

Coordination of pricing and cooperative advertising for perishable products in a two-echelon supply chain: A bi-level programming approach

Maryam Mokhlesian^{1*}, Seyed Hessameddin Zegordi¹

¹ *Department of Industrial Engineering, Tarbiat Modares University, Tehran, Iran.*

zegordi@modares.ac.ir, maryam.mokhlesian@modares.ac.ir

Abstract

In this paper the coordination of pricing and cooperative advertising decisions in one-manufacturer and one-retailer decentralized supply chain with different market power for channel members is studied. The products are both perishable and substitutable. The problem is modeled as a nonlinear bi-level programming problem to consider both retailer and manufacturer decisions about prices and advertisement expenditure as well as the amount of retailer's purchase. An Improved Particle Swarm Optimization (PSO) through combining by local search and diversification is proposed to solve the model. Finally, a numerical example is presented to analyze the effect of market scale and the coefficient of price elasticity on the decisions. Numerical results indicate that to raise profit when the consumers are more price-sensitive, both the manufacturer and the retailer should decrease their prices and increase their advertising expenditure. In the larger market scale, the manufacturer and the retailer are even permitted to increase their prices to gain more profit.

Key words: Bi-level programming; Pricing; Multi-product supply chain; Substitutable and perishable products; Cooperative advertising; Market power.

1- Introduction

In recent years, coordination between independent members of a supply chain has gained much attention. Without coordination, distribution channel members make their own decisions independently in order to maximize their own profits. It is well reviewed in marketing and economics literature that uncoordinated decisions lead to "double marginalization", which is one of the causes of channel inefficiency. A basic decision for supply chain coordination is pricing, which typically includes wholesale

*Corresponding author

ISSN: 1735-8272, Copyright c 2015 JISE. All rights reserved

price and retail price. Pricing is a core theme in the marketing research literature on distribution channels (Xie and Wei 2009).

Advertising is another marketing practice. Cooperative advertising is a practice that a manufacturer pays retailers a portion of the local advertising cost in order to induce sales. Cooperative advertising plays a significant role in marketing programs of channel members (Xie and Wei 2009).

In this study, perishable goods are considered. Some goods are perishable because of physical deterioration such as blood, fresh vegetables, and meats. Some of the deterioration reasons are because of deterioration of goods value, such as berth in plane, a bed in hotel, and a ticket in cinema. Some others are because of technological changes and shifts in consumer preferences. So, most firms dealing with perishable goods need to decide on the product's price and order quantity while confronted with a multi-period decision problem with a finite or infinite horizon. For a perishable product supply chain, it becomes more and more important to study how to control inventory and how to decide the prices (Jia and Hu 2011). Therefore, this study is encountered pricing for each period of products' life cycle.

Over the last few years, more and more manufacturers have increased product variety by differentiating one or several attributes of the product (such as technical attributes, appearance, and color) in order to compete in the market and to maximize their profit. To the end consumers, some of these products may be substitutable because of no distinct differentiation (Zhao et al.2012). Substitutability is the other characteristic of the considered supply chain's products which has effects on the decisions.

Often, members of a supply chain have a different power. Power is defined as "the ability of one channel member to control the decision variables in the marketing strategy of another member in a given channel at a different distribution level" (El-Ansary and Stern 1972). In this situation, each member makes its own decisions in a decentralized manner. In these hierarchical systems, one member may have a more dominant role than others. This member is named as "leader" and other downstream members are named "followers". A multidivisional bi-level programming model can be used for such issues (Lu et al. 2006). In the supply chain of this study, the manufacturer as a leader has more dominant power than the retailer as a follower.

We consider a single-manufacturer single-retailer channel in which the retailer sells the manufacturer's brand within a product class which is substitutable and perishable. It is assumed that channel members have different market power. Decision variables of this structure are their advertising expenditures, their prices (manufacturer's wholesale price and retailer's retail price), the volume of production for the manufacturer and the volume of purchase for the retailer as well as his inventory level. Supply chain models have concentrated on almost all facets related to pricing, purchasing, production, and inventories. However, models simultaneously dealing with at least two of the above facets are complex and sparse (Xie and Neyret 2009). This study aims to identify optimal pricing and cooperative advertising expenditure for a manufacturer and a retailer in a distribution channel.

However, in reality, the buyer and the vendor in a supply chain would have a competitive relationship. Naturally, they need to make decisions based on their own interests, while still considering the option of the other, as the other's decisions will have an influence on their own interests. Bi-level programming techniques intend to solve decision problems where each decision making unit independently optimizes its own objective, while it is affected by the actions of the other unit under a hierarchy. In a bi-level decision problem, a decision making unit at the upper-level is known as the leader and at the lower-level is known as the follower. The research on bi-level problems is strongly inspired by real world applications due to the existence of hierarchical organizations and systems. Bi-level programming techniques have been used in different fields such as decentralized resource planning,

electric power markets, logistics, civil engineering, and road network management (Gao et al. 2011). For this reason, bi-level programming is used in this paper to model the proposed problem.

Berger (1972), who was the first researcher to investigate cooperative advertising issues between a manufacturer and a retailer quantitatively, presented a mathematical model to yield improved managerial decisions and better performance for the whole channel. Researchers extended Berger's model in a variety of ways under different co-op advertising settings. Some studies were done on cooperative advertising which focused on a supply chain where the manufacturer was as leader and the retailer was as follower (see Hutchins 1953, Crimmins, 1970, Somers et al. 1990). Roslow et al. (1993) demonstrated that considering co-op advertising in the supply chain could increase the profit of the whole supply chain. Huang and Li (2001) discussed the cooperative advertising issue for the supply chain with one manufacturer and one retailer by using game theory concepts. They detected that the manufacturer always prefers the Stackelberg game to the synchronized move game while the preference of the retailer depends on the parameters of the model. Huang et al. (2002) developed independently a cooperative advertising model for a one-manufacturer-one-retailer supply chain with consideration of the national brand name investment. Khouja and Robbins (2003) studied the optimal local advertising expenditure and the ordering quantity under the newsboy framework. However, most of the above literature on co-op advertising considered only the impact of local advertising on the amount of sales, while the influence of other factors, such as the manufacturer's national brand name advertising, was left mislaid.

