Journal of Industrial and Systems Engineering
Vol. 8, No. 4,pp 18-37
Autumn 2015

JISE

Supplier selection with multi criteria group decision making based on interval valued intuitionistic fuzzy sets

(Case study on a project based company)

Ahmad Makui[1*], Mohammad Reza Gholamian[1], Seyed Erfan Mohammadi[1]

[1]Department of Industrial Engineering, Iran University of Science & Technology, Tehran, Iran

amakui@iust.ac.ir, gholamian@iust.ac.ir, erfanmohammadi@ind.iust.ac.ir

Abstract

Supplier selection can be considered as a complicated multi criteria decision-making problem. In this paper the problem of supplier selection is studied in the presence of conflicting evaluations and insufficient information about the criteria and different attitudes of decision makers towards the risk. Most of fuzzy approaches used in multi-criteria group decision making (MCGDM) are non-intuitionistic, which significantly restricts their application areas. Because of considering belongingness and non-belongingness of the issue in a same time, intuitionistic fuzzy sets can better encounter with a real supplier selection problem. Also to deal with different attitudes of decision makers toward the risk, the proposed approach in this paper employs a new decision function to participate this factor in decision process. In order to integrate fuzzy information, interval-valued intuitionistic fuzzy ordered weighted aggregation (IIFOWA) is applied to aggregate the obtained preferences. The influence of unfair arguments in final results can be reduced by assigning low weights to the "optimistic" or "pessimistic" discretions. Ranking process is based on the two indices, weighted score function and weighted accuracy function. To demonstrate the efficiency of the proposed approach, it is implemented to supplier selection in a project-based company.

Keywords: Multi-Criteria Group Decision Making, Supplier Selection, Interval-Valued Intuitionistic Fuzzy Set, Aggregation Operator, Risk Attitude, Decision Function.

---

*Corresponding author.
ISSN: 1735-8272,

1- Introduction

Supplier selection as an important activity in an effective supply chain has been embossed in recent years. Due to the complexity of product and the profusion of suppliers, a systematic process for selecting the best supplier is needed. Regarding to the conditions prevailing in organizations, such as ambiguity, insufficiency of information and different attitudes of decision makers (stakeholders) towards the risk, the traditional multi-criteria techniques seem not suitable for real type problems. Therefore, it seems that interval-valued intuitionistic fuzzy sets (IVIFSs) are a suitable tool to cover uncertainty and can define a new class of decision functions to response different attitudes of decision makers towards the risk. So this article makes an attempt to bring the intuitionistic fuzzy set (IFS) and multi-criteria group decision making together for introducing a new approach for supplier selection problem.

2- Literature survey

Supplier selection problem is one of the fundamental issues in any organization. Decisions on choosing appropriate suppliers for a company generally have long-term impact on its performance, and poor decisions could cause significant damage to company’s competitive advantage and profitability. Therefore, supplier selection problem has been traditionally treated as one of the most critical activities in the logistic and supply chain management (LSCM). There are many methods used in supplier selection such as cluster analysis and statistical models(De Boer, Labro, & Morlacchi, 2001), case based reasoning systems (Choy, Lee, & Lo, 2003), stochastic models (Hammami, Temponi, & Frein, 2014; Liao & Rittscher, 2007; Scott, Ho, Dey, & Talluri, 2014), decision support systems (Chan, Chan, Ip, & Lau, 2007), data envelopment analysis (Tavassoli, Faramarzi, & Saen, 2014), total cost of ownership models (Ramanathan, 2007), artificial intelligence (Ferreira & Borenstein, 2012; Tseng, 2011) and mathematical programming (Chang, Chen, & Zhuang, 2014; Li & Zabinsky, 2011; Yu, Goh, & Lin, 2012). By the way, the globalization has extended the supplier selection to international arena and makes it a complex and difficult multi-criteria decision-making(MCDM) task(Wang, Li, & Xu, 2011).To address the selection issue, different comparisons and tradeoff among multifarious factors have to be considered within the MCDM framework. Due to business reliability and other reasons, the evaluation of suppliers has to be conducted with uncertainty. Because of ambiguity and insufficient information in many decision situations, it is often difficult for a decision-maker (DM) to give her/ his assessments on the preference relations. Sometimes, due to knowledge limitation and time pressure, the decision making process confronts with hesitancy. In order to consider this matter, Atanassov (1986) introduced the intuitionistic fuzzy set (IFS), which is characterized by the membership function, non-membership function, and hesitancy function. IVFSs and IFSs are regarded as flexible and practical tools for dealing with fuzziness and uncertainty. Atanassov and Gargov (1989), later introduced the interval-valued intuitionistic fuzzy set (IVIFS), as a generalization of IVFSs and IFSs that provides the membership function and non-membership function with intervals rather than exact numbers.

However, the academic reports illustrate the growing research interests in IVIFSs, such as investigations on basic operations and relations of IVIFSs as well as their basic properties (K. T. Atanassov, 1994; Bustince & Burillo, 1995; Hong, 1998; Hung & Wu, 2002; Z. Xu & Chen, 2008), topological properties (Kumar Mondal & Samanta, 2001; Reiser & Bedregal, 2013), relationships between IFSs, L-fuzzy sets, interval-valued fuzzy sets and IVIFSs(Deschrijver, 2007, 2008; Deschrijver & Kerre, 2007), the entropy and subset hood (T.-Y. Chen, 2014; Liu, Zheng, & Xiong, 2005), and distance and similarity measures of IVIFSs (Baccour, Alimi, & John, 2013; Z. Xu & Chen, 2008). Xu and Chen (2007a), proposed some arithmetic aggregation operators including the interval-valued intuitionistic fuzzy weighted arithmetic aggregation (IIFWA) operator, the interval-valued intuitionistic fuzzy ordered weighted aggregation (IIFOWA) operator, and the interval- valued intuitionistic fuzzy hybrid aggregation (IIFHA) operator in order to aggregate interval-valued intuitionistic preference information. Meanwhile, Wei and Wang (2007), Ze-Shui (2007a) and Xu and Chen (2007b), presented some operators in terms of geometrics, such as the interval-valued intuitionistic fuzzy weighted geometric (IIFWG) operator, the interval-valued intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator, and the interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator. According the results, all these numbers are still IVIFSs. Obviously, there are different indications on arithmetic aggregation operators and geometric aggregation operators; Such that the former indicates the group's influence, whereas the later indicates the individual influence. To evaluate IVIFSs, score function and accuracy function have been developed with the capability of comparing criteria values expressed by IVIFSs in order to generate a permutation and give ordered weights to the corresponding criteria values. The accuracy functions proposed by Xu and Da (2003), do not give sufficient information about alternatives and may lead to difficult conditions in the decision making processes. So, more attentions to other measuring functions is made. Therefore Ze-Shui (2007a), proposed a score function and Lakshmana Gomathi Nayagam, Muralikrishnan, and Sivaraman (2011), proposed a new novel accuracy function to discriminate between the IVIFSs. With this enhanced studies on IVIFNs, researchers have turned their attention to decision problems whereas some input decision data are provided as IVIFNs (Wang, Li, & Wang, 2009; Z. Xu & Yager, 2008; Ze-Shui, 2007a). However, there are a few studies on MCDM involving multiple decision-makers in an interval-valued intuitionistic fuzzy environment. In order to avoid partial judgment caused by individual opinion's, group decision-making is used to integrate different opinions. Comparing with individual decision making, group decision making can elicit more complete information about the problem and provide more selective alternatives(T.-Y. Chen, Wang, & Lu, 2011).The risk attitude of a decision maker is an important parameter in the decision process, especially in multi-criteria group decision making (MCGDM) that can influences the process results, but it is generally neglected in the existing research on MCGDM with IVIFN assessments. In this paper, we proposed an approach to deal with the multi-criteria group decision making (MCGDM) problem based on the interval-valued intuitionistic fuzzy preference relation and the interval-valued intuitionistic fuzzy decision matrix, while the main focus is to contribute the risk attitude of decision group members in MCGDM process.

The remainder of this paper is organized as follows: Section 3, provides some preliminary background on IVIFSs. In Section 4, MCGDM based on interval-valued intuitionistic fuzzy information is described. In Section 5, the new approach was introduced to solve the MCGDM problem based on interval-valued intuitionistic fuzzy information. In Section 6, the results of the implementation of the proposed method in an Iranian company are given to demonstrate the efficiency of the proposed approach. In Section 7, the validity of the proposed approach is discussed. Finally a conclusion is given in Section 8.

3- Interval valued intuitionistic fuzzy set

3-1- Basic definitions

As a preparation for introducing the proposed approach, some relevant concepts are illustrated in this section.

3-1-1- Definition 1

$D[0,1]$ is the set of all closed subintervals of the interval $[0,1]$. ($X \neq \Phi$) is a given set. An interval-valued intuitionistic fuzzy set A in X is given by $A = \{(x, \mu_A(x), v_A(x)): x \in X\}$, where $\mu_A: X \rightarrow D[0,1]$, $v_A: X \rightarrow D[0,1]$and $0 \leq sup_x \mu_A(x) + sup_x v_A(x) \leq 1$.

