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Market power influential approach using game theory in a two competing supply chains with multi-echelons under centralized/decentralized environments

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Abstract

This paper is considering the competition between two multi-echelon supply-chains on price and service under balance and imbalance of market power between the chains which are analyzing through Nash and Stackelberg game approach. The problem is categorized as the centralized or decentralized structure of each chain, which means a few different possible scenarios are developing based on the Nash and Stackelberg games. The aim of the paper is to investigate the simultaneous effect of the chains' structure and market power on the decision variables. As a surprise result, we show that in the Stackelberg game, the chain will not always have the second-mover advantage. Furthermore, the results demonstrate that the leader's presence in the market may have different impacts on the situation depending on the structure of the chains. Also, when the chains take their decisions sequentially, the service and the price jointly play a strategic role in earning profits.
Keywords: Market power, Nash vs. Stackelberg game, pricing, service level, supply chain management.

1- Introduction

Supply chain in a rough viewpoint may be acknowledged as a collective effort of several companies, which leads to a satisfied reaction to the customer demands, and ultimately, the delivery of products to the final customers in the best manner. However, competition among companies has created a new era of competing among global supply chains, i.e.; depending on the kind of the role of the supply chains in the market (i.e.; Nash equilibrium and Stackelberg game), can create different results, which is the subject of the present research. On the other hand, based on the interactions existing in any supply chain (horizontal or vertical), vertical or horizontal competition can be defined. Simply, the horizontal competition is the one among the members of a certain level in a supply chain, whereas vertical competition is the one among various levels of a chain (Anderson and Bao, 2010). Consequently, two types of competition can be taken into account for supply chains:

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1) Competition among the members of a two- or three-echelon supply chain (vertical competition);
2) Competition between supply chains (horizontal competition);
The above two mentioned categories of competition are reviewed in the following sub-sections.

1-1-Competition among the members of a two- or three-echelon supply chain

Numerous studies have been conducted regarding competition within supply chains (Choi, 1991; Iyer, 1998; Pan et al., 2010; Sinha and Sarmah, 2010; Parthasarathi et al., 2011; Wu et al., 2012; Zhao et al., 2014; Xi and Lee, 2015). Tsay and Agrawal (2000) investigated a supply chain in which a manufacturer produces goods for two independent retailers. Both retailers compete on retailer price and service for final customers. Here, service means different types of efforts to increase demands including after-sales service, services before the purchase, in-store promotion, and advertising. Xiao and Qi (2008) investigated the disruption to production costs in a supply chain including a manufacturer and two competitive retailers in a Bertrand market. Two retailers, who compete on price, play a static game of complete information in a Bertrand market. Jaber and Goyal (2008) investigated the coordination of orders among the members of a three-echelon supply chain in which the first level includes several buyers, the second level is vendor (manufacturer), and the third level has several suppliers. Cai et al. (2009) focused on determining pricing strategy and price-only, buy-back, rebate in a supply chain which includes a manufacturer and a retailer. They assumed that, in addition to retailing channel for final customers, there is a direct channel. They studied the effect of discount contracts in three scenarios (Stackelberg Supplier, Stackelberg Retailer, and Nash Game) to calculate the decision variables. Seyed Esfahani et al. (2011) investigated a supply chain which includes a manufacture whose products were sold through a retailer. The manufacturer decides on the wholesale price, national advertising expenditures, and participation rate, while the retailer decides on retailer price and local advertising costs. Demand function depends on retailer price and advertising costs and has three modes (linear, convex and concave) compared to price. Lu et al. (2011) studied a supply chain which consists of two competitive manufacturers and a common retailer selling the products of both manufacturers.

Huang et al. (2011) investigated the coordination of decisions such as component and selection of suppliers, inventory, and pricing in a three-echelon supply chain including several suppliers, a manufacturer, and several retailers. Their numerical results show that increase in the market scale of a retailer reduces the profit of another retailer. Wei et al. (2013) and Giri et al. (2015) obtained significant results by considering three groups of customers who purchase the complementary products from two manufacturers and by considering the effect of competition on products quality among several manufacturers, respectively. Giri and Sarker (2015) investigated a supply chain with a manufacturer who may face a production disruption, and two independent retailers who compete with each other on price and service level. They showed that a linear wholesale price discount scheme can align the participating entities and coordinate the supply chain. Wang et al. (2017) studied the service and price-related decisions for complementary products in a two-echelon supply chain with two manufactures and a common retailer. The service is provided for both products by the retailer. One of the two manufacturers sell the products through the direct channel and retailing. Naimi Sadigh et al. (2016) investigated the coordination of pricing, inventory management, and marketing decisions in a multi-echelon supply chain with several products. The demand of each product was considered to be non-linear and dependent on retailer price and marketing costs. In this research, all the supply chain members have equal power in the chain. The results of a numerical example showed that if retailers offer greater retailer prices, then the related manufacturer and supplier sell their products at greater prices.

Lan et al (2018) analyzed a supply chain in which a manufacturer distributed a product through two distributors to a retailer whose demand was uncertain. The two channels differ in terms of their commitment to offering return credits, and they compete by charging different wholesale prices to the retailer. They showed that the dual-channel system benefits the manufacturer and the retailer if the level of demand uncertainty exceeds a threshold and that the competition between the two distributors leads to the coordination of the downstream supply chain. Giri and Dey (2019) developed the work by Jafari et al (2017) with a backup supplier considering uncertainty of collection of used products. Under various power structures of the supply chain entities, different game theoretic models are developed. As results, depending on the fractional part of the manufacturer's requirements of recyclable wastes supplied by the collector, the performance of the supply chain increases compared to that of Jafari et al.'s (2017). Zheng et al (2019) investigated a three-echelon closed-loop supply chain. Cooperative and non-cooperative game theoretic analyses were employed to characterize interactions among different parties. Results confirm the conventional wisdom: with the retailer's fairness concerns, the channel profits under the decentralized and partial-coalition models underperform that under the centralized model.

Other studies focused on the competition within the supply chain include: Chung et al. (2011), He & Zhao (2012), Chen et al. (2012), Jiang et al. (2014), Modak et al. (2016), Jafari et al (2017), Mokhlesian and Zegordi (2018) and Li and Chen (2018).

1-2- Competition between supply chains

The study by McGuire & Staelin (1983) on price competition between two suppliers whose products are sold through independent retailers in a duopoly market with two competitive supply chains showed that in deterministic model with substitutable products and competition on price, decentralized structure is preferred over the centralized structure by chains as the degree of substitution between products rises. Boyaci & Gallego (2004) studies three competitive scenarios between two supply chains: 1- Both chains are centralized; 2- Both chains are decentralized and 3- One chain is centralized whereas the other is decentralized. They assumed that both chains have selected similar prices for their products and compete based on customer service. Qian (2006) studied the price competition between Parallel Distribution Channels (PDCs). Each channel had a manufacturer and a retailer. This paper was, in fact, the first to consider the competition between two supply chains assuming that a chain was leader. Xiao and Yang (2008) investigated the price and service competition between two supply chains that each of them has a risk-neutral supplier and a risk-averse retailer. The demand of each supply chain is nondeterministic and depends on retail price and service. Wu et al. (2009) developed the competitive model with two supply chains by simultaneous decision making on price and quantity in the competitive model at once and indefinite number of time periods between two chains. Anderson and Bao (2010) investigated the competition between two-level supply chains where exclusive retailers compete for their final customers. In this study, supply chains were considered to be either integrated or decentralized. Li et al (2013) investigated contract selection by manufacturer to create coordination in competition between two supply chains. Two types of supply chain structures were taken into account: 1- Supply chains with two common retailers and 2- Supply chains with exclusive retailers. Mahmoodi & Eshghi (2014) studied the horizontal chain-to-chain competition based on price. The structure of industry was taken into account in three modes: 1- Both chains are integrated 2- One chain is centralized and the other is decentralized 3- Both chains are decentralized. Amin-Naseri & Azari Khojasteh (2015) investigated a competitive model between leader-follower supply chains, each of them having a risk-neutral manufacturer and a risk-averse retailer. They showed that in both leader and follower supply chains, increase in risk aversion of retailers may lead to decrease the total profit of the supply chain. Baron et al. (2016) developed the work by MacGuire and Staelin (1983). They studied the Nash Equilibrium on an industry with two competitive supply chains. They showed that when the demand is deterministic, both strategies of the Stackelberg manufacturer and vertical integration are particular modes of the Nash bargaining on wholesale prices. Price completion between two supply chains had been studied by Zheng et al (2016). They assumed that one of the supply chains is normal and the other is reverse. The supply chains may have the same or different structure. The impact of the degree of competition intensity between two chains and product return rate of the reverse supply chain on profit and price equilibrium is studied by them.

Hafezalkotob et al. (2017) formulated a competitive model in multi-product green supply chains under government supervision to reduce the environmental pollution cost. They provided a novel approach to construct a model that maximized the government tariffs and profits of the suppliers and manufacturers in all the green supply chains. The results demonstrated that the fiscal policy of the government greatly affected the reduction of environmental pollution costs. Taleizadeh and Sadeghi (2018) considered two competitive reverse supply chains that competed in collection and refurbishment of used products after their useful lives. One of the chains collected eligible obsolete products through the traditional and Internet channels, while the competitor used only the traditional channel. They showed that the e-channel proposed more appropriate rewards to the customer because it was less costly than the traditional channel, so the former channel achieved a more substantial share of the market. The works by Ha and Tong. (2008), Wu (2013), and Li and Li (2014) can be pointed in this regard.

This study aims to investigate the competition between two supply chains. Due to the fact that a real supply chain has more than two levels in the real world; therefore, to get closer to reality, two supply chains with three-echelons are considered. From managerial point of view, increase in the levels of a supply chain (i.e. adding a distributor in the second level) makes changes in vertical interaction as follows:

1- Difference in information gathering and analysis of information system;
2- Decrease in the distribution cost of manufacturer (because the distributor gets the customers' information and sends them to the manufacturer)

In each chain, the first level is considered to be a manufacturer, the second level is a distributor, and the third level is a retailer. The products of two chains are substitutable. The competition of supply chains is focused on retailer price and the service level given to the final customer by the manufacturer of each chain. Two games (Nash and Stackelberg) are considered between the chains based on the balance or imbalance of power between the chains in the market. In the Nash game, both chains make decisions simultaneously on the decision variables, while in the Stackelberg game, decisions are made sequentially due to the greater power of one chain in the market. Accordingly, based on the supply chains' structure, the mentioned games may result in three and four scenarios, respectively. In each chain, the manufacturer is considered to be the leader, and the distributor and retailer are the followers. Therefore, we model leader-follower relationship in each chain by a leader-several follower Stackelberg game. In fact, the purpose of this paper is to identify that when two rival chains with the specific structure determine their price and service simultaneously, what will happen to the service level, price, and profit if the chains take their decisions consecutively?

In other words, the main contribution of this paper is to analyze the simultaneous effect of market power structure, structure of each chain and product substitutability in each of price and service level dimensions in the horizontal competition between the supply chains on the equilibrium of decision variables. Our study completes the literature by examining the issue of three-level chains' movement in the market affecting the profit, service level, and price when the chains can have the same or different structure. Table 1 summarizes the studies of the current section based on competitive factors, structure of market power and the number of supply chain levels.

According to the study by Wang et al. (2017), Zhao et al. (2013), and Tsay & Agrawal (2000), service widely covers all efforts made to increase demand. These efforts include aftersales service, service before the sales, advertising, in-store promotions, product placement, and overall quality of the shopping experience. These factors, in turn, reveal the marketing and operational strategies of a certain company, which is considered as a decision variable for each of the manufacturers. Balance and imbalance of power in the market are also taken into account in the present study to specify the impact of type of movement on service level, price and profit.

This research has been conducted based on the activity of the two supply chains of home appliances industry in Iran (They are not referenced directly here due to the request of the above-mentioned chains). These two chains are the two main rivals in the Iranian audiovisual equipment market because of the production of substitutable products. Customers of these products are highly sensitive to both price and service level. These chains can use two structures to sell their goods. The manufacturers directly supply the final product to the market (centralized structure) or sell the product to the distributor. Then the distributor sells the product to a retailer and the product is supplied through a retailer (decentralized structure). For some reasons such as historical background, fame, innovation, etc., a leader may be created in the market. In the current study both of the considered brands have the same level of background and reputation on the market. Thus, a chain can, for example, act as a leader in the market by introducing a new product into the market, and the rival chain can also compete with the leader chain by producing similar and substitutable products.

