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Equilibrium pricing, routing and order quantity decisions in a three-level supply chain

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Abstract

In this paper, a supply chain, including a manufacturer, a distributor and some retailers, is considered. The manufacturer produces a single product and outsources distribution operations to a distributor in order to deliver products demanded by retailers. Each retailer has a time window to receive the products and faces the Newsvendor problem with stochastic demand. The manufacturer aims to serve retailers providing that the maximum lateness doesn't exceed a predetermined value. All players in the supply chain are willing to maximize their own profit. The model simultaneously includes pricing, order quantity and routing decisions. First, the manufacturer announces the whole sale price, then the distributor declares the unit transportation cost to the retailers, and finally each retailer decides on the amount of his order quantity. The profit functions of the players are formulated and linearized; then the solution is determined in three stages using game theory. Finally, a numerical example is presented and the equilibrium decisions of the players are determined using GAMS software.

Keywords: Three-level supply chain, pricing, routing, Newsvendor problem, game theory

1- Introduction and literature review

Inventory Routing Problem (IRP), the combination of inventory and routing decisions, is a complex optimization problem in logistics. The overall goal in IRP is to seek a solution which trades-off among the transportation costs, the inventory costs, lost sales costs at either a stochastic or deterministic environment (Andersson, 2006). For more information, please refer to the review paper by Coelho & Laporte (2013) on IRP models and Soysal et al. (2019) for a review on sustainable IRP. On the other hand, the pricing decisions can affect the inventory and order quantity decisions, which in turn affect the routing decisions. Such problems are in the field of inventory routing and pricing problems (IRPP) which are more complicated than IRPs (Sayarshad & Gao. 2018).

The inventory routing problem (IRP) was, firstly, introduced by Bell et al. (1983). They focused on gas distribution and determined routes, delivery quantities, visit times and used vehicles. Two excellent review articles on static and stochastic IRP are Kleywegt et al. (2002) and Coelho et al. (2014).

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The inventory routing problems were observed in various industries such as supermarket chains (Gaur and Fisher, 2004), maritime transportation industry (Al-Khayyal and Hwang, 2007), fuel and food distribution companies (Custódio and Oliveira, 2006, Vidović et al. 2014). According to Anderson et al (2010) and Etebari & Dabiri (2016), the criteria for IRPs include demand (deterministic or stochastic), fleet size (single or multiple), fleet (homogenous or heterogeneous), routing (direct or multiple), inventory (fixed or stocked out or lost sale or backorder), topology (one-to-one or one-to-many or many-to-many), planning horizon (single period or multi period).

There are few research articles in the literature that consider the inventory routing and pricing problems (IRPP). Adelman (2004) considered a stochastic inventory and routing problem that approximates the future costs of current actions by the optimal dual prices of a linear program. He obtained two such linear programs by formulating the control problem as a Markov decision process and then replacing the optimal value function with the sum of single-customer inventory value functions. Liu and Chen (2011) considered a pricing-routing-inventory problem and proposed a meta-heuristic algorithm for solving the problem according to each customer's offering prices. They considered a single period static pricing problem and did not incorporate the effects of supplying the demand during a period from other periods in the price optimization algorithm. Alaei & Setak (2015) studied a joint inventory-pricing-routing model in a two-level supply chain where a vendor distributes goods to buyers. They investigated the multi-objective coordination of the supply chain. The vendor determines wholesale price and routing decisions, while the retailers decide on the amount of order quantity. They proposed a revenue sharing contract for supply chain perfect coordination. Etebari & Dabiri (2016) studied an inventory routing problem under the dynamic regional pricing. They proposed a hybrid heuristic solution with five phases: initialization, demand generation, demand adjustment, inventory routing and neighborhood search which are embedded in a simulated annealing framework. They show that their proposed heuristic outperforms CPLEX by increasing problem size. Sayarshad & Gao. (2018), also, study a non-myopic dynamic inventory routing and pricing problem in a supply chain including suppliers and retailers. They propose a dynamic approach for a supplier who has to deliver products to a number of retailers while maximizing social welfare through dynamic pricing that accounts for customer waiting times, inventory holding, lost-sales costs, and delivery costs. They show that their proposed model increases the social welfare by up to 17% compared to the marginal pricing case.