The intervals$\mu_A(x)$ and $v_A(x)$ denote, respectively, the degree of belongingness and the degree of non-belongingness of the element x to the set$A$. Thus, for each $x \in X$,$\mu_A(x)$ and $v_A(x)$ are closed intervals whose lower and upper end points are respectively, denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$ and $v_{AL}(x)$, $v_{AU}(x)$. A can be denoted by,

$A = \{(x, [\mu_{AL}(x), \mu_{AU}(x)], [v_{AL}(x), v_{AU}(x)]) : x \in X\},$ (1)

Where $0 \leq \mu_{AU}(x) + v_{AU}(x) \leq 1$, $\mu_{AL}(x) \geq 0$ and $v_{AL}(x) \geq 0$. In addition the set of all the IVIFS in X is shown by IVIFS(X). For each element x the unknown degree (uncertainty degree) of an intuitionistic fuzzy interval of $x \in X$ in A can be defined as follows:

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x)$$
$$= [1 - \mu_{AU}(x) - v_{AU}(x), 1 - \mu_{AL}(x) - v_{AL}(x)] \quad (2)$$

An IVIFS value is denoted by $A = ([a, b], [c, d])$ for convenience.
(K. Atanassov & Gargov, 1989)

3-1-2- Definition 2

$A, B \in IVIFS(X)$. A subset relation is defined by:
$A \subset B \Leftrightarrow \mu_{AL}(x) \leq \mu_{BL}(x)$, $\mu_{AU}(x) \leq \mu_{BU}(x)$ and $v_{AL}(x) \geq v_{BL}(x)$, $v_{AU}(x) \geq v_{BU}(x)$, $\forall x \in X$.
(K. Atanassov & Gargov, 1989)

3-1-3- Definition 3

$A = B \Leftrightarrow A \subset B \; and \; B \subset A$. (3)
(K. Atanassov & Gargov, 1989)

3-1-4- Definition 4

$A = \{(x, \mu_A(x), v_A(x)) : x \in X\}$ then:
$A^c = \{(x, v_A(x), \mu_A(x)) : x \in X\}$ (4)
(K. Atanassov & Gargov, 1989)

3-2- Basic operations of interval-valued intuitionistic fuzzy number (IVIFN)

$([\mu_{AL}(x), \mu_{AU}(x)], [v_{AL}(x), v_{AU}(x)])$ is an interval-valued intuitionistic fuzzy number (IVIFN). For notational convenience, we denote an IVIFN $\tilde{\alpha} = ([a, b], [c, d])$ in the following, where: $\mu_{AL}(x) \in [0,1]$, $\mu_{AU}(x) \in [0,1]$, $v_{AL}(x) \in [0,1]$, $v_{AU}(x) \in [0,1]$, $\mu_{AU}(x) + v_{AU}(x) \leq 1$.

3-2-1- Definition 5

Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ be any two IVIFNs, then their operational laws can be defined as follows:

1) $\tilde{\alpha}_1{}^c = ([c_1, d_1], [a_1, b_1])$ (5)
2) $\tilde{\alpha}_1 + \tilde{\alpha}_2 = ([a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2], [c_1 c_2, d_1 d_2])$ (6)
3) $\tilde{\alpha}_1 . \tilde{\alpha}_2 = ([a_1 a_2, b_1 b_2], [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2])$ (7)
4) $\lambda \tilde{\alpha}_1 = \left(\left[1 - (1 - a_1)^\lambda, 1 - (1 - b_1)^\lambda\right], [c_1{}^\lambda, d_1{}^\lambda]\right)$, $\lambda \geq 0$ (8)
(Ze-Shui, 2007a).

Ze-Shui (2007a) showed that all the results of operations are also IVIFNs, and:

1) $\tilde{\alpha}_1 + \tilde{\alpha}_2 = \tilde{\alpha}_2 + \tilde{\alpha}_1$ (9)
2) $\tilde{\alpha}_1 . \tilde{\alpha}_2 = \tilde{\alpha}_2 . \tilde{\alpha}_1$ (10)
3) $\lambda(\tilde{\alpha}_1 + \tilde{\alpha}_2) = \lambda \tilde{\alpha}_1 + \lambda \tilde{\alpha}_2$, $\lambda \geq 0$ (11)
4) $\lambda_1 \tilde{\alpha}_1 + \lambda_2 \tilde{\alpha}_1 = (\lambda_1 + \lambda_2) \tilde{\alpha}_1, \lambda_1 \geq 0$, $\lambda_2 \geq 0$ (12)

3-3- Comparison operations of interval-valued intuitionistic fuzzy number (IVIFN)

3-3-1- Definition 6

Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. Then the score function (S) is defined by:
$S(\tilde{\alpha}) = 1/2(a - c + b - d)$ (13)
Where $S(\tilde{\alpha}) \in [-1,1]$. The greater the value of $S(\tilde{\alpha})$ denotes the greater IVIFN $\tilde{\alpha}$.
(Ze-Shui, 2007a)

3-3-2- Definition 7

Let $\tilde{\alpha} = ([a, b], [c, d])$ be IVIFN.A novel accuracy function (H) of an interval-valued intuitionistic fuzzy value, based on the unknown degree is defined by:
$H(\tilde{\alpha}) = (a + b - d(1 - b) - c(1 - a))/2$ (14)

Where $(\tilde{\alpha}\ ) \in [-1,1]$. The greater the value of $H(\tilde{\alpha}\ )$ denotes the greater IVIFN$\tilde{\alpha}$.

The relation between score function (S) and accuracy function (H)of IVIFN is similar to the relation between mean and variance in statistics(Hong & Choi, 2000). On the basis of the score function (S)and the accuracy function (H)in the following, an order relation between IVIFNs is introduced(Ze-Shui, 2007a).

Let $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ be any two IVIFNs, then:

- If $S(\tilde{\alpha}_1) \le S(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 \le \tilde{\alpha}_2$.
- If$S(\tilde{\alpha}_1) = S(\tilde{\alpha}_2)$, then
    1) If$H(\tilde{\alpha}_1) = H(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 = \tilde{\alpha}_2$.
    2) If $H(\tilde{\alpha}_1) \le H(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 \le \tilde{\alpha}_2$.
    (Lakshmana Gomathi Nayagam et al., 2011)

Hong & Choi (2000) introduced the following weighted score function and weighted accuracy function.

3-3-3- Definition 8

Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$,$\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$,…,$\tilde{\alpha}_n = ([a_n, b_n], [c_n, d_n])$ be n IVIFNs, and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of these IVIFNs. The weighted score function is defined as:

$$W_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \omega_1 S(\tilde{\alpha}_1) + \omega_2 S(\tilde{\alpha}_2) + \cdots + \omega_n S(\tilde{\alpha}_n) \quad (15)$$

3-3-4- Definition 9

Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$,$\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$,…,$\tilde{\alpha}_n = ([a_n, b_n], [c_n, d_n])$ be n IVIFNs, and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of these IVIFNs. The weighted accuracy function is defined as:

$$T_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \omega_1 H(\tilde{\alpha}_1) + \omega_2 H(\tilde{\alpha}_2) + \cdots + \omega_n H(\tilde{\alpha}_n) \quad (16)$$

From the operational laws of IVIFNs in Wang et al (2009), and the definitions of the score function and the accuracy function, we can conclude that the weighted score function is not the score function of the weighted sum of IVIFNs, and the weighted accuracy function is not the accuracy function of the weighted sum of IVIFNs(Z. Chen & Yang, 2012). Therefore, based on these two functions, a new class of decision functions is defined as follows:

3-3-5- Definition 10

Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$,$\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$,…,$\tilde{\alpha}_n = ([a_n, b_n], [c_n, d_n])$ be n IVIFNs, and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of these IVIFNs.$W_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$ and $T_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$are the weighted score function and the weighted accuracy function, respectively. The decision function $R(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$ is defined as:

$$R(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = W_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) + \lambda T_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)\ ;\ \ \lambda \in R \quad (17)$$

From the definitions of $W_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$and $T_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$, the decision function $R(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$ can be re-expressed as:

$$R(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = W_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) + \lambda T_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)\ ;\ \ \lambda \in R$$

$$= \sum_{j=1}^{n} \omega_j S(\tilde{\alpha}_j) + \lambda \sum_{j=1}^{n} \omega_j H(\tilde{\alpha}_j) = \sum_{j=1}^{n} \omega_j \left(S(\tilde{\alpha}_j) + \lambda H(\tilde{\alpha}_j) \right)$$

$$= \sum_{j=1}^{n} \omega_j \big((1/2(a_j - c_j + b_j - d_j)) + \lambda((a_j + b_j - d_j(1 - b_j) - c_j(1 - a_j))/2) \big)$$

$$= \sum_{j=1}^{n} \omega_j \left(\frac{\lambda + 1}{2} \left(a_j + b_j - c_j - d_j \right) + \frac{\lambda}{2} \left(a_j . c_j + b_j . d_j \right) \right) \tag{18}$$

In this definition, different λ can be used to specify different risk attitudes of decision makers. In fact, if $\lambda < 0$, the decision maker is risk-averse, and the larger $|\lambda|$indicates more risk-averseness of decision maker; if $\lambda = 0$, the decision maker only considers the weighted score function, she/he is thus risk-neutral; if $\lambda > 0$, the decision maker is risk-seeking, the larger λ indicates more risk seeking of decision maker. Therefore, the introduced decision function is rather generic and flexible. During the real decision process, the decision makers can choose suitable λ according to their risk attitudes.

3-4- Aggregation operation of interval-valued intuitionistic fuzzy number (IVIFN)

Ze-Shui (2007a), developed the interval-valued intuitionistic weighted arithmetic aggregation operator(IIFWA) and the interval-valued intuitionistic fuzzy ordered weighted aggregation operator (IIFOWA) to aggregate interval-valued intuitionistic fuzzy information.