Regarding to market condition and competitor performance, supply chains can use the centralized or decentralized structure and make their decisions simultaneously or consecutively. Hence, to analyze the outcomes of selected circumstance, we investigate the effect of concurrent or sequential movements of the chains for different scenarios of structure for supply chains on the price, service level, and profit. In fact, the proposed model will be functional in each duopoly market with two competitive supply chains with the above conditions.

Table 1. Summary of the literature on supply chain competition

Paper	**Competition between supply chains**					
	Structure of market power		Factor competition		Number of levels	
	Power balance	Power imbalance	Price	Non-price	2	>2
McGuire & Staelin (1983) Anderson & Bao (2010) Li et al. (2013) Mahmoodi & Eshghi (2014) Zheng et al. (2016) Baron et al. (2016)	*		*		*	
Qian (2006)	*	*	*		*	
Amin Naseri & Azari Khojasteh (2015)		*	*		*	
Xiao & Yang (2008)	*		*	*	*	
Wu et al. (2009) Wu (2013)	*		*	*	*	
Boyaci & Gallego (2004)	*			*	*	
Ha & Tong (2008) Li& Li (2014)	*			*	*	
This Paper	*	*	*	*		*

The rest of this paper is organized as follows: Section 2 is dedicated to the proposed mathematical model and its parameters along with all scenarios in the Nash and Stackelberg games. In Section 3, equilibrium solutions are extracted from the Game Theory for the manufacturer, distributor, and retailer prices and service levels under various scenarios. Section 4 analyzes the sensitivity of decision variables in each chain in various scenarios compared to the model parameters. Besides, numerical examples are provided to compare the price and service level equilibria and profit at various modes. At the end of this section, managerial insights are given. Finally, section 5 summarizes the findings and provides future research directions.

2- Model definition and Equilibrium analysis

In competition between two supply chains, it is considered that each supply chain consists of three levels (manufacturer, distributor, and retailer). It is assumed that the manufacturer is more influential than the two others and makes the first decision. Therefore, a Stackelberg game is followed between a leader and several followers in each supply chain. The manufacturer decides on both price and service while the distributors and retailers decide on distribution and retailer price, respectively. In each chain, all members try to maximize their own profit and decide based on complete information about demand. Similar to

Tsay & Agrawal (2000), Wu (2012) and Lu et al. (2011), we assumed that the cost of service, which is provided by the manufacturer, has a decreasing-return feature. Service diminishing returns can be written in a quadratic form of service fee. Table 2 shows the model parameters.

Table 2. Model parameters and indices

i	Index of supply chain
b_p	Responsiveness of each product's demand to its price
θ_p	Responsiveness of each product's demand to its competitor's price
b_s	Responsiveness of each product's demand to its service
θ_s	Responsiveness of each product's demand to its competitor's service
w_{Mi}	Manufacturer price in the ith chain
w_{Di}	Distributor price in the ith chain
p_{Ri}	Retailer price in the ith chain
s_i	Manufacturer service in the ith chain
α_i	Market base of product (supply chain) *i* or product *i*'s market base
η_i	Manufacturer *i* service cost factor
c_M	Manufacturer unit cost
c_D	Distributor unit cost
c_R	Retailer unit cost

α_i measures the size of product i's market (Lu et al. 2011).

For simplification, we consider: $c = c_M + c_D + c_R$.

The cost of service is considered as $\eta_i \frac{s_i^2}{2}$ in the objective function of manufacturer due to the diminishing return property of a service (Tsay & Agrawal, 2000; Zhao and wang, 2015). Demand is considered a function of service and price with the aim of focusing on competitive factors. Similar to the studies by Lu et al. (2011) and Tsay & Agrawal (2000), here, the demand structure is assumed to be symmetrical between two products. Therefore, demand function can be rewritten as follows:

$$D_i = \alpha_i - b_p p_{Ri} + \theta_p p_{Rj} + b_s s_i - \theta_s s_j \tag{1}$$

where,

$j = 3 - i$ و $i = 1,2$ ،$\theta_s > 0$ ،$b_s > 0$ ،$\theta_p > 0$ ،$b_p > 0$ ،$\alpha_i > 0$

Demand function, shown in equation (1), has been used by numerous researchers including Zhao et al. (2013), Zhao & Wang (2015), Modak et al. (2016), Wang et al. (2017) and Xiao & Yang (2008). In order to examine the effect of competition, all model parameters are assumed to be deterministic and common knowledge (Li and Li, 2014; Tsay and Agrawal, 2000). We also assume that:

$$b_p > \theta_p > b_s > \theta_s \tag{2}$$

Relation (2) is defined as follows:

The assumption of $b_p > \theta_p$ (taken into account in Modak et al. (2016), Jiang et al. (2014), Wang et al. (2017)) means that the customers of a certain product are more sensitive to its price than to the competitor price. Assumption (2) shows that b_p is greater than the other parameters (θ_p ,θ_s and b_s) because changes in the price of a product must have more effects on the demand than other factors (Wang et al., 2017). Finally, $\theta_p > b_s > \theta_s$ means that a change in the price of the competitor's product is more effective than a change in the service of a product and its competitor on the demand of product. The problem is taken into account in two modes:

Mode I: The Nash Game

Here, it is assumed that both chains make decisions at the same time. Therefore, they decide based on the Nash game. As the manufacture is the leader, a Stackelberg game is formed within each chain. Since both chains decide on their decision variables simultaneously, we will have three scenarios:

Scenario I (N-1): Supply chains have centralized structure.

In this scenario, both of the supply chains concurrently determine p^*_{Ri} and s^*_i to maximize their profits. Due to centralized structure of supply chains and by considering Eq. (1), the objective functions would be as follows (Tsay and Agrawal, 2000; Zhao and Wang, 2015):

$$\Pi^i_{sc} = (p_{Ri} - c_M - c_D - c_R)D_i - \eta_i \frac{s_i^2}{2} = (p_{Ri} - c)\big(\alpha_i - b_p p_{Ri} + \theta_p p_{Rj} + b_s s_i - \theta_s s_j\big) - \eta_i \frac{s_i^2}{2}$$

$$, i = 1,2 \ \ , j = 3 - i \tag{3}$$

Scenario II (N-2): Supply chains have decentralized structure.

Based on the decentralized structure for both chains, the following objective functions are defined for the manufacturer, distributor, and retailer, respectively:

$$\Pi^i_M = (w_{Mi} - c_M)\big(\alpha_i - b_p p_{Ri} + \theta_p p_{Rj} + b_s s_i - \theta_s s_j\big) - \eta_i \frac{s_i^2}{2} \tag{4}$$

$$\Pi^i_D = (w_{Di} - w_{Mi} - c_D)\big(\alpha_i - b_p p_{Ri} + \theta_p p_{Rj} + b_s s_i - \theta_s s_j\big) \tag{5}$$

$$\Pi^i_R = (p_{Ri} - w_{Di} - c_R)\big(\alpha_i - b_p p_{Ri} + \theta_p p_{Rj} + b_s s_i - \theta_s s_j\big) \tag{6}$$

Where, $i = 1,2 \ \ , j = 3 - i$.

Scenario III (N-3): Supply chain I has a decentralized structure, whereas supply chain II has a centralized one.

Given that the second chain has a centralized structure, the profit function is similar to equation (3), and the manufacturer, distributor, and retailer profit functions in the first chain with decentralized structures are similar to equations. (4) to (6).

Given that one chain has an integrated structure and the other has a decentralized structure, without the loss of generality, we assume that the first and second chains have decentralized and centralized structures, respectively. Boyaci & Gallego (2004) and Mahmoodi & Eshghi (2014) have also considered such an assumption. Therefore, the second chain is an integrated system with the following decision variables: retailer price and service. In the first chain, each member acts independently and follows its profit. In scenario III, decisions are made in the second supply chain and the manufacturer in the first supply chain as leader at the same time. Then the distributor and retailer determine their decision variables to optimize the profit in the first chain, respectively.

Mode II: The Stackelberg Game

Here, we assume that one chain is stronger than the other in the market, acting as a leader. In fact, the supply chains' decisions are made consecutively. First, the leader supply chain makes its decisions about price and service. Then, knowing the decision of the leader, the follower makes its decisions about these variables. Backward induction is used to solve the model with the Stackelberg game. Here, we will have four scenarios:

Scenario I (S-1): Leader and follower supply chains have an integrated structure.

Since the leader and follower structures are integrated, the corresponding objective functions are similar to (3) in this scenario.

Scenario II (S-2): Leader supply chain has an integrated structure and follower supply chain has decentralized structure.

In this scenario, the leader's objective function is as equation (3) and that of the follower is similar to equations (4) to (6).

Scenario III (S-3): Leader supply chain has a decentralized structure and follower supply chain has centralized structure.

In this scenario, the profit function in each chain is similar to equations (4) to (6) and the follower's profit function is similar to equation (3).

Scenario IV (S-4): Leader and follower supply chains have a decentralized structure.
Since both the leader and the follower have a decentralized structure in this scenario, their objective functions are similar to equations. (4) to (6).

Without the loss of generality, we assume that the first chain is the leader and the second acts as the follower.

To obtain the price and service equilibrium in different scenarios, stepwise and consecutive procedures are given in Appendix A.

5- Sensitivity analysis and numerical results

In this section, the results of the model will be analyzed. We assume that the market base and service cost factors are symmetric. A certain mode is taken into account where the parameters are symmetrical in both of the supply chains. It means that $\alpha_1 = \alpha_2 = \alpha$ and $\eta_1 = \eta_2 = \eta$. This assumption was used by several researchers including Lu et al. (2011) and Tsay & Agrawal (2000). We assume the supply chains have the same parameters in order to compare seven scenarios with each other. Asymmetry between the two chains may create problems in different scenarios. Therefore, all parameters are symmetrical in both chains. Comparison of the scenarios can distinguish the effects of different power structures of the chains and different structure (centralized or decentralized) of the chains from the effects of different parameters of the chains.

Here, Discussion 1-3 is dedicated to sensitivity analysis with respect to the parameters to compare the decision variables' behavior and the supply chains' profit in different scenarios. A comparison between profit and decision variables under different scenarios is presented in discussion 4.

Discussion 1) Sensitivity analysis with respect to market base

In this section, we investigate the behavior of each of the decision variables (retailer, distributor, and manufacturer prices, service level, and supply chain profit) in different scenarios with respect to the change of α. In fact, we intend to realize how market base changes the price, service, and profit in each chain under various conditions.

In Scenario (N-1), based on the assumption of symmetry between the two supply chains, the optimal values of service and price are, respectively as follows:

$$p_{R1}^* = p_{R2}^* = c + \frac{\eta(-b_p c + \alpha)}{\eta(2b_p + \theta_p) - b_s(b_s + \theta_s)}, \quad s_1^* = s_2^* = -\frac{(b_p c - \alpha)(b_s + \theta_s)}{\eta(2b_p + \theta_p) - b_s(b_s + \theta_s)}$$

As it is evident, both chains have equal values for service and price. Accordingly, the profit is equal in both chains. For the decision variables, we will have:

$$\frac{\partial p_{R1}^*}{\partial \alpha} = \frac{\partial p_{R2}^*}{\partial \alpha} = \frac{\eta}{\eta(2b_p + \theta_p) - b_s(b_s + \theta_s)} \quad , \frac{\partial s_1^*}{\partial \alpha} = \frac{\partial s_2^*}{\partial \alpha} = \frac{b_s + \theta_s}{\eta(2b_p + \theta_p) - b_s(b_s + \theta_s)}$$

According to relation (2), an increase in the market size causes the following changes in service and price:

$\frac{\partial p_{R1}^*}{\partial \alpha} = \frac{\partial p_{R2}^*}{\partial \alpha} > 0$ and $\frac{\partial s_1^*}{\partial \alpha} = \frac{\partial s_2^*}{\partial \alpha} > 0$.