In this paper, we formulate a three-level supply chain in order to simultaneously determine the pricing, routing and order quantity decisions. The model and the procedure to obtain the equilibrium are the main contributions of this research. The problem studied in this paper is a single product and a single period inventory-routing problem with pricing decisions. The supply chain includes a manufacturer, a distributor and some retailers. All players are willing to maximize their own profit: each retailer considers the revenue from sales, the lost sale costs and the leftover inventory costs; the manufacturer takes the revenue from sales and the lost sale costs into account; and the distributor considers the revenue from transportation and routing costs. The decision variables to be determined in the model are: the routes of vehicles, the arrival time to each retailer's location, the manufacturer's unit wholesale price, the retailers' order quantity and the distributor's unit transportation cost. The solution is determined in three stages using game theory. Finally, a numerical example is presented and the equilibrium decisions of the players are determined using GAMS software.

The remainder of this paper is organized as follows: in section 2, we formulate the profit functions of the retailers, the manufacturer and the distributor and their corresponding constraints. In section 3, we describe the solution approach to the problem. The solution is presented in three stages using game theory. A numerical study via an illustrative example is performed in section 4, and the equilibrium solution is presented. Finally, section 5 summarizes and concludes the paper.

2- Model description

In the following, we are going to introduce the profit functions of retailers, the manufacturer and the distributor.

2-1- Retailers' profit

Here, we assume that each retailer faces the Newsvendor problem and orders the amount of q_i to respond to the demand. The manufacturer incurs the procurement cost c per unit product, and then sells each product with wholesale price w to the distributor. The distributer adds the transportation cost c_t for each product and finally the retailers sell the product with retail price p. Moreover, we define g_r and g_s as the buyers' and the manufacturer's goodwill penalty cost per unit for lost sales, and v as the value earned per unit unsold at the end of the period. Suppose that the market demand of retailer i follows a uniform distribution with finite support $[a_i, b_i]$, i.e. $f_i(y) = 1/b_i - a_i$. For each retailer i, the expected sales, $S_i(q_i)$, the expected leftover inventory, $I_i(q_i)$, and the expected lost sales, $L_i(q_i)$, are as follows, respectively (Alaei & Setak, 2015).

$$S_i(q_i) = q_i - \int_{a_i}^{q_i} F_i(y)\,dy \tag{1}$$

$$I_i(q_i) = \int_{a_i}^{q_i} F_i(y)\,dy \tag{2}$$

$$L_i(q_i) = E(\mathrm{D}_i) - q_i + \int_{a_i}^{q_i} F_i(y)\,dy \tag{3}$$

Therefore, the retailer i's profit is given by:

$$\pi_i(q_i) = pS_i(q_i) + vI_i(q_i) - g_r L_i(q_i) - wq_i \tag{4}$$

Substituting (1-3) into (4) and considering $F_i(y) = (y - a_i)/(b_i - a_i);\ a_i \le y \le b_i$, each retailer i's profit can be simplified as:

$$\pi_i(q_i) = (p + g_r - w - c_t)q_i - (p - v + g_r)\frac{(q_i - a_i)^2}{2(b_i - a_i)} - g_r\frac{(a_i + b_i)}{2} \tag{5}$$

2-2- The manufacturer's profit

The manufacturer's profit can be calculated as a summation on the profit attained by all retailers minus the total lost sale costs. The manufacturer gains $(w-c)q_i$ from each retailer; so his profit function will be as follows.

$$\begin{aligned}\Pi_M &= (w-c)\sum_{i\in N} q_i - \sum_{i\in N} g_s L_i(q_i) \\ &= (w-c)\sum_{i\in N} q_i + \sum_{i\in N} g_s\left[q_i - \frac{(q_i - a_i)^2}{2(b_i - a_i)} - \frac{(a_i + b_i)}{2}\right]\end{aligned} \tag{6}$$

