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Economic Manufacturing Model under Partial Backordering and Sustainability Considerations

Vahid R. Soleymanfar[1], Ata Allah Taleizadeh[2*], Nadia Pourmohammad Zia[3]

[1]*School of Industrial Engineering, Islamic Azad University, South Tehran Branch, Tehran, Iran*
[2,3]*School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*

vrsoleymanfar@yahoo.com, Taleizadeh@ut.ac.ir, nadia.pmz@gmail.com

Abstract

Today, “Sustainable production” has attracted a great deal of interest by academic researchers and practitioners due to the raising environmental and social concerns. Sustainable EPQ model has been developed as a result of this interest and also necessity. This paper develops a novel sustainable EPQ (SEPQ) model under partial backordering consideration. The model converts all emission variations of inventory production lifecycle into economic tangible factors. A solution procedure to determine the optimal solution of the problem is developed for this SEPQ-PBO model. In order to demonstrate validity of the proposed model and applicability of the developed solution procedure, numerical examples accompanied by comprehensive sensitivity analysis of key parameters of the model are provided.

Keywords: Economic manufacturing model, Sustainability, Inventory, Shortage.

1- Introduction

Economic lot-sizing problems have been comprehensively studied since Harris (1913) introduced the basic model. Economic production quantity model is an extension of the basic model for manufacturing firms that tended to determine the most cost effective production quantity under rather stable conditions (Cheng, 1989). Although the basic models incorporated a number of simplistic assumptions, they offered a new study direction in academic researches. Lots of later studies which embedded other factors into the models were developed based upon these models.

Due to the government regulations, environmental concerns and social awareness, there is a great necessity to observe how business practices affect the environment. In this regard, sustainable development plays a key role in success of different firms. Sustainability is defined as “Meeting the need of present without compromising the ability of future generations to meet their own needs” (Mukhopadhyay and Goswami, 2014). As the result, different companies are enforced to employ rigorous policies to diminish their undesirable environmental impact as they try to enhance their economic performance. Sustainable production models such as sustainable economic quantity model are influential policies which aid firms in order to fulfill this aim.

Regarding the recent environmental and social concerns, Sustainable development is gaining a growing interest by academic researchers in different fields recently; among which manufacturing and production problems are of great significance. The impact of Sustainability on the financial performance of firms has been investigated by Ambec and Lanoie (2008). Rădulescu et al. (2009) formulated and studied a multi-objective programming approach for inventory production model which implemented suitable constraints on pollutant emissions. Two alternative optimization problems: (a) minimum pollution risk; (b) maximum expected return were considered in the paper. For each pollutant, they defined three different contamination levels and introduced penalties proportional to the amounts of pollutants that exceed these levels.

Tao et al. (2010) embedded the concept of green cost into EOQ and EPQ models. Later, Bonney and Jaber (2011) studied a group of inventory problems that were not covered appropriately by traditional inventory analysis such as designing responsible inventory systems. They examined the importance of inventory planning to the environment in greater detail. El Saadany et al. (2011) investigated emissions from manufacturing processes in a two-echelon supply chain model. They assumed demand as a function of the price and product's environmental quality.

Wahab et al. (2011) studied a two-level supply chain in order to determine the optimal production–shipment policy for items with imperfect quality in three different scenarios. Wang and Gupta (2011) addressed "blue ocean strategies (new competition opportunities)" as a result of green production. They claimed that it makes the firms able to present new type of competition to rivals. Glock et al. (2012) proposed a mathematical model that illustrated the trade-offs between sustainability, demand, costs, and profit in a supply chain with a single supplier and a single manufacturer. At a same time, Katz (2012) addressed diverse variety of harmful impacts of different sources of pollution resulted by various production practices and transportation policies.

Van der Veen and Venugopal (2011) developed a multi-objective framework based on LOT-SIZING model which considered Economic and Environmental Performance of the Firm. Later, Van der Veen and Venugopal (2014) tested the validity of two views including feasible synergy and trade-off between Economic and Environmental purposes by using a multi-objective approach to a variant of the well-known model. They demonstrated that both views are not contradictory but valid under different conditions.

