

Optimal length of warranty and burn-in periods considering different types of repair

Ehsan MoghimiHadji

*Industrial Engineering Department, Engineering Faculty, Islamic Azad University
(West-Tehran Branch), Tehran, Iran*

moqimi@yahoo.com

Abstract

Failure rate curve based on the failure rate function of many electrical and mechanical systems shows a bathtub-shape form. In the first phase of this curve, where the failure rate has a decreasing form with a high slope, manufacturers use the burn-in method to eliminate defective products before reaching the market. In this phase most of the failures are minor (since the component is completely new, this type of error generally takes happen because of bad assembling, displacement of a socket, and so on) or major type failures (for example because of wrong design, selecting unsuitable raw materials, and so on). In the second phase, where the failure rate curve shows a constant value, manufacturers offer warranty services to their customers to ensure them about the quality and performance of their products. In this paper, we investigate the total cost incurred during the burn-in and warranty periods from the manufacturer's point of view. We consider different types of repair services and obtain the expected total cost in each phase. We present an optimization example to illustrate the efficacy of the proposed model in finding optimal values for burn-in and warranty periods.

Keywords: Failure rate, burn-in, non-renewing warranty, minimal repair, general repair

1-Introduction

Nowadays, along with new and modern products, consumers expect better customer support from manufacturers to purchase their products with more confidence. To address this concern, manufacturers utilize different types of techniques and warranties to ensure their customers about the quality of their products. One of the methods used by manufacturers to decrease the probability of early failures during the useful life period of a product is the so-called burn-in method, which eliminates products with early failures in field operations (Cha, 2000).

Failure rate cure of many deteriorating systems based on the failure rate density function that plays a vital role in reliability theory, shows a bathtub-shape cure which can be roughly divided into three distinct regions namely, early failures region (region I), random failures region that sometimes called the normal operation region (region II), and the wear-out failures region (region III) (Brezavscek, 2013).

*Corresponding author

ISSN: 1735-8272, Copyright c 2021 JISE. All rights reserved

In the first region, the failure rate decreases rapidly. The main reasons for failure in this region are due to defects in materials and design or poor assembling or manufacturing quality. In the second region, which is known as the normal operation region, the failure rate is approximately constant.

In this region the failures are usually due to operate and/or fluctuation on operating environment (MoghimiHadji and Rangan, 2012). In the third region, which is known as the post warranty period, the failure rate is monotonically increasing. The main reason for failures in this region is the effect of aging on the components. Figure 1 depicts a typical bathtub-shape cure.

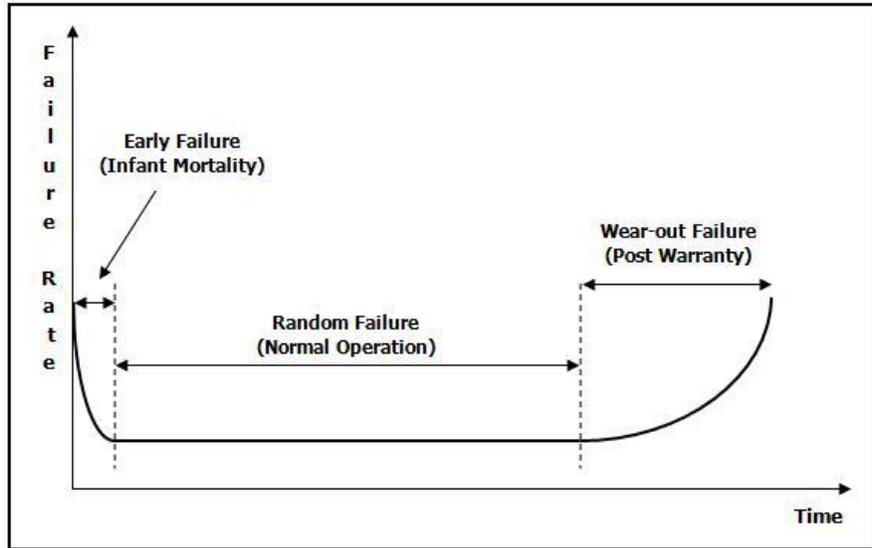


Fig 1. A typical bathtub failure rate with three distinct regions

There are some important issues that need to be addressed when using the burn-in method. The length of the burn-in period and the type of repair during it are important factors to manage costs for the manufacturer. Cha (2000) proposed a burn-in procedure for a repairable component in his paper. He assumed that the failed component is only minimally repaired. He gave the properties of optimal burn-in and block replacement policy. In another study (Cha, 2003), he extended his previous work by considering a time-dependent probability for type II failure and some mild conditions on the failure rate function. In addition, he determined optimal burn-in time and optimal replacement policy. In this study, we consider a repairable component, which has a bathtub shaped failure rate curve. During the burn-in period, the product faces minor failures with probability P_1 , which can be repaired minimally and the product can continue its burn-in test. The product faces a catastrophic failure with probability $1-P_1$. In this case, that manufacturer has to replace it with a new component and restart the burn-in test.