2-3- The distributor's profit

Most of the studies on inventory routing problems assume minimizing cost (Mirzapour & Rekik, 2013; Dehghani et al. 2017; Fattahi et al. 2017) and travel time (Li et al. 2014) as objective functions. However, some studies assume other types of objective functions such as maximizing the total profit, which equals to the selling revenue minus the supply chain cost (Li et al. 2017). We assume that the total profit to be the objective function of the considered problem. Suppose the set of retailers is given by set N. Each retailer $i \in N$ is characterized by demand q_i and time window $[et_i, lt_i]$. Serving retailer i cannot be started earlier than et_i, but it can be later than lt_i providing that the maximum lateness doesn't exceed L_{max}. The fixed cost for using each vehicle is denoted by F, and the manufacture's location is denoted by 0. The VRPTW is defined on a network created by the vertex set $V = 0 \cup N$ and the arc set A = (i, j), $\forall\, i, j \in V$. The traveling time and the traveling cost on arc $(i,j) \in A$ are denoted by t_{ij} and c_{ij}, respectively. To determine vehicle routes, a binary decision variable x_{ijk} is defined to take value 1 if vehicle $k \in K$ travels on arc $(i,j) \in A$. The decision variable c_t denotes the revenue gained for each unit transported to each retailer. Also, the decision variable s_{ik} denotes the arrival time of vehicle k to location $i \in V$. Hence, s_{i0} denotes the total travel time spent by vehicle k when it returns to the depot. Given this notation, the arc-flow formulation of the VRPTW is as follows.

$$Max \quad c_t \sum_{i \in N} q_i - \left[F \sum_{i \in N} \sum_{k \in K} x_{0i}^k + \sum_{(i,j) \in A} \sum_{k \in K} c_{ij} x_{ij}^k \right] \tag{7}$$

$$\sum_{i \in V} \sum_{k \in K} x_{ij}^k = 1 \qquad \forall j \in N \tag{8}$$

$$\sum_{i \in N} x_{0i}^k \le 1 \qquad \forall k \in K \tag{9}$$

$$\sum_{i \in V} x_{il}^k = \sum_{j \in V} x_{lj}^k \qquad \forall k \in K, l \in N \tag{10}$$

$$\sum_{i \in N} \sum_{j \in V} q_i x_{ij}^k \le Q \qquad \forall k \in K \tag{11}$$

$$t_{0i} - s_i^k \le M\left(1 - x_{0i}^k\right) \qquad \forall k \in K, i \in N \tag{12}$$

$$s_i^k + t_{ij} - s_j^k \le M\left(1 - x_{ij}^k\right) \qquad \forall k \in K, i \in N, j \in V \tag{13}$$

$$s_i^k \ge et_i \qquad \forall k \in K, i \in N \tag{14}$$

$$L_i \ge s_i^k - lt_i - M(1 - \sum_{j \in V} x_{ij}^k) \qquad \forall k \in K, i \in N \tag{15}$$

$$L_i \le L_{Max} \qquad \forall i \in N \tag{16}$$

$$c_t \le p - w \tag{17}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall k \in K, i, j \in V \tag{18}$$

$$c_t \ge 0 \tag{19}$$

$$s_i^k \ge 0 \qquad \forall k \in K, i \in V \tag{20}$$

$$L_i \ge 0 \qquad \forall i \in N \tag{21}$$

The objective function (7) maximizes sales revenue minus routing costs which includes vehicle fixed cost and traveling cost. Constraint (8) ensures that each customer is visited exactly once. By constraint (9), each vehicle is used at most once. The flow balance at each location is preserved by constraint (10), and the capacity of vehicles is controlled by constraints (11). Constraints (12) and (13) make the vehicle traveling on arc (i,j) to visit i before j. Constraint (14) guarantees that serving the retailer i cannot be started earlier than et_i; and the constraint (15) calculates each retailer's lateness. Constraint (16) limits the retailers' lateness to be smaller than L_{max}. Also, constraint (17) limits the transportation cost to be smaller than p-w. Finally, constraints (18-21) define the decision variables.

The distributer model in the current form is a mixed integer program (MIP) and contains a large number of variables and constraints even for small-size instances. The number of both binary variables and constraints are in the order of $|N|^2\,|K|$. Considering only 10 retailers and 6 vehicles, the problem will have 726 binary variables, 77 continuous variables, and 873 constraints which is hard to solve in a reasonable time for standard MIP solvers.

3- Solution approach

The assumed sequence of events is as follows. First, the manufacturer announces the whole sale price, w. Then, the distributor declares the unit transportation cost, c_t, to the retailers. Finally, each retailer decides on the amount of order quantity, q_i. Regarding the sequence of events, the solution for the model can be determined by backward induction. So, the distributor and the retailers play a Stackelberg game, where the distributor is the leader and the retailers act as followers. Another Stackelberg game is played between the manufacturer and the distributor, where the manufacturer is the leader and the distributor is the follower. The stages to determine the equilibrium are as follows.

