

Solving a Location-Allocation problem by a fuzzy self-adaptive NSGA-II

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Abstract

This paper proposes a modified non-dominated sorting genetic algorithm (NSGA-II) for a bi-objective location-allocation model. The purpose is to define the best places and capacity of the distribution centers as well as to allocate consumers, in such a way that uncertain consumers' demands are satisfied. The objectives of the mixed-integer nonlinear programming (MINLP) model are to (1) minimize the total cost of the network and (2) maximize the utilization of distribution centers. To solve the problem, a fuzzy modified NSGA-II with local search is proposed. To illustrate the results, computational experiments are generated and solved. The experimental results demonstrate that the performance metrics of the fuzzy modified NSGA-II is better than the original NSGA-II.

Keywords: Location-Allocation, fuzzy rule base, multi-objective evolutionary algorithm.

1- Introduction

Supply chain network design (SCND) models have been noticed by many researchers as an essential problem in determining the supply chain's structure. Multiplicities of decisions affect the SCND. Supply chain (SC) decisions involve strategic (i.e., location of facilities and capacity of facilities), tactical (i.e., flow of products, transportation mode and inventory) and operational (i.e., fulfilment of customer demands and pricing) decision levels. A key determination connected to the design and performance of a SC is the definition of the optimal or near-optimal locations for distribution centers (DC). Some investigations have been noticed inventory management models in the SCND as a location-inventory (L-I) problems (Mousavi et al., 2015; Salehi et al., 2015; Kaya and Urek, 2016; Puga and Tancrez, 2016; Zahiri et al., 2018).

Ahmadi et al. (2016) presented a bi-objective model that maximizes the total profit and minimizes the customer dissatisfaction. They proposed a three-level SC considering L-I decisions with capacitated in-house fleet, proactive lateral-transhipments between facilities and multiple products. Mehrabad et al. (2017) developed a four-echelon SC with multi-objective model in order to optimize the total cost of the network and the finishing rate. Variables and parameters of their research are related with transportation, manufacturer's location, distribution of the products from manufacturers

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to DCs, and inventories. They proposed a multi-objective evolutionary meta-heuristic algorithm based on the particle swarm. Soolaki and Arkat (2017) considered a location-allocation problem and integrated cellular manufacturing system into a three-level SC. Yadegari et al. (2018) extended a mathematical closed-loop SCND model with the multi-period, multi-echelon and considering inventory cost. They developed a Memetic algorithm by combinatorial local search method based on the nature of the multi-part solution representation.

Zheng et al. (2019) studied a single product L-I problem by considering routing in SC. In their model demands were considered independent and uncertain which follow the normal distribution. The objective function is to minimize the total cost (such as installation cost, inventory cost, and transportation cost). They introduced real-world constraints into the integrated model and developed an exact solution method based on the Generalized Benders Decomposition to solve the mathematical model. Zhen et al. (2018) developed a mathematical closed-loop SCND under uncertainty to determine the location of DCs and their capacities and define the flow of goods in forward and reverse directions. They converted the stochastic non-linear model to a conic quadratic mixed integer programming model and solved it by CPLEX.

In general, the SCND has been considered as a single-objective problem; however, it involves more than one objective. Multi-objective evolutionary algorithms (MOEA) have been considered for solving multi-objective mathematical models. One of the first MOEA is the non-dominated sorting genetic algorithm (NSGA) developed by Srinivas and Deb (1994) finding a set of Pareto-optimal solutions. Deb, et al., (2002) extended and improved the NSGA with the concept of the crowding distance, namely NSGA-II, which ranks and selects the population fronts. Like the genetic algorithm (GA), the performance of NSGA-II is affected by its parameters; therefore, a fuzzy modified NSGA-II with local search, namely FMNSGA-II, is proposed to solve the presented model. The remainder of this paper is organized as follows. Section 2 presents the multi-objective model for the SCND problem. Section 3 proposes the FMNSGA-II. The numerical outcome and computational results are presented in section 4. Finally, section 5 presents conclusions.

