

## **Designing a ride-sharing transportation system for assignment and transfer of passengers to a common destination**

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### **Abstract**

This paper proposes a mathematical model for ride-sharing vehicles with a common destination. A number of cars should assign to individuals by a company to pick up other participants in their way to the common destination. Traveling time as an important parameter is considered an uncertain parameter to enhance the applicability of the model which is formulated using fuzzy programming and necessity concept. Moreover, to have a better solution with better productivity, maximizing the earliest departure time of the individuals is considered beside of minimizing total traveling time. This helps to make justice among individuals for departure time. Goal programming is employed to work with objective functions and solve the model. Furthermore, a numerical example is implemented on the model to evaluate the applicability of the model which indicates the efficiency of employing fuzzy programming and considering both of the objective functions using goal programming. Results of the numerical example indicate the importance of considering both of the objective functions together in which ignoring each of them leads to inefficient solutions.

**Keywords:** Ride-sharing vehicles, mathematical modelling, fuzzy programming, goal programming.

### **1- Introduction**

The scheduling and the routing problem of vehicles play a key role in any system and these problems involve the practical aspects of the existed issues. Thus, considering these two factors are essential and also critical for any systems. It is worth mentioning that the scheduling and routing of service vehicles has a great impact on the quality of the provided services as well. Ridesharing is a type of transportation which passengers share a vehicle for a trip and it is a kind of system that considers flexibility and speed of vehicles at the same time (Furuhata et al., 2013). Generally, optimization of ridesharing vehicles involves solving a class of complex vehicle routing problems (VRP) together with pickup and delivery with time windows (VRPPDTW).

The passenger assignment problem considers the passenger behaviour to find the best choice in order whether to choose the current vehicle or wait for the next one (Binder et al., 2017). Transfer of passengers is an important market in many cases such as airlines, train stations which attempt to attract demand in a competitive market (de Barros et al., 2007). Employing ride-sharing can reduce costs, traveling time, and traffic which is important issues to decision makers. Moreover, companies can assign ride-sharing vehicles to their employees to increase their comfort.

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The rest of the paper is as follows. Section 2 review the related literature of the scheduling and routing problem of ridesharing vehicles and also the assignment and transfer of passengers. The problem statement is provided in section 3. The model description is given in section 4. In section 5, a numerical example is given in order to evaluate the model. The conclusion is presented in section 6, followed by some possible future research directions.

## 2- Literature review

This section reviews the literature by addressing the related problems of this field which are scheduling and routing problem of the ridesharing vehicles, the passenger assignment problem, and the problem of transferring of passengers.

Vehicle routing problem is a popular class of routing problems which is considered in many problems such as ride-sharing vehicles. As an example, an improvement in transportation efficiency of taxis has been given by Lin et al. (2012). In their research, a vehicle routing optimization problem of ridesharing vehicles is considered and then it is solved by a simulated annealing (SA) algorithm. The driver assignment vehicle routing problem is another type of the VRP in which drivers are assigned to customers before determining the demand of customers and after determining the demand a routing problem will be solved (Spliet & Dekker, 2016). Naoum-Sawaya et al. (2015) have proposed a stochastic model for the car placement problem in ridesharing systems. They have deployed mixed-integer linear programming (MILP) to model the issue which is transferring the participants to a common destination. Y. Hou et al. (2016) have investigated the application of electric taxis in a transportation network with transfer allowed ridesharing. They have introduced a mixed-integer programming (MIP) model together with a greedy heuristic algorithm to solve it.

Inventory routing problem is one of the expansions of the VRP which considers the routing problem together with the inventory management. Assigning vehicles to different routes and finding the optimal routes and also inventories are the other aspects of such a problem which can be found in Shaabani and Kamalabadi (2016) who have solved the model by a metaheuristic algorithm together with Lagrangian relaxation for the perishable products. X. Wang et al. (2016) have presented a new type of pickup and delivery problems with time windows in order to deal with passenger travel time under congestion. They also have considered the load-dependent toll cost. Their model was a zero-one integer programming which is solved by a heuristic algorithm. A new routing problem that considers environmental issues together with scheduling problem has been studied by Androutopoulos and Zografos (2017). Furthermore, to solve the model, they have proposed different techniques and also an ant colony optimization (ACO) algorithm. Furuhashi et al. (2013) have presented a comprehensive review of the ridesharing systems which can be useful for further studying about such a system.