There are many types of warranty introduced in the literature. In addition to the properties of the burn-in test, length and type of warranty have a vital role in defining the cost function. Mi (1999) considered random costs for both burn-in and renewable warranty and derived an average warranty cost. Moreover, he did some comparisons between replacement-free warranty, mixing renewable warranty and pure pro-rata warranty. Many systems consist of some components in series/parallel. Determining the warranty cost of such systems may require considering the whole system's reliability and additional calculations. Monga and Zuo (1998) considered burn-in, warranty and maintenance periods for a series-parallel system based on system reliability. They constructed an optimization model to minimize the cost of the unit time system life cycle. They employed genetic algorithms to solve this non-linear mixed-integer programming problem. Rangan and Khajoui (2007) constructed a new model that simultaneously considers burn-in, warranty and maintenance policies to design and manage strategies. In their stochastic model, the system maintained preventively on failure with general repair and minimal repair. Kwon et. al. (2010) considered burn-in and maintenance at the same time. They assumed a continuous non-decreasing function of t (time)

for the minimal repair cost at age t of the failed component. In another research, Moghimihadji and Rangan (2012) considered different types of cost during three different periods; burn-in, warranty and post-warranty. They also considered different options for the consumer to repair the failed system during the post-warranty period. In addition, they obtained three different cost functions in order to minimize the average total cost during the three mentioned periods. They also considered some constraints for their optimization model. For warranted products, Shafiee (2013) determined the optimal burn-in, the degree of preventive maintenance and the interval for preventive maintenance by deriving the expected cost function, which was time-dependent. Beside these researches, Park et. al. (2013) considered a renewable minimal repair-replacement warranty policy and proposed an optimal maintenance model after the warranty time is expired (post-warranty maintenance policy).

Although warranties ensure customers to be supported by the manufacturer and receive some services when the product fails, consumers' dissatisfaction due to product failure during its lifetime is not ignorable. In another study, Shafiee et. al. (2014) developed an optimization model to minimize the average total cost of product servicing by determining the length of the optimal burn-in and warranty periods. They also considered customer dissatisfaction during the post-warranty period by introducing a penalty cost. Podolyakina (2016) estimated of manufacturer's expected cost based on the method of providing warranty service for the broken product by considering either repair or replacement. Jiao and Zuo (2018) maximized the total profit by determining the optimal sale price. They considered a periodic preventive maintenance policy with minimal repair and replacement. In the replacement policy, they used a renewing warranty for the replaced product. In a more recent study, Wang et. al. (2019) considered a repairable system and obtained the optimal replacement policy by describing the time interval of the preventive repair and the failure correction with an extended geometric process. Park et. al. (2020) in their new study, determined optimal length of warranty period from the dealer's point of view by considering an optimal warranty period for second-hand products based on a two-stage repair-or-full-refund maintenance strategy. In order to propose the optimal warranty policy, they utilized the expected cost during the product maintenance cycle.

Existing models generally have been considered only one type of repair during each period but in this model, two types of repair during each period have been considered. In this study, we assume a burnt-in component with age b goes to the market with a non-renewable warranty of length W . During this period, it faces minor failure with probability P_2 , which can be repaired minimally, or a major failure with probability $1-P_2$, which requires a general repair by the manufacturer. Repair and replacement times are assumed negligible. The aim of this paper is to find the cost function during burn-in and warranty periods and minimize the average total cost during these periods from the manufacturer's perspective.