2- Problem description and formulation

The considered supply chain network consists of a fixed location manufactory, distribution centres (i.e., warehouses) and retailers (i.e., customers). The demands of customers are stochastic. The proactive lateral transshipment policy with partial pooling can be used through DCs. Accordingly, in the real world; the model is intended for supply chains of industries, such as clothing, pharmaceutical and food.

2-1- Notations

The following notations are used in order to model the given problem.

Indices:

- m Set of consumers ($m=1, \dots, M$)
- n, h Set of candidate DCs ($n, h=1, \dots, N$)
- P Set of products ($l=1, \dots, P$)
- k Set of capacity levels ($k=1, \dots, K$)

Parameters:

- T_{mnp} Transportation cost from DCs to consumers
- \bar{T}_{np} Transportation cost from the manufactory to DCs
- LC_{nhp} Transportation cost between DCs
- F_{nk} Installation cost of DCs
- d_{mp} Mean of the demand
- v_{mp} Variance of the demand
- L_{np} Lead time from the manufactory to DCs

H_{np}	Keeping cost
R_{np}	Fixed order cost
cap_{mk}	DCs' Capacity
s_p	Space requirement of products
PH	Planning horizon

Decision variables:

Z_{nk}	1 if the nk -th DC is opened; 0, otherwise
Y_{mnp}	1 if the m -th consumer is assigned to the n -th DC for the p -th product; 0, otherwise
\bar{Y}_{nhp}	1 if the j -th DC is served with the h -th DC for the p -th product; 0, otherwise
D_{np}	Mean demand of the product number p that covered to the n -th DC from the manufactory
DT_{np}	Mean demand of the p -th product that covered to the n -th DC from other DCs
V_{np}	Variance of demand of the product number p covering to the n -th DC
a_{nhp}	Percentage demands of the p -th product that covered to the n -th DC from the h -th DC

2-2- Mathematical model

Inventory is one of the significant elements of a SC. To determine the inventory cost of the network, the continuous inventory revision (r, Q) is considered. So, by considering the inventory system we can calculate the service level as:

$$Prob(\mu_D(L_{np}) \leq r_{np}) = 1 - \alpha \quad (1)$$

According to the (1), the reorder point of the product number p in the n -th DC is $r_{np} = \mu_D + SS_{np}$, where SS_{np} is the safety stock of the inventory system, and the mean demand of the product number p is μ_D , assigned to the n -th DC. When the demand is stochastic (based on a normal distribution function with a given probability $1 - \alpha$); SS_{np} can be determined by:

$$SS_{np} = Z_{1-\alpha} \cdot \sqrt{L_{np}} \cdot \sqrt{V_{np}} \quad (2)$$

The variance of demand of the product number p covering to the n -th DC is obtained by:

$$V_{np} = \sum_{m=1}^M \sum_{h=1}^N (1 - a_{nhp} \cdot \bar{Y}_{nhp})^2 \cdot v_{mp} \cdot Y_{mnp} + \sum_{m=1}^M \sum_{h=1}^N a_{hmp}^2 \cdot v_{mp} \cdot \bar{Y}_{hmp} \cdot Y_{mhp} \quad (3)$$

According to the basic economic ordering quantity (EOQ) inventory model, the inventory system's whole cost is calculated based on equation (4). The mean demand of the product number p that covered to the n -th DC from the manufactory (D_{np}) is calculated based on equation (5).

$$\sum_{n=1}^N \sum_{p=1}^P \sqrt{2 \cdot R_{np} \cdot \mu_{Dnp} \cdot H_{np}} + H_{np} \cdot SS_{np} \quad (4)$$

$$D_{np} = \sum_{m=1}^M d_{mp} \cdot Y_{mnp} + \sum_{m=1}^M \sum_{h=1}^N a_{hmp} \cdot d_{mp} \cdot \bar{Y}_{hmp} \cdot Y_{mhp} - DT_{np} \quad (5)$$

where DT_{np} is the mean demand of the p -th product that covered to the n -th DC from other DCs, and calculated by:

$$DT_{np} = \sum_{m=1}^M \sum_{h=1}^N a_{nhp} \cdot d_{mp} \cdot \bar{Y}_{nhp} \cdot Y_{mnp} \quad (6)$$