An event-based simulation approach for transit assignment models has been proposed by Nuzzolo et al. (2016). They have formulated the traveller choice behaviour by considering the choice of origin departure time and first boarding stop. Tong et al. (2017) have investigated the optimization of the passenger to vehicle assignment and vehicle routing problem, simultaneously. They have provided five main stages in operating procedure namely, travel request submission, demand matching, service optimization, seat booking, and trip implementation. Binder et al. (2017) have suggested a new algorithm for the passenger assignment problem in public transport system which is flexible and high speed. Additionally, the shortest path model is used for the implementation of their framework. A new method has been introduced for the passenger assignment problem by Cao and Wang (2017) which considers the waiting time and the penalty of delays. Their method considers travel time, waiting time, delay, and economic cost. They have employed branch and bound solution approach based on the shortest path to solve the optimization problem.

Transfer of passengers is considered for different contexts. The airlines are more important due to the level of their costs. In this regard de Barros et al. (2007) have applied regression analysis to identify the transfer passenger facilities and services at the airports. Three heuristic algorithms have been introduced by Coltin and Veloso (2014) for transferring of passengers to create more efficiency in the ridesharing services and also reducing greenhouse gases emissions. Si et al. (2016) have provided a case study in which a class of transit assignment problems is solved. In their research, a classification for passengers is proposed which significantly has improved the existed results.

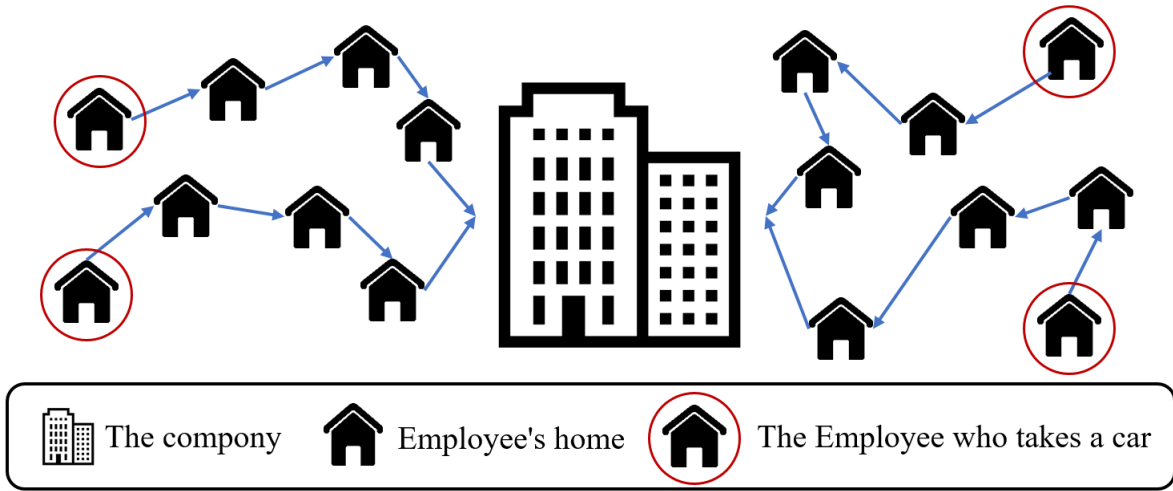
Jin et al. (2017) have considered the route choice behaviour of passengers in a metro system according to the discrete choice theory. Their route choice model consists of the level of service, socio-demographics, network knowledge, the path size factor, and the logical judgment. Y. Wang et al. (2018) have investigated the effects of taxi ride-sharing for individuals with similar origin and destination at nearly the same time of the day. Besides, scheduling has studied in a practical framework to reduce waiting and traveling times in the high demand periods. L. Hou et al. (2018) have studied the best routes for vehicles to match riders to drivers in a ridesharing problem aiming to maximization of the average loading ratio of the whole system. Besides a flow-dependent model is developed to analyse the effect of pick-up and drop-up congestion. Moreover, a large neighbourhood search algorithm is developed to deal with the complexity of the proposed model. Qadir et al. (2018) have proposed a framework for carpooling services named the highest aggregated score vehicular recommendation for the requesting passenger. Furthermore, they have proposed a heuristic to balance the incentives of both drivers and passengers. Peng et al. (2018) have suggested a stable matching model for ride-sharing aiming to the minimization of the commuters' cost of traveling. Designing payment of the system is performed according to the equity and incentive. Additionally, an algorithm is proposed to solve the model based on the deferred acceptance algorithm. Lokhandwala and Cai (2018) have studied taxi sharing by using agent-based modelling to investigate the potential benefits and drawbacks of it. Moreover, a case study of New York City is employed to evaluate the proposed model. Chen et al. (2019) have investigated a ride-sharing problem of scheduled commuter and business traffic with meeting points and return restrictions. A general integer linear program is developed for the problem, besides developing a constructive heuristic to solve it. Zhou (2019) has studied planning for ride-sharing service to supplement conventional transit services. Smartcard data is employed to utilize low-demand transit routes where ride-sharing services can be a supplement to or replacement of transit services along those routes. The current paper completes the mentioned paper and also contributes to the literature by considering the following points, simultaneously:

- 1) Considering the uncertainty of the traveling time
- 2) Dealing with the uncertainty using fuzzy programming
- 3) Considering a new objective function to enhance the productivity of model
- 4) Taking more than one individual in every point

Considering the uncertainty of the traveling time and dealing with it by fuzzy programming, increases the robustness of the proposed model. Besides, adding a new objective function to the proposed model for maximizing departure time of the participants reduces the waiting time of them which increases the satisfaction level of the participants. Additionally, the ability to take more than one individual in every point is considered in this research which besides of mentioned contributions makes the proposed model more capable to employ in the real world practices.

### **3- Problem statement**

Large companies tend to employ carpooling services due to cut costs and provide a suitable transportation system for employees. To achieve the optimal design of such an issue, the companies should find the best individuals to assign a car. It means that some employees take a car from the company to commute between their home and the company while they can take the other employees in their way to the company. Additionally, another issue is to find the best route for individuals who take a car from the company. The best route should be the shortest one while taking the most other employees to cut costs. To have a better understanding, figure 1 illustrates an example of the issue.



**Fig 1.** Example to illustrate the issue

This paper investigates the issue to find the optimal design which minimizes the total spending time of employees to commute between their home and the company. Additionally, maximizing the earliest departure time is considered to make justice and utility for participants. Ignoring this objective function may cause discontent with the departure time. Furthermore, scheduling of the vehicles is another issue of such systems which is studied in this paper. As traveling time is not certain and clears in the real world, this study considers traveling time uncertainty to enhance the practicability. Besides, fuzzy programming is employed to deal with uncertainty.

#### 4- Model description

In this section, first, fuzzy programming is discussed to have better insight about the uncertainty. After that, goal programming is explained to deal with having two objective functions. Finally, the mathematical model of the problem is proposed.

##### 4-1- Fuzzy programming

Due to living in an uncertain world, it is necessary to consider uncertainties in the mathematical programming models. This leads to having a better result in implementing mathematical models in real-world applications. Therefore, many researchers focused on dealing with uncertainty in mathematical models. Fuzzy programming is one of the approaches to work with uncertainties to obtain an optimal solution in decision-making problems (Bellman & Zadeh, 1970). It enables researchers to work with the uncertainties that can be defined by an interval considering probabilities. In other words, in fuzzy programming, the uncertain parameter can take a value in a certain interval which taking each value has a certain possibility.

Several studies are conducted towards enhancing the concept. Defining triangular fuzzy membership function is one of them which consider a specific value in the interval as the most possible value. The probability of values decreases from this probable value toward both the start and the end point of the interval (Buckley, 1988a). The necessity and possibility concepts are defined to adjust the constraints in a mathematical programming model and are utilized to create solutions (Buckley, 1988b). Having a better understanding of necessity and possibility concepts, they are discussed as follows.