2-Notations used

c_1 : Raw material and manufacturing cost/ purchasing cost

c_2 : Installation and setting cost

c_3 : Operating cost during burn-in period $(0, b]$

c_4 : Minimal repair cost in the factory

c_5 : Minimal repair cost in the customer place ($c_5 > c_2 + c_4$)

c_6 : Extra cost more than c_5 for a general repair

$f(t)$: Failure time density function

$F(t)$: Failure time distribution function

$\bar{F}(t)$: Survivor function

$h(t)$: Bathtub failure rate function which is given by $h(t) = \frac{f(t)}{\bar{F}(t)}$

$N(t)$: Number of failures during the interval $(0, t)$

$M(t)$: Average number of failures during the interval $(0, t)$

δ : Degree of repair

3-The model

In this study, a repairable component with failure density $f(t)$ and bathtub failure rate function $h(t)$ is considered. After producing the component, the manufacturer puts it through a burn-in process with length b (burn-in period). Manufacturers use this method to eliminate defective components before reaching the market. In this process, the component is tested under some conditions similar to field operation. If a component can pass this period of time successfully, it reaches the market.

3-1-Cost during the burn-in period $(0, b]$

The related costs during this period are the cost of buying raw material and manufacturing or purchasing cost c_1 , the cost of installation and setting of the component c_2 , the cost of operating each component per unit time c_3 during the burn-in period, and the cost of each minimal repair c_4 for manufacturer during this period. The replacement cost of a component is considered to be $c_1 + c_2$ (it can be any other value). Let b be the length of the period, $N(b)$ be the number of failures during the period, and P_1 be the probability of having a minor failure during this period. The total cost during the period $(0, b]$ is given by

$$TC_b = P_1 [c_1 + (c_2 + c_4)N(b) + c_3 b] + (1 - P_1) [(c_1 + c_2)N(b) + c_3 \left(\sum_{i=1}^{N(b)-1} x_i + b \right)] \quad (1)$$

Where $N(b)-1$ is the number of component replacements before the successful completion of the burn-in process by a component and x_i is the lifetime of the i^{th} unsuccessful component which fails before completing the burn-in period b . Based on the definition of the burn-in process, it is worth mentioning that $N(b)$ has a geometric distribution given by

$$P_r[N(b) = m] = F(b)^{m-1} \bar{F}(b) \quad (2)$$

Thus, based on the properties of the geometric distribution, the expected value of $N(b)$ can be written as

$$E[N(b)] = \frac{1}{\bar{F}(b)} \quad (3)$$

Using Wald's identity, we have

$$E\left(\sum_{i=1}^{N(b)-1} x_i\right) = E[N(b)]E(x_i) - E(x_{N(b)}) = \frac{\int_0^b \bar{F}(t) dt}{\bar{F}(b)} - b \quad (4)$$

Hence, the expected total cost during the burn-in period is given by

$$E(TC_b) = P_1 \left[c_1 + (c_2 + c_4) \int_0^b h(t) dt + c_3 b \right] + (1 - P_1) \left[(c_1 + c_2) \frac{1}{\bar{F}(b)} + c_3 \frac{\int_0^b \bar{F}(t) dt}{\bar{F}(b)} \right] \quad (5)$$

The component which has passed the burn-in test is sold in the market with a non-renewable warranty offer of period W . If a component fails during this period, based on the type of failure (minor failure with probability P_2 or major failure with probability $1-P_2$) it will require minimal repair or general repair.

3-2-Cost during the warranty period $(b, b+W]$

The relevant costs during this period are the cost of minimal repair in the customer place c_5 and the extra cost of general repair c_6 more than minimal repair proportional to the degree of repair δ , with $0 \leq \delta \leq 1$. If a general repair is done on a component of age x , then the age of the component after this general repair will be δx . $\delta=1$ means that the component was minimally repaired whereas $\delta=0$ indicates that the failed component was replaced by a new one. Thus, in general repair, δ could be any value between 0 and 1. During the warranty period $(b, b+W]$, a minor failure takes place with probability P_2 and a major failure happens with probability $1-P_2$. Thus, the total cost during this period is given by

$$TC_W = P_2[c_5 N_b(W)] + (1 - P_2)\{[c_5 + c_6(1 - \delta)]N_g(W)\} \quad (6)$$

Where $N_b(W)$ is the number of minor failures of a burnt-in component during $(b, b+W]$ and $E[N_b(W)]$ is given by

$$E[N_b(W)] = \int_b^{b+W} h(t) dt \quad (7)$$

And $N_g(W)$ is the number of major failures during the warranty period and its expected value is $M_g(W)$. When a general repair is performed on a failed component, it is known as a g-renewal function introduced by Kijima (1989) given by

$$M_g(t) = Q(t|0) + \int_0^t Q(t-x|x) m(x) dx \quad (8)$$

Where

$$Q(x) = \int_0^t \frac{f(y+x)}{\bar{F}(x)} dy \quad (9)$$

Thus, the expected total cost during the warranty period is given by

$$E(TC_W) = P_2 \left(c_5 \int_b^{b+W} h(t) dt \right) + (1 - P_2)\{[c_5 + c_6(1 - \delta)]M_g(W)\} \quad (10)$$

Finding an explicit solution for equation (10) is not possible. Thus, an approximation method proposed by Rangan and Moghimihadji (2011) is utilized to evaluate $M_g(W)$ numerically.

