic adopted from the swarm intelligence which shows the social behavior of organisms. The logic of the algorithm is based on the search of an area to find the food by the animals that are equal to search for the solution in solution space (Talbi, 2009). Each particle possesses two characteristics of position and velocity. The best finding for each particle that is ever found is maintained in the local best (Pbest). Additionally, the best solution among all particles is called the global best (Gbest) (Prasanna Venkatesan & Kumanan, 2012). A solution is generated randomly that updates the local and the global best repositories in each iteration. Also, the velocity factor is calculated for each particle that determines the new position. Figure 2 shows the procedure of the algorithm and velocity update mechanism.

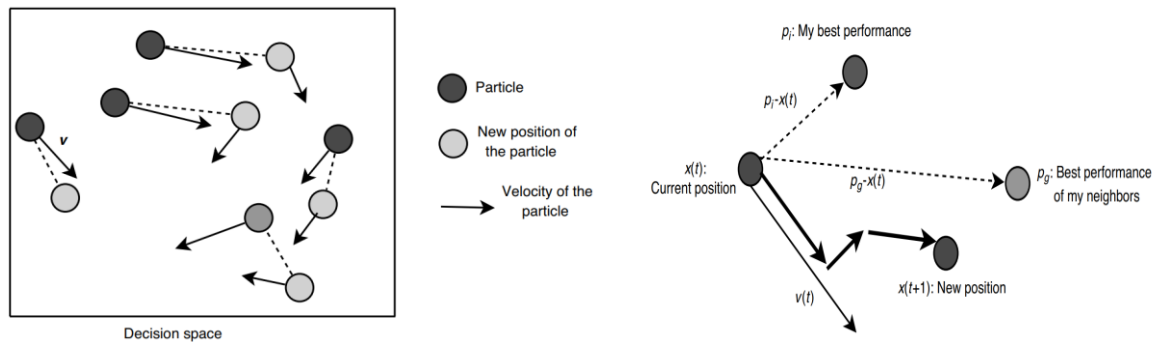


Fig 2. Particle swarm procedure and velocity update

Equation (46) calculates the new position and velocity respectively. Parameters c_1 and c_2 are the local and global learning coefficients that give weight to local and global best repositories. In this paper, a novel mechanism is used for updating the solutions that implement the vector of similarity. The solutions are considered as vectors in this method that computes the vector resemblance-degree among the current position (x_{ij}^t) and Pbest and Gbest repositories. The resemblance-degree between the current particle, Pbest, and Gbest are denoted by $r(p_i, x_i)$ and $r(g, x_i)$, respectively.

$$\begin{aligned} v_{ij}^t &= v_{ij}^{t-1} + c_1 r_1 (p_{ij} - x_{ij}^{t-1}) + c_2 r_2 (G_j^{t-1} - x_{ij}^{t-1}), \\ x_{ij}^t &= x_{ij}^{t-1} + v_{ij}^t, \end{aligned} \quad (46)$$

The magnitude of vector is calculated by the following formula in equation (47). The resemblance-degree vector for Pbest and Gbest are shown in equation (48).

$$\|x_i\| = \sqrt{\sum_{j=1}^m x_{ij}^2}, i=1,\dots,m \quad (47)$$

$$m(g, x_i) = \frac{\sum_{j=1}^m x_{ij} \times g_j}{\sqrt{\sum_{j=1}^m g_j^2 \times \sum_{j=1}^m x_{ij}^2}}, i=1,\dots,m \quad (48)$$

$$m(p_i, x_i) = \frac{\sum_{j=1}^m x_{ij} \times p_{ij}}{\sqrt{\sum_{j=1}^m g_j^2 \times \sum_{j=1}^m x_{ij}^2}}, i=1,\dots,m$$

5-2-Solution representation

In this section, we present the encoding procedure that shows the representation of the solution. Suppose we deal with the network shown in figure 3 which is composed of two types of resources and depots that become A+B nodes. In this case, the representation will be a permutation of A+B according to priority-based encoding presented on the left side. Indeed, each element in permutation shows the priority of that node in the network.