Some researchers consider the coordination of pricing and cooperative advertising. Xie and Ai (2006) assumed that the demand of a retailer was positively related to both local advertising expenditures and brand name investment while being negatively related to the sales price set by the retailer. Yue et al. (2006) and Szmerekovsky and Zhang (2009) developed a price discount model to coordinate the advertising expenditures of the two parties under the assumption that the consumer demand is dependent on retail price and co-op advertising efforts by channel members. Xie and Neyret (2009) identified the optimal pricing and co-op advertising strategies by discussing four different classical types of relationships between a manufacturer and a retailer using game theory models. Xie and Wei (2009) further studied the co-op advertising and pricing problems in a one-manufacturer one-retailer channel by employing a sales response function with respect to advertising expenditures and retail price. Yan (2010) studied both cooperative advertising and pricing in a manufacturer-e-retailer supply chain for categories of products. The leader-follower Stackelberg and strategic alliance model was established. Aust and Buscher (2012) expanded the existing research which deals with advertising and pricing decisions in a manufacturer-retailer supply chain simultaneously, by means of game theory. Zhang et al. (2013) proposed a dynamic cooperative advertising model for a manufacturer-retailer supply chain and analyzed how the reference price effect would influence the decisions of all the channel members.

There are only several models developed in the literature which was combined pricing and ordering problems for products with deterministic or random demand. Rajan et al. (1992) and Abad (1996) developed continuous time pricing and ordering policies for deterministic demand. Gallego and van Ryzin (1994) proposed a pricing model for a perishable product with random demand and no inventory replenishment. Burnetas and Smith (2000) considered the combined problem of pricing and ordering for a perishable product with uncertain price-sensitive demand. Bisi and Dada (2007) investigated the optimal dynamic ordering and pricing policies for a retailer with a mixed (additive and multiplicative) price-sensitive demand. Chew et al. (2009) investigated the joint pricing and inventory allocation decisions for a perishable product with predetermined life time. Jia and Hu (2011) studied the combined problem of pricing and ordering for a perishable product supply chain including one supplier and one retailer in a finite horizon. The life time of the product is two periods and demand in each period is random and price-sensitive. Li et al. (2013) considered a retailer dynamic pricing problem with uncertain demand which was modeled as a hybrid of random and fuzzy variable. Chew et al. (2014) determined the order quantity

and prices for a perishable product with multiple life time periods. Mokhlesian and Zegordi (2014) investigated coordination of pricing and inventory decisions in a one-manufacturing-several competitive-retailers supply chain who presents some substitutable products.

Table 1. The outline of studied researches

Decisions				Competition		Products type		Products number		References
Inventory	Ordering policy	Advertising	Pricing	with	without	Substitutable	Perishable	Multi	Single	
		*	*		*				*	Aust and Buscher, 2012; Szmerekovsky and Zhang, 2009
		*	*	*					*	Berger, 1972
*	*		*		*		*		*	Bisi and Dada, 2007; Rajan et al., 1992
	*		*		*		*		*	Burnetas and Smith, 2002
*			*		*		*		*	Chew et al., 2009; Jia and Hu, 2011
*			*		*		*	*		Chew et al., 2014; Gallego, G and van Ryzin, 1994
		*			*				*	Huang and Li, 2001; Huang et al., 2002
	*	*			*				*	Khouja and Robbins, 2003
			*		*		*	*		Li et al., 2013
*			*	*		*		*		Mokhlesian and Zegordi, 2014
		*	*		*				*	Xie and Neyret, 2009; Xie and Wei, 2009; Yue et al., 2006; Zhang et al., 2013
		*	*		*			*		Yan, 2010

According to the literature, simultaneous decisions on pricing and advertising in supply chains attract many researchers. These decisions are influenced by the structure of the channel, i.e. power structure, the number of products, the kinds of products, limitations to the channel members, etc. Our main interest is to investigate how the manufacturers and the retailer make their own decisions when facing different market power structures in a decentralized supply chain. One of these occasions is considered in this study in which the manufacture has more power than the retailer. A kind of informative advertising called “cooperative advertising” is used to introduce the products which are substitutable and perishable. Bi-level programming model is used to depict this decentralized situation.

2- Problem description

The problem studied here is described as follows. One manufacturer and one retailer trade several perishable products with equal life cycle time. Each product has two period life cycles. Fresh goods can be sold for two periods, so those not sold in the first period can be sold at the next period. The products, whose life cycles are expired, must be discarded. The retailer orders only fresh goods once each period according to its demand, inventory, and the manufacturer's wholesale price. Since the consumers prefer the fresh goods, the retailer offers lower prices to persuade consumers for the unsold products than the fresh products to prevent deterioration of its products. Figure 1 demonstrates the inventory system of a product in the planning horizon.

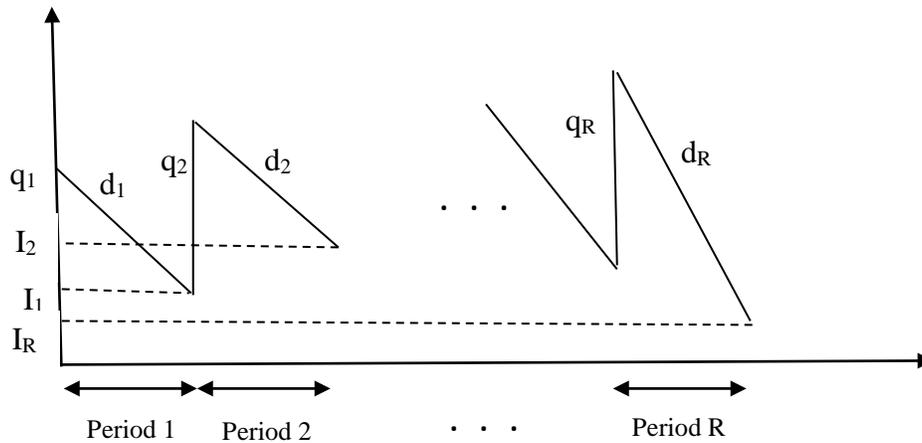


Figure 1. The inventory system of the retailer

Some of these products are substitutable. It means that if a product is not available when it is demanded, its substitute will be offered. Therefore, increasing the price of each product increases the demand of its substitute.