Consider  $\tilde{a}$  as a fuzzy subset of  $\mathfrak{R}$  which is a set of real numbers. A fuzzy number is a fuzzy subset which has a membership function  $\mu_{\tilde{a}}: \mathfrak{R} \rightarrow [0, 1]$ . In the same way, consider  $\tilde{b}$  with the membership function  $\mu_{\tilde{b}}$  as another fuzzy number. Possibility and necessity can be defined as follows:

$$Pos(\tilde{a} * \tilde{b}) = \sup\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)), x, y \in \mathfrak{R}, x * y\} \quad (1)$$

$$Nes(\tilde{a} * \tilde{b}) = 1 - Pos(\overline{\tilde{a} * \tilde{b}}) \quad (2)$$

Where  $*$  can be one of the relations  $\leq, \geq, =, <, >$  (Dubois & Prade, 1983; Zadeh, 1978).

Dealing with constraints which have a triangular fuzzy number, an equivalent constraint can be written. Consider constraint  $A \geq \tilde{\xi}$  which  $\tilde{\xi}$  is a triangular fuzzy number and the least, the most possible, and the most values of it are  $\xi^{min}$ ,  $\bar{\xi}$ , and  $\xi^{max}$ , respectively. For satisfying the constraint at a confidence level  $\alpha$ , the following constraint is defined (Liu & Iwamura, 1998):

$$A \geq \alpha \xi^{max} + (1 - \alpha) \bar{\xi} \quad (3)$$

Equation (3) is employed in this study to deal with the uncertain triangular fuzzy number of the proposed model.

### 4-3- Goal programming

Due to having two objective functions, goal programming is employed to make an objective function which contains both of them. Goal programming was suggested by Charnes et al. (1955) to deal with multi-objective functions and was enhanced by Charnes and Cooper (1957). Goal programming methods attempt to minimize deviations of objective functions from their best value (Kornbluth, 1973). As goal programming is capable of managing a wide range of problems, it is chosen to deal with having two objective functions in the presented issue.

To calculate the deviations of objective functions from their best value, normalization is an important part and necessary to scale the goals onto a similar unit of measurement (Jones & Tamiz, 2010). Consider  $Z_1$  and  $Z_2$  as first and second objective functions, and  $Z_1^*$  and  $Z_2^*$  as the optimum value of the first and second objective function when the model is solved to optimize each of them. Solving the model to optimize  $Z_1$ ,  $Z_2$  reaches its nadir which is defined by  $Z_2^-$ .  $Z_1^-$  can be defined in the same way. The equations (4) and (5) calculate the deviations of objective functions from their best value when the first and second objective functions should minimize and maximize, respectively:

$$\frac{Z_1 - Z_1^*}{Z_1^- - Z_1^*} = d_1 \quad (4)$$

$$\frac{Z_2^* - Z_2}{Z_2^* - Z_2^-} = d_2 \quad (5)$$

Where  $d_1$  and  $d_2$  are the deviations of first and second objective functions from their best value, respectively which are positive variables. A new objective function can be defined as equation (6) to consider both mentioned objective functions.

$$Z^{new} = wd_1 + (1 - w)d_2 \quad (6)$$

Where  $w$  is the importance percent of the first objective function in regard to the second one. Minimizing  $Z^{new}$  optimizes both of the objective functions.

### 4-2- Mathematical model

At first, Notations which are used in the proposed model defines as follows:

**Table 1.** Indices and sets

$i$	Node index ( $i = 1, \dots, I$ )
$i, j$	Index pair, referring to arc from node $i$ to node $j$
$k$	Vehicles ( $k = 1, \dots, K$ )
$V$	Set of participants home locations with the exception of the vertex 0 which is set to the common destination
$E$	Set of available routes

**Table 2.** Parameters

$C$	Capacity of each vehicle
$t_i$	The number of individuals in node $i$ waiting to pick up
$E_i$	Earliest departure time for participant $i$
$L_i$	Latest allowed arrival time for participant $i$
$\bar{D}_{ij}$	Nominal travel time from node $i$ to node $j$
$D_{ij}^{max}$	Maximum travel time from node $i$ to node $j$
$\alpha$	Confidence level for fuzzy constraint
$Z_1^*$	Best value of first objective function when the model is solved to optimize $Z_1$
$Z_2^*$	Best value of second objective function when the model is solved to optimize $Z_2$
$w$	Importance percent of the first objective function in regard to the second one
$M$	A big number