Bouchery et al. (2012) also contributed the existing literature by revisiting classical inventory methods taking sustainability concerns into account. They believed that reducing all aspects of sustainable development to a single objective was not desirable. Absi et al. (2013) incorporated carbon emissions constraints in multi-sourcing inventory problem. The constraints aimed at limiting carbon emission per unit of product provided by different modes.

In Jaber at al. (2013) A two-level (vendor–buyer) supply chain model with a coordination mechanism was presented while accounting for greenhouse gas (GHG) emissions from manufacturing processes. Despeisse et al. (2013) presented a tactics library to provide a connection between those generic sustainability concepts and more specific examples of operational practices for resource efficiency in factories. They introduced factory modelling approach to support the use of tactics by combining the analysis of building energy and manufacturing process resource flows.

Benjaafar et al. (2013) modified traditional models by incorporating carbon emission parameters into the model in order to analyze the trade-off between optimal cost and carbon foot-print. Later, Andriolo et al. (2014) explored and discussed the evolution of inventory models during one hundred years of history, starting from the basic model developed by Harris in 1913, up to today. According to this comprehensive study, they outlined that future key challenge in the inventory replenishment problems would be expected for sustainable production and inventory models.

Mukhopadhyay and Goswami (2014) addressed a scenario of a production inventory system where the items produced were either perfect, or imperfect or defective. Imperfect items were of comparatively less quality than standard product which could be sold at a discounted price. They discussed different cases for pollution preventive model. Nouira et al. (2014) demonstrated that the optimization models for manufacturing systems should evolve to consider the environmental impacts of manufacturing activities and to integrate the environmental performance of finished products. Zhu et al. (2014) investigated the application of a Model Predictive Controller, equipped with linear-programming based optimizer, with application to energy management in production environments.

Although sustainability issues are becoming active directions of academic study, they have been rarely investigated in quantitative models such as inventory production problems and further studies are needed to cover this area. In this paper, a new sustainable EPQ model is developed by applying a direct accounting approach which converts all emission variations of inventory production lifecycle into economic tangible factors. The inventory holding, obsolescence and emission of obsolescence costs are considered to be as same as the Battini et al. (2014). Except that instead of emission of transportation, emission of production is considered and shortages are allowed in the form of partial backordering. The approach of Pentico and Drake (2009) is incorporated for modeling this EPQ problem with partial backordering. Figure (1) depicts the graph of partial backorder case in an EPQ problem with FIFO backorder filling that assumes the existing backorders will be filled before any new demands.

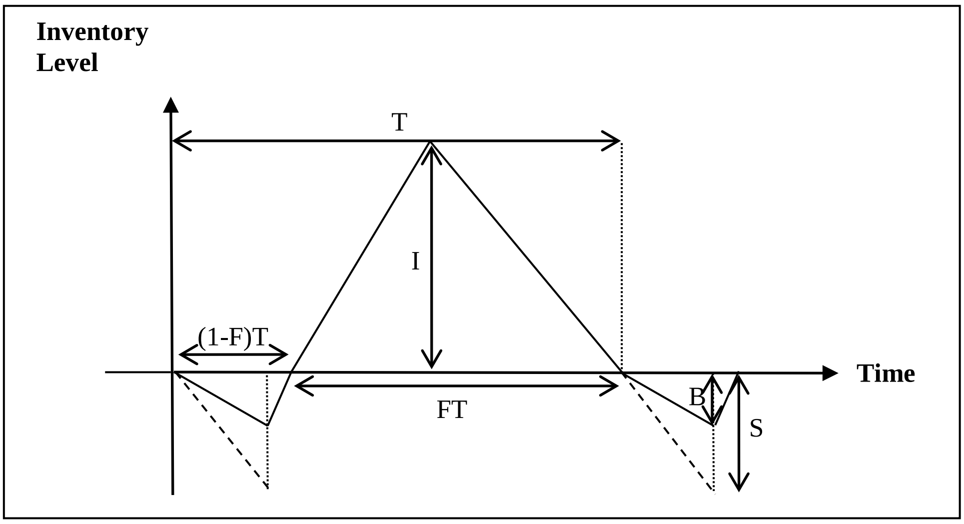

Figure 1. Graph of Partial Backorder Case for EPQ with FIFO backorder filling

2- Notation

In order to have a standard set of parameters and variables, the following notations are applied throughout the paper.