This result cannot be easily obtained for the supply chain profit and the profit in other scenarios due to the rough and complex calculations. Therefore, it is difficult to analyze the changes of these variables. As a result, numerical examples are used to show the effects of these parameters on the optimal values of decision variables and profit in the decision-making models. The analysis of figures 1-3 is done in two categories based on the supply chains, which may have the same structures or not. The first category covers the scenarios in which the structure of supply chains is similar (N-1, N-2, S-1, and S-4). The second category covers N-3, S-2, and S-3 scenarios in which the structure of supply chains is different.

Here, we assume that the parameters have the following values (The same values are taken by Xiao and Yang (2008)):

$b_p = 0.7 , \theta_p = 0.5 , \theta_s = 0.2 , b_s = 0.3 , \eta = 0.4 , c_M = 15 , c_D = 6 , c_R = 5 , \alpha \in \{100,200, \dots 700\}$

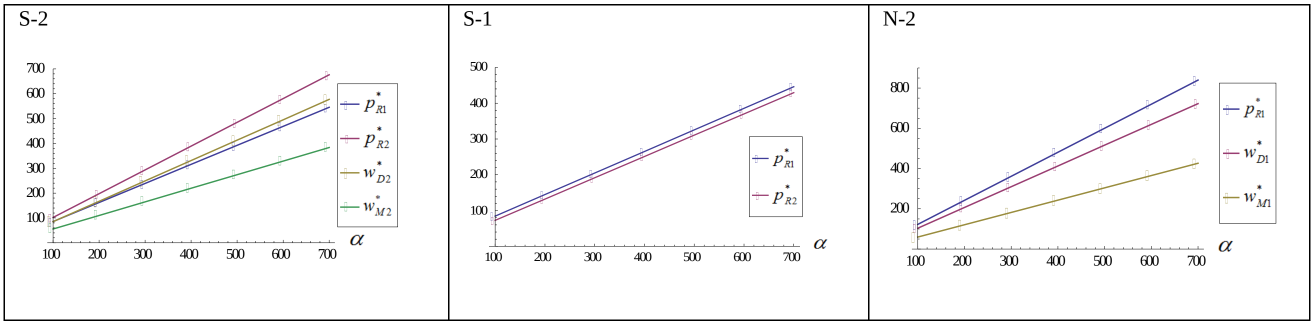

Fig 1. Changes in price relative to α in different scenarios

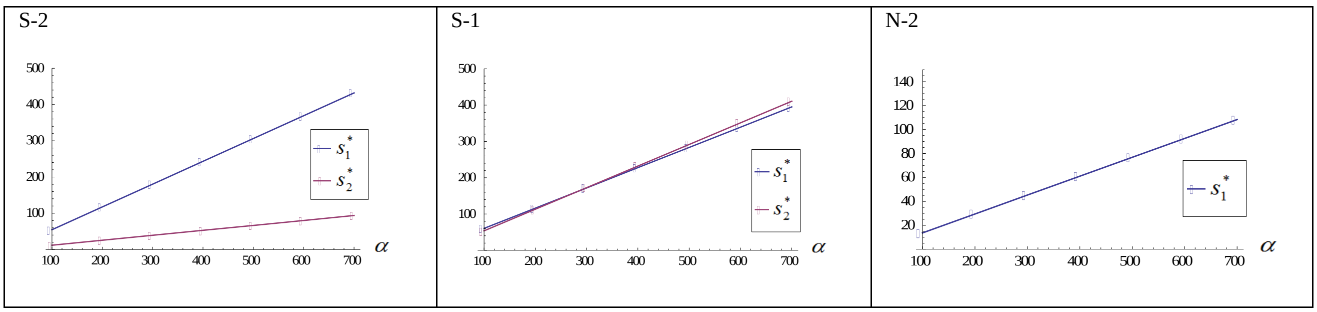

Fig 2. Changes in service with regard to α in different scenarios

As shown in figures 1-3, in all scenarios, the service and profit in each chain are increased by an increase in the market base. Due to the increase in price with respect to market base, the manufacturer has to promote his/her service to compensate the increase in price. In general, increase in the market base is beneficial for all supply chain members caused by increased demand in the market. Therefore, regardless of the structure of the supply chains and power structure between them, the optimal values of decision variables and profit would increase as the market base expands.

First category: As the figures 1-3 show, in all scenarios, service level, retailer price, and profit growth rates are almost similar in both chains relative to the market size regardless of their power structures. On the other hand, the imbalance in market power increases the difference between the equilibrium values in the two chains when α increases.

Second category: Regardless of the power structure between two chains, retailer price, service level, and profit growth rates are greater in centralized chain than in the decentralized chain. Therefore, the increase of α is responsible for the greater difference between the values in the two chains compared to the mode with similar structure.

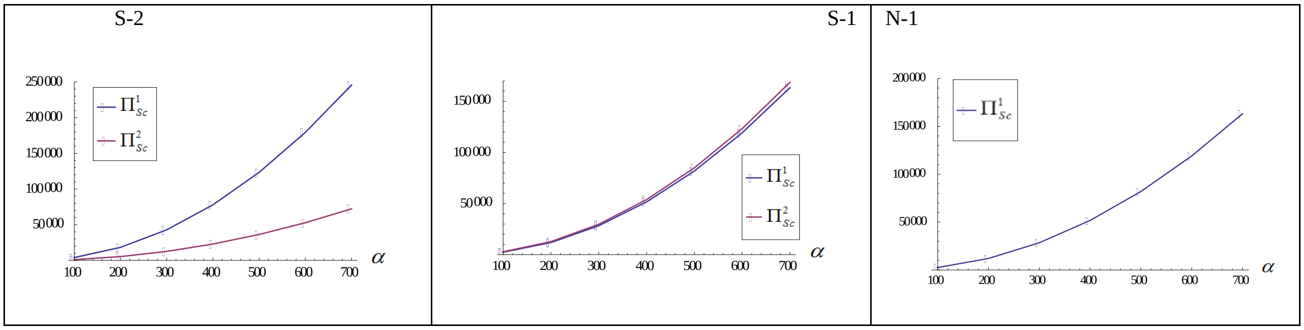

Fig 3. Changes in the supply chains' profit relative to α in different scenarios

(Because of similarity of some figure, all figures are not shown in discussion 1 and 2.)

Discussion 2) Sensitivity analysis with respect to η

Since service and price equilibrium and profit in each chain are complicated, numerical examples are used. The default values are:

$\alpha = 100 \, , b_p = 0.7 \, , \theta_p = 0.5 \, , \theta_s = 0.2 \, , b_s = 0.3 \, , c_M = 15 \, , c_D = 6 \, , c_R = 5 \, , \eta \in \{0.2, 0.5, \ldots, 1.7, 2\}$

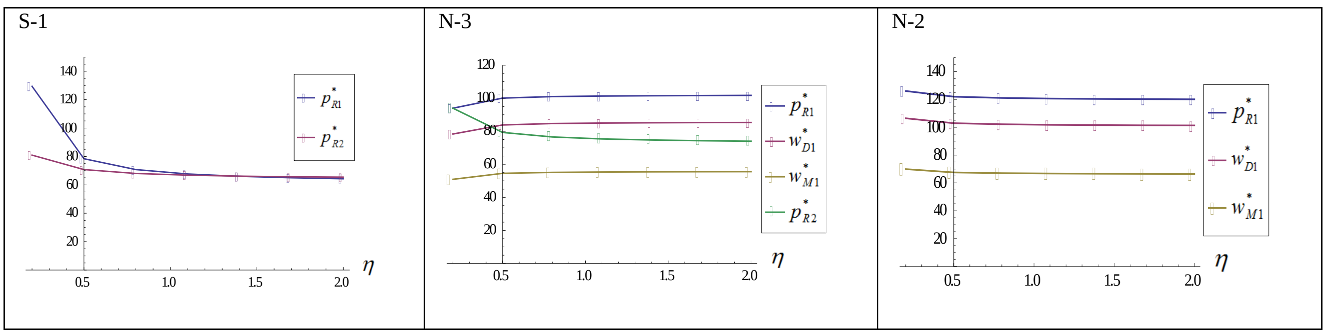

Fig 4. Changes in price with regard to η in different scenarios

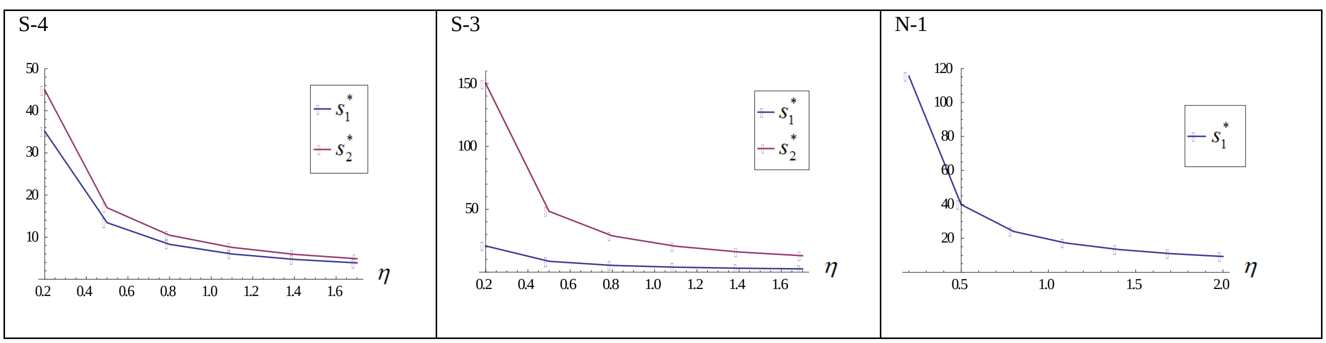

Fig 5. Changes in service with regard to η in different scenarios

As it can be seen in figures 4-6, increase of η has the greatest impact on the service, which ultimately leads to slight changes in price and profit. Therefore, the more is η ($\eta > 0.5$), the less is the effect of service reduction on price and profit so that there is almost no significant change in price and profit.

But for a small amount of η ($\eta < 0.5$), the service performs as a more important and influential factor on price and profit. Evidently, the structure of both chains is influential as well. For example, if the supply chains have the same structure, the rate of service reduction in structure II is greater than in structure DD, while the rate of decline in price and profit in both structures is roughly similar. In DI structure, the speed of service reduction in an integrated chain is greater than in decentralized chain, while the rate of change in price and profit is almost the same but in the opposite direction.

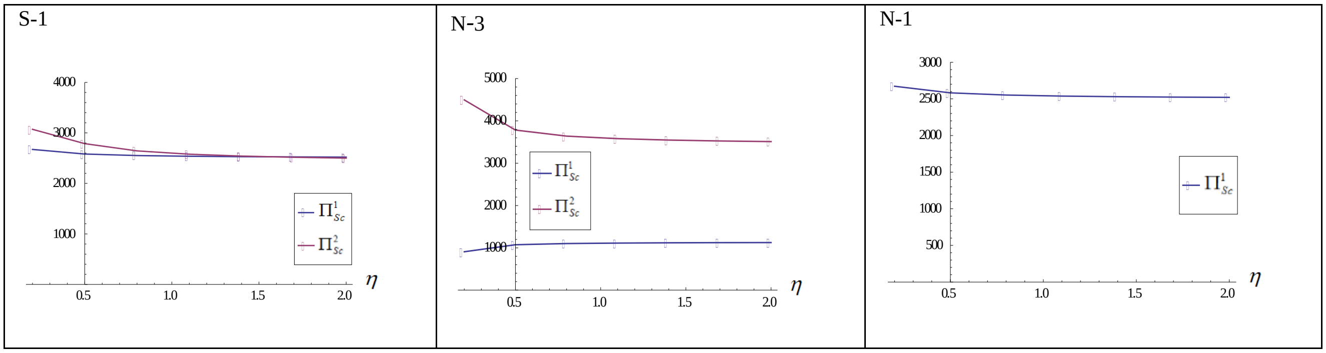

Fig 6. Changes in supply chains profit relative to η in different scenarios

Result 1). Impression of supply chain structure and power structure in market under $(\alpha_1 = \alpha_2, \eta_1 = \eta_2)$ condition

- In DI structure, the supply chains in the Stackelberg scenario maintain the same values and trends as in the Nash scenario because of the properties of centralized and decentralized structures that imbalance of power in the market also has no effect on it.
  But in II or DD structure, with changes in α and η, the values of profit, service, and price of the chains differ from each other due to the sequence of movement between the chains. As the two chains with similar structure compete in the market, the presence of a leader in the market affects their trend in the Nash game.
- If both chains have the same structures (either centralized or decentralized), the decision variable and profit changes will be almost similar; however, if the structure of two chains is different, changes in the centralized structure will occur more meaningfully compared to the decentralized one.
  In general, growth (reduction) rate is greater in the supply chain with centralized structure than in the decentralized one.
- Due to the changes in the parameters, changes for decision variables and profit will be in one direction in both chains when there is either balance or imbalance in power. However, balance or imbalance of power can make differences in the change rates (more reduction or increase) or the values taken by the decision variables or profit.