**Table 3.** Decision variables

$Z_1$	First objective function to minimize expected travelling time
$Z_2$	Second objective function to maximize departure time of every participant
$Z^{new}$	New objective function to consider both objective functions
$Y_{ij}$	Binary variable indicates whether route $(i, j)$ is traveled by a vehicle or not
$Z_{ik}$	Binary variable indicates whether vehicle $k$ is assigned to node $i$ ( $i \neq 0$ ) or not
$\gamma_i$	Binary variable indicates whether participant $i$ takes own car to common destination or not
$u_i$	The number of individuals in the car when arrives to node $i$
$v_i$	The time when car departs from node $i$
$a_i$	The time when participant $i$ arrives to the destination
$d_1$	The deviation of first objective function from its best value
$d_2$	The deviation of second objective function from its best value

The objective functions of the proposed model are to minimize the total expected travel time and to maximize the earliest departure time of participants. Employing goal programming equations (7)-(11) are formulated to achieve goals as follows:

$$Z^{new} = wd_1 + (1 - w)d_2 \quad (7)$$

$$\frac{Z_1 - Z_1^*}{Z_1^- - Z_1^*} = d_1 \quad (8)$$

$$\frac{Z_2^* - Z_2}{Z_2^* - Z_2^-} = d_2 \quad (9)$$

$$Z_1 = \sum_{(i,j) \in E} \bar{D}_{ij} Y_{ij} \quad (10)$$

$$Z_2 \leq v_i, \quad \forall i \in V - \{0\} \quad (11)$$

Equation (12) guarantees that every participant departs from his/her home location.

$$\sum_{j|(i,j) \in E} Y_{ij} = 1, \quad \forall i \in V - \{0\} \quad (12)$$

Equation (13) states that each car can assign to just one individual.

$$\sum_{i \in V - \{0\}} Z_{ik} \leq 1, \quad \forall k \in K \quad (13)$$

Every individual should go toward the destination using his/her own car or vehicle of another individual or a vehicle that the company assigned to him/her which is declared in equation (14).

$$\gamma_i + \sum_{j|(j,i) \in E} Y_{ji} + \sum_{k \in K} Z_{ik} = 1, \quad \forall i \in V - \{0\} \quad (14)$$

Every individual that takes his/her own car must go directly to the destination which is shown in equation (15).

$$\gamma_i \leq Y_{i0}, \quad \forall i \in V - \{0\} \quad (15)$$

The capacity of vehicles is limited and is controlled by equations (16) and (17).

$$u_j \geq u_i + t_i - M(1 - Y_{ij}), \quad \forall (i,j) \in E, j \neq 0 \quad (16)$$

$$u_i \leq C - t_i, \quad \forall i \in V - \{0\} \quad (17)$$

Calculating the departure time and applying the earliest departure time are considered in equations (18) and (19). Equation (18) as a fuzzy constraint is formulated based on fuzzy programming which is defined by equation (3).

$$v_j \geq v_i + \alpha D_{ij}^{max} + (1 - \alpha) \bar{D}_{ij} - M(1 - Y_{ij}), \quad \forall (i,j) \in E, j \neq 0 \quad (18)$$

$$v_i \geq E_i, \quad \forall i \in V - \{0\} \quad (19)$$

Equations (20)-(22) are defined to enforce the latest arrival time of individuals.

$$a_i \geq v_i + \alpha D_{i0}^{max} + (1 - \alpha) \bar{D}_{i0} - M(1 - Y_{i0}), \quad \forall i \in V - \{0\} \quad (20)$$

$$a_i \geq a_j - M(1 - Y_{ij}), \quad \forall (i,j) \in E, j \neq 0 \quad (21)$$

$$a_i \leq L_i, \quad \forall i \in V - \{0\} \quad (22)$$

Equation (23) imposes binary and non-negativity conditions of decision variables.

$$Y_{ij}, Z_{ik}, \gamma_i \in \{0, 1\}, \quad u_i, v_i, a_i \geq 0, \quad \forall i \in V, \quad \forall (i, j) \in E, \quad \forall k \in K \quad (23)$$