Parameters:

d	annual demand
P	maximum production rate per year
s	unit selling price
s'	unit scrap price
C_p	unit production price
A	fixed ordering cost
h	annual unit holding cost
π	annual unit backordering cost
$\hat{\pi}$	unit goodwill loss of unsatisfied demand
$\pi_{(1)} = (p - c_p) + \hat{\pi}$	Lost sale cost, including the lost profit and goodwill loss

β the partial backordering rate [percent]
θ annual average inventory obsolescence rate [percent]
l space occupied by a product unit [m3/unit]
w weight of an obsolete unit stored in the warehouse [ton/ unit]
h_e average carbon emission cost to hold inventory [currency/m3]
C_{eo} average carbon emission cost of inventory obsolescence (waste) for collection and disposal [currency/ton]
C_{ep} average carbon emission cost of producing a unit [currency/ unit]

Decision Variables

T replenishment cycle
F Percentage of demand that will be filled from stock

Dependent Variables

Q production quantity
I the maximum inventory level, and $\bar{I}$ is the annual average inventory level
S the maximum stock-out level, including both backorders and lost sales
B the maximum backorder level, and $\bar{B}$ is the annual average backorder level (That can be calculated with this relation: $B = \beta S$)
TP total profit function (that illustrated by $\Pi(T, F)$ for SEPQ-PBO model and $\Pi_{SEPQ-Basic}$ for basic SEPQ model)

3-Modeling of the SEOQ with partial backordering

In the first step, we define a total profit (TP) function as below (Pentico, Drake and Toews 2009, Battini, Persona and Sgarbossa 2013):

TP = Total sale – production cost –emission cost from production –setup cost– holding cost – emission cost from inventory holding – inventory obsolescence cost- emission of inventory obsolescence cost– backordering cost – good will loss (1)

We named TP as *Π(T,F)* That is a function of T and F variables, and then:

$$\Pi(T,F) = sd\,[F + \beta(1-F)] - C_p d\,[F + \beta(1-F)] - C_{ep}\,d\,[F + \beta(1-F)] - \frac{A}{T} - h\bar{I} - h_e l\bar{I} - \alpha(s-s')\bar{I} - \theta w\bar{I}C_{eo} - \pi\bar{B} - \hat{\pi}d\,(1-\beta)(1-F) \tag{2}$$

That $\bar{I}$ *and* $\bar{B}$ determines with these relations (Pentico, Drake and Toews 2009):

$$\bar{I} = \frac{dTF^2}{2}\left(1 - \frac{d}{P}\right) \tag{3}$$

$$\bar{B} = \frac{\beta d\,T(1-F)^2}{2}(1 - \beta d\,/P) \tag{4}$$

Substituting these expressions (Equations 3 and 4) into Equation (2), total profit function can be written as

$$\Pi(T,F) = \left(s - C_p - C_{ep}\right)d\,[F + \beta(1-F)] - \frac{A}{T} - \frac{hd\,TF^2}{2}(1 - \frac{d}{P}) - \frac{h_e ld\,TF^2}{2}(1 - \frac{d}{P}) - \theta\left(s - s'\right)\frac{d\,TF^2}{2}(1 - \frac{d}{P}) - \theta w C_{eo}\frac{d\,TF^2}{2}(1 - \frac{d}{P}) - \frac{\pi\beta d\,T(1-F)^2}{2}(1 - \beta d\,/P) - \hat{\pi}d\,(1-\beta)(1-F) \tag{5}$$