3-4-1- Definition 11

Let $\tilde{\alpha}_j (j = 1,2, \dots, n)$be a collection of IVIFNs. An IIFWA operator of dimension n is a mapping IIFWA:$\Omega^n \to \Omega$ which has associated with it a weighting vector$\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$with the conditions $\omega_j \geq 0$ and $\sum_{j=1}^{n} \omega_j = 1$, such that: (Ze-Shui, 2007a)

$$IIFWA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \sum_{j=1}^{n} \omega_j \tilde{\alpha}_j \tag{19}$$

In particular, if $\omega = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^T$, the function IIFWA is reduced to the interval-valued intuitionistic fuzzy arithmetic aggregation (IIFA) operator defined by:

$$IIFA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \frac{1}{n} \sum_{j=1}^{n} \tilde{\alpha}_j \tag{20}$$

Based on the basic operations (6) and (8) of IVIFSs, (19) can be further transformed as follows:

3-4-2- Definition 12

Let$\tilde{\alpha}_j = \left(\left[a_j, b_j \right], \left[c_j, d_j \right] \right)$, $j = (1,2, \dots, n)$be a collection of IVIFNs. The IIFWA operator is further defined by: (Ze-Shui, 2007a)

$$IIFWA_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \left(\left[1 - \prod_{j=1}^{n} \left(1 - a_j \right)^{\omega_j}, 1 - \prod_{j=1}^{n} \left(1 - b_j \right)^{\omega_j} \right], \left[\prod_{j=1}^{n} c_j^{\omega_j}, \prod_{j=1}^{n} d_j^{\omega_j} \right] \right) \tag{21}$$

Where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of$\tilde{\alpha}_j (j = 1,2, \dots, n)$;$\omega_j \in [0,1]$, and $\sum_{j=1}^{n} \omega_j = 1$.

(Z.-S. Xu & Chen, 2007a)proposed the interval-valued intuitionistic fuzzy ordered weighted aggregation (IIFOWA) operator to aggregate IVIFNs. The operator is characterized by reordering the IVIFNs in descending order. A weight,w_j , is associated with a particular ordered position. The arguments are endowed with new weightsw_j rather than the initial weights ω_j .

3-4-3- Definition 13

(Z.-S. Xu & Chen, 2007a). Let $\tilde{\alpha}_j = \left(\left[a_j, b_j \right], \left[c_j, d_j \right] \right)$, $j = (1,2, \dots, n)$ be a collection of IVIFNs, and$\left(\tilde{\alpha}_{\sigma(1)}, \tilde{\alpha}_{\sigma(2)}, \dots, \tilde{\alpha}_{\sigma(n)} \right)$be a permutation of $(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$, such that$\tilde{\alpha}_{\sigma(j-1)} \geq \tilde{\alpha}_{\sigma(j)}$ for all j, and let$\tilde{\alpha}_{\sigma(j)} = \left(\left[a_{\sigma(j)}, b_{\sigma(j)} \right], \left[c_{\sigma(j)}, d_{\sigma(j)} \right] \right)$, then the IIFOWA operator can be defined by:

$$IIFOWA_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \left(\left[1 - \prod_{j=1}^{n} \left(1 - a_{\sigma(j)}\right)^{w_j}, 1 - \prod_{j=1}^{n} \left(1 - b_{\sigma(j)}\right)^{w_j} \right], \left[\prod_{j=1}^{n} c_{\sigma(j)}{}^{w_j}, \prod_{j=1}^{n} d_{\sigma(j)}{}^{w_j} \right] \right) \quad (22)$$

Where $w = (w_1, w_2, \dots, w_n)^T$ is the weight vector of the IIFOWA operator, $w_j \in [0,1]$,and $\sum_{j=1}^{n} w_j = 1$. The weight vector of the IIFOWA operator can be determined by the method of (Z. Xu, 2005) , which uses the perspective of normal distribution to gain weights. In this way, it can reduce the influence of unfair arguments in the final results by assigning low weights to the “optimistic” or “pessimistic” discretions.

4- Multiple criteria group decision making based on interval-valued intuitionistic fuzzy information

Decision information is generally composed of criteria values for alternatives and criteria weights. In normal conditions, criteria values are unable to be precisely given by decision makers, on account of limited attention and information processing capabilities (Weber, 1987); instead, values within an interval are conceivable. Information about criteria weights may also appear as imprecise values, rather than as exact numbers. Xu (2007b) indicates that preference relations, also called pairwise comparison matrices or judgment matrices, are powerful tools used by decision makers to provide their preference information in the process of decision making. Xu (2007c) established some models to derive criterion weights from different formats of preference relations such as interval utility values, interval fuzzy preference relations, and interval multiplicative preference relations. Since preference relations are more understandable for decision makers to determine the relative importance of criteria, Chen et al. (2011) established an interval-valued intuitionistic fuzzy preference relation matrix on criteria and the criterion values are also measured by means of IVIFSs.

The preference relation on criteria, based on IVIFSs can be concisely expressed in a pairwise comparison matrix. Suppose that there is a set of criteria $G = \{g_1, g_2, \dots, g_p\}$, and a set of decision makers (experts) $E = \{e_1, e_2, \dots, e_m\}$. Each expert has to compare the relative importance of each pair of criteria with IVIFSs.

4-1- Definitions

4-1-1- Definition 14

If an interval-value intuitionistic fuzzy preference relation matrix $\tilde{G}_k$ on the set X is defined as $\tilde{G}_k = \left(\tilde{g}_{ij}{}^{(k)}\right)_{p\times p} \subset X \times X$, then: (T.-Y. Chen et al., 2011)

$$\tilde{G}_k = \begin{bmatrix} \tilde{g}_{11}{}^{(k)} & \cdots & \tilde{g}_{1p}{}^{(k)} \\ \vdots & \ddots & \vdots \\ \tilde{g}_{p1}{}^{(k)} & \cdots & \tilde{g}_{pp}{}^{(k)} \end{bmatrix} \quad (23)$$

Where $\tilde{g}_{ij}{}^{(k)} = \left([a_{ij}, b_{ij}]^{(k)}, [c_{ij}, d_{ij}]^{(k)}\right)$, $i, j = (1,2, \dots, p)$ is an IVIFS. $[a_{ij}, b_{ij}]^{(k)}$ Indicates the expert e_k's interval-valued intuitionistic fuzzy preference degree for the criterion g_i when the criteria $g_i$and g_j are compared and criterion g_i is preferred in comparison with the other one ;also $\left[c_{ij}, d_{ij}\right]^{(k)}$indicatesthe expert e_k's interval-valued intuitionistic fuzzy preference degree for the criterion g_i when the criteria $g_i$and g_j are compared and criterion g_j is preferred in comparison with the other one ; $\left[a_{ij}, b_{ij}\right]^{(k)} \subset [0,1]$, $\left[c_{ij}, d_{ij}\right]^{(k)} \subset [0,1]$,$\left[a_{ji}, b_{ji}\right]^{(k)} = \left[c_{ij}, d_{ij}\right]^{(k)}$, $\left[c_{ji}, d_{ji}\right]^{(k)} = \left[a_{ij}, b_{ij}\right]^{(k)}$, $[a_{ii}, b_{ii}]^{(k)} = [c_{ii}, d_{ii}]^{(k)} = [0.5,0.5]$, and ${b_{ij}}^{(k)} + {d_{ij}}^{(k)} \leq 1$, $i, j = (1,2, \dots, p)$ and $k = 1,2, \dots, m$.

4-1-2- Definition 15

Let $\tilde{G}_k = \left({\tilde{g}_{ij}}^{(k)}\right)_{p \times p}$be an interval-valued intuitionistic fuzzy preference relation matrix. If ${\tilde{g}_{ij}}^{(k)} = {\tilde{g}_{ik}}^{(k)} . {\tilde{g}_{kj}}^{(k)}$for all i, j, k, then $\tilde{G}_k$ is called the consistent interval-valued intuitionistic fuzzy preference relation matrix (Z. Xu, 2007b).

The criteria values can also be expressed in a decision matrix based on IVIFSs to discern the performance of each alternative with respect to criteria. Now, suppose there exists a set of alternatives $Y = \{y_1, y_2, \dots, y_n\}$which consist of n non-inferior decision-making alternatives, a set of criteria $G = \{g_1, g_2, \dots, g_p\}$, and a set of experts $E = \{e_1, e_2, \dots, e_m\}$.

4-1-3- Definition

If an interval-value intuitionistic fuzzy decision matrix $\widetilde{D}_k$on the set X is defined as $\widetilde{D}_k = \left({\tilde{d}_{ij}}^{(k)}\right)_{n \times p} \subset X \times X$, then: (T.-Y. Chen et al., 2011).

$$\widetilde{D}_k = \begin{matrix} & g_1 & \cdots\cdots\cdots & g_p \\ \begin{matrix} y_1 \\ \vdots \\ y_n \end{matrix} & \multicolumn{3}{c}{\begin{bmatrix} {\tilde{d}_{11}}^{(k)} & \cdots & {\tilde{d}_{1p}}^{(k)} \\ \vdots & \ddots & \vdots \\ {\tilde{d}_{n1}}^{(k)} & \cdots & {\tilde{d}_{np}}^{(k)} \end{bmatrix}} \end{matrix} \tag{24}$$

Where ${\widetilde{d}_{ij}}^{(k)} = \left(\left[a_{ij}, b_{ij}\right]^{(k)}, \left[c_{ij}, d_{ij}\right]^{(k)}\right)$, $i = (1,2, \dots, n)$, $j = (1,2, \dots, p)$, is an IVIFS.$\left[a_{ij}, b_{ij}\right]^{(k)}$indicates the extent to which the expert$e_k$considers the alternative $y_i$to satisfy the criterion g_j of the fuzzy concept ‘‘excellence.’’ Also $\left[c_{ij}, d_{ij}\right]^{(k)}$ indicates the extent to which the expert e_k considers the alternative $y_i$does not satisfy the criterion g_j of the fuzzy concept ‘‘excellence.’’ In addition,$\left[a_{ij}, b_{ij}\right]^{(k)} \subset [0,1]$, $\left[c_{ij}, d_{ij}\right]^{(k)} \subset [0,1]$, $0 \leq {b_{ij}}^{(k)} + {d_{ij}}^{(k)} \leq 1$, $i = (1,2, \dots, n)$, $j = (1,2, \dots, p)$, $k = 1,2, \dots, m$.