Stage 1. *Determine the retailers' best response, as* $q_i^*(w, c_t)$, *for an arbitrate amount of* w *and* c_t.

Stage 2. *Optimize the distributor model taking the retailers' best response into account and determine the distributor's best response, as* $c_t^*(w)$.

Stage 3. *Optimize the manufacturer model regarding to* $q_i^*(w, c_t)$ *and* $c_t^*(w)$.

Note that the first stage can be determined analytically and it can be easily substituted in the distributor's model. But, the second and third stages need numerical study in which we search w and $c_t^*(w)$ to optimize the manufacturer's profit function.

- ***Stage 1***

In the following, **Proposition 1** determines the solution for stage 1.

Proposition 1. *Given the wholesale price, w, and unit transportation cost,* c_t, *the optimal response of each retailer is as below:*

$$q_i^* = A_i + B_i c_t \quad , i \in N,$$

where

$$A_i = \frac{a_i\left(w-v\right)+b_i\left(p+g_r-w\right)}{p+g_r-v}$$

$$B_i = \frac{-\left(b_i-a_i\right)}{p+g_r-v}.$$

Considering equation (5), it can be easily proved by the first order necessary condition, $\partial \pi_i / \partial q_i = 0$, which fulfills the second order condition for the maximization function, $\partial^2 \pi_i / \partial q_i^2 = 0$.

- ***Stage 2***

In the stage 2, for an arbitrary amount of w, the distributor model should be maximized in order to determine the optimal value of the transportation cost, $c_t^*(w)$. So, substituting q_i in **Proposition 1** into the model (7-21) will result in a non-linear objective function and capacity constraint that are rewritten as follows, respectively.

$$Max \quad \sum_{i\in N}\left[A_i c_t + B_i c_t^2\right] - \left[F\sum_{i\in N}\sum_{k\in K} x_{0i}^k + \sum_{(i,j)\in A}\sum_{k\in K} c_{ij} x_{ij}^k\right] \tag{22}$$

$$\sum_{i\in N}\sum_{j\in V} A_i x_{ij}^k + B_i c_t x_{ij}^k \le Q \qquad \forall k \in K \tag{23}$$

The objective function (22) and constraint (23) includes c_t^2 and $c_t x_{ij}^k$, respectively that make the model non-linear. As previously discussed, the distributor model, (7-21), is hard to solve in a reasonable time even in the linear form. The constraint (23) can be easily linearized by defining a new variable Q_{ij}^k, as the product of variables c_t and x_{ij}^k, and the following constraints.

$$\sum_{i \in N}\sum_{j \in V} A_i x_{ij}^k + B_i Q_{ij}^k \le Q \qquad \forall k \in K \tag{24}$$

$$Q_{ij}^k \ge c_t - M\left(1 - x_{ij}^k\right) \qquad \forall k \in K, i \in N, j \in V \tag{25}$$

Also, we assume that the transportation cost, c_t, to be a discrete variable. Since c_t is bounded to *p-w*, it can be rewritten as the summation of binary variables as below:

$$c_t = \sum_{m=0}^{\lambda} 2^m y_m \tag{26}$$

Where, the value of λ can be easily determined as $\lambda = \left[Ln(P-w)/Ln(2)\right]$. Also, the value of c_t^2 can be determined by defining new variables sc_t and $Y_{mm'}$ through the following constraints.

$$sc_t = \sum_{m=0}^{\lambda}\sum_{m'=0}^{\lambda} 2^{m+m'} Y_{mm'} \tag{27}$$

$$Y_{mm'} \le \frac{1}{2}\left(y_m + y_{m'}\right) \qquad \forall m, m' = 0,1,\ldots,\lambda \tag{28}$$

$$\frac{1}{2}\left(y_m + y_{m'} - 1\right) \le Y_{mm'} \qquad \forall m, m' = 0,1,\ldots,\lambda \tag{29}$$

Finally, the non-linear equation (22) can be rewritten as below.

$$Max \quad \sum_{i \in N}\left[A_i c_t + B_i \times sc_t\right] - \left[F \sum_{i \in N}\sum_{k \in K} x_{0i}^k + \sum_{(i,j) \in A}\sum_{k \in K} c_{ij} x_{ij}^k\right] \tag{30}$$

So, the new formulation of the distributor model with regard to the retailers' best responses will be as follows.