To simplify the modelling, we define:

$$h' = h(1 - \frac{d}{P}) \tag{6}$$

$$h_e{}' = h_e(1 - \frac{d}{P}) \tag{7}$$

$$s'' = (s - s')(1 - \frac{d}{P}) \tag{8}$$

$$C'_{eo} = C_{eo}(1 - \frac{d}{P}) \tag{9}$$

$$\pi' = \pi(1 - \frac{\beta d}{P}) \tag{10}$$

That gives the following relation for profit function

$$\Pi(T,F) = \left(s - C_p - C_{ep}\right)d\,[F + \beta(1-F)] - \frac{A}{T} - \frac{h'd\,TF^2}{2} - \frac{h_e{}'ld\,TF^2}{2} - \theta s''\frac{d\,TF^2}{2} - \theta w C'_{eo}\frac{d\,TF^2}{2} - \frac{\pi'\beta d\,T(1-F)^2}{2} - \hat{\pi}d\,(1-\beta)(1-F) \tag{11}$$

In the next step, we must find the optimum values for T and F by maximizing the $\Pi(T,F)$ function. For this reason we must take the partial derivative of $\Pi(T,F)$ in respect of F and T. First we take the partial derivative of $\Pi(T,F)$ in respect of F.

$$\frac{\partial \Pi}{\partial F} = d\left(s - C_p + \hat{\pi} - C_{ep}\right)(1-\beta) + d\beta\pi T - dFT\left[h' + lh_e{}' + \theta s'' + \theta w C'_{eo} + \beta\pi'\right] = 0 \tag{12}$$

For simplification of the relation we can define a new parameter (λ) that:

$$\lambda = h' + lh_e{}' + \theta s'' + \theta w C'_{eo} \tag{13}$$

Also we know

$$\pi_{(1)} = \left(s - C_p\right) + \hat{\pi} \tag{14}$$

Finally

$$\Rightarrow F = \frac{(1-\beta)\left(\pi_{(1)} - C_{ep}\right) + \beta\pi' T}{T(\lambda + \beta\pi')} \tag{15}$$

Next we take the partial derivative of $\Pi(T,F)$ in respect of T.

$$\frac{\partial \Pi}{\partial T} = \frac{A}{T^2} - \frac{h'd\,F^2}{2} - \frac{h_e{}'ld\,F^2}{2} - \theta s''\frac{d\,F^2}{2} - \theta w C'_{eo}\frac{d\,F^2}{2} - \frac{\pi'\beta d\,(1-F)^2}{2} = 0 \tag{16}$$

$$\Rightarrow \frac{2A}{T^2} = d\,F^2\lambda - \pi'\beta d\,(1-F)^2 \tag{17}$$

Then

$$T = \sqrt{\frac{2A}{d\,F^2\lambda - \pi'\beta d\,(1-F)^2}} \tag{18}$$

Substituting Equation (15) into this expression, we get after some algebra

$$T^* = \sqrt{\frac{2A(\lambda + \pi'\beta)}{\pi'\beta d\,\lambda} - \frac{(1-\beta)^2\left(\pi_{(1)} - C_{ep}\right)^2}{\pi'\beta\lambda}} \tag{19}$$

T^* for the EPQ model with partial backordering (SEPQ-PBO) must be at least as large as T^* for the basic SEPQ that is determined in the next section.

3-1- Modeling of the Basic SEPQ

T^* for the basic SEPQ model is obtained by maximizing total annual profit function as below;

$$\Pi_{SEPQ-Basic} = sd - C_p d - C_{ep}\, d - \frac{A}{T} - \frac{h' d\, T}{2} - \frac{h_e{}' l d\, T}{2} - \theta s'' \frac{d\, T}{2} - \theta w C'_{eo} \frac{d\, T}{2} \tag{20}$$