Since advertising can inform and persuade people to purchase, expending more costs for advertising can lead to more profit through more sales. The goal of advertising is to increase the demand of the final consumers.

The aim of each channel member is to determine lot size, prices and advertising expenditures to maximize its profit.

Since the structure of a bi-level programming problem (BLPP) facilitates the formulation of the problems that involve a hierarchical decision making process, the considered problem will be modeled as a bi-level programming model in contrast to a single level model. Also, bi-level programming is practical for consecutive decision making. In the literature, it is mostly assumed that the manufacturer is the dominant player as the leader (e.g. Wang 2002, Corebtt et al. 2004). In some cases, it is recognized that the dominant manufacturer is not appropriate (see Chu and Messinger 1997). In this case, no member is dominant. In the case of powerful international retailer chain (such as Wal-Mart, Tesco, etc) the retailer is dominant (see Tsay 2002, Ertek and Griffin 2002). Because it is assumed that the manufacture plays a more significant role than the retailer, the manufacturer is considered as the first level (leader) and the retailer as the second one (follower).

The advantage to formulate this problem as a bi-level programming model is that both retailer and manufacturer are allowed to decide independently, according to the decisions of each other. Hence, each member of the channel tries to maximize its profit.

This structure is applicable to some situations like distribution channel of vegetables, dairy and other foods as well as other perishable goods in which the manufacturer produces several products and the retailer sells them.

2-1- Assumptions

Some of the other assumptions considered to build the model are as follows:

1. Life cycles of all products are considered to be the same.
2. It is assumed that the planning horizon is equal to the products' life cycle.
3. Demand of each fresh product is deterministic and price sensitive (function of its price, price of substitutable products, price of old products as well as advertising expenditure).
4. The retailer purchases only fresh products from the manufacturer.
5. The retail price of each fresh product and unsold product are different, but the price of fresh products is the same in all periods.
6. To prevent loss of reputation, no shortage is permitted.
7. The inventory of each product at the retailer stage is considered. The manufacturer has no inventory and produces equal to the orders.
8. The manufacturer has finite supply capacity, i.e. has a capacity constraint.
9. The retailer has a limited warehousing space.
10. Each stage has a limited budget.

2-2- Notations

The following parameters are common for both manufacturer and retailer:

- j Index of periods.
- i Index of products.
- N Number of products.
- R Number of periods.
- J_i Set of products that can be substituted by product i .

Parameters and variables for the manufacturer (upper-level) are as follow:

- P_i Production capacity for product i .
- C_i Production cost per unit of product i .
- g Fixed costs of the facilities to manufacture the products.
- BM Available budget of the manufacturer.
- t Decision variable, the manufacturer participation rate.
- A Decision variable, the manufacturer's national advertising expenditure.
- w_i Decision variable, wholesale price of product i charged to the retailer.

The parameters and variables for the retailer (lower-level) are presented in the following:

- α_{ij} A constant in product i 's demand function of the retailer in period j which represents its market scale.
- β_{ik}^j Coefficient of demand elasticity between products i and k for the retailer in the period j .
- d_{ij} Demand of product i in the period j from final consumers.
- h_i The retailer's holding cost per unit of product i 's inventory per unit of time.
- O_i Logistic cost for retailer per order of product i .
- l_i Required space to store a unit of product i .
- f Retailer's fixed costs for the facilities to carry.
- M Available warehouse space for the retailer.
- BR Available budget of the retailer.
- a Decision variable, the retailer's local advertising expenditure.
- p_{ij} Decision variable, retail price of product i charged to the customer in the period j .
- q_{ij} Decision variable, the retailer's order quantity of product i in the j th period.
- I_{ij} Decision variable, the retailer's inventory level of product i at the end of j th period.

2-3- Problem formulation

We consider a single-manufacturer, single-retailer channel in which the manufacturer produces several substitutable and perishable products and the retailer sells only the manufacturer's brand. The demand of the retailer $d_{ij}(p_{ij}, a, A)$ is a function of the retail price p_{ij} and the advertising expenditures a and A . Hence, the demand function is as follows (Xie and Wei 2009):

$$d_{ij}(p_{ij}, a, A) = u_{ij}(p_{ij}) \cdot K(a, A), \quad i = 1, 2, \dots, N, \forall i; \quad j = 1, 2, \dots, R, \forall j. \quad (1)$$

Where $u_{ij}(p_{ij})$ indicates the impact of the retail price on the retailer's demand and $K(a, A)$ displays the impact of the advertising expenditures on the demand, also known as the sale response function.

A linear function, which is a well known demand function in the literature (e.g. Xie and Wei 2009, Xie and Neyret 2009), is assumed for $u_{ij}(p_{ij})$ as follows:

$$u_{ij}(p_{ij}) = \alpha_{ij} - \beta_{ii}^j p_{ij} + \sum_{k \in J_i} \beta_{ik}^j p_{kj} + \beta_{ii}^{j-1} p_{i(j-1)}, \quad i = 1, 2, \dots, N, \forall i; \quad j = 1, 2, \dots, R, \forall j. \quad (2)$$

Where $\beta_{ii}^0, p_{i0} = 0$. In the equations (2), the substitutability of products is shown by $\sum_{k \in J_i} \beta_{ik}^j p_{kj}$. This means that if the price of each product increases, then the demand of its substitute will increase (Ingene and Parry 2007). The impact of unsold products prices' on the demand of the fresh products is demonstrated by $\beta_{ii}^{j-1} p_{i(j-1)}$. It is denoted that the less prices of the unsold products causes the lower demand of the fresh products.