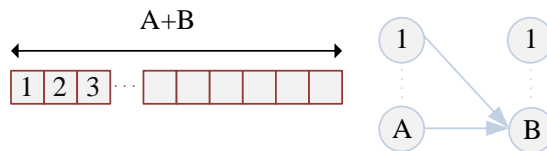


Fig 3. The priority-based representation

5-3-Solution update

PSO updates the positions (solutions) by adding the velocity to the current positions. Since we use the permutation-based procedure for the solutions, a novel position update mechanism is presented that adapted to this procedure, shown in figure 4.

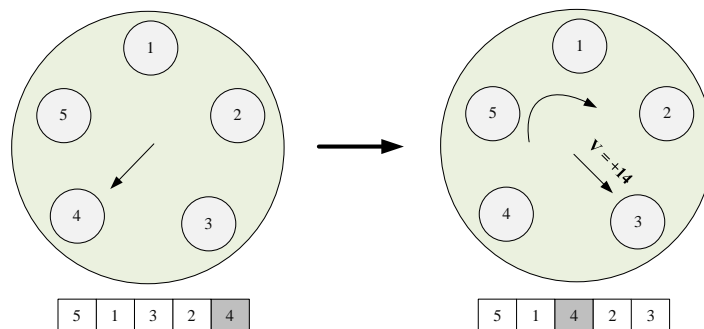


Fig 4. Position update

A priority-based decoding algorithm decodes the solution using the steps shown below. This procedure gets the solution encoding as the input. Also, the required parameters such as the number of vertices, demand, capacity, and transportation costs are initialized. Vertices in the solution vector are selected based on the priority and the transportation costs. This procedure continues until all the vertices are selected.

A: inspection centers, B: end-users,
d: end-users demand
tc: travel cost from end-users to inspection centers,
cap: Capacity of inspection centers,
sol(A*B): initial solution,
x: flow between inspection centers and end-users
While all(sol(:)) \neq 0
1. **Solution generation:** $random \leftarrow argmax\{sol(i), i \in (A + B)\}$;
2. **Select vertex:** $r^* = \left\lceil \frac{random}{(A+B)} \right\rceil$,
3. **Select inspection center and end-user**
 $r^* = argmin\{tc | sol \neq 0\}$, selecting a pair of vertices with minimum cost
4. **Determine the flow**
 $x = min\{d, cap\}$,
5. **Update demand and capacity**
 $d = d - x, cap = cap - x$
end

6-Results and discussions

Integrated network design and planning models are categorized as np- hard problems since location problems complicate solving the model. Therefore, the exact methods would be very complicated for such models, especially in big-size problems so, a hybrid priority-based particle swarm algorithm is proposed as a solution method. We used MATLAB R2019a to solve the sample problems by PC with Intel® Core™ i5-9400F CPU @ 2.90GHz and 16.00 GB RAM. Also, parameter tuning for PSO is conducted. Random data are generated by the parameters shown in table 2 used to solve the problem in small, medium, and large-scale instances. Additionally, the parameters of the algorithm are illustrated in three levels in table 3.

Table 2. Equipment and spare parts

f_r	Uniform [2000, 5000]
$t_{sll'}$	Uniform [50, 120]
$o_{ss'w_i}$	Uniform [500, 800]
d_{sc}	Uniform [0, 10]
rw_{sr}	Uniform [5, 30]
cap_r	Uniform [2000, 5000]
rs_{rs}	Uniform [5, 30]
$sc_{ss'}$	Uniform [0, 50]
v_{sd}	Uniform [0, 1]
ψ_{si}	Uniform [0, 1]
h_{sw}	Uniform [250, 1000]
bc_{sw}	Uniform [700, 1000]
rc_{sr}	Uniform [500, 2500]