5 - An approach to multi-criteria group decision making

The proposed multi-criteria group decision making approach based on interval-valued intuitionistic fuzzy environment is as follows:

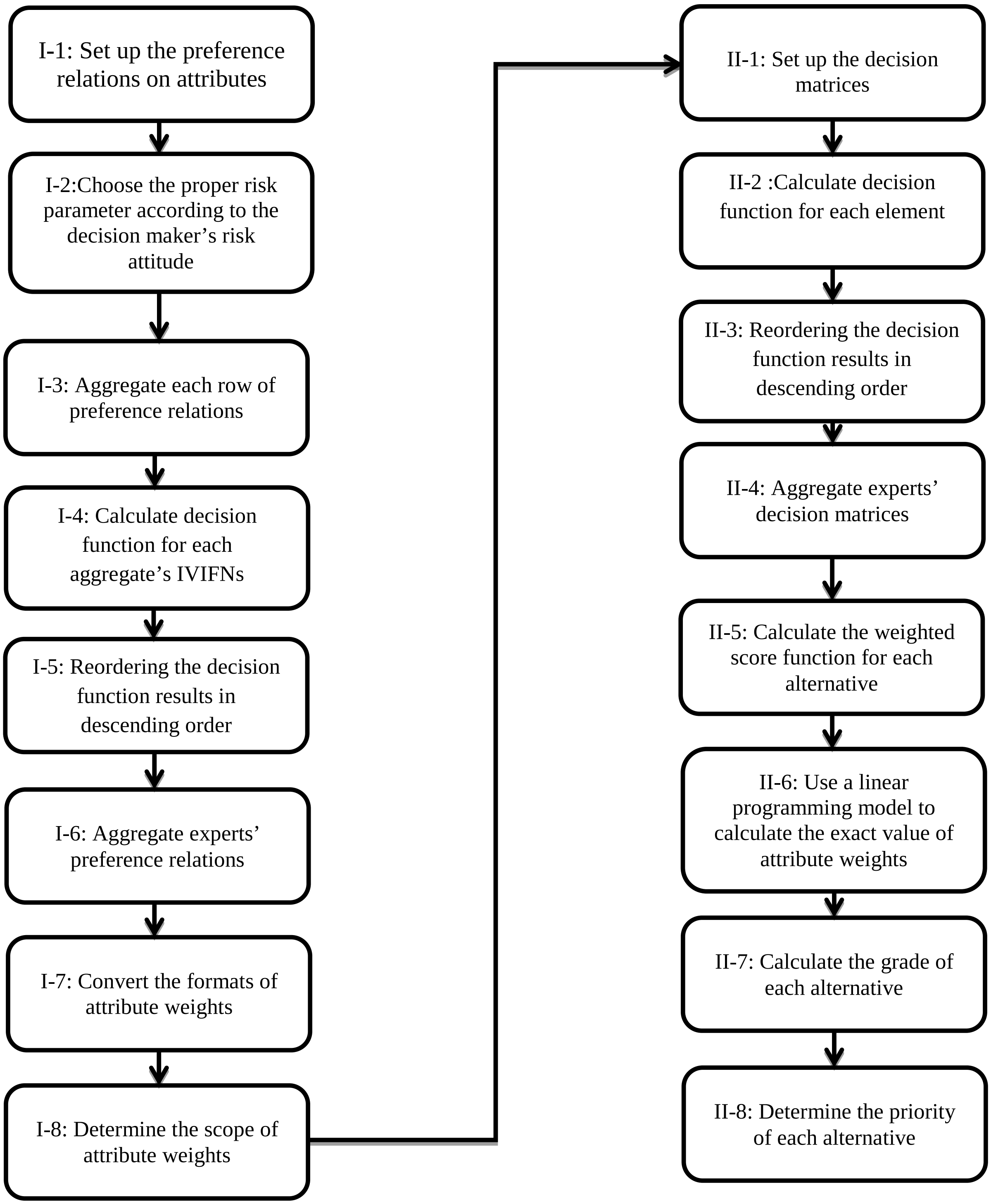

Fig.1. The diagram for the proposed approach.

In the first stage, the preference relation matrices for criteria weights are required. Experts use the IVIFNs to express their performances. In the condition where the criterion weights are unknown, experts deliver the preference relations on criteria by pairwise comparison.

Step I-1: Use (23) to set up the interval-valued intuitionistic fuzzy preference relation on criteria as follows:
$\tilde{G}_k = \left(\tilde{g}_{ij}{}^{(k)}\right)_{p\times p}, \qquad i,j = 1,2,\dots,p\,; \;\; k = 1,2,\dots,m.$
Step I-2: According to the decision maker's risk attitude, choose the specific decision function. If the decision maker is risk-neutral, then set $\lambda = 0$; if the decision maker is risk-averse, then select a proper value with $\lambda < 0$; if the decision maker is risk-seeking, then select a proper value with $\lambda > 0$.

Step I-3: Apply the operation in (20) to aggregate each row of preference relations as follows:
$\tilde{g}_i{}^{(k)} = IIFA_\omega\left(\tilde{g}_{i1}{}^{(k)}, \tilde{g}_{i2}{}^{(k)}, \dots, \tilde{g}_{ip}{}^{(k)}\right), \qquad i = 1,2,\dots,p\,; \;\; k = 1,2,\dots,m.$

Step I-4: Use (18) to calculate decision function for each aggregate's IVIFNs as follows:

$R_i{}^{(k)} = W_\omega\left(\tilde{g}_i{}^{(k)}\right) + \lambda^{(k)} T_\omega\left(\tilde{g}_i{}^{(k)}\right); \;\; \lambda^{(k)} \in R, \qquad i = 1,2,\dots,p\,; \;\; k = 1,2,\dots,m.$

Step I-5: Reorder the decision function results in descending order based on previous step, such that $\tilde{g}_{i_{\sigma(j-1)}} \geq \tilde{g}_{i_{\sigma(j)}}$ for all, and let $\tilde{g}_{i_{\sigma(j)}} = \left(\left[a_{\sigma(j)}, b_{\sigma(j)}\right], \left[c_{\sigma(j)}, d_{\sigma(j)}\right]\right)$.

Step I-6: Apply the operation in (22) to integrate experts' opinions on criteria, and express the criteria weights in the interval-valued intuitionistic fuzzy format as follows:
$\tilde{g}_i = IIFOWA_w\left(\tilde{g}_i{}^{(1)}, \tilde{g}_i{}^{(2)}, \dots, \tilde{g}_i{}^{(m)}\right), \qquad i = 1,2,\dots,p.$

Step I-7: Convert the criteria weights from interval-valued intuitionistic fuzzy formats $\tilde{g}_i$, $i = 1,2,\dots,p$ into interval-valued fuzzy formats $\tilde{\tilde{g}}_i$; $i = 1,2,\dots,p$.

Step I-8: Determine the scope of attribute weights, and achieve the criteria weights g_i, $i = 1,2,\dots,p$ within the lower and upper boundaries.

In the second stage, the decision matrices of criteria values are another required input for the proposed approach. Again experts use the IVIFNs to express their opinions.

Step II-1: Use (24) to set up the decision matrices of criteria values as follows:
$\tilde{D}_k = \left(\tilde{d}_{ij}{}^{(k)}\right)_{n\times p}, \qquad i = 1,2,\dots,n \quad ; j = 1,2,\dots,p \quad ; k = 1,2,\dots,m.$

Step II-2: Use (18) to calculate decision function for each element in the decision matrices as follows:

$R_{ij}{}^{(k)} = W_\omega\left(\tilde{d}_{ij}{}^{(k)}\right) + \lambda^{(k)} T_\omega\left(\tilde{d}_{ij}{}^{(k)}\right); \;\; \lambda^{(k)} \in R;$

Step II-3: Reorder the decision function results in descending order based on previous step, such that $\tilde{d}_{ij_{\sigma(j-1)}} \geq \tilde{d}_{ij_{\sigma(j)}}$ for all, and let $\tilde{d}_{ij_{\sigma(j)}} = \left(\left[a_{\sigma(j)}, b_{\sigma(j)}\right], \left[c_{\sigma(j)}, d_{\sigma(j)}\right]\right)$.

Step II-4: Apply the operation in (22) to integrate experts' opinions on criteria values, and establish the aggregated decision matrix of criterion values as follows:

$\dot{\tilde{D}} = \left(\dot{\tilde{d}}_{ij}\right)_{n\times p}$

Where $\dot{\tilde{d}}_{ij} = IIFOWA_w\left(\tilde{d}_{ij}{}^{(1)}, \tilde{d}_{ij}{}^{(2)}, \dots, \tilde{d}_{ij}{}^{(m)}\right); i = 1,2,\dots,n\,; j = 1,2,\dots,p$.