There are considerable estimations of the sales response function with respect to the advertising expenditures in the literature. Some studies do not discriminate the impacts on the sales between local (retailer) and national (manufacturer) advertising expenditures (Xie and Wei 2009). Both types of advertising efforts could influence sales, so their effects should be evaluated separately. Hence, advertising effects on the consumer demand are considered as:

$$K(a, A) = k_r \sqrt{a} + k_m \sqrt{A} \quad (3)$$

Where k_r and k_m are positive constants reflecting the efficiency of each type of advertising in generating sales. The $K(a, A)$ is an increasing and concave function with respect to a and A because of the ‘‘advertising saturation effect’’, i.e., additional advertising spending continuously weakens demand. According to the literature, it can be inferred that weakening returns characterize the shape of the advertising-sales response function (Xie and Wei 2009).

Lemma. $K(a, A)$ is a concave function.

Proof. See Appendix A.

Given the above lemma, the consumer demand function can be written as follows:

$$d_{ij}(p_{ij}, a, A) = (\alpha_{ij} - \beta_{ii}^j p_{ij} + \sum_{k \in J_i} \beta_{ik}^j p_{kj} + \beta_{ii}^{j-1} p_{i(j-1)}) \cdot (k_r \sqrt{a} + k_m \sqrt{A}), \quad i = 1, 2, \dots, N, \forall i; \quad (4)$$

$$j = 1, 2, \dots, R, \forall j.$$

Based on the mentioned assumptions, notations and definitions, the bi-level programming model to determine the decision variables of each level which maximize its own profit is as follows:

$$\max_{w, A, t} \pi_m = \sum_i \sum_j w_i q_{ij} - ta - A - \sum_i \sum_j C_i q_{ij} - g \quad (5)$$

s.t.

$$ta + A + \sum_i C_i q_{ij} + g \leq BM \quad j = 1, 2, \dots, R, \forall j \quad (6)$$

$$q_{ij} \leq P_i \quad j = 1, 2, \dots, R, \forall j \quad (7)$$

$$w_i, A \geq 0, \quad 0 \leq t \leq 1 \quad i = 1, 2, \dots, N, \forall i \quad (8)$$

$$j = 1, 2, \dots, R, \forall j$$

$$\max_{p, q, a, I, d} \pi_r = \sum_i \sum_j (p_{ij} - w_i) q_{ij} - (1-t)a - \sum_i \sum_j h_i I_{ij} - \sum_i \sum_j O_i q_{ij} - f \quad (9)$$

s.t.

$$d_{ij}(p_{ij}, a, A) = (\alpha_{ij} - \beta_{ii}^j p_{ij} + \sum_{k \in J_i} \beta_{ik}^j p_{kj} + \beta_{ii}^{j-1} p_{i(j-1)}) \cdot (k_r \sqrt{a} + k_m \sqrt{A}) \quad i = 1, 2, \dots, N, \forall i \quad (10)$$

$$\sum_i w_i q_{ij} + (1-t)a + \sum_i h_i I_{ij} + \sum_i O_i q_{ij} - f \leq BR \quad j = 1, 2, \dots, R, \forall j \quad (11)$$

$$\sum_i I_{ij} \leq M \quad j = 1, 2, \dots, R, \forall j \quad (12)$$

$$\sum_{k \in J_i} q_{kj} \geq \sum_{k \in J_i} d_{kj} - \sum_{k \in J_i} I_{k, j-1} \quad i = 1, 2, \dots, N, \forall i \quad (13)$$

$$j = 1, 2, \dots, R, \forall j$$

$$I_{i2} = q_{i2} - d_{i2} \quad i = 1, 2, \dots, N, \forall i \quad (14)$$

$$p_{ij}, I_{ij}, d_{ij}, q_{ij}, a \geq 0 \quad \begin{matrix} i = 1, 2, \dots, N, \forall i \\ j = 1, 2, \dots, R, \forall j \end{matrix} \quad (15)$$

Objective function (5) maximizes the net profit of the manufacturer over the planning horizon. The profit of the manufacturer is equal to revenue gained by sales minus the costs including advertising, production and fixed costs. Constraint (6) indicates the limited budget in each period. Constraint set (7) demonstrates the limited production capacity in each period. Constraint set (8) imposes non-negativity for the manufacturer's decision variables. Objective function (9) maximizes the net profit of the retailer in the planning horizon. The profit of the retailer is equal to the revenue gained by sales minus the costs including advertising, purchasing, holding the inventory and logistics as well as fixed costs. Constraint set (10) displays the retailer's demand of each product in each period that is a function of its retail price, retail prices of substitutable products and the retail price of unsold products as well as advertising expenditures. Constraint sets (11) and (12) demonstrate the limited budget and warehouse in each period for the retailer, respectively. Constraint set (13) prevents shortage. Constraint set (14) indicates the amount of the retailer's inventory. Constraint set (15) imposes non-negativity for the retailer's decision variables.

It is assumed that the starting inventory of each product at the retailer is zero; i.e. $I_{i0} = 0, \forall i, i = 1, 2, \dots, N$.

3- Improved particle swarm optimization algorithm

Since it is proved in the following proposition, the proposed bi-level programming model is a NP-hard problem and a heuristic or metaheuristic approach must be used to solve the large scale problems.

Proposition 3-1: The proposed bi-level programming model is a NP-hard problem.

Proof: See appendix B.

There are some approximate and metaheuristic algorithms that can be used to solve the bi-level programming problem (BLPP). Rajesh et al. (2003) presented a tabu search based algorithm to solve the BLPP. Lan et al. (2007) combined neural network and tabu search algorithms to propose a method to solve BLPP. Wang et al. (2007) described an adaptive genetic algorithm for the linear bi-level programming problem to overcome the difficulty of determining the probabilities of crossover and mutation. Kuo and Huang (2009) developed a method based on Particle Swarm Optimization (PSO) algorithm with swarm intelligence. Hu et al. (2010) proposed a novel neural network approach. Their proposed neural network is capable to generate an optimal solution to the linear BLPP. Gao et al. (2011) developed a PSO based algorithm to solve pricing problems in a supply chain that was modeled as a BLPP.