Discussion 3) Sensitivity analysis with respect to competitive parameters

Tables 3-4 are used to evaluate the changes of decision variables and profit with respect to competitive factors. Assume that the numerical values for the parameters are as follows: $\alpha = 100$, $\eta = 1$, $c_M = 15$, $c_D = 6$, $c_R = 5$

In order to reduce the calculations, the values of θ_s and b_s are derived by the following relation: $\theta_s = b_s - 0.04$.

According to relation (2), based on different values of competition parameters, equilibrium price, and service level and profit of supply chains are calculated in all scenarios as shown in tables 3-5.

Table 3. Comparison of profit in different scenarios vs. different values of competition parameters

b_p	θ_p	b_s	N-1	N-2	N-3		S-1		S-2		S-3		S-4	
			Π^{1}_{Sc-I}	Π^{1}_{Sc-D}	Π^{1}_{Sc-D}	Π^{2}_{Sc-I}	Π^{1-L}_{Sc-I}	Π^{2-F}_{Sc-I}	Π^{1-L}_{Sc-I}	Π^{2-F}_{Sc-D}	Π^{1-L}_{Sc-D}	Π^{2-F}_{Sc-I}	Π^{1-L}_{Sc-D}	Π^{2-F}_{Sc-D}
0.5	0.1	0.05	4065.3	2037.83	1780.31	4618.74	4066.09	4070.45	4618.23	1793.87	1770.46	4620.06	2010.05	2048.65
	0.3	0.05	3882.94	2423.46	1749.08	5159.13	3888.93	3940.61	5170.48	1845.43	1696.91	5175.26	2253.19	2511.52
		0.2	3963.91	2475.82	1674.84	5525.21	3969.59	4028.7	5546.08	1781.85	1633.16	5540.73	2295.89	2570.77
1	0.1	0.05	1637.48	767.79	718.82	1755.58	1637.44	1638.04	1755.04	718.23	715.82	1755.58	764.22	767.43
	0.3	0.05	1615.31	858.06	713.03	1913.03	1615.61	1619.23	1915.61	726.21	706.53	1917.01	835.41	867.49
		0.2	1626.38	862.54	693.36	1998.67	1626.36	1634.06	1998.54	703.92	685.04	1999.41	842.29	875.62
	0.8	0.05	1508.61	1030.38	690.39	2116.54	1514.41	1546.42	2127.82	756.19	662.5	2129.14	928.02	1088.18
		0.2	1526.21	1042.61	682.64	2181.42	1530.39	1565.96	2195.26	747.51	653.67	2192.79	938.58	1100.47
		0.5	1614.45	1108.44	511.12	2636.08	1616.09	1659.33	2673.67	675.32	577.94	2642.83	990.63	1177.84
		0.7	1626.83	1193.27	368.84	3452	1625.61	1661.85	3615.66	496.58	380.74	3463.92	1056.44	1282.01
1.5	0.1	0.05	852.98	389.91	374.22	890.33	852.33	852.97	891.05	374.15	372.85	892.93	387.29	389.48
	0.3	0.05	847.77	422.04	372.38	955.27	847.31	848.09	956.03	376.61	371.84	956.28	417.74	424.31
		0.2	850.63	424.53	362.17	984.07	849.15	850.98	984.73	362.42	361.79	986.41	417.86	428.77
	0.8	0.05	815.54	491.13	366.72	1053.41	818.07	825.12	1057.02	385.48	358.65	1058.57	460.99	504.47
		0.2	822.42	493.6	363.68	1080.65	821.9	832.87	1082.24	380.98	356.52	1084.08	464.82	509.66
		0.5	848	512.07	322.98	1248.49	847.64	858.33	1254.22	344.85	319.36	1249.96	481.53	530.95
		0.7	850.32	537.76	243.32	1522.15	860.12	859.08	1550.55	278.64	265.04	1526.65	501.46	561.37
	1.3	0.05	776.375	545.34	358.95	1105.62	778.58	799.43	1110.51	396.47	342.71	1110.03	485.03	578.35
		0.2	781.94	549.26	354.47	1127.36	785.28	805.52	1134.97	391.55	339.42	1134.12	486.83	583.28
		0.5	816.86	571.51	332.16	1266.19	818.37	842.55	1280.25	376.72	320.68	1271.68	505.04	610.73
		0.7	845.41	600.94	227.38	1570.55	848.07	866.68	1577.35	339.88	260.47	1577.11	526.05	644.79
		0.9	862.02	643.92	174.58	1875.43	883.29	893.11	1983.43	253.94	182.08	1883.38	560.64	694.46
		1	875.76	670.76	69.3	2215.76	902.45	914.96	2462.6	164.14	78.37	2233.71	579.55	729.62

Π^{1-L}_{Sc-I}: Profit of Leader supply chain with integrated structure, Π^{2-F}_{Sc-D}: Profit of Follower supply chain with decentralized structure

Table 4. Comparison of decision variables in different scenarios vs. different values of competition parameters

b_p	θ_p	b_s	N-1		N-2				S-1			
			$Sc_1(Sc_2)$		$Sc_1(Sc_2)$				Sc_1^L		Sc_2^F	
			p_{R1}	s_1	w_{M1}	w_{D1}	p_{R1}	s_1	p_{R1}	s_1	p_{R2}	s_2
0.5	0.1	0.05	102.01	8.94	96.14	145.53	174.06	2.15	102.84	6.52	101.38	8.02
	0.3	0.05	89.32	6.84	92.63	142.11	167.93	1.98	91.33	6.58	89.96	7.03
		0.2	93.34	29.05	95.3	144.21	169.81	7.83	95.04	28.08	93.8	29.93
1	0.1	0.05	57.97	4.75	48.07	74.05	89	1.05	57.71	3.82	57.66	3.88
	0.3	0.05	55.06	3.57	47.27	72.87	87.37	0.91	54.77	3.39	54.3	3.55
		0.2	55.85	14.62	47.52	73.33	88.64	3.69	55.76	14.07	55.55	14.67
	0.8	0.05	48.42	2.94	45.75	70.4	84.42	0.91	49.5	2.71	48.29	2.94
		0.2	48.96	11.83	46.06	70.8	84.72	3.42	50.22	11.2	49.17	12.09
		0.5	54.54	35.77	47.54	73.24	87.56	9.22	54.73	33.69	54.66	35.63
		0.7	63.58	62.39	50.11	76.7	91.95	13.55	64.22	60.75	63.92	63.09
1.5	0.1	0.05	42.82	2.65	31.66	49.46	60.51	0.74	42.11	2.32	42.12	2.33
	0.3	0.05	41.35	2.78	31.35	48.72	60.02	0.58	40.83	2.11	40.7	2.18
		0.2	41.89	8.86	31.42	49.16	60.19	2.27	41.26	8.64	41.2	8.91
	0.8	0.05	37.9	1.89	30.65	47.87	58.3	0.5	38.4	1.79	37.94	1.9
		0.2	39.31	7.68	30.78	48.09	58.53	2.07	38.75	7.3	38.35	7.72
		0.5	41.42	21.79	31.48	49.2	59.89	5.36	40.84	20.93	40.77	21.83
		0.7	44.3	35.87	32.38	50.73	61.48	7.54	44.5	34.93	44.35	35.66
	1.3	0.05	35.36	1.75	30.13	47.12	57.2	0.47	36.67	1.55	35.83	1.7
		0.2	36.23	6.77	30.23	47.66	57.61	2.04	36.88	6.38	36.14	6.86
		0.5	37.83	18.9	30.82	48.08	58.43	5.21	38.43	17.77	38.07	19.14
		0.7	40.78	30.17	31.74	49.57	60.11	7.53	41.73	29.09	40.8	30.53
		0.9	44.6	45	32.4	52.36	62.32	10.22	46.36	42.63	44.78	45.57
		1	47.56	47.15	33.71	52.21	63.28	11.7	59.5	43.12	47.76	47.53

Table 4. Continued

b_p	θ_p	b_s	S-2						S-3					
			Sc_1^L		Sc_2^F				Sc_1^L				Sc_2^F	
			p_{R1}	s_1	w_{M2}	w_{D2}	p_{R2}	s_2	w_{M1}	w_{D1}	p_{R1}	s_1	p_{R2}	s_2
0.5	0.1	0.05	108.17	8.28	90.76	138.1	163.65	2.06	90.71	138.03	163.6	1.29	107.1	8.8
	0.3	0.05	105.07	7.18	80.55	122.86	146.04	1.83	80.15	122.24	145.31	1.14	99.71	8.06
		0.2	111.65	31.03	79.83	121.76	144.71	7.19	78.76	120.16	142.83	6.56	106.72	35.11
1	0.1	0.05	59.18	3.87	46.64	71.94	86.54	0.94	46.6	71.91	86.57	0.93	59.09	4.01
	0.3	0.05	58.42	3.56	43.7	67.54	81.47	0.91	43.66	67.51	81.41	0.82	57.53	3.84
		0.2	60.27	14.88	43.3	66.96	80.78	3.54	43.15	66.76	80.56	3.37	59.51	16.17
	0.8	0.05	56.84	3.05	38.87	60.35	73.05	0.8	38.45	59.74	72.37	0.7	53.46	3.41
		0.2	58.1	12.63	38.82	60.22	72.91	3.06	38.29	59.44	72.	2.72	54.71	14.28
		0.5	66.89	38.74	37.83	58.73	71.17	7.47	36.29	56.43	68.5	6.65	64.14	45.13
		0.7	68.13	74.21	34.07	53.13	69.64	9.14	30.33	47.49	88.08	8.25	83.94	90.86
1.5	0.1	0.05	42.7	2.3	31.09	48.81	59.39	0.57	31.05	48.57	59.39	0.59	42.69	2.34
	0.3	0.05	42.38	2.15	29.95	46.75	57.12	0.56	29.77	46.72	57.15	0.52	42.13	2.3
		0.2	43.19	8.99	29.6	46.41	56.83	2.17	29.62	46.36	56.78	2.1	42.95	9.58
	0.8	0.05	41.73	1.92	27.45	43.19	53.05	0.52	27.4	43.1	52.95	0.47	40.5	2.16
		0.2	42.3	7.86	27.44	43.11	53.	1.97	27.28	42.92	52.76	1.8	41.13	8.82
		0.5	46.18	23.2	26.78	42.1	51.8	4.71	26.29	41.4	50.97	4.37	45.39	26.35
		0.7	49.92	40.75	25.03	39.56	50.85	5.99	23.9	37.89	53.84	5.59	52.09	47.6
	1.3	0.05	41.18	1.75	25.91	40.85	50.34	0.44	25.61	40.43	49.82	0.37	38.98	1.96
		0.2	41.6	7.18	25.88	40.83	50.27	1.77	25.55	40.33	49.7	1.55	39.44	8.16
		0.5	43.63	20.47	25.64	40.48	49.95	4.44	24.96	39.46	48.75	3.87	42.53	23.53
		0.7	44.22	33.85	25.01	39.55	48.8	9.98	23.81	37.72	47.64	8.32	47.52	39.91
		0.9	45.09	57.94	22.9	36.33	46.09	16.68	20.28	32.42	49.49	14.93	48.87	71.78
		1	45.8	80.31	20.11	32.2	45.96	26.72	15.64	25.48	52.38	19.11	71.68	105.31