To find $T^*_{SEPQ-Basic}$ we must take the derivative of $\Pi_{SEPQ-Basic}$ in respect of T

$$\frac{d\Pi}{dT} = \frac{A}{T^2} - \frac{d}{2}\left(h' + l h_e{}' + \theta s'' + \theta w C'_{eo}\right) = 0 \tag{21}$$

$$\Rightarrow\ T^*_{SEPQ-Basic} = \sqrt{\frac{2A}{d\,\lambda}} \tag{22}$$

The same result obtained if we minimize total cost function that usually did not include production and emission of production costs because they are not depended to time. Total cost function can be determined as below (rely on Equations 20 and 13)

$$\begin{aligned} TC_{SEPQ-Basic} &= \frac{A}{T} + \frac{h' d\, T}{2} + \frac{h_e{}' l d\, T}{2} + \frac{\theta s'' d\, T}{2} + \frac{\theta a C'_{eo} d\, T}{2} \\ &= \frac{A}{T} + \frac{d\, T}{2}\left(h' + l h_e{}' + \theta s'' + \theta w C'_{eo}\right) = \frac{A}{T} + \frac{d\, T\lambda}{2} \end{aligned} \tag{23}$$

Substituting Equation (22) into this expression, we get after some algebra

$$TC_{SEPQ-Basic} = \sqrt{2dA\lambda} \tag{24}$$

Considering Equations (19) and (22) in order to satisfy the inequality $T^*_{SEPQ-PBO} \geq T^*_{SEPQ-Basic}$ finally gives the bound for β:

$$\frac{2A(\lambda + \pi'\beta)}{\pi'\beta d\lambda} - \frac{(1-\beta)^2\left(\pi_{(1)} - C_{ep}\right)^2}{\pi'\beta\lambda} \geq \frac{2A}{d\lambda} \tag{25}$$

$$\Rightarrow \frac{2A(\lambda + \pi'\beta) - d\,(1-\beta)^2\left(\pi_{(1)} - C_{ep}\right)^2}{\pi'\beta d\,\lambda} \geq \frac{2A}{d\,\lambda} \tag{26}$$

$$\Rightarrow 2A\lambda \geq d\,(1-\beta)^2\left(\pi_{(1)} - C_{ep}\right)^2 \tag{27}$$

$$\Rightarrow 1 - \beta \leq \sqrt{\frac{2A\lambda}{d\left(\pi_{(1)} - C_{ep}\right)^2}} \tag{28}$$

$$\Rightarrow \beta \geq 1 - \sqrt{\frac{2A\lambda}{d\left(\pi_{(1)} - C_{ep}\right)^2}} \tag{29}$$

We named the right hand of the final inequality β^*

$$\beta^* = 1 - \sqrt{\frac{2A\lambda}{d\left(\pi_{(1)} - C_{ep}\right)^2}} \tag{30}$$

3-2- Solution procedure

Based on Pentico et al. (2009), the procedure for determining the optimal values for *T*, *F*, *Q*, *I*, *S*, and *B* is:

Step 1. Calculate values of h', h_e', C'_{eo} and s'' from Equations (6) to (9) and determine λ from Equation (13).

Step 2. Determine β^* the critical value for β, from Equation (30).

Step 3. a. If $\beta \le \beta^*$, determine T^* from the basic SEPQ model using Equation (22), and using Equation (24), determine the total cost of basic SEOQ model that allows no stock-outs. Compare it with the cost of losing all demand, $(\pi_{(1)} - C_{ep})\, d$, to determine whether it is optimal to allow no stock-outs or all lost. We can calculate total profit (*TP*) for the basic SEOQ model from Equation (20).

b. If $\beta > \beta^*$, determine π' from Equation (10), then use Equation (19) to determine the value of T^* and then Equation (15) to calculate the value of F^*. We can calculate total profit (TP) from Equation (5).

Step 4. Determine the optimal values of the other variables as follows

a. for $\beta > \beta^*$, get T^* and F^* from *Step 3.b.* and use this relations:

$I^* = F^*\, d\, T^*(1 - d/P)$, $S^* = (1 - F^*)\, d\, T^*(1 - \beta\, d/P)$, $B^* = \beta\, S^*$, $Q^* = [F^* + \beta(1 - F^*)]\, d\, T^*$.

b. for $\beta \le \beta^*$, let $F^* = 1$ and get T^* from *Step 3.a.* then determine optimal values from relations above.