A PSO-based algorithm is applied to solve the proposed model in this paper. In the basic model of PSO, a swarm of particles includes n particles moving around in a d dimensional search space. Each particle k is a candidate solution of the problem, and is represented by the vector x_k in the decision space. A particle has its own position and velocity, which indicates the movement direction and step of the particle, respectively. Each particle moves from its position to its neighbor by its own velocity. The fitness of each particle represents the quality of its position. The velocity and position of each particle are updated using the following equations:

$$v^{i+1} = W.v^i + c.r_1^i(p_o^i - x_k^i) + c.r_2^i(x^{*i} - x_k^i) \quad (16)$$

$$x_k^{i+1} = x_k^i + v^{i+1} \quad (17)$$

Where po^i is the best position previously visited by the swarm and x^{*i} is the current best position. c , r_1 and r_2 are the coefficients of the algorithm representing the attraction of a particle either toward its own success or toward the success of its neighbors. The inertia weight W will control the impact of the previous velocity on the current one. A larger value of the inertia weight leads to a higher impact on the current one by the previous one. Hence, at each iteration, each particle will change its position according to its own experience and that of neighboring particles (Talbi 2009).

In this study, we improve the PSO which is intensified by a local search method and diversified by a mutation approach. The local search improves the quality of the solution. The diversification prevents the fast convergence.

PSO is a population-based algorithm that is commenced from an initial population. Often, the initial population is generated randomly. So, we initialize the population randomly.

The quality of each particle is represented by the fitness value. We consider sum of the objective function value of channel members as the fitness of each particle.

The PSO can be extended to solve bi-level programming models, so that, in an iterative manner, first the best solution of upper-level (leader) is obtained; then based on the solution of the upper-level, the solution of lower-level (follower) is achieved. In this study, we apply a local search method to intensify the search space and a mutation approach to diversify the solutions. We named the proposed algorithm as Improved Particle Swarm Optimization (IPSO).

- Local search method (intensification): it is applied for the best solution (particle) found at each iteration. In this method, one of the components is selected, randomly. Then, its value is changed, randomly.
- Mutation approach (diversification): in this approach, a percentage of the best particles ($mrate \times \text{particle size}$) are selected and copied. Then, the insertion mutation is performed on them. Finally, the worst particles are replaced with these obtained particles.

Table 2 shows the notations used in IPSO.

Table 2. Notations used in the IPSO

N_l	The number of particles for the leader.
N_f	The number of particles for the follower.
x_k	The k^{th} particle for the leader.
po_{x_k}	The best previously visited position of x_k .
x_k^*	Current best position for particle x_k .
v_{x_k}	The velocity of x_k .
K_l	Current iteration number for the upper-level problem.
y_i	The i^{th} particle for the follower.
po_{y_i}	The best previously visited position of y_i .
y_i^*	Current best position for particle y_i .
K_f	Current iteration number for the lower-level problem.
v_{y_i}	The velocity of y_i .
$\max K_l$	The predefined max iteration number for K_l .
$\max K_f$	The predefined max iteration number for K_f .
W_l, W_f	Inertia weights for the leader and the follower respectively (coefficients for PSO).
c_l, c_f	Acceleration constants for the leader and the follower respectively (coefficients for PSO).
r_1, r_2	Random numbers uniformly distributed in $[0,1]$ for the leader and the follower (coefficients for PSO).

The procedure of the proposed IPSO can be outlined as follows.

Step 1: Sample N_l particles of x_k , and the corresponding velocities v_{x_k} , randomly;

Step 2: Initiate the leader's loop counter $K_l = 0$;

Step 3: For the k^{th} particle, $k = 1, \dots, N_l$, generate the response from the follower;

Step 3.1: Sample N_f candidates y_i and the corresponding velocities v_{y_i} , $i = 1, \dots, N_f$;

Step 3.2: Initiate the follower's loop counter $K_f = 0$;

Step 3.3: Record the best particles po_{y_i} and y^* from po_{y_i} , $i = 1, \dots, N_f$;

Step 3.4: Use local search approach around the y^* ;

Step 3.5: Update the follower's velocities and positions using

$$v_{y_i}^{k+1} = W_f v_{y_i}^k + c_f r_{1l}^k (po_{y_i} - y_i^k) + c_f r_{2l}^k (y^* - y_i^k)$$

$$y_i^{k+1} = y_i^k + v_{y_i}^{k+1}$$

Step 3.6: Use the diversification approach for particles;

Step 3.7: $K_f = K_f + 1$;

Step 3.8: If $K_f \geq \max K_f$ or the solution changes for several consecutive generations are small enough, then go to Step 4, otherwise go to Step 3.5;

Step 3.9: Output y^* as the response from the follower.

Step 4: Record po_{x_k} , x_k^* , $k = 1, \dots, N_l$ for each x_k , $k = 1, \dots, N_l$;

Step 4.1: Use the local search approach around the x_k^* ;

Step 5: Update leader's velocities and positions using

$$v_{x_k}^{k+1} = W_l v_{x_k}^k + c_l r_{1l}^k (po_{x_k} - x_k^k) + c_l r_{2l}^k (x^* - x_k^k)$$

$$x_k^{k+1} = x_k^k + v_{x_k}^{k+1}$$

Step 5.1: Use the diversification approach for particles;

Step 6: $K_l = K_l + 1$;

Step 7: If $K_l \geq \max K_l$, algorithm is terminated, otherwise go to Step 3

4- Computational experiment

4-1- Design of experiments

Choosing appropriate algorithm parameters' affects both the running times and the quality of solutions. Hence, considering a method to tune the parameters of an algorithm is very important. One of the most popular methods is ANOVA. Initial implementation of the test problems shows that N_i, N_f , $\max k_i$ and $\max k_f$ have a greater effect on both the running times and the quality of solutions compared to other parameters. Therefore, the values of other parameters are set as follows:

$$r_{1l} = 0.2, \quad r_{2l} = 0.5, \quad c_i = 30, \quad c_f = 20,$$

$$W_i = 8, \quad W_f = 10, \quad \text{Mrate} = 0.4$$

In order to study the effect of these parameters, we suggest a general factorial design of experiment (DOE). Four factors are considered in the experimentation and each factor is given two levels. The values of these levels are identified in Table 3. These levels are selected by experimentation.