Table 3. PSO parameters

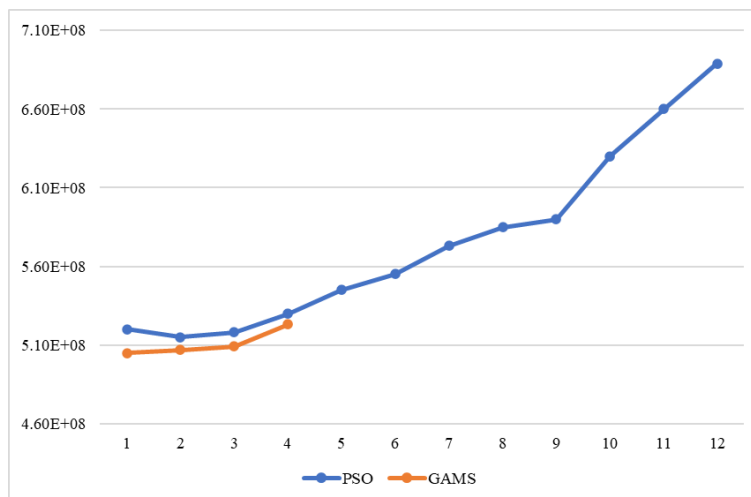
Parameters	Level		
	L1	L2	L3
Local set coefficient	0.6	0.7	0.9
Global set coefficient	0.9	0.7	0.6
Repository	2	5	10
Iteration	70	100	150
Population	5	10	15
Inertia weight	0.2	0.4	0.6

The instances and their results are presented in table 4 that shows the comparison of the exact and meta-heuristic solutions. Three small, medium and large-scale problems are organized as 12 instances that can be seen in table 4. Small size instances are solved by the exact method using GAMS and compared with the results of hybrid PSO. In this table, we can see that the difference between the exact and hybrid methods is not significant that shows a slight deviation between the two methods. Other instances cannot be solved by GAMS due to an increase in complexity. The comparison of the exact and hybrid methods is also illustrated in figure 5.

We observe a trade-off between the number of the warehouses and total costs. To minimize stock level, it is necessary to reduce inventory level, since maintaining an optimal stock level of spare parts and equipment can satisfy demands while reducing holding costs.

Table 4. Hybrid and exact method results

	Instances	# spare parts	# Central warehouses	# Local warehouses	# End-users	Hybrid PSO cost	Exact method cost	Error (%)
Small	I1	10	2	5	10	5.12×10^8	5.05×10^8	2.88
	I2	15				5.15×10^8	5.07×10^8	1.55
	I3	20				5.18×10^8	5.09×10^8	1.74
	I4	25				5.30×10^8	5.23×10^8	1.32
Medium	I5	30	3	10	15	5.45×10^8	-	-
	I6	40	5	15	20	5.55×10^8	-	-
	I7	50	7	20	30	5.73×10^8	-	-
	I8	60	10	25	40	5.85×10^8	-	-
Large	I9	80	20	50	50	5.9×10^8	-	-
	I10	150	30	70	100	6.3×10^8	-	-
	I11	200	50	100	150	6.6×10^8	-	-
	I12	250	70	150	250	6.89×10^8	-	-

**Fig 5.** hybrid PSO vs. exact method

Instance 1 is considered for sensitivity analyses. The stock level of spare parts in central warehouses are presented in table 5. Also, we performed sensitivity analyses regarding the total cost when holding costs change, illustrated in table 5 and figure 6. It can be seen that total costs increase by growth in holding costs that lead to the integration of stocks as presented in table 6. In case the holding costs increase, it is economical to integrate the stocks to decrease the number of warehouses to optimize costs.

Also, the costs decrease when there is a prominent cut down in holding costs. It is notable that as the number of warehouses decreases, it can affect the lead-time and expected shortage conversely i.e. as we have a spread network of facilities, we should expect more lead-time than the usual that may increase the expected shortage.