Table 4. Continued

b_p	θ_p	b_s	N-3						S-4							
			Sc_1				Sc_2		Sc_1^L				Sc_2^F			
			w_{M1}	w_{D1}	p_{R1}	s_1	p_{R2}	s_2	w_{M1}	w_{D1}	p_{R1}	s_1	w_{M2}	w_{D2}	p_{R2}	s_2
0.5	0.1	0.05	90.68	138.02	163.55	1.35	107.11	8.8	96.67	146.98	174.18	2.06	96.42	146.69	173.82	2.2
	0.3	0.05	80.16	122.2	144.63	1.33	99.59	8.08	93.79	142.65	169.06	1.77	92.76	141.08	167.36	2.14
		0.2	78.83	120.17	142.97	6.71	106.64	35.01	94.94	144.42	171.2	7.28	94.13	143.5	169.12	8.65
1	0.1	0.05	46.6	71.9	86.57	0.92	59.06	4.01	48.08	74.11	89.09	0.94	48.06	74.05	89.03	1.02
	0.3	0.05	43.62	67.53	81.43	0.86	57.5	3.84	47.3	72.92	87.75	0.89	47.12	72.7	87.49	0.95
		0.2	43.15	66.75	80.61	3.42	59.49	16.15	47.62	73.37	88.27	3.56	47.5	73.23	88.08	3.97
	0.8	0.05	38.51	59.76	73.01	0.7	53.34	3.4	45.79	70.67	85.15	0.74	45.2	69.82	84.17	0.99
		0.2	38.3	59.45	72.64	2.87	54.67	14.23	46.07	71.99	88.57	3.09	45.58	70.35	84.63	3.76
		0.5	36.33	56.53	68.6	6.85	64.1	45.05	47.56	73.35	88.23	8.01	47.45	73.15	87.16	9.86
		0.7	30.32	47.46	58.07	8.3	83.75	90.	50.42	76.78	91.58	11.83	50.03	76.08	91.15	14.75
1.5	0.1	0.05	31.04	48.59	59.3	0.57	42.69	2.33	31.66	49.5	60.46	0.58	31.62	49.48	60.37	0.6
	0.3	0.05	29.76	46.7	57.11	0.51	42.13	2.3	31.33	48.94	59.81	0.56	31.29	48.67	59.78	0.59
		0.2	29.58	46.36	56.73	2.12	42.98	9.6	31.49	49.23	60.05	2.14	31.43	49.15	59.98	2.31
	0.8	0.05	27.42	43.11	52.94	0.49	40.48	2.11	30.7	48.03	58.75	0.46	30.44	47.71	58.34	0.58
		0.2	27.26	42.97	52.77	1.85	41.1	8.79	30.8	48.21	58.91	1.92	30.63	47.9	58.54	2.27
		0.5	26.27	41.41	50.96	4.44	45.33	26.32	31.7	49.51	59.98	4.94	31.44	49.13	59.73	5.85
		0.7	23.9	37.85	46.88	5.69	52.04	47.51	32.7	50.88	61.4	7.25	32.41	50.42	61.17	8.55
	1.3	0.05	25.62	40.43	49.92	0.47	38.91	1.94	30.15	47.22	57.78	0.41	29.78	46.64	57.14	0.58
		0.2	25.54	40.34	49.71	1.7	39.37	8.14	30.24	47.38	57.95	1.75	29.87	46.82	57.33	2.16
		0.5	25.03	39.53	48.73	4.08	42.46	23.45	30.92	48.27	58.92	4.58	30.65	47.91	58.42	5.68
		0.7	23.81	37.74	46.65	9.46	47.48	39.85	31.97	49.78	60.08	6.62	31.52	49.33	59.25	8.29
		0.9	20.25	32.5	40.46	15.97	58.91	71.71	32.45	50.76	62.87	8.88	32.08	50.04	62.04	11.21
		1	15.62	25.6	32.45	19.35	71.5	105.23	33.84	51.68	63.94	10.16	33.74	51.35	63.07	12.9

-According to tables (3) and (4), increase in the value of b_p (or sensitivity of demand for a product compared to its price) leads to a decrease in the retailer, distributor, and manufacturer prices, service level, and profit in each supply chain. In this mode, the manufacturer, as the leader, has to reduce the price to attract more customers due to the increased sensitivity of customers to the product's price. Therefore, the manufacturer considers less service to save the investment. As a result, the optimal values of price, service, and profit decline as b_p increases regardless of the structure of each supply chain and their power structure.

-Table (4) shows that increase in the value of θ_p leads to decrease in the retailer, distributor, and manufacturer price, and service. In fact, when θ_p increases, the chains need to attract more customers by reducing the price. Remarkably, reduced price is far less than the increase in b_p. In fact, as θ_p rises, the manufacturers, and consequently, the retailers must reduce the price to attract more customers. On the other hand, reduced price decreases the service level so that the manufacturer considers lower service to save the investment. Therefore, regardless of the structure of each chain and power structure between the chains, the optimal values of price and service decrease when θ_p increases. However, increased value of θ_p causes profit behavior to change in various scenarios (Table 3). When both supply chains have centralized (decentralized) structures, regardless of power structure between the chains, increased value of θ_p is responsible for the decrease (increase) in supply chains' profit. On the other hand, if the structure of two chains is different:

(1) When there is power balance between the two chains, increased value of θ_p is responsible for profit reduction in the decentralized chain and profit increase in the centralized chain.

(2) When there is imbalance of power, increased value of θ_p is always responsible for the increased profit of centralized structure chain. However, by increase in the value of θ_p in decentralized chain, if the supply chain acts as the leader, then the profit reduces, and if it acts as the follower, then the profit will increase.

-The following points can be extracted from tables (3) and (4):
Regardless of the power structure between two chains, if the chains have similar structures, simultaneous increase in θ_s and b_s is always responsible for the increased profit (as well as service and price) of chains, associated with increased sensitivity of customers in service followed by increased price. When the two chains have different structures, the scenario is similar for the chain with centralized structure; however, in the chain with decentralized structure, simultaneous increase of the mentioned two parameters is responsible for the decrease in price and increase in profit and service. In other words, in the decentralized structure, service increases but price decreases, this is mainly associated with the fact that price reduction can be due to the decentralized structure of the chain. Table 3 shows that despite having different power structures, the rate of changes is greater in the chain with integrated structure than in the chain with decentralized structure.

Result 2). Impression of competition under ($\alpha_1 = \alpha_2$, $\eta_1 = \eta_2$) condition

- As b_p increases, profit, price, and service decrease in all scenarios.
- Increase of θ_p is responsible for the decrease of the in price and service of chains in all scenarios.
- The existence of a DD structure in the supply chains brings about greater profit for both chains by increasingθ_p; however, this is not true in the II structure. Therefore, in the DD structure, both chains prefer an environment with greater competition, but the supply chains in the II market prefer a less competitive market.
- The consequence of increase in b_p is far more than that of increase of θ_s (b_s) on profit increase or decrease.
- In S-1, S-3 and S-4 scenarios, in all cases, the follower supply chain's profit is greater than that of the leader.
- In S-2 scenario, the leader supply chain's profit is greater than that of the follower supply chain due to the integrated structure of the leader chain against a rival with a decentralize structure. As a result, the follower cannot obtain more profit versus the leader with integrated structure.
- As a general result, changes in the market base will cause considerable changes in profit, price, and service.

Discussion 4) Consequence of imbalance of market power under ($\alpha_1 = \alpha_2$,$\eta_1 = \eta_2$) condition

The effect of the presence of leader in a market with two competitive supply chains and symmetrical parameters in different scenarios is shown in table (5).

Table 5. Effect of imbalance of market power on profit and decision variables

Scenarios	Structure of supply chain	Price	Service	Profit
First Category (N-1, N-2, S-1, S-4)	**Both centralized**	$p \leq p^F < p^L$	$s^L < s \leq s^F$	$\Pi \leq \Pi^L < \Pi^F$
	Both decentralized	$w_M^F < w_M < w_M^L$ $w_D^F < w_D < w_D^L$ $p^F < p < p^L$	$s^L < s < s^F$	$\Pi^L < \Pi < \Pi^F$
Second Category (N-3, S-2, S-3)	**Centralized supply chain is leader**	$p^L < p^F$	$s^F < s^L$	$\Pi^F < \Pi^L$
	Decentralized supply chain is leader	$w_M^F < w_M^L$ $w_D^F < w_D^L$ $p^F < p^L$	$s^L < s^F$	$\Pi^L < \Pi^F$

p*:* Retailer price in a market without leader, p^F*:* Retailer price of follower chain, p^L: Retailer price of leader chain,
Π*:* Profit of supply chain in a market without leader, Π^L: Profit of leader chain-Π^F*:* Profit of follower chain,
s*:* Service in a market without leader, s^L:Service of leader chain, s^F*:* Service of follower chain

According to table (5), the following points are extracted:

- Structure II (DD)
  In a market where two supply chains operate with the same structure, if a leader is created (the chains make their decisions sequentially), the follower chain supplies its products at lower price and higher service than the product of the leader chain to gain more profit and attract more customer attention. In fact, both service and price have a strategic role for the follower. Thus, by offering a product at lower price and higher service, the follower can obtain more profit than the leader does. When the supply chains take their decisions sequentially, in the II structure, both chains gain more profit than the Nash scenario, whereas the follower and leader obtain more and less profits than the Nash scenario in the DD structure, respectively. The less sensitive a product's demand is to the price of the rival goods (θ_p), the profit of the leader and the follower becomes very close to each other, and with increasingθ_p, the difference between their profits increases too. Additionally, with the increase of the sensitivity of demand to service level (b_s, θ_s), the difference of service level supplied by the leader and follower chains will also increase.

- Structure DI
  In this market, when the two chains make their decisions simultaneously, the integrated chain has lower price and higher service than the non-integrated chain, which ultimately leads to more profits to be gained by that chain (The existence of an integrated structure can be a reason of it). In this market, if the chains make their decisions consecutively:
  - The integrated chain is the leader: In this case, the integrated chain will have a higher price and less service than in the Nash scenario. The decentralized chain offers a higher price and service than in the Nash scenario. Ultimately, both chains gain more profit than in the Nash scenario. Therefore, consecutive decision making of chains can be beneficial to both of them. The integrated chain (leader) offers lower price and higher service than the decentralized chain (follower) - similar to the Nash scenario - mainly due to integrated structure of the leader chain. Therefore, the dominant influence of the integrated structure can be easily seen, which makes the leader gain more profit than the follower does. The follower chain, despite having the second-

mover advantage, is not able to gain more profit than the leader with a centralized structure due to its decentralized structure.

- The decentralized chain is the leader: In this case, the price and service in the integrated chain are either fixed or a small increase than in the Nash scenario. The service level in the decentralized chain decreases compared to the Nash scenario, but the price remains constant or undergoes a slight decline. Therefore, compared to the Nash scenario, decentralized and centralized structures earn less and more profit, respectively. Thus, the leadership of the decentralized chain in the market will only be in the interest of the follower. By the leadership of the decentralized chain, this chain will have a higher price and less service than the centralized chain (follower), and the follower will eventually gain more profit than the leader.

Result 3).

-Qian (2006) (and also Eric (1994), Dastidar (2004)) have mentioned that in the price Stackelberg game, as the consecutive movement of supply chains is considered and price is a decision variable, the supply chain with the Second-Mover advantage (follower) gains more profit than the leader. This point is true for the supply chains with the same structure; therefore, according to Table 5:

- In scenarios s-1 and s-2, because the supply chains have the same structure and their decision making is based on the Stackelberg game (i.e. they make their decisions sequentially and also price and service are the decision variables of this game), the follower as the second mover can obtain higher profit than the first mover (leader) by offering lower price and higher service, so the supply chain has the Second-Mover advantage.
- Whenever the structure of chains is diverse, then if the decentralized chain is the leader (scenario s-3), the follower will gain higher profit in comparison with leader due to the fact that follower chain has a centralized structure (because of the features of the centralized structure in comparison with the decentralized structure) and that it is a Second-Mover. In other words, there is a Second-Mover advantage.

-In the leadership position of an integrated chain in a market, despite that the follower is Second-Mover in the price and service Stackelberg game, it cannot gain more profit than the leader does due to the activity of the leader with an integrated structure against the follower chain with non-integrated structure; so, there is the First-Mover advantage. Consequently, in this scenario, the structure has a prominent role than in the Second-Mover (Table 6).

-In the DI structure, the integrated chain always obtains benefit with the entry of the leader into the market.