Step II-5: Calculate the weighted score function for each alternative in the condition of unknown criteria weights.

Step II-6: Use a linear programming model to calculate the exact criterion weights. The optimization is defined by a sum of weighted score function grades of each alternative, and subject to the weight assumption$H(x)$.

$$Max \sum_{i=1}^{n}\sum_{j=1}^{p} W_j S(\dot{\tilde{d}}_{ij})$$

St. $H(x)$

Step II-7: Apply the results of criteria weights in weighted score function grades of each alternative in Step II-5 to calculate the interval-valued intuitionistic fuzzy grades of each alternatives.

Step II-8: Determine the priority of the alternatives.

6 - Case study

In order to demonstrate how the proposed procedure can be applied in practice, it is implemented in an Iranian manufacturing and engineering company. In order to increase market share over the long-term planning they wanted to select the best supplier for establishing an effective supply chain management. Four suppliers were identified ($y_1, y_2, y_3 and\ y_4$) and five experts ($e_1, e_2, e_3, e_4\ and\ e_5$), who are technically competent and experienced, participated in our study. The expert weight vector is given by$\omega = (0.30,0.25,0.15,0.15,0.15)^T$and also their risk attitude vector is given by$\lambda = (0.00,0.25,1.00,-0.50,-0.25)^T$.These experts were trained to use the proposed approach and asked them to apply this procedure to select the most appropriate supplier. The criteria considered in the selection process were: production ability (g_1), financial issues (g_2), delivery time (g_3) and services (g_4).The experiment results and survey findings indicated that proposed approach was satisfying and helped the group to make the best decision. The selected supplier by using the proposed method was also approved from the purchasing manager. The results are brought in the following.

Step I-1: Construct the interval-valued intuitionistic fuzzy preference relation matrices on the criteria based on pairwise comparison.

$$\tilde{G}_1 = \begin{bmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.4,0.7],[0.1,0.2]) & ([0.5,0.6],[0.2,0.3]) & ([0.3,0.5],[0.2,0.4]) \\ ([0.1,0.2],[0.4,0.7]) & ([0.5,0.5],[0.5,0.5]) & ([0.5,0.6],[0.1,0.2]) & ([0.6,0.7],[0.1,0.3]) \\ ([0.2,0.3],[0.5,0.6]) & ([0.1,0.2],[0.5,0.6]) & ([0.5,0.5],[0.5,0.5]) & ([0.3,0.4],[0.5,0.6]) \\ ([0.2,0.4],[0.3,0.5]) & ([0.1,0.3],[0.6,0.7]) & ([0.5,0.6],[0.3,0.4]) & ([0.5,0.5],[0.5,0.5]) \end{bmatrix}$$

$$\tilde{G}_2 = \begin{bmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.4,0.6],[0.3,0.4]) & ([0.5,0.7],[0.2,0.3]) & ([0.5,0.7],[0.2,0.3]) \\ ([0.3,0.4],[0.4,0.6]) & ([0.5,0.5],[0.5,0.5]) & ([0.4,0.6],[0.1,0.3]) & ([0.4,0.5],[0.1,0.2]) \\ ([0.2,0.3],[0.5,0.7]) & ([0.1,0.3],[0.4,0.6]) & ([0.5,0.5],[0.5,0.5]) & ([0.5,0.7],[0.1,0.2]) \\ ([0.2,0.3],[0.5,0.7]) & ([0.1,0.2],[0.4,0.5]) & ([0.1,0.2],[0.5,0.7]) & ([0.5,0.5],[0.5,0.5]) \end{bmatrix}$$

$$\tilde{G}_3 = \begin{bmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.7,0.8],[0.1,0.2]) & ([0.6,0.7],[0.1,0.2]) & ([0.6,0.7],[0.2,0.3]) \\ ([0.1,0.2],[0.7,0.8]) & ([0.5,0.5],[0.5,0.5]) & ([0.5,0.7],[0.2,0.3]) & ([0.4,0.6],[0.2,0.3]) \\ ([0.1,0.2],[0.6,0.7]) & ([0.2,0.3],[0.5,0.7]) & ([0.5,0.5],[0.5,0.5]) & ([0.3,0.4],[0.5,0.6]) \\ ([0.2,0.3],[0.6,0.7]) & ([0.2,0.3],[0.4,0.6]) & ([0.5,0.6],[0.3,0.4]) & ([0.5,0.5],[0.5,0.5]) \end{bmatrix}$$

$$\tilde{G}_4 = \begin{bmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.5,0.6],[0.3,0.4]) & ([0.3,0.4],[0.5,0.6]) & ([0.7,0.8],[0.1,0.2]) \\ ([0.3,0.4],[0.5,0.6]) & ([0.5,0.5],[0.5,0.5]) & ([0.3,0.4],[0.5,0.6]) & ([0.5,0.6],[0.3,0.4]) \\ ([0.5,0.6],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4]) & ([0.5,0.5],[0.5,0.5]) & ([0.5,0.6],[0.3,0.4]) \\ ([0.1,0.2],[0.7,0.8]) & ([0.3,0.4],[0.5,0.6]) & ([0.3,0.4],[0.5,0.6]) & ([0.5,0.5],[0.5,0.5]) \end{bmatrix}$$

$$\tilde{G}_5 = \begin{bmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.3,0.4],[0.4,0.6]) & ([0.5,0.6],[0.3,0.4]) & ([0.4,0.5],[0.3,0.4]) \\ ([0.4,0.6],[0.3,0.4]) & ([0.5,0.5],[0.5,0.5]) & ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.1,0.3]) \\ ([0.3,0.4],[0.5,0.6]) & ([0.2,0.3],[0.6,0.7]) & ([0.5,0.5],[0.5,0.5]) & ([0.3,0.4],[0.2,0.3]) \\ ([0.3,0.4],[0.4,0.5]) & ([0.1,0.3],[0.6,0.7]) & ([0.2,0.3],[0.3,0.4]) & ([0.5,0.5],[0.5,0.5]) \end{bmatrix}$$

Step I-2: According to the decision maker's risk attitude, choose the appropriate risk parameter.

$$\lambda^{(1)} = 0.00 \qquad \lambda^{(2)} = 0.25 \qquad \lambda^{(3)} = 1.00 \qquad \lambda^{(4)} = -0.50 \qquad \lambda^{(5)} = -0.25$$

Step I-3: Aggregate each row of preference relations.

$$\begin{aligned} \tilde{g}_1{}^{(1)} = {} & ([1-(1-0.5)^{0.25}\times(1-0.4)^{0.25}\times(1-0.5)^{0.25}\times(1-0.3)^{0.25}, 1 \\ & -(1-0.5)^{0.25}\times(1-0.7)^{0.25}\times(1-0.6)^{0.25}\times(1-0.5)^{0.25}], [0.5^{0.25}\times 0.1^{0.25} \\ & \times 0.2^{0.25}\times 0.2^{0.25}, 0.5^{0.25}\times 0.2^{0.25}\times 0.3^{0.25}\times 0.4^{0.25}]) \\ & = ([0.4308,0.5838],[0.2115,0.3310]) \end{aligned}$$

$\tilde{g}_1{}^{(1)} = ([0.4308,0.5838],[0.2115,0.3310])$,
$\tilde{g}_2{}^{(1)} = ([0.4523,0.5319],[0.2115,0.3807])$,
$\tilde{g}_3{}^{(1)} = ([0.2915,0.3598],[0.5000,0.5733])$,
$\tilde{g}_4{}^{(1)} = ([0.3486,0.4616],[0.4054,0.5144])$,
$\tilde{g}_1{}^{(2)} = ([0.4767,0.6337],[0.2783,0.3663])$,
$\tilde{g}_2{}^{(2)} = ([0.4042,0.5051],[0.2115,0.3663])$,
$\tilde{g}_3{}^{(2)} = ([0.3486,0.4793],[0.3162,0.4527])$,
$\tilde{g}_4{}^{(2)} = ([0.2455,0.3120],[0.4729,0.5916])$,
$\tilde{g}_1{}^{(3)} = ([0.6064,0.6920],[0.1778,0.2783])$,
$\tilde{g}_2{}^{(3)} = ([0.3938,0.5319],[0.3440,0.4356])$,
$\tilde{g}_3{}^{(3)} = ([0.2915,0.3598],[0.5233,0.6192])$,
$\tilde{g}_4{}^{(3)} = ([0.3675,0.4405],[0.4356,0.5384])$,
$\tilde{g}_1{}^{(4)} = ([0.5213,0.6064],[0.2943,0.3936])$,
$\tilde{g}_2{}^{(4)} = ([0.4084,0.4820],[0.4401,0.5180])$,
$\tilde{g}_3{}^{(4)} = ([0.5000,0.5771],[0.3409,0.4229])$,
$\tilde{g}_4{}^{(4)} = ([0.3147,0.3840],[0.5439,0.6160])$,
$\tilde{g}_1{}^{(5)} = ([0.4308,0.5051],[0.3663,0.4681])$,
$\tilde{g}_2{}^{(5)} = ([0.5319,0.6337],[0.2340,0.3663])$,
$\tilde{g}_3{}^{(5)} = ([0.3346,0.4042],[0.4162,0.5051])$,
$\tilde{g}_4{}^{(5)} = ([0.2915,0.3808],[0.4356,0.5144])$.