Table 3. Levels of DOE.

Parameters	Label	Levels	
N_i and N_f	A	1000	2000
$\max K_i$ and $\max K_f$	B	50	100

For each combination of the four factors under study 30 randomly selected problems are solved and the responses are measured. These problems are selected from a given data set shown in Table 4.

Table 4. Problem parameters.

Levels of supply chain	Parameters
Manufacturer	$P \sim U(0, 100000)$
	$C_i \sim U(1, 50)$
	$g \sim U(1, 7000)$
	$k_m \sim U(0, 20)$
	$BM \sim U(0, 100000000)$
Retailer	$J_i \sim DU(1, N)$
	$a_{ij} \sim U(0, 20000)$
	$b_{ij} \sim U(0, 10)$
	$h_i \sim U(1, 10)$
	$O_i \sim U(1, 50)$
	$f \sim U(1, 7000)$
	$k_r \sim U(0, 20)$
	$BR \sim U(0, 100000000)$
	$M \sim U(1, 10000)$

Table 5 shows the analysis of variance (ANOVA) while the response is the value of the fitness function.

Table 5. ANOVA results.

Sources	df	SS	MS	F	P-value
A	1	2.78E+223	2.78E+223	1.50E+00	0.223
B	1	2.78E+223	2.78E+223	1.50E+00	0.223
A×B	1	1.1E+224	1.11E+224	5.98E+00	0.016
Error	116	2.16E+225	1.86E+223		
Total	119	2.24E+225			

The first column in Table 5 indicates the factors (parameters) which effects on the response function (the value of the fitness function). The second column means the degrees of freedom for each factor. The third column demonstrates the sum of squares. The MS column shows the deviations from the sample mean. The F column presents the results of F-test. The last column indicates the significance level. According to Table 5, the interaction between parameters has a significant effect at the 0.05 level. Student-Newman-Keules test which is a test on means is selected, after implementing experiment to choose the best level for the significant parameters (Hicks 1993). The results of parameter tuning are as follow:

$$N_i \text{ and } N_f = 50 \quad \max K_i \text{ and } \max K_f = 2000$$

4-2- Performance measurement

To evaluate the performance of the proposed algorithm, it is compared with an existing PSO algorithm in the literature that was proposed by Gao et al. in 2011. The best solution obtained for each instance by each of the two algorithms (PSO or IPSO) is named $Best_{sol}$ and the solution obtained for each instance by IPSO (the proposed algorithm) is named Alg_{sol} . So, the Relative Percentage Deviation (RPD) as a comparison criterion is calculated as follows:

$$RPD = \frac{Best_{sol} - Alg_{sol}}{Best_{sol}} \times 100$$

Another criterion to compare these two algorithms is the percentage of times that the proposed algorithm has equal or better solution than the PSO. We show this criterion by PTB.

The results of the comparison are shown in Table 6.

Table 6. Results of performance measurement

Problem size (N)	Average of RPD	PTB criterion	Average of IPSO run time (seconds)
Small (10)	1.56%	83%	90
Medium (50)	1.15%	90%	450
Large (100)	0.96%	96%	1020

According to the average of RPD column in Table 6, the proposed algorithm is 1.56% below the best obtained solution on average in the small size, 1.15% in the medium size and 0.96% in the large size. The PTB criterion column indicates that in the small-size problems, 83% of time, the proposed IPSO operates better than the PSO. This criterion is equal to 90% and 96% for medium and large size problems, respectively.

5- Numerical example

Since demand effect on the chain, its parameters such as the market scale (α_{ij}) and the coefficient of demand elasticity (β_{ik}^j) have effect on the channel decisions. In this section, these effects are examined based on the results of a5 products example with 2 periods of life cycle.

Table 7 investigates the impact of the market scale on the decisions. The values are the average of each decision variable.

Table 7. Impacts of α_{ij}

Problem	Period	Channel member							
		Manufacturer				Retailer			
		w	A	t	Profit	p	q	a	Profit
$\alpha_{ij} = 10,000$	j=1	4,618	15,628	0.726	6,745,245	8,861	1,067	16,135	31,062,461
	j=2					8,289	9,137		
$\alpha_{ij} = 15,000$	j=1	7,330	57,884	0.793	15,145,412	12,597	13,332	60,065	43,716,899
	j=2					12,342	13,321		
$\alpha_{ij} = 20,000$	j=1	12,282	72,346	0.835	18,023,788	13,858	17,437	150,285	96,768,200
	j=2					10,102	2,021		

According to Table 7, when the market scale is enlarged, the manufacturer and the retailer can grow the sale amount through larger advertising expenditure to inform the consumers. Also, the manufacturer may increase its participation rate to encourage the retailer for more local advertising expenditure. In this case, both the manufacturer and the retailer allow increasing their prices. Increasing the prices and sale amount leads to raise the profit of the manufacturer and the retailer.

Table 8 shows how the coefficient of demand elasticity variations influences on the channel members' decisions.

Table 8. Impacts of β_{ik}^j

Problem	Period	Channel member							
		Manufacturer				Retailer			
		w	A	t	Profit	p	q	a	Profit
$\beta_{ik}^j = 2$	j=1	973	40,323	0.382	2,295,144	25,029	2,328	10,335	5,369,670
	j=2					23,294	1,208		
$\beta_{ik}^j = 5$	j=1	417	53,056	0.429	5,754,649	17,210	2,437	27,625	6,191,933
	j=2					15,539	2,013		
$\beta_{ik}^j = 10$	j=1	235	60,665	0.698	6,903,480	10,623	2,523	85,277	7,898,664
	j=2					10,102	2,021		

Table 8 demonstrates if the consumers are more price-sensitive, the retailer and the manufacturer should decrease their prices to attract consumers for purchase. Moreover, the more advertising expenditure is needed to persuade consumers. More participation rate leads to more local advertising expenditure. These cause more profit for both the manufacturer and the retailer.