DC model:

$$Max \quad \sum_{i \in N}\left[A_i c_t + B_i \times sc_t\right] - \left[F \sum_{i \in N}\sum_{k \in K} x_{0i}^k + \sum_{(i,j) \in A}\sum_{k \in K} c_{ij} x_{ij}^k\right]$$

$$\sum_{i \in V}\sum_{k \in K} x_{ij}^k = 1 \qquad \forall j \in N$$

$$\sum_{i \in N} x_{0i}^k \le 1 \qquad \forall k \in K$$

$$\sum_{i \in V} x_{il}^k = \sum_{j \in V} x_{lj}^k \qquad \forall k \in K, l \in N$$

$$t_{0i} - s_i^k \le M\left(1 - x_{0i}^k\right) \qquad \forall k \in K, i \in N$$

$$s_i^k + t_{ij} - s_j^k \le M\left(1 - x_{ij}^k\right) \qquad \forall k \in K, i \in N, j \in V$$

$$s_i^k \ge et_i \qquad \forall k \in K, i \in N$$

$$L_i \geq s_i^k - lt_i - M(1-\sum_{j\in V} x_{ij}^k) \qquad \forall k \in K, i \in N$$

$$L_i \leq L_{Max} \qquad \forall i \in N$$

$$\sum_{i\in N}\sum_{j\in V} A_i x_{ij}^k + B_i Q_{ij}^k \leq Q \qquad \forall k \in K$$

$$Q_{ij}^k \geq c_t - M\left(1-x_{ij}^k\right) \qquad \forall k \in K, i \in N, j \in V$$

$$c_t = \sum_{m=0}^{\lambda} 2^m y_m$$

$$sc_t = \sum_{m=0}^{\lambda}\sum_{m'=0}^{\lambda} 2^{m+m'} Y_{mm'}$$

$$Y_{mm'} \leq \frac{1}{2}\left(y_m + y_{m'}\right) \qquad \forall m, m' = 0,1,\ldots,\lambda$$

$$\frac{1}{2}\left(y_m + y_{m'} - 1\right) \leq Y_{mm'} \qquad \forall m, m' = 0,1,\ldots,\lambda$$

$$c_t \leq p - w$$

$$x_{ij}^k \in \{0,1\} \qquad \forall k \in K, i, j \in V$$

$$y_m \in \{0,1\} \qquad \forall m = 0,1,\ldots,\lambda$$

$$Y_{mm'} \in \{0,1\} \qquad \forall m, m' = 0,1,\ldots,\lambda$$

$$Q_{ij}^k \geq 0 \qquad \forall k \in K, i, j \in V$$

$$sc_t, c_t \geq 0$$

$$s_i^k \geq 0 \qquad \forall k \in K, i \in V$$

$$L_i \geq 0 \qquad \forall i \in N$$

- ***Stage 3***

At this stage, for any *w* such that *c*<*w*<*P*, we solve the DC MODEL and determine $c_t^*(w)$. Then, using **Proposition 1**, we can calculate the order quantity $q_i^*(w, c_t^*)$ and the manufacturer's profit as $\Pi_M = (w-c)\sum_{i\in N} q_i$. Now, we can numerically determine the optimum value of the wholesale price *w*.

4- Illustrative example

In this section, we consider a supply chain including a manufacturer (depot), a distributor and 10 retailers. There are K=6 vehicles available to the distributer to serve the retailers. The information such as the depot's and retailers' coordinate, the retailers' demand as a uniform distribution, (a_i,b_i), and the retailers' time window, [et_i,lt_i], are given in table 1. Moreover, the maximum lateness, L_{max}, is 20 minutes; the fixed cost for using each vehicle, F, is 10000 monetary units; each vehicle's capacity, *Q*, is 450 units. The traveling time, t_{ij}, and the traveling cost, c_{ij}, on arc $(i,j) \in A$ are calculated as $c_{ij} = 500 + 5d_{ij}$ and $t_{ij} = 0.5d_{ij}$, respectively, based on the distance between nodes *i* and *j*, where d_{ij} is the Euclidean distance between *i* and *j*. Table 2 gives the complementary information of the market.