Step I-4: Calculate decision function for each aggregate's IVIFNs.

$$\begin{aligned} R_1{}^{(1)} = 0.30\bigg(& \frac{0+1}{2}(0.4308+0.5838-0.2115-0.3310) \\ & + \frac{0}{2}(0.4308\times 0.2115+0.5838\times 0.3310)\bigg) = 0.0708 \end{aligned}$$

$R_1{}^{(1)} = 0.0708$,
$R_2{}^{(1)} = 0.0588$,
$R_3{}^{(3)} = -0.0455$,
$R_4{}^{(3)} = 0.0049$,

$R_3{}^{(1)} = -0.0633,$
$R_4{}^{(1)} = -0.0164,$
$R_1{}^{(2)} = \;\; 0.0842,$
$R_2{}^{(2)} = \;\; 0.0603,$
$R_3{}^{(2)} = \;\; 0.0195,$
$R_4{}^{(2)} = -0.0698,$
$R_1{}^{(3)} = \;\; 0.1489,$
$R_2{}^{(3)} = \;\; 0.0495,$
$R_1{}^{(4)} = \;\; 0.0018,$
$R_2{}^{(4)} = -0.0186,$
$R_3{}^{(4)} = -0.0038,$
$R_4{}^{(4)} = -0.0326,$
$R_1{}^{(5)} = -0.0017,$
$R_2{}^{(5)} = \;\; 0.0251,$
$R_3{}^{(5)} = -0.0164,$
$R_4{}^{(5)} = -0.0217.$

Step I-5: Reordering the decision function results in descending order based on previous step.

$R_1{}^{(3)} = 0.1489 > R_1{}^{(2)} = 0.0842 > R_1{}^{(1)} = 0.0708 > R_1{}^{(4)} = 0.0018 > R_1{}^{(5)} = -0.0017$
Then extract the new the permutation of arguments as follows:

$\tilde{g}_{1_{\sigma(1)}} = ([0.6064, 0.6920], [0.1778, 0.2783]),$
$\tilde{g}_{1_{\sigma(2)}} = ([0.4767, 0.6337], [0.2783, 0.3663]),$
$\tilde{g}_{1_{\sigma(3)}} = ([0.4308, 0.5838], [0.2115, 0.3310]),$
$\tilde{g}_{1_{\sigma(4)}} = ([0.5213, 0.6064], [0.2943, 0.3936]),$
$\tilde{g}_{1_{\sigma(5)}} = ([0.4308, 0.5051], [0.3663, 0.4681]).$

Step I-6: Integrate experts' opinions on the criteria, and express the criterion weights in the interval-valued intuitionistic fuzzy format.

$$\begin{aligned}\tilde{g}_1 = \; & ([1 - (1 - 0.6064)^{0.1117} \times (1 - 0.4767)^{0.2365} \times (1 - 0.4308)^{0.3036} \times (1 - 0.5213)^{0.2365} \\ & \times (1 - 0.4308)^{0.1117}, 1 - (1 - 0.6920)^{0.1117} \times (1 - 0.6337)^{0.2365} \\ & \times (1 - 0.5838)^{0.3036} \times (1 - 0.6064)^{0.2365} \times (1 - 0.5051)^{0.1117}], [0.1778^{0.1117} \\ & \times 0.2783^{0.2365} \times 0.2115^{0.3036} \times 0.2943^{0.2365} \times 0.3663^{0.1117}, 0.2783^{0.1117} \\ & \times 0.3663^{0.2365} \times 0.3310^{0.3036} \times 0.3936^{0.2365} \times 0.4681^{0.1117}]) \\ & = ([0.4860, 0.6071], [0.2545, 0.3601])\end{aligned}$$

$\tilde{g}_1 = ([0.4860, 0.6071], [0.2545, 0.3601]),$
$\tilde{g}_2 = ([0.4459, 0.5505], [0.2725, 0.4050]),$
$\tilde{g}_3 = ([0.3659, 0.4451], [0.4148, 0.5079]),$
$\tilde{g}_4 = ([0.3148, 0.4014], [0.4555, 0.5480]).$

Where the weight vector of arguments is derived by the normal distribution which is $w = (0.1117, 0.2365, 0.3036, 0.2365, 0.1117)^{\mathrm{T}}$.

Step I-7: Convert the criteria weights from interval-valued intuitionistic fuzzy formats into interval-valued fuzzy formats.

$$\begin{aligned}\tilde{g}_1 = \; & ([0.4860/(0.6071 + 0.5505 + 0.4451 + 0.4014), 0.6071/(0.4860 + 0.4459 + 0.3659 \\ & + 0.3148)], [0.2545/(0.3601 + 0.4050 + 0.5079 + 0.5480), 0.3601/(0.2545 \\ & + 0.2725 + 0.4148 + 0.4555)]) = ([0.2425, 0.3765], [0.1397, 0.2577])\end{aligned}$$

$\tilde{g}_1 = ([0.2425,0.3765],[0.1397,0.2577])$, $\tilde{g}_3 = ([0.1826,0.2760],[0.2278,0.3635])$,
$\tilde{g}_2 = ([0.2225,0.3414],[0.1496,0.2898])$, $\tilde{g}_4 = ([0.1571,0.2489],[0.2501,0.3922])$.

Because of the IFSs and IVFSs are mathematically equivalent, the above criteria weights can be converted into the interval-valued fuzzy format. Then $\tilde{g}_i$is converted as follows:

$$\tilde{g}_1 = ([0.2425,0.3765],[0.1397,0.2577]) = ([0.2425,0.3765],[1-0.2577,1-0.1397]) \\ = ([0.2425,0.3765],[0.7423,0.8603])$$

$\tilde{g}_1 = ([0.2425,0.3765],[0.7423,0.8603])$, $\tilde{g}_3 = ([0.1826,0.2760],[0.6365,0.7722])$,
$\tilde{g}_2 = ([0.2225,0.3414],[0.7102,0.8504])$, $\tilde{g}_4 = ([0.1571,0.2489],[0.6078,0.7499])$.

Step I-8: Determine the scope of criteria weights, and achieve the criteria weights g_i within the lower and upper boundaries.

$0.2425 \le g_1 \le 0.8603$, $0.1826 \le g_3 \le 0.7722$,
$0.2225 \le g_2 \le 0.8504$, $0.1571 \le g_4 \le 0.7499$.

Step II-1: Set up the decision matrixes of criterion values as follows:

$$\tilde{D}_1 = \begin{bmatrix} ([0.3,0.5],[0.4,0.5]) & ([0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.2,0.3]) & ([0.4,0.7],[0.0,0.1]) \\ ([0.6,0.8],[0.1,0.2]) & ([0.6,0.7],[0.2,0.3]) & ([0.6,0.8],[0.1,0.2]) & ([0.5,0.7],[0.1,0.3]) \\ ([0.7,0.8],[0.1,0.2]) & ([0.7,0.8],[0.0,0.1]) & ([0.5,0.7],[0.2,0.3]) & ([0.6,0.8],[0.1,0.2]) \\ ([0.2,0.3],[0.4,0.5]) & ([0.5,0.7],[0.1,0.3]) & ([0.4,0.6],[0.3,0.4]) & ([0.4,0.5],[0.1,0.3]) \end{bmatrix}$$

$$\tilde{D}_2 = \begin{bmatrix} ([0.5,0.6],[0.3,0.4]) & ([0.4,0.6],[0.1,0.2]) & ([0.6,0.7],[0.2,0.3]) & ([0.5,0.6],[0.1,0.2]) \\ ([0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.3,0.4]) & ([0.4,0.5],[0.3,0.4]) & ([0.5,0.7],[0.1,0.2]) \\ ([0.6,0.8],[0.1,0.2]) & ([0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.3,0.4]) & ([0.7,0.9],[0.0,0.1]) \\ ([0.4,0.6],[0.3,0.4]) & ([0.4,0.5],[0.0,0.1]) & ([0.4,0.5],[0.2,0.4]) & ([0.4,0.6],[0.1,0.2]) \end{bmatrix}$$

$$\tilde{D}_3 = \begin{bmatrix} ([0.5,0.7],[0.2,0.3]) & ([0.5,0.6],[0.1,0.2]) & ([0.5,0.6],[0.2,0.4]) & ([0.4,0.6],[0.1,0.3]) \\ ([0.5,0.6],[0.1,0.2]) & ([0.5,0.7],[0.2,0.3]) & ([0.3,0.6],[0.2,0.4]) & ([0.6,0.8],[0.0,0.1]) \\ ([0.5,0.8],[0.1,0.2]) & ([0.5,0.8],[0.1,0.2]) & ([0.4,0.7],[0.2,0.3]) & ([0.5,0.8],[0.0,0.2]) \\ ([0.4,0.6],[0.1,0.3]) & ([0.4,0.6],[0.0,0.1]) & ([0.3,0.5],[0.2,0.4]) & ([0.4,0.6],[0.2,0.3]) \end{bmatrix}$$

$$\tilde{D}_4 = \begin{bmatrix} ([0.3,0.4],[0.4,0.6]) & ([0.6,0.7],[0.1,0.2]) & ([0.7,0.8],[0.1,0.2]) & ([0.5,0.6],[0.0,0.1]) \\ ([0.5,0.8],[0.1,0.2]) & ([0.6,0.7],[0.2,0.3]) & ([0.4,0.6],[0.1,0.4]) & ([0.5,0.6],[0.1,0.3]) \\ ([0.7,0.8],[0.1,0.2]) & ([0.7,0.8],[0.0,0.1]) & ([0.5,0.7],[0.2,0.3]) & ([0.6,0.7],[0.1,0.2]) \\ ([0.2,0.3],[0.4,0.5]) & ([0.5,0.7],[0.1,0.3]) & ([0.5,0.6],[0.1,0.3]) & ([0.4,0.5],[0.1,0.3]) \end{bmatrix}$$

$$\tilde{D}_5 = \begin{bmatrix} ([0.3,0.4],[0.4,0.6]) & ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) & ([0.5,0.7],[0.0,0.1]) \\ ([0.4,0.6],[0.2,0.4]) & ([0.6,0.8],[0.1,0.2]) & ([0.5,0.6],[0.3,0.4]) & ([0.7,0.8],[0.1,0.2]) \\ ([0.5,0.7],[0.1,0.3]) & ([0.7,0.9],[0.0,0.1]) & ([0.5,0.6],[0.2,0.4]) & ([0.6,0.9],[0.0,0.1]) \\ ([0.2,0.3],[0.5,0.6]) & ([0.5,0.7],[0.1,0.3]) & ([0.4,0.6],[0.1,0.2]) & ([0.4,0.6],[0.1,0.3]) \end{bmatrix}$$