-In the DI structure, the relationships between the price and service of the centralized and decentralized chains do not change with the entry of the leader into the market, i.e. the integrated chain always provides a higher service and a lower price than the decentralized chain, which will not change with the entry of the leader into the market.

Table 6. Advantage of mover in the Stackelberg game

Supply chains' structure	**II**	**DI**		**DD**
Scenarios	**S-1**	**S-2**	**S-3**	**S-4**
Advantage of mover	Second-Mover	Second-Mover	First-Mover	Second-Mover

Result (4). Managerial insights

A general conclusion is drawn to determine the most appropriate power structure at various conditions and structures. In other words, according to the supply chains' and competitors' parameters and structure, the best power structure is selected in the market for obtaining the greatest profit. Therefore, the following table (Table 7) is used as a solution for managerial decisions at various conditions at a duopoly market with two competitive supply chains.

Table 7. Managerial decisions at various conditions

Supply chains' conditions	Supply chains' structure	Appropriate strategy to obtain maximum profit	Advantage of mover
$\alpha_1 = \alpha_2$, $\eta_1 = \eta_2$	Both centralized	Presence of leader in the market: When there is a leader in the market, both chains gain greater profit comparing to power balance between the chains in the market.	Second-Mover
$\alpha_1 = \alpha_2$, $\eta_1 = \eta_2$	Both decentralized	Absence of leader in the market: In the market with the presence of leader, compared to power balance, the follower chain obtains greater profit while the leader chain earns less profit. The presence of leader in the market is in the favor of the chain acting as follower. Hence, Presence of leader in the market is not beneficial for both of them.	Second-Mover
$\alpha_1 = \alpha_2$, $\eta_1 = \eta_2$	Centralized supply chain is leader	Presence of leader in the market: When centralized supply chain is leader in the market, both chains gain greater profit compared to power balance in the market.	First-Mover
	Decentralized supply chain is leader	When decentralized supply chain is leader in the market, follower chain with integrated structure obtains greater profit while the leader chain earns less profit. Therefore, leadership of decentralized chain, is not beneficial for both of them.	Second-Mover

As it can be seen in Table 7, if the centralized chains operate in a power balanced market, then, as an example, offering a new product in the market will cause benefiting of both chains from the process. Thus, both chains will benefit if any of them becomes a leader.

In the DD structure, the imbalance of power in the market leads to higher and lower profit for the follower and leader chains, respectively, in comparison with the Nash scenario. Therefore, in this market, the advent of the leader is in the favor of follower chain. Thus, in such a market, becoming a leader will benefit its rival. In this regard, it is better for chains to make simultaneous decisions.

In the DI structure, becoming a leader for the integrated chain leads to more profits for both chains at the time of simultaneous decisions. Whereas, if the decentralized chain becomes the leader, then this chain and the centralized chain will gain lower and higher profit, respectively, in comparison with the Nash scenario. Thus, when the two chains make their decisions simultaneously, it is better that the decentralized chain never plays the role of the leader because only its rival will benefit from it. But if the integrated chain is introduced as the market leader (for example, by offering a new product), then both chains will have a better condition in terms of profit.

Finally, the last column of this table points out that if a leader is created in a market, which of the first or second mover can obtain a higher profit than its competitor. If the two chains are in the same structure, or if the supply chain with decentralize structure be leader when the structure of the chains is different, there is a Second-Mover advantage. In the DI structure, if the supply chain with centralize structure be leader, there is a First-Mover advantage.

6- Conclusion and future works

We investigated a competitive model between two three-echelon supply chains consisting of a manufacturer, distributor, and retailer. In this model, the competition was considered between the two supply chains on price and service level in different market conditions. Seven scenarios were developed based on the power structure of the two chains in the market. The results show that when there is power balance between two chains in the market, three scenarios are developed, which are dependent on the structure of each chain. The other four scenarios capture the market condition with imbalance of power between the chains. The model, indeed, attempts to analyze the impact of the market's power imbalance in price, service, and profit of the supply chains under different combinations of structure for the chains. It was found that in the Stackelberg game with price and service as the decision variables, in the DI structure, the existence of a leader with integrated structure versus a follower with non-integrated structure leads to a First-Mover advantage while in the II (DD) structure, and in DI structure when the decentralized supply chain is the leader, there is a Second-Mover advantage. The existence of the leader in the II structure and the leadership of the integrated chain in the DI structure are the only scenarios in which both chains benefit from sequential decision making. In all structures, when a leader is created in the market, the greatest benefit is gained by the chain that provides a higher service. In other words, offering lower price by a chain does not solely cause earning more profit than the competitor.

The above results were drawn based on the assumptions for this competitive model. However, the model can be developed by other methods as well. In the present paper, service and price were taken into account as the competition parameters, while other factors such as the product quality, product stability degree, etc. can be considered as well. This competitive model can be investigated for complementary products at various conditions of demand function. Demand uncertainty can be taken into account as well. Also we assumed that all chain members have symmetric information on demand; in future models, asymmetric information for all members can be developed. The model was analyzed based on the Stackelberg manufacturer. Future studies can consider supply chain with different power structures. Another assumption was that there is only one member in each level. More members can be considered in each level in future studies. Finally, we assumed that all members in both chains are risk neutral. In future studies, risk-averse members can be considered.

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Appendix A. Solving procedure of scenarios

N-1)

Solving procedure is as follows:

(1) Differentiation of the objective function with respect to the decision variables ($p_{Ri}$and s_i)

(2) Solve the equations $\frac{\partial \Pi_{sc}^1}{\partial p_{R1}} = 0$, $\frac{\partial \Pi_{sc}^1}{\partial s_1} = 0$, $\frac{\partial \Pi_{sc}^2}{\partial p_{R2}} = 0$ and $\frac{\partial \Pi_{sc}^2}{\partial s_2} = 0$ simultaneously and determine the retailer price and service equilibrium in each chain.

Thus, the decision variable equilibrium values are as follows in each chain:

$\mathrm{p}_{R1}^* = \frac{A_1}{A}$, $\mathrm{p}_{R2}^* = \frac{A_2}{A}$, $\mathrm{s}_1^* = \frac{A_3}{A}$, $\mathrm{s}_2^* = \frac{A_4}{A}$

For more information about the above parameters, please see Appendix B (A, A_1 , … are abbreviations for calculated equilibriums).

Theorem 1. Supply chain's profit in equation (3) is strictly concave.

Proof. In the integrated mode for the i^{th} supply chain, Eq. (3), the Hessian matrix of chain profit function is as follows:

$$Hessian\ Matrix = H = \begin{pmatrix} \frac{\partial^2 \Pi_{sc}^i}{\partial p_{Ri}^2} & \frac{\partial^2 \Pi_{sc}^i}{\partial p_{Ri} \partial s_i} \\ \frac{\partial^2 \Pi_{sc}^i}{\partial s_i \partial p_{Ri}} & \frac{\partial^2 \Pi_{sc}^i}{\partial s_i^2} \end{pmatrix}$$

Where,

$$\frac{\partial^2 \Pi_{sc}^i}{\partial \mathrm{p}_{Ri}^2} = -2(b_p + \theta_p) < 0 \quad , \frac{\partial^2 \Pi_{sc}^i}{\partial p_{Ri} \partial s_i} = \frac{\partial^2 \Pi_{sc}^i}{\partial s_i \partial p_{Ri}} = (b_s + \theta_s) > 0 \quad , \frac{\partial^2 \Pi_{sc}^i}{\partial \mathrm{s}_i^2} = -\eta_i < 0$$

The determinant of the Hessian matrix is calculated as follows:

$|H| = 2(b_p + \theta_p)\eta_i - (b_s + \theta_s)^2$. Since all parameters are positive, we will have $\frac{\partial^2 \Pi_{sc}^i}{\partial \mathrm{p}_{Ri}^2} < 0$. On the other hand, according to relation (2), we always have: $2(b_p + \theta_p)\eta_i - (b_s + \theta_s)^2 > 0$. Therefore, the above Hessian matrix is a negative definite matrix. Thus, profit function which is expressed by Eq. (3), is strictly concave. ■

Equation (3) is strictly concave; consequently, it can be concluded that the values obtained for p_{Ri}^* and s_i^* are optimal and unique.

N-2)

Solving procedure is as follows:

(1) Differentiation of Function (6) with respect to p_{Ri}

(2) Simultaneously solve the equations $\frac{\partial \Pi_R^1}{\partial p_{R1}} = 0$ and $\frac{\partial \Pi_R^2}{\partial p_{R2}} = 0$ and determine the retailer prices:

$$p_{R1} = f(w_{D1}, w_{D2}, s_1, s_2) = A_5\{2(b_p + \theta_p)^2 w_{D1} + \theta_p(b_p + \theta_p)w_{D2} + A_6 s_1 + A_7 s_2 + A_8\} \quad (7)$$

$$p_{R2} = f(w_{D1}, w_{D2}, s_1, s_2) = A_5\{\theta_p(b_p + \theta_p)w_{D1} + 2(b_p + \theta_p)^2 w_{D2} + A_7 s_1 + A_6 s_2 + A_9\} \quad (8)$$

Here, $f(.)$ means that retailer price is a function of certain decision variables. The same symbol has been used in this paper.

Theorem 2. Retailer's profit function is strictly concave.

Proof. For Π_R^i, we have:

$$\frac{\partial \Pi_R^i}{\partial p_{Ri}} = b_s s_i + b_p(c_R - 2p_{Ri} + w_{Di}) + \alpha_i + \theta_p(c_R - 2p_{Ri} + p_{Rj} + w_{Di}) + \theta_s(s_i - s_j)$$

On the other hand, $\frac{\partial^2 \Pi_R^i}{\partial (p_{Ri})^2} = -2(b_p + \theta_p)$. Because $b_p, \theta_p > 0$, therefore, $\frac{\partial^2 \Pi_R^i}{\partial (p_{Ri})^2} < 0$. As a result, Π_R^i is strictly concave. ■

Since the objective function of retailer is concave, the obtained value for p_{Ri} is optimal and unique.

(3) Replace (7) and (8) in the profit function of distributors, then:
$\Pi_D^1 = f(w_{M1}, w_{D1}, w_{D2}, s_1, s_2)$ and $\Pi_D^2 = f(w_{M2}, w_{D1}, w_{D2}, s_1, s_2)$

(4) Determine the first derivative of distributor profit function with respect to w_{D1} and w_{D2}

(5) Simultaneously solve the equations $\frac{\partial \Pi_D^1}{\partial w_{D1}} = 0$ and $\frac{\partial \Pi_D^2}{\partial w_{D2}} = 0$ and determine the prices of distributors:

$$w_{D1} = f(w_{M1}, w_{M2}, s_1, s_2) = A_{10}\{A_{11}w_{M1} + A_{12}w_{M2} + A_{13}s_1 + A_{14}s_2 + A_{15}\} \quad (9)$$

$$w_{D2} = f(w_{M1}, w_{M2}, s_1, s_2) = A_{10}\{A_{12}w_{M1} + A_{11}w_{M2} + A_{14}s_1 + A_{13}s_2 + A_{16}\} \quad (10)$$

Theorem 3. Distributor's profit function (equation 5) is strictly concave.
Proof. From the supplier profit function in Step (3), we will have:

$$\frac{\partial^2 \Pi_D^1}{\partial (w_{D1})^2} = \frac{\partial^2 \Pi_D^2}{\partial (w_{D2})^2} = -2(b_p + \theta_p)\{\frac{2b_p^2 + 4b_p\theta_p + \theta_p^2}{(2b_p + \theta_p)(2b_p + 3\theta_p)}\}$$

Since all parameters are positive, the relationship is always negative. Therefore, we will have $\frac{\partial^2 \Pi_D^1}{\partial (w_{D1})^2} = \frac{\partial^2 \Pi_D^2}{\partial (w_{D2})^2} < 0$, and Π_D^1 and Π_D^2 are also concave. ■
Eq. (5) is strictly concave; consequently, it can be concluded that the value obtained for w_{Di} is optimal and unique.