So the mean demand of the p -th product, assigned to the n -th DC is calculated by $\mu_{Dnp} = D_{np} + DT_{np}$ and with the change in the notations, the inventory system's whole cost can be rewritten by:

$$\begin{aligned} & \sum_{n=1}^N \sum_{p=1}^P \sqrt{2 \cdot H_{np} \cdot R_{np}} \cdot \sqrt{(D_{np} + DT_{np})} \\ & + \sum_{n=1}^N \sum_{p=1}^P H_{np} \cdot Z_{1-\alpha} \cdot \sqrt{L_{np}} \cdot \sqrt{V_{np}} \end{aligned} \quad (7)$$

By adding the costs of DC installing with a planning horizon (PH) to variable costs and considering the average utilization DCs as the second objective, the mathematical multi-objective model of the SCND problem through the lateral transshipment policy is as follows:

$$\text{Min } W_1 = \sum_{n=1}^N \sum_{k=1}^K F_{nk} \cdot Z_{nk} + PH \cdot \sum_{n=1}^N \sum_{p=1}^P \bar{T}_{np} \cdot D_{np} + PH \cdot \sum_{m=1}^M \sum_{n=1}^N \sum_{p=1}^P T_{mnp} \cdot d_{mp} \cdot Y_{mnp} \quad (8)$$

$$\begin{aligned} & + PH \cdot \sum_{m=1}^M \sum_{n=1}^N \sum_{h=1}^N \sum_{p=1}^P LC_{nhp} \cdot a_{nhp} \cdot d_{mp} \cdot \bar{Y}_{nhp} \cdot Y_{mnp} + PH \cdot \sum_{n=1}^N \sum_{p=1}^P \sqrt{2 \cdot H_{np} \cdot R_{np}} \cdot \sqrt{(D_{np} + DT_{np})} \\ & + PH \cdot \sum_{n=1}^N \sum_{p=1}^P H_{np} \cdot Z_{1-\alpha} \cdot \sqrt{L_{np}} \cdot \sqrt{V_{np}} \\ \text{Max } W_2 & = \sum_{n=1}^N \frac{\sum_{p=1}^P s_p \cdot (D_{np} + DT_{np})}{\sum_{k=1}^K cap_{nk} \cdot Z_{nk}} / \sum_{n=1}^N \sum_{k=1}^K Z_{nk} \end{aligned} \quad (9)$$

s.t.

$$\sum_{n=1}^N Y_{mnp} = 1 \quad ; \forall m, p \quad (10)$$

$$\sum_{h=1}^N \bar{Y}_{nhp} \leq \sum_{m=1}^M Y_{mnp} \quad ; \forall n, p \quad (11)$$

$$\sum_{n=1}^N \sum_{p=1}^P \bar{Y}_{mnp} = 0 \quad (12)$$

$$\sum_{p=1}^P s_p \cdot (D_{np} + DT_{np}) \leq \sum_{k=1}^K cap_{nk} \cdot Z_{nk} \quad ; \forall n \quad (13)$$

$$\sum_{k=1}^K X_{nk} \leq 1 \quad ; \forall n \quad (14)$$

$$Z_{nk}, Y_{mnp}, \bar{Y}_{nhp} \in \{0, 1\} \quad ; \forall m, n, p, k \quad (15)$$

$$DT_{np} \in Z \quad ; \forall n \quad (16)$$

Equations (10) and (11) show the single source assumption. Equation (12) ensures that there is no internal transshipment in any DCs. Equation (13) shows the capacity level constraint. Equation (14) warrants that each DC can be established on a unique capacity level. Finally, constraints (15) and (16) state the integer and real number variables, respectively.

3- Solution algorithm

The main components of the FMNSGA-II are described below.