4- Numerical example

To illustrate the application of the solution procedure that has been given above, we will use the numerical example, which

d = 40 unit/year, P = 100 unit/year , s = 10 \$/unit, s' = 5 \$/unit, C_p = 7 \$/unit, A = 20 \$/order, h = 2.5 \$/unit, π = 3 \$/unit, $\hat{\pi}$ = 1 \$/unit, θ = 10 %, l = 1.7 m3/unit, w = 2 ton/unit, h_e = 0.55 \$/m3, C_{eo} = 13 \$/ton, C_{ep} = 0.3 \$/unit, $\pi_{(1)} = (s - C_p) + \hat{\pi}$ = 4 \$/unit

Step 1. First we must calculate values of C'_h, C'_{eh}, C'_{eo} and s'' from Equations (6) to (9) as below

h' = 2.5(1-40/100)=1.5

h_e' = 0.55(1-40/100)=0.33

C'_{eo} = 13(1-40/100)=7.8

s''= (10-5)(1-40/100)=3

And from Equation (13) we get

$\lambda = 1.5 + 1.7 \times 0.33 + 0.1\,(3) + 0.1 \times 2 \times 7.8 = 3.921$ \$/unit

Step 2. Applying Equation (30), we get $\beta^* = 1 - \sqrt{\frac{2*20*3.921}{40\,(4-0.3)^2}} = 0.465.$

Step 3.a. For $\beta \leq 0.465$: T^* determined using Equation (22) $T^* = \sqrt{\frac{2\times 20}{40\times 3.921}} = 0.505$. Then using Equation (13), if there are no stock-outs, the cost is calculated as $TC = \sqrt{2\times 40\times 20\times 3.921} =$ 79.2 \$ and if all sales are lost, the cost is (4-0.3) × 40 = 148 \$. Hence the optimal policy is to allow no stock-outs. Finally from Equation (20) total profit is calculated $\Pi_{SEOQ-Basic}$ = 28.794 \$.

b. For $\beta > 0.465$, for example $\beta = 0.5$, first we determine π' from Equation (10)*:*

π' = 3(1-0.5×40/100)=2.4

Then we determine the values of T^* and F^* from Equations (19) and (15)

$$T^* = \sqrt{\frac{2\times 20(3.921 + 2.4\times 0.5)}{2.4\times 0.5\times 40\times 3.921} - \frac{(1-0.5)^2(4-\ 0.3)^2}{2.4\times 0.5\times 3.921}} = 0.601$$

$$F^* = \frac{(1-0.5)(4-\ 0.3) + 0.5\times 2.4\times .0601}{0.601(3.921 + 0.5\times 2.4)} = 0.836$$

Step 4.

a. For $\beta > 0.465$, for example $\beta = 0.5$:

Total demand during a cycle = 40*0.601 = 24.33,

I^* = 0.836×40×0.601× (1-40/100) = 12.049,

S^* = (1 – 0.836) ×40×0.601× (1- 0.5×40/100) = 3.161,

B^* = 0.5×3.161 = 1.580

Q^* = 0.601×40(0.836+0.164×0.5) = 22.057.

b. For $\beta \leq 0.465$ let F^* = 1 and T^* = 0.505, for which

$B^* = S^*$ = 0,

I^* = 1×40×0.505× (1-40/100) = 12.12,

Q^* = Total demand during a cycle = $d\ T^*$ = 40 × 0.505 = 20.2.

5- Sensitivity analysis

At the next step, we have prepared a comprehensive sensitivity analysis for all parameters of SEPQ-PBO model in the range of -50% to +50% based on our numerical example. Results of sensitivity analysis are represented in Table (1). In exception of β parameter, all of other parameters sensitivity is analyzed by assuming $\beta=0.5$.

The highlighted rows in Table (1) show the conditions that β^* has a negative value, but still can be considered as a critical value for β and other values can be computed regularly.

As we see in Table (1) this model is very sensitive to s, C_p and P parameters and at the second place is sensitive to D and C_O. Also sensitive manner of S and B respect to β shows the importance of backordering strategy in SEPQ-PBO model.