Table 5. Stock levels

Spare part		1	2	3	4	5	6	7	8	9	10
Warehouse	1	3	2	1	2	5	10	5	2	1	2
	2	1	5	2	1	3	2	10	5	1	3

Table 6. stock level and cost fluctuations vs. change in holding costs

Spare part No. 1								
Holding cost fluctuation		-30%	-20%	-10%	0	10%	20%	30%
Stock level in warehouse	1	4	4	3	3	2	4	4
	2	2	1	1	1	1	0	0
Total cost		5.02E+08	5.03E+08	5.04E+08	5.05E+08	5.07E+08	5.09E+08	5.30E+08
Spare part No. 3								
Holding cost fluctuation		-30%	-20%	-10%	0	10%	20%	30%
Stock level in warehouse	1	2	2	1	1	3	3	3
	2	7	5	3	2	1	1	0
Total cost		4.90E+08	5.00E+08	5.01E+08	5.05E+08	5.08E+08	5.10E+08	5.35E+08
Spare part No. 7								
Holding cost fluctuation		-30%	-20%	-10%	0	10%	20%	30%
Stock level in warehouse	1	6	6	5	5	10	12	15
	2	15	13	12	10	5	3	0
Total cost		4.60E+08	4.80E+08	5.00E+08	5.05E+08	5.20E+08	5.50E+08	5.70E+08

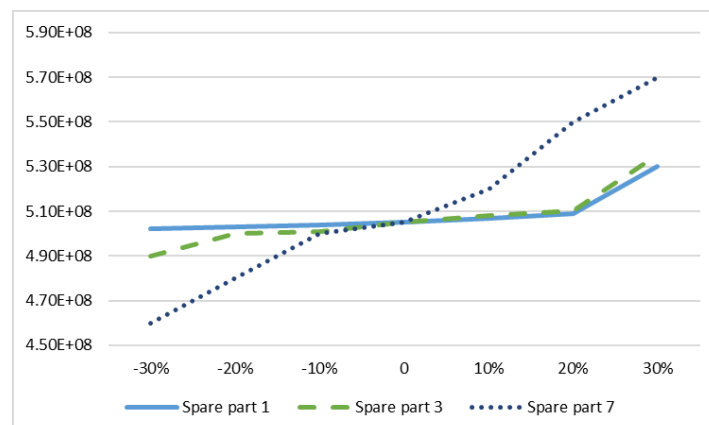


Fig 6. Total costs fluctuations when holding costs change

7-Conclusions and research opportunities

Spare parts are the crucial resources used in maintenance and repair operations in every industry that may have high or low value either high or low demand. In this paper, we focus on repairable expensive spare parts that are used in operational bases that would be disrupted in case of spare part failure. Therefore, a proper network design and planning help not only reduce the probability of spare part shortage but also it can optimize total costs and prevent unexpected failures. One of the issues in this research area is to find the optimized stock level since it can impose high inventory costs on the supply chain, but the dimension of the problems in the real world is a big obstacle. In this study, we deal with a repairable spare part supply chain that includes repair centers, warehouses, and inspection centers. The network flow begins from the end-users. The defective spare parts are replaced with a new one then moved to inspection centers for technical inspection that may be qualified for repair or dispose. The repair requires repair capability, repair time, and budget that affect the repair assignment. A METRIC model is used to handle the inventory management decisions in warehouses such as reorder point and the stock level. Also, two types of suppliers such as manufacturer and vendors are considered that includes local and foreign supplies. An important research gap in this paper is to present integrated inventory management, and network design and planning model. Also, the model covers simultaneous decisions such as stock level of warehouses, network flow, location of facilities, order assignment to suppliers, and reorder level. Due to the complexity of the real-world problems and NP-hardness of location problem besides the inventory management, we present a hybrid priority-based Particle Swarm Optimization (PSO) that uses permutation representation. Also, a novel position update for PSO is described. The comparison of the exact and hybrid methods shows a slight deviation of the proposed meta-heuristic algorithm. Future researches can compare other meta-heuristics to testify the performance of the algorithms. Additionally, researchers may investigate the operations to accelerate the processes.

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