### 5- Numerical example

Assessing the applicability of the proposed model, a numerical example is provided in this section. It is considered 40 places as pick-up points and 10 cars to commute between places and the common destination. The confidence level for fuzzy constraint ( $\alpha$ ) is considered 0.95 and importance percent of the first objective function in regard to the second one ( $w$ ) is considered 0.1. The earliest departure time and the latest allowed arrival time for all of the individuals are considered zero and 100, respectively. Nominal traveling time and maximum traveling time are generated randomly in an interval of [5, 10] and [6, 12], respectively, using uniform distribution function. The capacity of each vehicle is considered as four and the number of individuals in each point to pick up is generated randomly in an interval of [1, 4], employing uniform distribution function. Solving the model using an optimization software, gives us the following results:

**Table 4.** Results of the numerical example

Variable / Objective Function	$Z_1$	$Z_2$	$d_1$	$d_2$
$Z_1$	364.496	0		
$Z_2$	389.768	88.112		
$Z^{new}$	364.496	88.112	0	0

Solving the model considering  $Z_1$  and  $Z_2$  as the objective function, causes the earliest departure time of zero and 88.112 by spending 364.768 and 389.768 unit of time for traveling all of the individuals. This results in  $Z_1^* = 364.496$ ,  $Z_2^* = 88.112$ ,  $Z_1^- = 389.768$ , and  $Z_2^- = 0$ . On the other side, considering  $Z^{new}$  as the objective function leads to the earliest departure time of 88.112 by spending 364.768 unit of time for traveling all of the individuals. Obviously, considering  $Z^{new}$  as the objective function leads to the best departure time and total traveling time at the same time.

Furthermore, a realization of situations is investigated to evaluate the productivity of employing the fuzzy approach. In this case, two models have considered which one of them is the proposed model and the other one is a model with certain traveling time. The first and second model can be called fuzzy and deterministic model, respectively. Simulation is employed to evaluate the solutions of both models by defining two indexes. The first index is the total waiting time of individuals which can be calculated from the difference between the real and nominal departure time. The second index is the total delay time which can be calculated from the difference between the arrival and latest allowed arrival time. The results of the simulation are illustrated as follows:

**Table 5.** Results of realization

Model: Variable / Objective Function	Fuzzy		Deterministic	
	total waiting time	total delay time	total waiting time	total delay time
$Z_1$	0.326	19.652	22.639	98.003
$Z_2$	0	29.644	0	55.9
$Z^{new}$	0	29.644	0.414	56.944



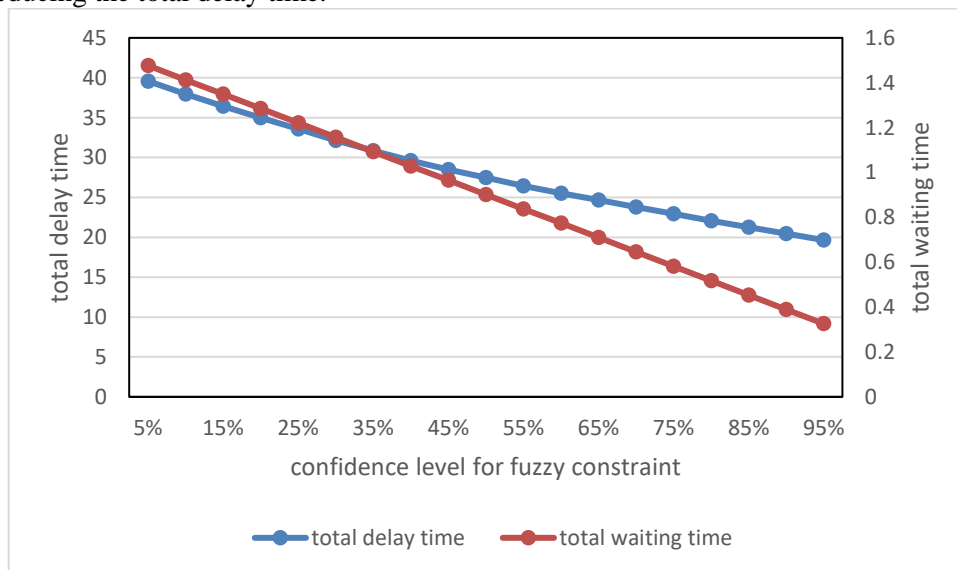
According to table 5, employing fuzzy programming model leads to the less total waiting time and total delay time, obviously. This can confirm the validity and efficiency of the proposed model.