Table 1. Sensitivity Analysis of SEPQ-PBO model parameters (based on numerical example)

		Variation Percent						
		T	F	I	S	B	Q	TP
β	-0.50	-15.95%	19.67%	0.59%	-100.00%	-100.00%	-8.42%	-1.59%
	-0.25	-15.95%	19.67%	0.59%	-100.00%	-100.00%	-8.42%	-1.59%
	+0.25	30.74%	-28.73%	-6.82%	201.54%	276.92%	20.84%	18.36%
	+0.50	44.28%	-42.47%	-17.00%	298.75%	498.13%	36.79%	45.74%
d	-0.50	95.46%	-46.78%	-30.65%	312.63%	312.63%	-23.08%	-102.07%
	-0.25	44.18%	-30.22%	-11.97%	242.82%	242.82%	-6.74%	-60.06%
	+0.25	-17.64%	19.67%	2.66%	-100.00%	-100.00%	12.16%	85.11%
	+0.50	-15.95%	19.67%	0.59%	-100.00%	-100.00%	37.37%	182.97%
P	-0.50	45.59%	19.67%	190.38%	-100.00%	-100.00%	58.62%	88.65%
	-0.25	-4.69%	19.67%	90.10%	-100.00%	-100.00%	3.84%	28.49%
	+0.25	6.24%	-3.95%	70.07%	59.46%	59.46%	4.33%	393.39%
	+0.50	8.96%	-5.42%	71.74%	73.75%	73.75%	6.27%	469.04%
s	-0.50	67.46%	-86.25%	-61.62%	1026.97%	1026.97%	1.71%	-471.66%
	-0.25	68.01%	-56.80%	20.96%	716.35%	716.35%	24.57%	-260.51%
	+0.25	-17.51%	19.67%	64.53%	-100.00%	-100.00%	-10.12%	335.06%
	+0.50	-18.99%	19.67%	61.58%	-100.00%	-100.00%	-11.73%	671.80%
s'	-0.50	2.38%	-4.47%	63.01%	57.07%	57.07%	0.30%	-4.16%
	-0.25	1.22%	-2.30%	64.82%	41.31%	41.31%	0.16%	-2.11%
	+0.25	-1.28%	2.44%	68.53%	8.11%	8.11%	-0.19%	2.19%
	+0.50	-2.64%	5.03%	70.43%	-9.38%	-9.38%	-0.41%	4.46%
Cp	-0.50	-15.95%	19.67%	67.65%	-100.00%	-100.00%	-8.42%	476.90%
	-0.25	-15.95%	19.67%	67.65%	-100.00%	-100.00%	-8.42%	237.65%
	+0.25	56.69%	-47.75%	36.44%	571.31%	571.31%	22.63%	-190.44%
	+0.50	73.47%	-69.71%	-12.44%	885.20%	885.20%	18.42%	-350.75%
A	-0.50	-40.56%	19.67%	18.55%	-100.00%	-100.00%	-35.24%	77.70%
	-0.25	-27.21%	19.67%	45.19%	-100.00%	-100.00%	-20.69%	34.68%
	+0.25	32.43%	-17.62%	81.82%	213.80%	213.80%	21.81%	-24.47%
	+0.50	58.35%	-26.52%	93.94%	364.71%	364.71%	39.24%	-44.04%
h	-0.50	-6.53%	19.67%	86.43%	-100.00%	-100.00%	1.84%	25.67%
	-0.25	-7.18%	13.90%	76.22%	-65.97%	-65.97%	-1.30%	11.79%
	+0.25	5.56%	-10.35%	57.71%	101.38%	101.38%	0.58%	-9.90%
	+0.50	10.00%	-18.48%	49.45%	166.69%	166.69%	0.75%	-18.37%
π	-0.50	13.73%	-12.45%	65.95%	132.14%	132.14%	7.29%	1.16%
	-0.25	4.78%	-4.71%	66.41%	62.32%	62.32%	2.53%	0.42%
	+0.25	-2.98%	3.18%	66.84%	1.70%	1.70%	-1.58%	-0.28%
	+0.50	-5.02%	5.47%	66.96%	-14.26%	-14.26%	-2.65%	-0.47%
$\hat{\pi}$	-0.50	22.79%	-21.28%	61.11%	219.48%	219.48%	10.90%	8.97%
	-0.25	12.39%	-12.26%	64.35%	128.01%	128.01%	6.12%	3.74%
	+0.25	-15.24%	18.67%	67.65%	-94.59%	-94.59%	-8.03%	-1.59%