The average total cost of the component up to the end of the warranty period is the sum of the burn-in cost given in (5) and the warranty cost given in (10). Thus, the average total cost per unit time is given by

$$E(TC) = \frac{E(TC_b) + E(TC_W)}{W} \quad (11)$$

The aim of this study is to minimize the above cost function to determine the optimal system design decision variables, namely the optimal burn-in period b^* and the optimal warranty period W^* . We present a numerical example in the next section to show how the derived formulas can be utilized to find the optimal burn-in and warranty periods.

4-Numerical illustration

Suppose that the component has the bathtub failure rate function given by the below five-parameter failure rate function introduced by Dhillon (1979).

$$h(t) = KCLt^{C-1} + (1 - K)bt^{b-1}Be^{Bt^b} \quad (12)$$

Where L and B are the scale parameters and C and b are the shape parameters and K is a number between 0 and 1, inclusive. In this example, the values of parameters are as follows: $K=0.5$, $L=1$, and $B=1$. By

changing the values for the shape parameters C and b , one can obtain different shapes of the failure rate curve. In our example, we assume $C=0.3$ and $b=2.5$ to have a relatively long normal operation region (region II). It is possible to consider any other positive values for shape parameters C and b . Figure 2 shows the failure rate curve of the above failure rate function using the specified parameters.

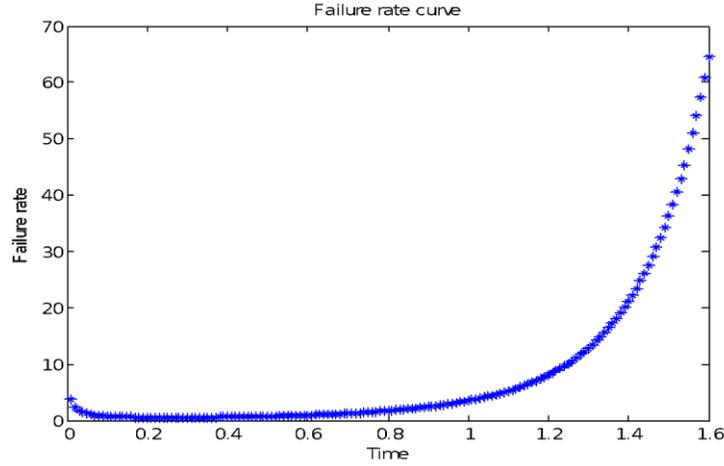


Fig 2. Failure rate curve based on the following parameters; $K=0.5$, $L=1$, $B=1$, $b=2.5$ and $C=0.3$

For our example, the cost parameters are chosen to be $c_1=100$, $c_2=2$, $c_3=1$, $c_4=5$, $c_5=10$, and $c_6=30$. Moreover, suppose that δ , the degree of repair, is equal to 0.7 . Thus, the general repair cost is $c_5+c_6(1-\delta) = 10+30(1-0.7)=19$. It is worth mentioning that it is also possible to find the optimal value for the degree of repair but for simplicity, we fix it at 0.7 . Let P_1 , the probability of minor failure during the burn-in period, be equal to 0.8 and P_2 , the probability of minor failure during the warranty period, be equal to 0.5 . Table 1 shows the amount of the average total cost using different values for the burn-in and warranty periods.

Table 1. Amount of average total cost using different values for b and W

Burn-in period	Warranty period	Average total cost	Burn-in period	Warranty period	Average total cost
0.02	0.2	535.78	0.04	0.2	539.52
	0.4	272.41		0.4	274.37
	0.6	185.92		0.6	187.39
	0.8	144.50		0.8	145.82
	1.0	122.04		1.0	123.45
	1.2	110.37		1.2	112.04
	1.4	108.22		1.4	110.75
	1.6	124.35		1.6	129.95
0.06	0.2	542.44	0.08	0.2	544.97
	0.4	275.93		0.4	277.32
	0.6	188.62		0.6	189.75
	0.8	147.02		0.8	148.14
	1.0	124.74		1.0	125.999
	1.2	113.63		1.2	115.24
	1.4	113.34		1.4	116.11
	1.6	136.09		1.6	142.94

As can be seen in table 1, generally, the average total cost increases by increasing the length of the burn-in period for the same warranty periods. However, when a fixed burn-in period is considered, the total cost goes down by increasing the warranty period up to a point. Further increasing the warranty period will increase the total cost. It is because of the high slope of failure rate curve in that area. For this example, the optimal burn-in period, b^* is equal to 0.02 and the optimal warranty period, W^* is equal to 1.4. In our example, δ , the degree of repair was fixed at 0.7. It is also possible to find the optimal value for the degree of repair.