3-1- Initialization

The proposed chromosome structure is shown in figure 1. Each chromosome consists of two sections. As shown in this figure, the first section consisting of J genes is related to location and capacity level decisions. Those genes can obtain content in $[0, k]$. The second section is related to allocation and lateral transshipment decisions and consists of L sub-sections for products, in which each sub-section consists of I part for customers whom each part has three genes. The first gene value

in each part (taking a value between $[0, J]$) is related to allocation variables. The second gene (taking a value between $[0, J]$) is related to lateral transshipment variables, and the third gene (taking a value between zero to 1) refers a constant of a_{jhl} variables.

3-2- Non-dominated sorting

To prepare a diverse front, a crowding distance (CD) is calculated for each solution as given in (17). The individuals with a lower value of a CD are preferred over the individuals with a higher value of a CD in a selection procedure.

$$CD_i = \sum_{k=1}^K \frac{|Z_k^{i+1} - Z_k^{i-1}|}{Z_k^{Max} - Z_k^{Min}} \quad (17)$$

The notation K in equation (17) is the number of objectives, Z_k^i is the k -th objective of the i -th individual, Z_k^{Max} is the maximum value of the k -th objective in the front, and Z_k^{Min} is the minimum value of the k -th objective in the front.

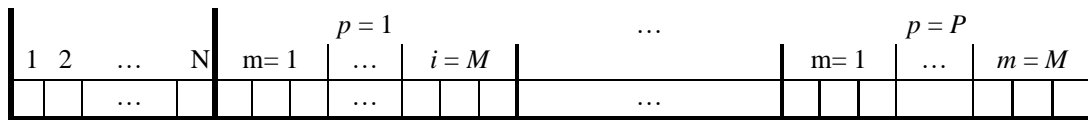


Fig. 1. Chromosome presentation

1. Start
2. Consider LS selected by a selection operator.
3. $i = 2$.
4. Until $i \leq 3$
 5. If $i = 2$ then
 6. Select randomly two genes from the second section of chromosome ($G1$ and $G2$).
 7. Replace the values of $G1, G1+1, G1+2$ by $G2, G2+1, G2+2$ and vice versa.
 8. If $i = 3$, then
 9. Select randomly three genes from the second section of chromosome ($G1, G2$ and $G3$).
 10. Replace Values of $G1, G1+1, G1+2$ by $G3, G3+1, G3+2$ and $G2, G2+1, G2+2$ by $G1, G1+1, G1+2$ and $G3, G3+1, G3+2$ by $G2, G2+1, G2+2$.
 11. Evaluation objective functions of new chromosome.
 12. If LS dominated by a new chromosome, then
 13. Replace LS by a new chromosome.
 14. Else
 15. $i = i+1$
 16. Go to line 4
17. LS is the new solution that generate by local search operator.
18. End.

Fig. 2. Local search algorithm

3-3- Crossover, mutation and local search

The tournament selection operator is used to create the offspring population. Then, the crossover operator with respect to the crossover rate (P_c) generates new children. After applying the crossover operator, more solutions are created by using the mutation operator with respect to the mutation rate (P_m). In contrast to the standard NSGA-II, one of the differences between the FMNSGA-II and the

NSGA-II is the 2-opt and 3-opt local search operators as described in Fig. 2. A local search rate (P_{ls}) indicates how often this operator will be performed.

3-4- Fuzzy adaptive operators

To improve the performance and quality of the proposed algorithm, a fuzzy inference system (FIS) is used to adapt P_c , P_m and P_{ls} dynamically. Thus, a three-input, three-output, eight-rule FIS is developed. The Mamdani type is applied as an inference process. Three functions are proposed as the inputs of the FIS (Sardou and Ameli, 2016). The first function is the total normalized fitness value of the best comprehensive solution (μ_{BCS}). The total normalized fitness value of the k -th solution (i.e., μ_k) is computed by:

$$\mu_k = \frac{\sum_{z=1}^Z \mu_k^z}{Z} \quad (18)$$

where μ_k^z is the normalized value of the z -th objective function for the k -th solution, which is calculated by:

$$\mu_k^z = \begin{cases} \frac{\text{Max}(F^z) - F_k^z}{\text{Max}(F^z) - \text{Min}(F^z)} & \text{For minimum objectives} \\ \frac{F_k^z - \text{Min}(F^z)}{\text{Max}(F^z) - \text{Min}(F^z)} & \text{For maximum objectives} \end{cases} \quad (19)$$