(6) Replace equations (7) – (10) in the profit function of manufacturers. As a result:
$\Pi_M^1 = f(w_{M1}, w_{M2}, s_1, s_2)$ and $\Pi_M^2 = f(w_{M1}, w_{M2}, s_1, s_2)$

(7) Determine the first derivative of manufacturer's profit function (Π_M^i) with respect to w_{Mi} and s_i

(8) Solve the following equations simultaneously:

$$\frac{\partial \Pi_M^1}{\partial w_{M1}} = 0\,, \ \frac{\partial \Pi_M^1}{\partial s_1} = 0\,, \ \frac{\partial \Pi_M^2}{\partial w_{M2}} = 0 \ \text{ and } \ \frac{\partial \Pi_M^2}{\partial s_2} = 0$$

Finally, the manufacture's price and service equilibria is are follows:

$$w_{M1}^* = c_M - \frac{A_{17}}{A_{18}} + \frac{A_{19}}{A_{20}}, \quad w_{M2}^* = c_M - \frac{A_{17}}{A_{18}} - \frac{A_{19}}{A_{20}}$$

$$s_1^* = A_{21}\{(b_p + \theta_p)(2b_p^2 + 4b_p\theta_p + \theta_p^2)(b_s(b_p + \theta_p)(8b_p^2 + 16b_p\theta_p + 5\theta_p^2) + (2b_p + \theta_p)(4b_p^2 + 7b_p\theta_p + \theta_p^2)\theta_s)A_{22}\}$$

$$s_2^* = A_{21}\{(b_p + \theta_p)(2b_p^2 + 4b_p\theta_p + \theta_p^2)(b_s(b_p + \theta_p)(8b_p^2 + 16b_p\theta_p + 5\theta_p^2) + (2b_p + \theta_p)(4b_p^2 + 7b_p\theta_p + \theta_p^2)\theta_s)A_{23}\}$$

Theorem 4. Function (4) is strictly concave.
Proof. For this function:

$$\frac{\partial^2 \Pi_M^1}{\partial (w_{M1})^2} = -\frac{2(b_p + \theta_p)(2b_p^2 + 4b_p\theta_p + \theta_p^2)(8b_p^4 + 32b_p^3\theta_p + 39b_p^2\theta_p^2 + 14b_p\theta_p^3 + \theta_p^4)}{(2b_p + \theta_p)(2b_p + 3\theta_p)(4b_p^2 + 7b_p\theta_p + \theta_p^2)(4b_p^2 + 9b_p\theta_p + 3\theta_p^2)}, \quad \frac{\partial^2 \Pi_M^1}{\partial s_1^2} = -\eta_1$$

$$, \frac{\partial^2 \Pi_M^1}{\partial w_{M1}\partial s_1} = \frac{\partial^2 \Pi_M^1}{\partial s_1 \partial w_{M2}} = ((b_p + \theta_p)(2b_p^2 + 4b_p\theta_p + \theta_p^2)(b_s(b_p + \theta_p)(8b_p^2 + 16b_p\theta_p + 5\theta_p^2) + (2b_p + \theta_p)(4b_p^2 + 7b_p\theta_p + \theta_p^2)\theta_s))/((2b_p + \theta_p)(2b_p + 3\theta_p)(4b_p^2 + 7b_p\theta_p + \theta_p^2)(4b_p^2 + 9b_p\theta_p + 3\theta_p^2))$$

Thus, determinant of the Hessian matrix is calculated as follows:

$$|H| = ((b_p + \theta_p)(2b_p^2 + 4b_p\theta_p + \theta_p^2)\left(2\eta_1(2b_p + \theta_p)(2b_p + 3\theta_p)(4b_p^2 + 7b_p\theta_p + \theta_p^2)(4b_p^2 + 9b_p\theta_p + 3\theta_p^2)(8b_p^4 + 32b_p^3\theta_p + 39b_p^2\theta_p^2 + 14b_p\theta_p^3 + \theta_p^4) - (b_p + \theta_p)(2b_p^2 + 4b_p\theta_p + \theta_p^2)(b_s(b_p + \theta_p)(8b_p^2 + 16b_p\theta_p + 5\theta_p^2) + (2b_p + \theta_p)(4b_p^2 + 7b_p\theta_p + \theta_p^2)\theta_s)^2\right))/((2b_p + \theta_p)^2(2b_p + 3\theta_p)^2(4b_p^2 + 7b_p\theta_p + \theta_p^2)^2(4b_p^2 + 9b_p\theta_p + 3\theta_p^2)^2)$$

In order to have a concave manufacture profit function, it is enough to have $\frac{\partial^2 \Pi_M^1}{\partial (w_{M1})^2} < 0$ and $|H| > 0$. Since all parameters are positive, it is obvious that $\frac{\partial^2 \Pi_M^1}{\partial (w_{M1})^2} < 0$. However, according to relation (2), $|H| > 0$. Therefore, the Hessian Matrix is a negative definite matrix and Π_M^1 is strictly concave. These issues are also true for Π_M^2 . ■
As a result of Theorem 4, it can be stated that the values obtained for w_{Mi}^* and s_i^* are optimal and unique.

Therefore, optimal distributor and retailer prices are as follows:

$w_{D1}^{*} = A_{10}\{A_{11}w_{M1}^{*} + A_{12}w_{M2}^{*} + A_{13}s_1^{*} + A_{14}s_2^{*} + A_{15}\}$, $\quad w_{D2}^{*} = A_{10}\{A_{12}w_{M1}^{*} + A_{11}w_{M2}^{*} + A_{14}s_1^{*} + A_{13}s_2^{*} + A_{16}\}$

And

$p_{R1}^{*} = A_5\{2\left(b_p + \theta_p\right)^2 w_{D1}^{*} + \theta_p\left(b_p + \theta_p\right)w_{D2}^{*} + A_6 s_1^{*} + A_7 s_2^{*} + A_8\}$

$p_{R2}^{*} = A_5\{\theta_p\left(b_p + \theta_p\right)w_{D1}^{*} + 2\left(b_p + \theta_p\right)^2 w_{D2}^{*} + A_7 s_1^{*} + A_6 s_2^{*} + A_9\}$

N-3)

Solving procedure is as follows:

(1) Determine the first derivative of profit function in the second chain and retailer in the first chain with respect to p_{R1}, p_{R2}, and s_2 respectively.

(2) Solve the equations: $\frac{\partial \Pi_R^1}{\partial p_{R1}} = 0, \frac{\partial \Pi_{Sc}^2}{\partial p_{R2}} = 0$ and $\frac{\partial \Pi_{Sc}^2}{\partial s_2} = 0$ simultaneously

Finally, determine the variable values as follows:

$$p_{R1} = A_{24}\{A_{25}w_{D1} + A_{26}s_1 + A_{27}\} \quad (11)$$

$$p_{R2} = A_{24}(-\theta_p)\{(\eta_2\theta_p(b_p + \theta_p))w_{D1} + (\eta_2 A_7)s_1 + A_{28}\} \quad (12)$$

$$s_2 = A_{24}\left(\theta_p(b_s + \theta_s)\right)\{\left(\theta_p\left(b_p + \theta_p\right)\right)w_{D1} + A_7 s_1 + A_{29}\} \quad (13)$$

(3) Replace equations (11)-(13) in the profit function of the first distributor. As a result, $\Pi_D^1 = f(w_{M1}, w_{D1}, s_1)$.

(4) Determine the first derivative of Π_D^1 with respect to w_{D1}

(5) Solve Eq. $\frac{\partial \Pi_D^1}{\partial w_{D1}} = 0$ and calculate w_{D1}:

$$w_{D1} = A_{30}\{\left(\frac{1}{2A_{30}}\right)w_{M1} + A_{31}s_1 + A_{32}\} \quad (14)$$

(6) Replace equations (11)-(14) in the profit function of the first manufacturer

(7) Determine the first derivative of $\Pi_M^1 = f(w_{M1}, s_1)$ with respect to w_{M1} and s_1 , respectively

(8) Simultaneously solve the Eqs. $\frac{\partial \Pi_M^1}{\partial w_{M1}} = 0$ and $\frac{\partial \Pi_M^1}{\partial s_1} = 0$, and determine the equilibria of decision variables: $w_{M1}^{*} = c_M - \frac{A_{33}}{A_{34}}$ and $s_1^{*} = \frac{A_{35}}{A_{36}}$

Therefore, the equilibria values of other decision variables are as follows:

$w_{D1}^{*} = A_{30}\{(\frac{1}{2A_{30}})w_{M1}^{*} + A_{31}s_1^{*} + A_{32}\}$, $\quad p_{R2}^{*} = A_{24}(-\theta_p)\{(\eta_2\theta_p(b_p + \theta_p))w_{D1}^{*} + (\eta_2 A_7)s_1^{*} + A_{28}\}$

$p_{R1}^{*} = A_{24}\{A_{25}w_{D1}^{*} + A_{26}s_1^{*} + A_{27}\}$, $\quad s_2^{*} = A_{24}\left(\theta_p(b_s + \theta_s)\right)\{\left(\theta_p\left(b_p + \theta_p\right)\right)w_{D1}^{*} + A_7 s_1^{*} + A_{29}\}$

Mode II)

S-1)

Stepwise solving procedures of leader-follower problem are as follows:

(i) Follower problem

(1) Differentiation of Function (3) with respect to p_{R2} and s_2

(2) Simultaneously solve the Eqs. $\frac{\partial \Pi_{Sc}^2}{\partial s_2} = 0$ and $\frac{\partial \Pi_{Sc}^2}{\partial p_{R2}} = 0$, and determine the price and service equilibria:

$$p_{R2} = f(p_{R1}, s_1) = B\{\left(-\eta_2\theta_p\right)p_{R1} + (\eta_2\theta_s)s_1 + B_1\} \quad (15)$$

$$s_2 = f(p_{R1}, s_1) = B\{\left(-\theta_p(b_s + \theta_s)\right)p_{R1} + \left(\theta_s(b_s + \theta_s)\right)s_1 + B_2\} \quad (16)$$

(ii) Leader problem

(3) Replace equations (15) and (16) in the profit function

(4) Determine the first derivative of function with respect to p_{R1} and s_1

(5) Simultaneously solve equations. $\frac{\partial \Pi_{Sc}^1}{\partial s_1} = 0$ and $\frac{\partial \Pi_{Sc}^1}{\partial p_{R1}} = 0$, and determine the equilibrium of price and service: $p_{R1}^{*} = c - \frac{B_3}{B_4}$ and $s_1^{*} = \frac{B_5}{B_6}$

Therefore, the decision variable equilibrium in the follower chain is as follows:
$p_{R2}^{*} = B\{(-\eta_2\theta_p)p_{R1}^{*} + (\eta_2\theta_s)s_1^{*} + B_1\}$ $\quad ,s_2^{*} = B\{\left(-\theta_p(b_s+\theta_s)\right)p_{R1}^{*} + \left(\theta_s(b_s+\theta_s)\right)s_1^{*} + B_2\}$
For more information about the above parameters, please see Appendix C.