The members of a chain must be aware to the consumers' factors. When these factors have a change, the demand is changed, consequently. Hence, the channel members should be cope with this new demand which changes the decisions such as prices, amount of production and purchase as well as advertising expenditure.

6- Conclusion

In this study, a two echelon supply chain consisting of one manufacturer and one retailer is considered. The manufacturer produces several perishable products, some of them being substitutable. The demand of each product in each stage depends on the advertising expenditure, its price and the price of substitutable products. This demand has a linear function. Coordination of dynamic pricing and cooperative advertising decisions in a bi-level system is the aim of this paper.

In order to achieve this goal, first the problem is formulated as a bi-level programming problem. Since the proposed bi-level programming model is proved to be a NP-hard problem, an algorithm based on improved particle swarm optimization (IPSO) approach in combination with a diversification approach and a local search method is proposed. Then, this proposed algorithm is compared with an existing PSO method. The implementation of test problems demonstrates an acceptable priority to the PSO.

The results show that in larger market scale, an increase in the prices, can lead to increase the profit through more advertising to inform consumers. Furthermore, when the consumers' sensitivity to the price

is enlarged, the retailer and the manufacturer should decrease their prices and increase advertising expenditure to raise their sales and their profit.

This paper has several limitations which can be extended in further studies. In this study, competition is not investigated; hence, it can be extended to a competitive environment. Only one manufacturer and one retailer in a supply chain are considered in this study; however, other components of the supply chain could also be considered. The cooperative advertising strategy is assumed to be the main strategy in this article but it is also suggested to consider other strategies.

References

- Abad, P.L.(1996). Optimal pricing and lot-sizing under conditions of perishability and partial backordering.*Management Science*,42(8), 1093–1104.
- Aust, G., &Buscher, U.(2012). Vertical cooperative advertising and pricing decisions in a manufacturer-retailer supply chain: A game-theoretic approach.*European Journal of Operational Research*,223, 473-482.
- Berger, P.D.(1972). Vertical cooperative advertising ventures.*Journal of Marketing Research*,9,309–312.
- Bisi, A.,& Dada, M.(2007). Dynamic learning, pricing, and ordering by a censored newsvendor.*Naval Research Logistics*,54(4), 448–461.
- Burnetas, A. N.,& Smith, C.E.(2000). Adaptive ordering and pricing for perishable products.*Operations Research*,48(3),436–443.
- Chew, E.P., Lee, C.,& Liu, R.(2009). Joint inventory allocation and pricing decisions for perishable products.*International Journal of Production Economics*,120, 139–150.
- Chew, E.P., Lee, C., Liu, R., Hong, K.S.,& Zhang, A.(2014). Optimal dynamic pricing and ordering decisions for perishable products. *International Journal of Production Economics*. 157, 39-48.
- Chu, W.,&Messinger, P.R.(1997). Information and channel profits.*Journal of Retailing*,73(4), 487–499.
- Corbett, C.J., Zhou, D.& Tang, C.S.(2004). Designing supply contracts: Contract type and information asymmetry.*Management Science*,50(4), 550–559.
- Crimmins, E.C.(1970).*A management guide to cooperative advertising*. Association of national advertisers: New York.
- EI-Ansary, A.L.&Stern, L.W.(1972). Power measurement in the distribution channel.*Journal of Marketing Research*,9(1), 47–52.
- Ertek, G.,& Griffin, P.M.(2002). Supplier- and buyer-driven channels in a two-stage supply chain.*IIE Transactions*,34(8), 691–700.
- Gallego, G.,&van Ryzin, G.(1994). Optimal dynamic pricing of inventories with stochastic demand over finite horizons.*Management Science*,40(2),999–1020.
- Gao, Y., Zhang, G., Lu, J.,& Wee, H.M.(2011). Particle swarm optimization for bi-level pricing problems in supply chains.*Journal of Global Optimization*,51,245-254.

- Hicks, C.R.(1993).*Fundamental concepts in the design of experiments*.4th ed., Oxford University Press: New York.
- Hu, T., Guo, X., Fu, X.,&Lv, Y.(2010). A neural network approach for solving linear bilevel programming problem.*Knowledge-Based Systems*,23,239–242.
- Huang, Z.,&Li, S.X.(2001). Co-op advertising models in manufacturer–retailer supply chains: A game theory approach.*European Journal of Operational Research*,135,527–544.
- Huang, Z., Li, S.X.,& Mahajan, V.(2002). An analysis of manufacturer–retailer supply chain coordination in cooperative advertising.*Decision Sciences*,33,469–494.
- Hutchins, M.S.(1953).*Cooperative advertising*. Ronald Press: New York.
- Ingene, C.A.,& Parry, M.E.(2007). Bilateral monopoly, identical distributors, and game-theoretic analyses of distribution channels.*Journal of the Academy of Marketing Science*,35, 586-602.
- Jia, J.,&Hu, Q.(2011). Dynamic ordering and pricing for a perishable goods supply chain.*Computers & Industrial Engineering*,60,302–309.
- Khouja, M.,&Robbins, S.S.(2003). Linking advertising and quantity decisions in the single-period inventory model.*International Journal of Production Economics*,86,93–105.
- Kuo, R.J.,&Huang, C.C.(2009). Application of particle swarm optimization algorithm for solving bi-level linear programming problem.*Computers and Mathematics with Applications*,58,678-685.
- Lan, K. M., Wen, U. P., Shih, H. S.,& Lee, E.S.(2007).A hybrid neural network approach to bilevel programming problems.*Applied Mathematics Letters*,20,880–884.
- Li, G., Xiong, Z., Zhou, Y.,&Xiong, Y.(2013). Dynamic pricing for perishable products with hybriduncertainty in demand.*Applied Mathematics and Computation*,219(20), 10366-10377.
- Lu, J., Shi, C.,&Zhang, G.(2006). On bilevel multi-follower decision making: General framework and solutions.*Information Sciences*,176,1607–1627.
- Mokhlesian, M., &Zegordi, S.H. (2014). Application of multidivisional bi-level programming to coordinate pricing and inventory decisions in a multiproduct competitive supply chain. *Int J AdvManufTechnol*, 71, 1975–1989.
- Rajan, A., Steinberg, R.,&Richard, S.(1992). Dynamic pricing and ordering decisions by a monopolist.*Management Science*,38(2), 240–262.
- Rajesh, J., Gupta, K., Kusumakar, H.S., Jayaraman, V.K.,&Kulkarni, B.D.(2003). A tabu search based approach for solving a class of bilevel programming problems in chemical engineering.*Journal of Heuristics*,9,307–319.
- Roslow, S., Laskey, H.A.,& Nicholls, J.A.F.(1993). The enigma of cooperative advertising.*Journal of Business & Industrial Marketing*,8,70–79.
- Sahni, S. (1974). Computationally related problems. *SIAM Journal on Computing*, 3, 262–279.
- Somers, T. M., Gupta, Y.P.,&Herriott, S.R.(1990). Analysis of cooperative advertising expenditures: A transfer-function modeling approach.*Journal of Advertising Research*, (October-November), 35-45.