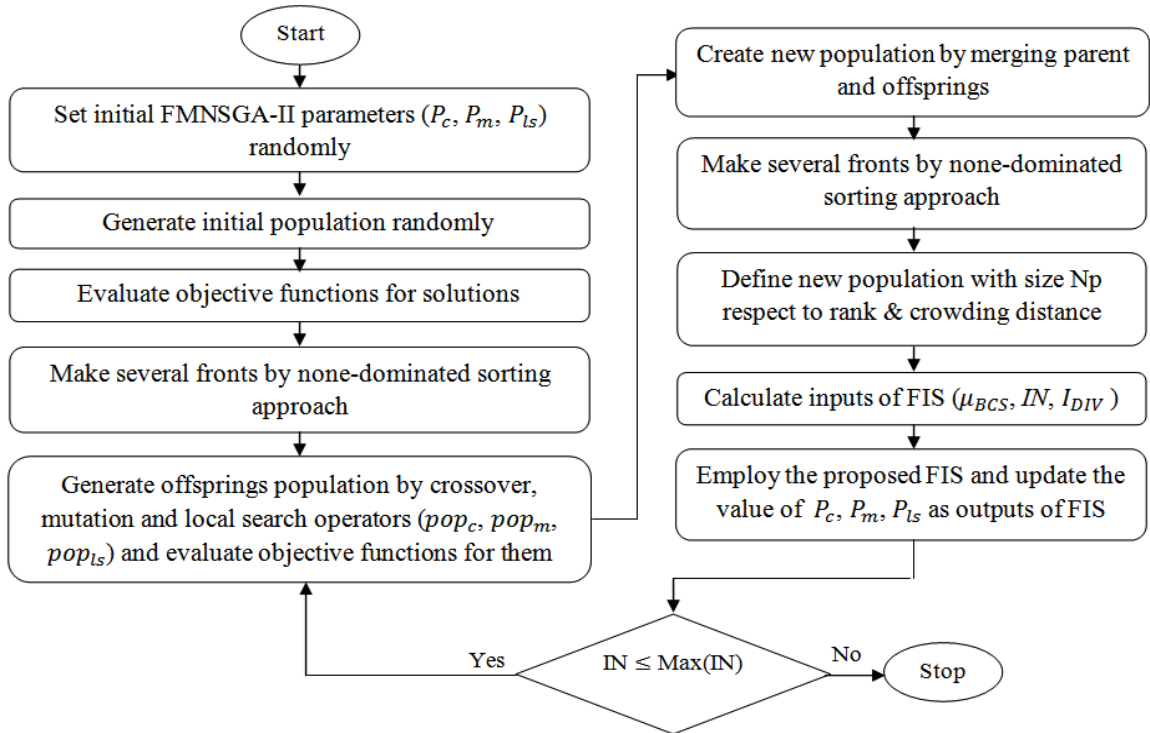


Fig. 3. Flowchart of the FMNSGA-II

where F_k^z denotes the real value of the z -th objective function in the k -th efficient solution. The second function (as the input of the FIS) is the iteration number (IN), and the third function is the index of diversity (I_{DIV}) that is equal to the summation of the CD of each solution. The value of μ_{BCS} is expected to improve in each iteration. Therefore, if μ_{BCS} does not get better impressively over a number of iterations (IN), then for the next iteration, IN is considered to affect the value of p_c , p_m and p_{ls} . Along with convergence to the Pareto-optimal set, it is also desired that the algorithm ensures the

diversity along the non-dominated front. Hence, I_{DIV} is taken into account to make changes in p_c , p_m and p_{ls} . Fig. 3 shows a graphical representation of the FMNSGA-II for the multi-objective SCND. The membership functions of three inputs and three outputs shown in the figures 4 to 9, respectively. The fuzzy rules are shown in Table 1. In addition, the centroid calculation is applied as a defuzzification method.