This example clarifies that concentrating on just one of the objective functions, which was studied in previous researches, is not efficient. Furthermore, changing importance percent of the first objective function in regard to the second one does not change the solution. Optimality of the goal programming solution in both of the objection functions ( $d_1, d_2 = 0$ ) is the reason for solution dependency regarding  $w$ . Besides, employing fuzzy programming, at a confidence level of 95%, the participants arrive at the common destination at the right time. Moreover, the realization of situations clarified the efficiency of employing fuzzy models which helps individuals in time.

### 5-1- Sensitivity analysis

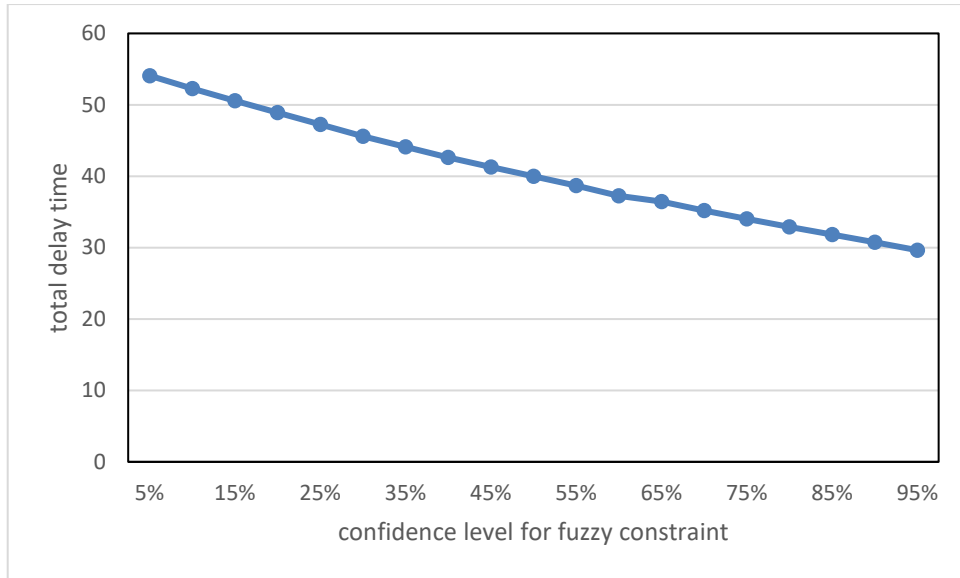
Having better insight about the influence of confidence level for fuzzy constraint on the optimal results, the sensitivity analysis is prepared in this section. The confidence level for the fuzzy constraint is chosen because of its direct relationship with scheduling vehicles and individuals and also being adjustable. It should be noted that travel time was another important parameter to care about its change which was considered as an uncertain parameter in the proposed model. Changing the confidence level for the fuzzy constraint is investigated in an interval of  $[0.05, 0.95]$  to solve the proposed model by considering  $Z_1, Z_2$ , and  $Z^{new}$  as the objective function, respectively. Additionally, every step in the interval is considered 0.05. Total delay time and total waiting time are chosen because they are the important factors to company and individuals, and other decision variables like  $Z_1, Z_2$ , and  $Z^{new}$  cannot make any insight to decision makers.

Figure 2 illustrates the effect of the confidence level for fuzzy constraint on total delay and total waiting time when the proposed model is solved considering  $Z_1$  as the objective function. This figure illustrates the point that the increase in confidence level for fuzzy constraint causes a decrease in total delay time and total waiting time which shows the validity of the model. Furthermore, by increasing the confidence level for fuzzy constraint, the slope of reducing total waiting time is more than the slope of reducing the total delay time.