Step II-2: Calculate decision function for each element in the decision matrices.

$$R_{11}{}^{(1)} = 0.30\left(\frac{0+1}{2}(0.3+0.5-0.4-0.5)+\frac{0}{2}(0.3\times0.4+0.5\times0.5)\right) = -0.0150$$

$R_{11}{}^{(1)} = -0.0150,$
$R_{11}{}^{(2)} = \ \ 0.0747,$
$R_{11}{}^{(3)} = \ \ 0.1282,$
$R_{11}{}^{(4)} = -0.0247,$
$R_{11}{}^{(5)} = -0.0236.$

Step II-3: Reorder the decision function results in descending order based on previous step.

$\tilde{d}_{11_{\sigma(1)}} = ([0.5,0.7],[0.2,0.3]),$
$\tilde{d}_{11_{\sigma(2)}} = ([0.5,0.6],[0.3,0.4]),$
$\tilde{d}_{11_{\sigma(3)}} = ([0.3,0.5],[0.4,0.5]),$
$\tilde{d}_{11_{\sigma(4)}} = ([0.3,0.4],[0.4,0.6]),$
$\tilde{d}_{11_{\sigma(5)}} = ([0.3,0.4],[0.4,0.6]).$

Step II-4: Integrate experts' opinions on criteria values, and establish the aggregated decision matrix of criterion values.

$$\begin{aligned}\dot{\tilde{d}}_{11} \ = \ &([1-(1-0.5)^{0.1117}\times(1-0.5)^{0.2365}\times(1-0.3)^{0.3036}\times(1-0.3)^{0.2365}\\ &\times(1-0.3)^{0.1117}, 1-(1-0.7)^{0.1117}\times(1-0.6)^{0.2365}\times(1-0.5)^{0.3036}\\ &\times(1-0.4)^{0.2365}\times(1-0.4)^{0.1117}],[0.2^{0.1117}\times0.3^{0.2365}\times0.4^{0.3036}\times0.4^{0.2365}\\ &\times0.4^{0.1117}, 0.3^{0.1117}\times0.4^{0.2365}\times0.5^{0.3036}\times0.6^{0.2365}\times0.6^{0.1117}])\\ &= ([0.3774,0.5226],[0.3458,0.4774])\end{aligned}$$

$$\dot{\tilde{D}} = \begin{bmatrix} ([0.3774,0.5226],[0.3458,0.4774]) & ([0.5231,0.6496],[0.1178,0.2201]) & ([0.5630,0.6651],[0.1851,0.3069]) & ([0.4607,0.6381],[0.0000,0.1645]) \\ ([0.5279,0.7064],[0.1081,0.2161]) & ([0.5612,0.7026],[0.1920,0.2974]) & ([0.4304,0.6039],[0.2132,0.3702]) & ([0.5678,0.7310],[0.0000,0.2190]) \\ ([0.5769,0.7799],[0.1000,0.2201]) & ([0.6306,0.8080],[0.0000,0.1454]) & ([0.4897,0.6496],[0.2262,0.3504]) & ([0.5917,0.8356],[0.0000,0.1571]) \\ ([0.2763,0.4239],[0.3374,0.4677]) & ([0.4672,0.6504],[0.0000,0.2046]) & ([0.3903,0.5677],[0.1777,0.3288]) & ([0.4000,0.5612],[0.1178,0.2867]) \end{bmatrix}$$

Step II-5: Calculate the weighted score function for each alternative in the condition of unknown criteria weights.

$$\begin{aligned}W_\omega\left(\dot{\tilde{d}}_1\right) = \ &g_1((1/2(0.3774-0.3458+0.5226-0.4774)))\\ &+g_2((1/2(0.5231-0.1178+0.6496-0.2201)))\\ &+g_3((1/2(0.5630-0.1851+0.6651-0.3069)))\\ &+g_4((1/2(0.4607-0.0000+0.6381-0.1645)))\end{aligned}$$

$$W_\omega\left(\dot{\tilde{d}}_2\right) = g_1((1/2(0.5279 - 0.1081 + 0.7064 - 0.2161)))$$
$$+ g_2((1/2(0.5612 - 0.1920 + 0.7026 - 0.2974)))$$
$$+ g_3((1/2(0.4304 - 0.2132 + 0.6039 - 0.3702)))$$
$$+ g_4((1/2(0.5678 - 0.0000 + 0.7310 - 0.2190)))$$
$$W_\omega\left(\dot{\tilde{d}}_3\right) = g_1((1/2(0.5769 - 0.1000 + 0.7799 - 0.2201)))$$
$$+ g_2((1/2(0.6306 - 0.0000 + 0.8080 - 0.1454)))$$
$$+ g_3((1/2(0.4897 - 0.2262 + 0.6496 - 0.3504)))$$
$$+ g_4((1/2(0.5917 - 0.0000 + 0.8356 - 0.1571)))$$
$$W_\omega\left(\dot{\tilde{d}}_4\right) = g_1((1/2(0.2763 - 0.3374 + 0.4239 - 0.4677)))$$
$$+ g_2((1/2(0.4672 - 0.0000 + 0.6504 - 0.2046)))$$
$$+ g_3((1/2(0.3903 - 0.1777 + 0.5677 - 0.3288)))$$
$$+ g_4((1/2(0.4000 - 0.1178 + 0.5612 - 0.2867)))$$

Step II-6: Use a linear programming model to calculate the exact criteria weights.

$$Max\ W_\omega\left(\dot{\tilde{d}}_1\right) + W_\omega\left(\dot{\tilde{d}}_2\right) + W_\omega\left(\dot{\tilde{d}}_3\right) + W_\omega\left(\dot{\tilde{d}}_4\right)$$

$$s.t.\quad \begin{cases} 0.2425 \le g_1 \le 0.8603, \\ 0.2225 \le g_2 \le 0.8504, \\ 0.1826 \le g_3 \le 0.7722, \\ 0.1571 \le g_4 \le 0.7499, \\ g_1 + g_2 + g_3 + g_4 = 1. \end{cases}$$

$g_1 = 0.2425,$ $g_3 = 0.1826,$
$g_2 = 0.2225,$ $g_4 = 0.3525.$

Step II-7: Apply the results of criteria weights in the weighted score function grades of each alternative in Step II-5 to calculate the interval-valued intuitionistic fuzzy grades of each alternative.

$W_\omega\left(\dot{\tilde{d}}_1\right) = 0.4487,$ $W_\omega\left(\dot{\tilde{d}}_3\right) = 0.5975,$
$W_\omega\left(\dot{\tilde{d}}_2\right) = 0.5020,$ $W_\omega\left(\dot{\tilde{d}}_4\right) = 0.3521.$

Step II-8: Determine the priority of the alternatives.

$$W_\omega\left(\dot{\tilde{d}}_3\right) > W_\omega\left(\dot{\tilde{d}}_2\right) > W_\omega\left(\dot{\tilde{d}}_1\right) > W_\omega\left(\dot{\tilde{d}}_4\right)$$
$$y_3 \succ y_2 \succ y_1 \succ y_4$$

Thus, according to calculations made in previous stages, the weighted score function grades of each alternative is obtained. The third alternative has the greatest degree of acceptance and on the other hand the fourth alternative has the least degree of acceptance. The results of applying the proposed model were presented to the group of decision makers. Received opinions and feedback indicate the efficiency and effectiveness of the proposed approach.

7 - Validity of the proposed approach

In this section, in order to prove the validity of the proposed approach some of the features of the model are relaxed. Then the problem is solved again and the results are compared together. Therefore, initially relaxing risk parameter from decision function, the decision maker only considers the weighted score function. Subsequently, interval-valued fuzzy set (IVFS) is used instead of interval-valued intuitionistic fuzzy sets (IVIFSs) to cover the uncertainty. Accordingly, the final results from the application of both approaches are presented.

Table 1. The final results from the application of both approaches (Proposed approach & Simplified model)

criterion weights		grades of each alternative		priority of the alternatives	
Proposed approach	Simplified model	Proposed approach	Simplified model	Proposed approach	Simplified model
0.2425	0.2559	0.4487	0.6683	3	3
0.2225	0.4018	0.5020	0.7005	2	2
0.1826	0.1856	0.5975	0.7611	1	1
0.3525	0.1566	0.3521	0.5674	4	4

Due to the convergence observed in the results and the similarity in ranking process in two approaches, it can be argued that the proposed approach is valid.