Table 1. Fuzzy rule database for the rate of operators

Rule number	μ_{BCS}	IN	I_{DIV}	P_c	P_m	P_{ls}
1	-	High	High	High	Low	Low
2	-	High	Medium	Low	High	Medium
3	-	High	Low	Low	High	High
4	High	Medium	-	Medium	Medium	Low
5	Medium	Medium	-	High	Medium	Low
6	High	Low	-	High	Low	Low
7	Medium	Low	-	High	Medium	Low
8	Low	-	-	High	Low	Low

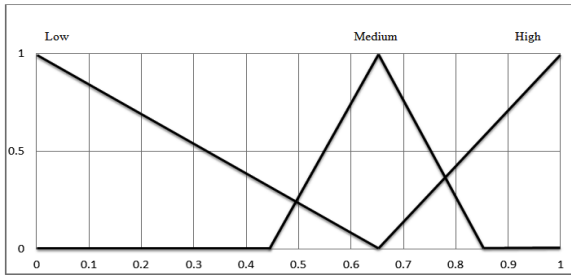


Fig. 4. Membership function of μ_{BCS}

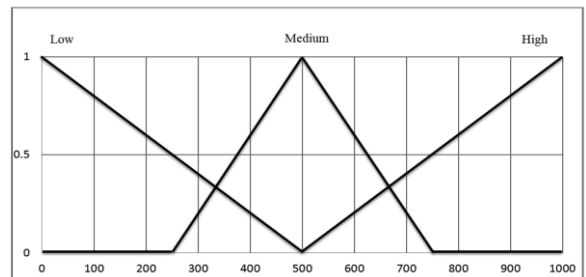


Fig. 5. Membership function of IN

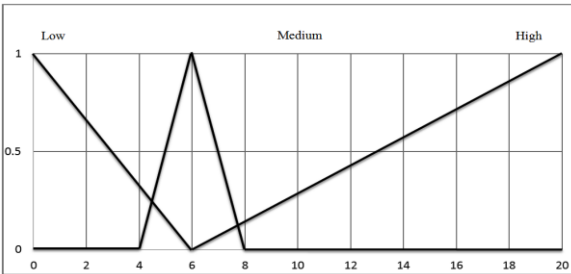


Fig. 6. Membership function of I_{DIV}

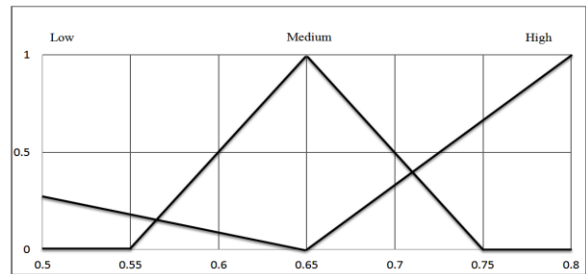


Fig. 7. Membership function of P_c

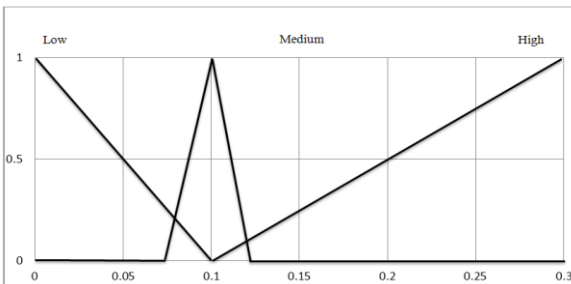


Fig. 8. Membership function of P_m

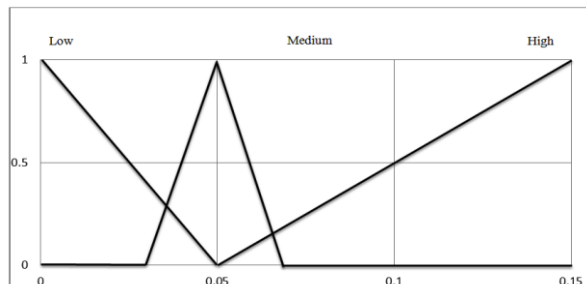


Fig. 9. Membership function of P_{ls}

Table 2. Test problems

Problem number	Consumers	Products	DCs	Constraints	Integer variables	Nonlinear variables
1	40	2	10	1451	1070	1310
2	40	3	10	2141	1580	1940
3	40	5	10	3521	2600	3200
4	45	2	12	1951	1452	1788
5	45	3	12	2884	2148	2652
6	45	5	12	4750	3540	4380
7	50	2	15	2756	2055	2565
8	50	3	15	4081	3045	3810
9	50	5	15	6731	5025	6300
10	75	2	20	5091	3940	4820

4- Computational results

As described at table 2 several different test problems, with various sizes developed as the computational experiments. Consumer's value and potential DC zones are from 40 to 75 and 10 to 20, respectively. The quantities of products are from {2,3,5} and capacity levels are from {1:5}. To obtain the performance of the proposed algorithm, the results of FMNSGA-II are compared with the original NSGA-II. The parameters of NSGA-II are defined by Taguchi technique (Sadeghi et al., 2014). Two algorithms are coded with Matlab2013, and run 10 times on a PC (i5-CPU at 2.67 GHz, 4.00 GB of RAM), in which the best run of each test problem is selected for comparison.

Table 3 shows comparison between the results. According to this table, three different performance metrics are used for the experiments (Deb, 2001) such as spacing (SP), generational distance (GD), spread (Δ). It is observed that the performance of the FMNSGA-II, except the execution times, is better than the NSGA-II with respect to the mean values of metrics.