Table 1. The information of the depot and retailers

Node	Coordinates		Demand		Time Window	
	x	**y**	a_i	b_i	et_i	lt_i
Depot	25	30	-	-	-	-
1	95	32	30	700	58	464
2	81	23	45	950	6	466
3	25	51	60	800	84	472
4	14	59	25	780	23	342
5	61	52	80	1200	27	391
6	27	61	20	950	98	307
7	12	15	30	900	120	364
8	16	76	80	560	90	387
9	50	3	30	150	86	363
10	95	72	25	750	1	326

Table 2. Market information of the supply chain

Parameter	p	g_s	g_r	v	c
value	110	5	10	20	40

According to stage 3 of the previous section, for any w such that $40<w<110$, we solve the DC model and determine $c_t^*(w)$. Then, we calculate the values of decision variables as well as the players' profits. Note that, we limit the search procedure to the discrete values of the wholesale price, w. Table 3 shows the results. In the first row, for $40<w<45$, the manufacturer's profit is negative; similarly, the distributor's profit is negative for $69<w<110$; so, the optimal decisions will be inside the range of $45<w<69$. As shown in table 3, the manufacturer's profit takes its maximum value for w=60, which is the optimal decision for the manufacturer. The distributor's optimal decision for w=60 is ct^*=37.

The solution procedure is implemented in GAMS 23.5.1 with CPLEX solver. The experiment is executed on a computer with a 2.10 GHz processor and 4 GB RAM. Note that for each discrete value of the wholesale price, w, $40<w<69$, the average CPU time is 600 seconds. However, for each discrete value in 68<w<110, the problem is infeasible and the average CPU time is 0.1 seconds. Overall, the total CPU time for solving the problem is 17400 seconds.

Table 3. The results of the search procedure

w	ct^*	Distributor's Profit	Manufacture's Profit
40-45	-	-	Negative
46	51	43249	1806
47	51	40521	3954
48	49	32234	8808
49	48	37298	16201
50	48	33786	25101
51	46	29183	38883
52	45	27095	39679
53	45	29587	41654
54	43	23209	41642
55	42	23904	43252
56	42	27697	43373
57	40	31588	45462
58	40	33463	46925
59	40	31391	48328
60	**37**	**23729**	**49216**
61	37	27671	47006
62	35	25904	44978
63	35	23983	43777
64	32	22079	40181
65	32	28382	37977
66	31	12753	36503
67	31	5592	33848
68	31	1275	32047
69-110	-	Negative	-

Table 4 shows the optimal order quantities of the retailers and the arrival time to retailers' location. Moreover table 5 shows the optimal routes of vehicles.

Table 4. The optimal order quantities and arrival times

Node	Order quantity	Arrival time	Delay
1	184	484	20
2	253	476	0
3	230	492	20
4	199	362	20
5	338	411	20
6	234	327	20
7	230	384	20
8	190	407	20
9	58	383	20
10	192	334	0

Table 5. The optimal routes

Depot $\rightarrow$ 4 $\rightarrow$ 3 $\rightarrow$ Depot
Depot $\rightarrow$ 2 $\rightarrow$ 1 $\rightarrow$ Depot
Depot $\rightarrow$ 6 $\rightarrow$ 8 $\rightarrow$ Depot
Depot $\rightarrow$ 9 $\rightarrow$ 5 $\rightarrow$ Depot
Depot $\rightarrow$ 10 $\rightarrow$ 7 $\rightarrow$ Depot

5- Conclusion

In this paper, we studied the pricing, order quantity and routing decisions in a supply chain, including a manufacturer, a distributor and some retailers. The problem was investigated in a single period for a single product. The retailers face the Newsvendor problem with stochastic demand, and they demand the product in a particular time window. The manufacturer outsources distribution operations to a distributor in order to serve the retailers without exceeding a predetermined amount of time. The problem was formulated and solved using game theory in three stages, and equilibrium decisions were determined. Finally, a numerical example was presented and the equilibrium decisions of the players were determined using GAMS software. There are some directions for future research. In this paper, the discrete search procedure, to find the equilibrium, gives the approximate solution; so, one can design an evolutionary algorithm to get better solutions (refer to Azuma et al. 2011). We assume that each player independently aims to maximize his or her profit function; so, one can study the centralized setting in order to maximize the overall profit of the supply chain. Furthermore, designing a coordination contract can be very interesting.

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