8 - Conclusions

In this paper a new multi-criteria group decision making (MCGDM) approach based on interval-valued intuitionistic fuzzy sets is proposed. We have investigated the participation of risk attitude of decision makers as an important parameter in the decision results, and define a new decision function to participate this primary human's factor in decision process. Then, in order to integrate interval-valued intuitionistic fuzzy information, interval-valued intuitionistic fuzzy ordered weighted aggregation (IIFOWA) is applied to aggregate the obtained preferences, which uses the perspective of normal distribution to gain the weights. Ranking process is used based on the two indices, weighted score function and weighted accuracy function, to rank the alternatives. Finally, to demonstrate how the proposed procedure can be applied in practice, it is implemented to choose the suppliers in a project-based company. This paper moves us one step closer to the usage of MCGDM in real world situations.

References:

Atanassov, K., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. Fuzzy sets and systems, 31(3), 343-349.

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. Fuzzy sets and systems, 20(1), 87-96.

Atanassov, K. T. (1994). Operators over interval valued intuitionistic fuzzy sets. Fuzzy sets and systems, 64(2), 159-174.

Baccour, L., Alimi, A. M., & John, R. I. (2013). Similarity measures for intuitionistic fuzzy sets: State of the art. Journal of Intelligent and Fuzzy Systems, 24(1), 37-49.

Bustince, H., & Burillo, P. (1995). Correlation of interval-valued intuitionistic fuzzy sets. Fuzzy sets and systems, 74(2), 237-244.

Chan, F., Chan, H., Ip, R., & Lau, H. (2007). A decision support system for supplier selection in the airline industry. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 221(4), 741-758.

Chang, C.-T., Chen, H.-M., & Zhuang, Z.-Y. (2014). Integrated multi-choice goal programming and multi-segment goal programming for supplier selection considering imperfect-quality and price-quantity discounts in a multiple sourcing environment. International Journal of Systems Science, 45(5), 1101-1111.

Chen, T.-Y. (2014). The Inclusion-Based LINMAP Method for Multiple Criteria Decision Analysis Within an Interval-Valued Atanassov's Intuitionistic Fuzzy Environment. International Journal of Information Technology & Decision Making, 1-36.

Chen, T.-Y., Wang, H.-P., & Lu, Y.-Y. (2011). A multicriteria group decision-making approach based on interval-valued intuitionistic fuzzy sets: A comparative perspective. Expert Systems with Applications, 38(6), 7647-7658.

Chen, Z., & Yang, W. (2012). A new multiple criteria decision making method based on intuitionistic fuzzy information. Expert Systems with Applications, 39(4), 4328-4334.

Chou, S.-Y., Chang, Y.-H., & Shen, C.-Y. (2008). A fuzzy simple additive weighting system under group decision-making for facility location selection with objective/subjective attributes. European Journal of Operational Research, 189(1), 132-145.

Choy, K. L., Lee, W., & Lo, V. (2003). Design of a case based intelligent supplier relationship management system—the integration of supplier rating system and product coding system. Expert Systems with Applications, 25(1), 87-100.

De Boer, L., Labro, E., & Morlacchi, P. (2001). A review of methods supporting supplier selection. European Journal of Purchasing & Supply Management, 7(2), 75-89.
Deschrijver, G. (2007). Arithmetic operators in interval-valued fuzzy set theory. Information Sciences, 177(14), 2906-2924.

Deschrijver, G. (2008). A representation of t-norms in interval-valued< i> L</i>-fuzzy set theory. Fuzzy sets and systems, 159(13), 1597-1618.

Deschrijver, G., & Kerre, E. E. (2007). On the position of intuitionistic fuzzy set theory in the framework of theories modelling imprecision. Information Sciences, 177(8), 1860-1866.

Ferreira, L., & Borenstein, D. (2012). A fuzzy-bayesian model for supplier selection. Expert Systems with Applications, 39(9), 7834–7844.

Hammami, R., Temponi, C., & Frein, Y. (2014). A scenario-based stochastic model for supplier selection in global context with multiple buyers, currency fluctuation uncertainties, and price discounts. European Journal of Operational Research, 233(1), 159-170. doi: http://dx.doi.org/10.1016/j.ejor.2013.08.020

Hong, D. H. (1998). A note on correlation of interval-valued intuitionistic fuzzy sets. Fuzzy sets and systems, 95(1), 113-117.

Hong, D. H., & Choi, C.-H. (2000). Multicriteria fuzzy decision-making problems based on vague set theory. Fuzzy sets and systems, 114(1), 103-113.

Hung, W.-L., & Wu, J.-W. (2002). Correlation of intuitionistic fuzzy sets by centroid method. Information Sciences, 144(1), 219-225.

Kumar Mondal, T., & Samanta, S. (2001). Topology of interval-valued intuitionistic fuzzy sets. Fuzzy sets and systems, 119(3), 483-494.

Lakshmana Gomathi Nayagam, V., Muralikrishnan, S., & Sivaraman, G. (2011). Multi-criteria decision-making method based on interval-valued intuitionistic fuzzy sets. Expert Systems with Applications, 38(3), 1464-1467.

Lee, E. K. (2001). Development of a Supplier Selection and Management System and Safety Stock Policies in Supply Chain. Thesis for the Degree of Doctor of Philosophy, Department of Industrial Engineering Graduate School, Korea University, 45–55.

Li, L., & Zabinsky, Z. B. (2011). Incorporating uncertainty into a supplier selection problem. International Journal of Production Economics, 134(2), 344–356.

Liao, Z., & Rittscher, J. (2007). A multi-objective supplier selection model under stochastic demand conditions. International Journal of Production Economics, 105(1), 150-159.

Liu, X.-d., Zheng, S.-h., & Xiong, F.-l. (2005). Entropy and subsethood for general interval-valued intuitionistic fuzzy sets Fuzzy systems and knowledge discovery (pp. 42-52): Springer.

Ramanathan, R. (2007). Supplier selection problem: integrating DEA with the approaches of total cost of ownership and AHP. Supply Chain Management: An International Journal, 12(4), 258-261.

Reiser, R. H., & Bedregal, B. (2013). Interval-valued intuitionistic fuzzy implications–construction, properties and representability. Information Sciences, 248, 68-88.

Scott, J., Ho, W., Dey, P. K., & Talluri, S. (2014). A decision support system for supplier selection and order allocation in stochastic, multi-stakeholder and multi-criteria environments. International Journal of Production Economics.

Tavassoli, M., Faramarzi, G. R., & Saen, R. F. (2014). A joint measurement of efficiency and effectiveness for the best supplier selection using integrated data envelopment analysis approach. International Journal of Mathematics in Operational Research, 6(1), 70-83.

Tseng, M. (2011). Green supply chain management with linguistic preferences and incomplete information. Applied Soft Computing Journal, 11(8), 4894–4903.

Wang, Z., Li, K. W., & Wang, W. (2009). An approach to multiattribute decision making with interval-valued intuitionistic fuzzy assessments and incomplete weights. Information Sciences, 179(17), 3026-3040.

Wang, Z., Li, K. W., & Xu, J. (2011). A mathematical programming approach to multi-attribute decision making with interval-valued intuitionistic fuzzy assessment information. Expert Systems with Applications, 38(10), 12462-12469.

Weber, M. (1987). Decision making with incomplete information. European Journal of Operational Research, 28(1), 44-57.

Wei, G.-W., & Wang, X.-R. (2007). Some geometric aggregation operators based on interval-valued intuitionistic fuzzy sets and their application to group decision making. Paper presented at the Computational Intelligence and Security, 2007 International Conference.

Xu, Z.-S., & Chen, J. (2007a). Approach to group decision making based on interval-valued intuitionistic judgment matrices. Systems Engineering-Theory & Practice, 27(4), 126-133.

Xu, Z. (2005). An overview of methods for determining OWA weights. International journal of intelligent systems, 20(8), 843-865.

Xu, Z. (2007b). A survey of preference relations. International journal of general systems, 36(2), 179-203.

Xu, Z. (2007c). Multiple-attribute group decision making with different formats of preference information on attributes. Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on, 37(6), 1500-1511.

Xu, Z., & Chen, J. (2007b). On geometric aggregation over interval-valued intuitionistic fuzzy information. Paper presented at the Fuzzy Systems and Knowledge Discovery, 2007. FSKD 2007. Fourth International Conference.

Xu, Z., & Chen, J. (2008). An overview of distance and similarity measures of intuitionistic fuzzy sets. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 16(04), 529-555.

Xu, Z., & Da, Q.-L. (2003). An overview of operators for aggregating information. International journal of intelligent systems, 18(9), 953-969.

Xu, Z., & Yager, R. R. (2008). Dynamic intuitionistic fuzzy multi-attribute decision making. International Journal of Approximate Reasoning, 48(1), 246-262.

Yu, M., Goh, M., & Lin, H. (2012). Fuzzy multi-objective vendor selection under lean procurement. European Journal of Operational Research, 219(2), 305–311.

Zadeh, L. A. (1965). Fuzzy sets. Information and control, 8(3), 338-353.

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. Information Sciences, 8(3), 199-249.

Ze-Shui, X. (2007a). Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making [J]. Control and Decision, 2, 019.

Zhang, X., Liu, J., Lei, J., & Yang, B. a. (2008). The weak consistency of an interval-valued intuitionistic fuzzy matrix. Paper presented at the Fuzzy Systems, 2008. FUZZ-IEEE 2008.(IEEE World Congress on Computational Intelligence). IEEE International Conference.