Table 3. Comparison of the performance metrics

Problem number	NSGA-II				FMNSGA-II			
	SP	GD	Δ	Time	SP	GD	Δ	Time
1	0.0136	0.0181	0.9824	577	0.0111	0.0146	0.8248	673
2	0.0078	0.0391	1.0577	719	0.0061	0.0244	0.9112	871
3	0.0024	0.0318	1.0101	987	0.0006	0.0014	1.0021	1199
4	0.0049	0.0437	1.0337	755	0.0086	0.0422	1.0058	906
5	0.0049	0.0130	0.9787	994	0.0061	0.0133	1.0390	1221
6	0.0050	0.0570	1.0254	1435	0.0031	0.0085	1.0191	1778
7	0.0113	0.0204	0.9670	1088	0.0058	0.0110	1.0145	1321
8	0.0027	0.0057	1.0145	1462	0.0027	0.0181	1.0147	1759
9	0.0078	0.0250	0.6852	2218	0.0050	0.0056	0.8436	2866
10	0.0076	0.0225	1.0126	2356	0.0119	0.0202	0.7503	2937
Ave.	0.0068	0.0276	0.9767	1259	0.0061	0.0159	0.9425	1553

5- Conclusions

A multi-objective SCND problem was considered to determine the location of DC sites and flows of goods through the chain. In addition, the proactive lateral transshipment policy with partial pooling was considered. It is found that the performance of the NSGA-II is dependent on its parameters. So a fuzzy inference system can be used to create a more efficient algorithm. Therefore, FMNSGA-II proposed to solve the MINLP model. The FMNSGA-II parameters were self-adaptive via the fuzzy inference system. To determine the efficiency of the proposed algorithm, 10 different problems with various sizes were generated randomly. These test problems are solved by the FMNSGA-II and standard

NSGA-II, in which the results showed that the performance metrics which obtained by the FMNSGA-II are better than NSGA-II. It can be concluded that by improving the quality of the results in this method, in comparison with the previous method, the decision variables are determined, which will increase the utilization of the DCs and reduce the total cost of the chain. Reactive lateral transshipment policy can be considered as an extension of this paper for future research.

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