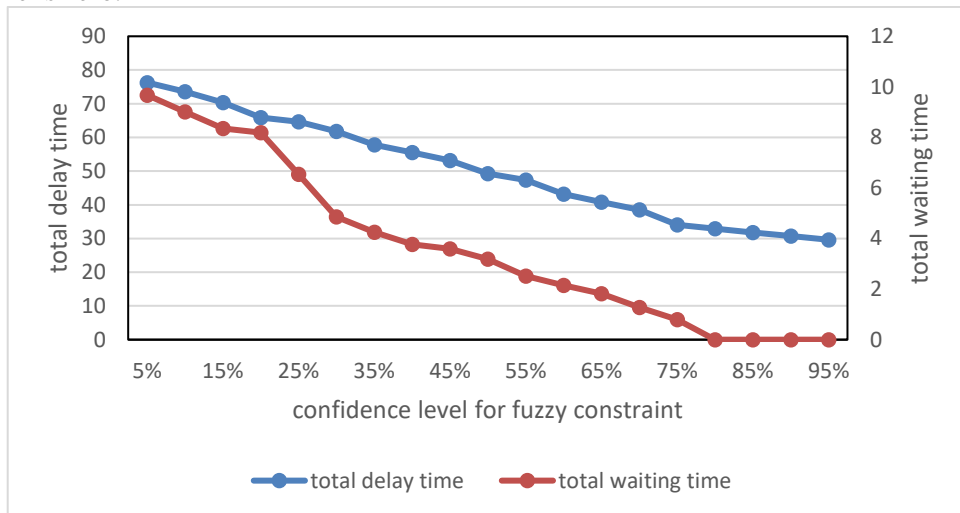
**Fig 2.** Effect of confidence level on total delay and total waiting time (by solving  $Z_1$ )

Figure 3 illustrates the effect of the confidence level for fuzzy constraint on total delay time when the proposed model is solved considering  $Z_2$  as the objective function. Total waiting time is zero in this state for all values of  $\alpha$ . This is because of solving the model using  $Z_2$  as the objective function to maximize departure time of every participant which causes minimizing total waiting time. According to this figure, total delay time decreases by increasing the confidence level of fuzzy constraint which can show the validity of the model.



**Fig 3.** Effect of confidence level on total delay time (by solving  $Z_2$ )

Figure 4 illustrates the effect of the confidence level for fuzzy constraint on total delay and total waiting time when the proposed model is solved considering  $Z^{new}$  as the objective function. As it is seen in figure 2 and figure 3, increasing in values of  $\alpha$  causes less total delay and total waiting time which can show the validity of the proposed model. According to the figure slope of decreasing total waiting time on an interval of  $\alpha = [0.2, 0.3]$  is steeper than other  $\alpha$  values and for  $\alpha \geq 0.8$ , the total waiting time is zero.



**Fig 4.** Effect of confidence level on total delay and total waiting time (by solving  $Z^{new}$ )

## 6- Conclusion

This study investigated scheduling and routing issue with a common destination to assign individuals ride-sharing vehicles. In this regard, traveling time was considered as a fuzzy parameter to deal with its uncertainty. Necessity concept employed to the constraints containing the fuzzy parameter. Furthermore, goal programming method employed to work with two objective functions which were minimizing expected total traveling time and maximizing the earliest departure time of participants.

Evaluating the applicability of the proposed model, a numerical example is implemented. The numerical example illustrated the importance of considering both of the objective functions. As a matter of fact, ignoring one of the objective functions leads to an inefficient solution while both of

them can take the best solution at the same time. Additionally, the efficiency of employing fuzzy model is investigated in a realization. This realization confirms the validity and productivity of the proposed model which can reduce the total waiting time and the total delay time, particularly. Furthermore, sensitivity analysis is done on the numerical example to have a better understanding of the behavior of the proposed model. Besides, the behavior of the model according to changes in the confidence level for fuzzy constraint can show the validity of the proposed model. Some managerial insights which are obtained from the results are as following:

- Considering just one aspect of optimizing the ride-sharing systems is not helpful.
- As it is seen in the results of this study, considering another goal along with the main goal can optimize both of them at the same time, without worsening the main goal.
- Considering the uncertainty of the important parameters such as traveling time is a good idea to increase the utility of both employees and the company.
- Fuzzy programming approach is a good method to deal with uncertainties in such problems and is helpful for decision makers.

Due to being NP-hard, the proposed model can be solved using decomposition methods or heuristic and meta-heuristic approaches in future studies. Furthermore, it can be a good idea to evaluate the issue from cost aspect which is important to companies. Considering multi destinations is another issue which can discuss in future researches.

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