In our example, based on the current failure rate curve, we consider $b \geq 0.02$ with a step size of 0.02 and $W \geq 0.2$ with a step size of 0.2. Since the burn-in test is an expensive procedure, the model tries to keep the length of the burn-in period at a minimum. It is clear that by changing the value of failure rate function parameters or by changing the cost parameters, the optimum values of burn-in period and warranty period will change.

5-Concluding remarks

In modern trade, offering services like suitable warranty becomes an integral part of selling products. In this paper, the expected total cost function during burn-in and warranty periods from the manufacturer's perspective is obtained. In order to consider different conditions similar to the field conditions, two types of failures are considered; minor failure and major failure. Based on the type of failure, different types of repair such as minimal repair, general repair, and replacement are also considered.

In order to extend this study, one can investigate post-warranty period costs, which should be paid by the consumer. Indeed, it is possible to extend this problem from a component to a system, which consists of some components in series or parallel. It is also possible to consider some constraints for the system like the minimum amount of required reliability, the maximum amount of available budget or maximum acceptable weight/volume of the system.

References

- Brezavscek, A. (2013). A simple discrete approximation for the renewal function. *Business System Research*, 4 (1), 65-75.
- Cha, J.H. (2000). On a better burn-in procedure. *Journal of applied probability*, 37(4), 1099-1103.
- Cha, J.H. (2003). A further extension of the generalized burn-in model. *Journal of applied probability*, 40(1), 264-270.
- Dhillon, B. (1979). A hazard rate model. *IEEE transactions on reliability*, 28(2), 150.
- Jiao, C., & Zhu, X. (2018). Optimal design of sales and maintenance under the renewable warranty. *RAIRO operations research*, 52(2), 529-542.
- Kijima, M. (1989). Some results for repairable systems with general repair. *Journal of applied probability*, 26(1), 89-102.
- Kwon, Y.M., Wilson, R., & Na, M.H. (2010). Optimal burn-in with random repair cost. *Journal of the Korean statistical society*, 39(2), 245-249.
- Mi, J. (1999). Comparisons of renewable warranties. *Naval research logistics*, 46(1), 91-106.
- MoghimHadji, E., & Rangan, A. (2012). Optimal burn-in, warranty and maintenance decisions in system design. *International journal of modeling in operations management*, 2(3), 266-278.

- Monga, A., & Zuo, M.J. (1998). Optimal system design considering maintenance and warranty. *Computers and operations research*, 25(9), 691-705.
- Park, M., Jung, K.M., & Park, D.H. (2013). Optimal post-warranty maintenance policy with repair time threshold for minimal repair. *Journal of reliability engineering and system safety*, 111 (C), 147-153.
- Park, M., Jung, K.M., & Park, D.H. (2020). Warranty cost analysis for second-hand products under a two-stage repair-or-full refund policy, *Journal of reliability engineering and system safety*, 193 (C), 106596.
- Podilyakina, N. (2016). Product reliability and warranty period as a cost-forming factors. *Business: Theory and Practice*, 17(4), 361-369.
- Rangan, A., & Khajoui, S. (2007). Optimal system design based on burn-in, warranty and maintenance. *1st annual RFID Eurasia conference*, Istanbul, Turkey.
- Rangan, A., & MoghimiHadji, E. (2011). Approximation to g-renewal functions. *International journal of quality and reliability management* , 28(7), 773-780.
- Shafiee, M. (2013). Optimal burn-in and preventive maintenance warranty strategies with time dependent maintenance costs. *IIE transactions*, 45(9), 1024-1033.
- Shafiee, M., Chukova, S., & Yun, W.Y. (2014). Optimal burn-in and warranty for a product with post-warranty failure penalty. *International journal of advanced manufacturing technology*, 70(1-4), 297-307.
- Wang, J., Ye, J., & Xie, P. (2019). New repairable system model with two types repair based on extended geometric process. *Journal of systems engineering and electronics*, 30(3), 613-623.