- Szmerekovsky, J.G.,&Zhang, J.(2009). Pricing and two-tier advertising with one manufacturer and one retailer.*European Journal of Operational Research*,192(3),904–917.
- Talbi, E.G.(2009).*Metaheuristics: from design to implementation*. John Wiley & Sons, Inc., Hoboken: New Jersey.
- Tsay, A.A.(2002). Risk sensitivity in distribution channel partnerships: Implications for manufacturer return policies.*Journal of Retailing*,78(2), 147–160.
- Wang, G., Wang, X., Wan, Z.,&Jia, S.(2007). An adaptive genetic algorithm for solving bilevel linear programming problem. *Applied Mathematics and Mechanics*, (English Edition)28(12),1605–1612.
- Wang, Q.(2002). Determination of suppliers' optimal quantity discount schedules with heterogeneous buyers.*Naval Research Logistics*,49(1), 46–59.
- Xie, J.,&Ai, S.(2006). A note on “Cooperative advertising, game theory and manufacturer–retailer supply chains”.*Omega*,34,501–504.
- Xie, J.,&Neyret, A.(2009). Co-op advertising and pricing models in manufacturer–retailer supply chains.*Computers & Industrial Engineering*,56,1375–1385.
- Xie, J.,&Wei, J.(2009). Coordinating advertising and pricing in a manufacturer–retailer channel.*European Journal of Operational Research*,197,785–791.
- Yan, R.(2010). Cooperative advertising, pricing strategy and firm performance in the e-marketing age.*Journal of the Academy of Marketing Science*,38,510–519.
- Yue, J., Austin, J., Wang, M.C.,& Huang, Z.(2006). Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount.*European Journal of Operational Research*,168,65–85.
- Zhang, J., Gou, Q., Liang, L.,& Huang, Z.(2013). Supply chain coordination through cooperative advertising with reference price effect.*Omega*,41(2),345-353.
- Zhao, J., Tang, W.,& Wei, J.(2012). Pricing decision for substitutable products with retail competition in a fuzzy environment.*International Journal of Production Economics*,135(1),144-153.

Appendix A

Proof. The concavity of $K(a, A)$ can be examined by Hessian matrix.

$$H = \begin{bmatrix} \frac{\partial^2 K(a, A)}{\partial a^2} & \frac{\partial^2 K(a, A)}{\partial a \partial A} \\ \frac{\partial^2 K(a, A)}{\partial A \partial a} & \frac{\partial^2 K(a, A)}{\partial A^2} \end{bmatrix} = \begin{bmatrix} -0.25k_m a^{-\frac{3}{2}} & 0 \\ 0 & -0.25k_m A^{-\frac{3}{2}} \end{bmatrix} \quad (\text{A.1})$$

Since $k_m, A > 0$, the Hessian matrix is negative definite, $K(a, A)$ is concave.

Appendix B

Consider the below non convex quadratic programming (QP) model:

$$-\min -F(X) = \frac{1}{2} X Q X^T + d X^T \equiv \max F(X) \quad (\text{B.1})$$

s.t.

$$A X \leq b \quad (\text{B.2})$$

Where $X = (x_1, x_2, x_3, x_4, x_5, x_6)$, $d = (0, C, 0, 0, 1, 1)$ and

$$Q = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & C & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} BM - g \\ P \\ 0 \\ 1 \end{bmatrix}$$

The mentioned QP can be rewritten as follow based on the above definitions:

$$- \min \quad \pi_m = -x_1 x_2 + x_6 + x_5 + Cx_2 + g \quad (\text{B.3})$$

s.t.

$$x_6 + x_5 + Cx_2 \leq BM - g \quad (\text{B.4})$$

$$x_2 \leq P \quad (\text{B.5})$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \quad (\text{B.6})$$

$$x_3 \leq 1 \quad (\text{B.7})$$

Since Q is not PSD, it is interpreted that the above non convex quadratic programming problem is NP-Hard (Sahni 1974).

Through the below transformations for the parameters and decision variables, the above mentioned QP is reduced to the manufacturer's problem (p1) as a sub-problem of the proposed problem (main problem):

$$x_1 = w_i \quad i = 1, 2, \dots, N \quad (\text{B.8})$$

$$x_2 = q_{ij} \quad i = 1, 2, \dots, N ; j = 1, 2, \dots, R. \quad (\text{B.9})$$

$$x_3 = t \quad (\text{B.10})$$

$$x_4 = a \quad (\text{B.11})$$

$$x_5 = A \quad (\text{B.12})$$

$$x_6 = ta \quad (\text{B.13})$$

$$C = C_i \quad i = 1, 2, \dots, N. \quad (\text{B.14})$$

$$P = P_i \quad i = 1, 2, \dots, N. \quad (\text{B.15})$$

Since this non convex quadratic programming problem is NP-hard, the manufacturer's problem is NP-hard, too. Because the degree of the complexity of a sub-problem is not less than the main problem, the main problem being NP-hard causes(p1) to be NP-hard, too.