	+0.50	-15.95%	19.67%	67.65%	-100.00%	-100.00%	-8.42%	-1.59%
h_e	-0.50	-5.17%	9.95%	73.77%	-41.41%	-41.41%	-0.88%	8.61%
	-0.25	-2.46%	4.68%	70.18%	-7.07%	-7.07%	-0.38%	4.16%
	+0.25	2.23%	-4.20%	63.24%	55.05%	55.05%	0.28%	-3.90%
	+0.50	4.28%	-7.99%	59.91%	83.29%	83.29%	0.48%	-7.56%
Ceo	-0.50	-6.09%	19.67%	87.31%	-100.00%	-100.00%	2.32%	26.83%
	-0.25	-7.51%	14.57%	76.61%	-69.98%	-69.98%	-1.38%	12.31%
	+0.25	5.75%	-10.71%	57.37%	104.18%	104.18%	0.60%	-10.26%
	+0.50	10.32%	-19.06%	48.82%	171.51%	171.51%	0.75%	-18.99%
Cep	-0.50	-8.71%	10.06%	67.46%	-44.27%	-44.27%	-4.53%	19.23%
	-0.25	-4.21%	4.69%	67.13%	-8.80%	-8.80%	-2.17%	9.51%
	+0.25	3.96%	-4.15%	66.09%	57.35%	57.35%	2.00%	-9.32%
	+0.50	7.71%	-7.86%	65.41%	88.40%	88.40%	3.85%	-18.47%
θ	-0.50	-3.76%	19.67%	91.95%	-100.00%	-100.00%	4.86%	32.68%
	-0.25	-9.23%	18.03%	78.57%	-90.54%	-90.54%	-1.78%	14.98%
	+0.25	6.71%	-12.47%	55.67%	117.96%	117.96%	0.65%	-12.05%
	+0.50	11.84%	-21.82%	45.74%	194.83%	194.83%	0.73%	-22.02%
w	-0.50	-6.09%	19.67%	87.31%	-100.00%	-100.00%	2.32%	26.83%
	-0.25	-7.51%	14.57%	76.61%	-69.98%	-69.98%	-1.38%	12.31%
	+0.25	5.75%	-10.71%	57.37%	104.18%	104.18%	0.60%	-10.26%
	+0.50	10.32%	-19.06%	48.82%	171.51%	171.51%	0.75%	-18.99%
l	-0.50	-5.17%	9.95%	73.77%	-41.41%	-41.41%	-0.88%	8.61%
	-0.25	-2.46%	4.68%	70.18%	-7.07%	-7.07%	-0.38%	4.16%
	+0.25	2.23%	-4.20%	63.24%	55.05%	55.05%	0.28%	-3.90%
	+0.50	4.28%	-7.99%	59.91%	83.29%	83.29%	0.48%	-7.56%

Conclusions and Further Research

In this work we have developed a new sustainable EPQ model that considered partial backordering. The model was named as SEPQ-PBO. We applied a direct accounting approach for considering sustainability issues like emission cost of inventories (including: holding, obsolescence and transportation) in addition of ordinary inventory costs. Also we used the approach of Pentico et al. (2009) for modeling the partial backordering in EPQ problem. This new model can be used widely because of its sensible design as well as its simple application procedure and computations.

There are a number of future research directions to enhance the model. First it can be compared with other sustainability approaches like Carbon Tax, Direct Cap, Cap & Trade and Carbon Offsets. Different transportation modes can be taken into account. Finally, briefly speaking, all of past pure economic inventory models can be changed to a sustainable model by considering sustainability issues.

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