S-2)

Solving procedure is as follows:

(i) Follower problem

(1) Calculate the first derivative of retailer function with respect to p_{R2} and solve $\frac{\partial\Pi_R^2}{\partial p_{R2}} = 0$. Therefore, we will have:

$$p_{R2} = f(p_{R1}, s_1, w_{D2}, s_2) = \frac{1}{2(b_p+\theta_p)}\{\theta_p p_{R1} - \theta_s s_1 + (b_p+\theta_p)w_{D2} + (b_s+\theta_s)s_2 + \alpha_2 + c_R(b_p+\theta_p)\} \tag{17}$$

(2) Replace (17) in the distributor's profit function. As a result, we will have: $\Pi_D^2 = f(w_{M2}, w_{D2}, p_{R1}, s_1, s_2)$

(3) Determine the first derivative of this function and solve $\frac{\partial\Pi_D^2}{\partial w_{D2}} = 0$

$$w_{D2} = f(p_{R1}, s_1, w_{M2}, s_2) = \frac{1}{2(b_p+\theta_p)}\{\theta_p p_{R1} - \theta_s s_1 + (b_p+\theta_p)w_{M2} + (b_s+\theta_s)s_2 + \alpha_2 + (c_D - c_R)(b_p+\theta_p)\} \tag{18}$$

(4) Replace equations (17) and (18) in the manufacturer's profit function. As a result: $\Pi_M^2 = f(w_{M2}, p_{R1}, s_1, s_2)$

(5) Determine the first derivative of Π_M^2 with respect to w_{M2} and s_2, and simultaneously solve the equations $\frac{\partial\Pi_M^2}{\partial w_{M2}} = 0$ and $\frac{\partial\Pi_M^2}{\partial s_2} = 0$:

$$w_{M2} = f(p_{R1}, s_1) = \frac{1}{-8\eta_2(b_p+\theta_p)+(b_s+\theta_s)^2}\left\{(-4\eta_2\theta_p)p_{R1} + (4\eta_2\theta_s)s_1 + b_s^2 c_M + 4\eta_2\left(-\alpha_2 + (c_D - c_M + c_R)(b_p+\theta_p)\right) + 2b_s c_M\theta_s + \theta_s^2 c_M\right\} \tag{19}$$

$$s_2 = f(p_{R1}, s_1) = \frac{(b_s+\theta_s)}{-8\eta_2(b_p+\theta_p)+(b_s+\theta_s)^2}\{(-\theta_p)p_{R1} + (\theta_s)s_1 - \alpha_2 + c(b_p+\theta_p)\} \tag{20}$$

(ii) *Leader problem*

(6) Replace equation (18) in equation (17). As a result, $p_{R2} = f(p_{R1}, s_1, w_{M2}, s_2)$

(7) Replace equations (19) and (20) in $p_{R2} = f(p_{R1}, s_1, w_{M2}, s_2)$. As a result: $p_{R2} = f(p_{R1}, s_1)$

(8) Replace $p_{R2} = f(p_{R1}, s_1)$ and (20) in the supply chain's profit function

(9) Calculate the first derivative of chain profit function with respect to p_{R1} and s_1, and solve $\frac{\partial\Pi_{Sc}^1}{\partial p_{R1}} = 0$ and $\frac{\partial\Pi_{Sc}^1}{\partial s_1} = 0$ simultaneously: $p_{R1}^{*} = c - \frac{B_7}{B_8}$ and $s_1^{*} = \frac{B_9}{B_{10}}$

Therefore, service and price equilibria in the follower chain are as follows:

$$w_{M2}^{*} = \frac{1}{-8\eta_2(b_p+\theta_p)+(b_s+\theta_s)^2}\left\{(-4\eta_2\theta_p)p_{R1}^{*} + (4\eta_2\theta_s)s_1^{*} + b_s^2 c_M + 4\eta_2\left(-\alpha_2 + (c_D - c_M + c_R)(b_p+\theta_p)\right) + 2b_s c_M\theta_s + \theta_s^2 c_M\right\}$$

$$s_2^{*} = \frac{(b_s+\theta_s)}{-8\eta_2(b_p+\theta_p)+(b_s+\theta_s)^2}\{(-\theta_p)p_{R1}^{*} + (\theta_s)s_1^{*} - \alpha_2 + c(b_p+\theta_p)\}$$

$$w_{D2}^{*} = \frac{1}{2(b_p+\theta_p)}\{\theta_p p_{R1}^{*} - \theta_s s_1^{*} + (b_p+\theta_p)w_{M2}^{*} + (b_s+\theta_s)s_2^{*} + \alpha_2 + (c_D - c_R)(b_p+\theta_p)\}$$

$$p_{R2}^* = \frac{1}{2(b_p + \theta_p)} \{\theta_p p_{R1}^* - \theta_s s_1^* + (b_p + \theta_p) w_{D2}^* + (b_s + \theta_s) s_2^* + \alpha_2 + c_R (b_p + \theta_p)\}$$

S-3)

Solving procedure is as follows:

(i) *Follower problem*

In this section, the decision variable values are similar to equations (15) and (16).

(ii) *Leader problem*

(1) Replace equations (15) and (16) in the retailer's profit function

(2) Determine the first derivative $\Pi_R^1 = f(w_{D1}, p_{R1}, s_1)$ with respect to p_{R1} and solve $\frac{\partial \Pi_R^1}{\partial p_{R1}} = 0$, and determine retailer price as follows:

$$p_{R1} = f(w_{D1}, s_1) = B_{11}\{(1/(2B_{11}))w_{D1} + B_{12}s_1 + B_{13}\} \quad (21)$$

(3) Replace equations (15), (16) and (21) in the distributor's profit function. As a result: $\Pi_D^1 = f(w_{D1}, w_{M1}, s_1)$

(4) Determine the first derivative with respect to w_{D1} and solve $\frac{\partial \Pi_D^1}{\partial w_{D1}} = 0$:

$$w_{D1} = f(w_{M1}, s_1) = B_{11}\{(1/(2B_{11}))w_{M1} + B_{12}s_1 + B_{14}\} \quad (22)$$

(5) Replace Eqs. (15) and (16) in the manufacture’s profit function. As a result: $\Pi_M^1 = f(w_{M1}, s_1, p_{R1})$

(6) Replace Eq. (22) in equation (21). As a result: $p_{R1} = f(w_{M1}, s_1)$

(7) Replace $p_{R1} = f(w_{M1}, s_1)$ in Π_M^1

(8) Determine the first derivative of the manufacturer's profit function ($\Pi_M^1 = f(w_{M1}, s_1)$) with respect to w_{M1} and s_1

(9) Simultaneously solve the equations $\frac{\partial \Pi_M^1}{\partial w_{M1}} = 0$ and $\frac{\partial \Pi_M^1}{\partial s_1} = 0$, and determine the manufacture's price and service equilibrium as follows: $w_{M1}^* = c_M + \frac{B_{15}}{B_{16}}$ and $s_1^* = \frac{B_{17}}{B_{16}}$

Therefore, the decision variables equilibria are as follows:

$w_{D1}^* = B_{11}\{(1/(2B_{11}))w_{M1}^* + B_{12}s_1^* + B_{14}\}$, $\quad p_{R1}^* = B_{11}\{(1/(2B_{11}))w_{D1}^* + B_{12}s_1^* + B_{13}\}$

$p_{R2}^* = B\{(-\eta_2\theta_p)p_{R1}^* + (\eta_2\theta_s)s_1^* + B_1\}$, $\quad s_2^* = B\{(-\theta_p(b_s + \theta_s))\, p_{R1}^* + (\theta_s(b_s + \theta_s))s_1^* + B_2\}$

S-4)

Stepwise solving procedures of leader-follower problem are as follows:

(i) *Follower problem*

In this section, the decision variable values are similar to equations (17) - (20).

(ii) *Leader problem*

(1) Replace equation (18) in equation (17), as a result: $p_{R2} = f(p_{R1}, s_1, s_2, w_{M2})$

(2) Replace equations (19) and (20) in $p_{R2} = f(p_{R1}, s_1, s_2, w_{M2})$, as a result: $p_{R2} = f(p_{R1}, s_1)$ (23)

(3) Replace equations (23) and (20) in the retailer’s profit function. As a result, $\Pi_R^1 = f(w_{D1}, p_{R1}, s_1)$

(4) Determine the first derivative Π_R^1 with respect to p_{R1} and solve $\frac{\partial \Pi_R^1}{\partial p_{R1}} = 0$:

$$p_{R1} = f(w_{D1}, s_1) = B_{18}\{(1/(2B_{18}))w_{D1} + B_{19}s_1 + B_{20}\} \quad (24)$$

(5) Replace equations (23), (20) and (24) in the distributor's profit function. As a result, we will have: $\Pi_D^1 = f(w_{M1}, w_{D1}, s_1)$

(6) Determine the first derivative Π_D^1 with respect to w_{D1} and solve $\frac{\partial \Pi_D^1}{\partial w_{D1}} = 0$:

$w_{D1} = f(w_{M1}, s_1) = B_{18}\{(1/(2B_{18}))w_{M1} + B_{19}s_1 + B_{21}\}$ (25)

(7) Replace equation (25) in equation (24), as a result: $p_{R1} = f(w_{M1}, s_1)$ (26)

(8) Replace equations (23), (20) and (26) in the manufacture's profit function, then $\Pi_M^1 = f(s_1, w_{M1})$

(9) Determine the first derivative of this function with respect to w_{M1} and s_1

(10) Simultaneously solve the equations $\frac{\partial \Pi_M^1}{\partial w_{M1}} = 0$ and $\frac{\partial \Pi_M^1}{\partial s_1} = 0$, and finally, determine the manufacture's price and service equilibria as follows: $w_{M1}^* = c_M + \frac{B_{22}}{B_{23}}$ and $s_1^* = \frac{B_9}{B_{24}}$

Therefore, the equilibria of other decision variables would be as follows:

$$w_{D1}^* = B_{18}\{(1/(2B_{18}))w_{M1}^* + B_{19}s_1^* + B_{21}\} \quad , p_{R1}^* = B_{18}\{(1/(2B_{18}))w_{D1}^* + B_{19}s_1^* + B_{20}\}$$

$$s_2^* = \frac{(b_s+\theta_s)}{-8\eta_2(b_p+\theta_p)+(b_s+\theta_s)^2}\{(-\theta_p)p_{R1}^* + \theta_s s_1^* - \alpha_2 + c(b_p+\theta_p)$$

$$w_{M2}^* = \frac{1}{-8\eta_2(b_p+\theta_p)+(b_s+\theta_s)^2}\Big\{(-4\eta_2\theta_p)p_{R1}^* + (4\eta_2\theta_s)s_1^* + b_s^2 c_M + 4\eta_2\Big(-\alpha_2 + (c_D - c_M + c_R)(b_p+\theta_p)\Big) + 2b_s c_M\theta_s + \theta_s^2 c_M\Big\}$$

$$w_{D2}^* = \frac{1}{2(b_p+\theta_p)}\{(\theta_p)p_{R1}^* - (\theta_s)s_1^* + (b_p+\theta_p)w_{M2}^* + (\theta_s+b_s)s_2^* + \alpha_2 + (c_D - c_R)(b_p+\theta_p)\}$$

$$p_{R2}^* = \frac{1}{2(b_p+\theta_p)}\{(\theta_p)p_{R1}^* - (\theta_s)s_1^* + (b_p+\theta_p)w_{D2}^* + (\theta_s+b_s)s_2^* + \alpha_2 + c_R(\theta_p+b_p)\}$$

Appendix B. Parameters for the Nash game

$$A = (b_s^4 + 4b_s^3\theta_s - b_s(\eta_1+\eta_2)(4b_p+3\theta_p)\theta_s + 2b_s\theta_s^3 + b_s^2(-2(\eta_1+\eta_2)(b_p+\theta_p) + 5\theta_s^2) + (2b_p+\theta_p)(2b_p\eta_1\eta_2 + 3\eta_1\eta_2\theta_p - (\eta_1+\eta_2)\theta_s^2))$$

$$A_1 = (b_s^4 c + \eta_1\eta_2(2b_p(b_p c + \alpha_1) + (5b_p c + 2\alpha_1 + \alpha_2)\theta_p + 3c\theta_p^2) + 4b_s^3 c\theta_s - b_s(b_p c(\eta_1 + 4\eta_2) + \eta_1(2\alpha_1+\alpha_2) + 3c(\eta_1+\eta_2)\theta_p)\theta_s - \Big(\eta_1(\alpha_1+\alpha_2) + c(2b_p\eta_2 + (\eta_1+\eta_2)\theta_p)\Big)\theta_s^2 + 2b_s c\theta_s^3 + b_s^2(-b_p c(\eta_1 + 2\eta_2) - \eta_1\alpha_1 - 2c(\eta_1+\eta_2)\theta_p + 5c\theta_s^2))$$

$$A_2 = c + (\eta_2\Bigg(-2b_p^2 c\eta_1 - b_s^2\alpha_2 + \eta_1(\alpha_1+2\alpha_2)\theta_p - b_s(\alpha_1+2\alpha_2)\theta_s - (\alpha_1+\alpha_2)\theta_s^2 + b_p\Bigg(2\eta_1\alpha_2 + c\Bigg(-3\eta_1\theta_p + (b_s+\theta_s)(b_s+2\theta_s)\Bigg)\Bigg)\Bigg))$$

$$A_3 = ((b_s+\theta_s)\Bigg(-2b_p^2 c\eta_2 - b_s^2\alpha_1 + \eta_2(2\alpha_1+\alpha_2)\theta_p - b_s(2\alpha_1+\alpha_2)\theta_s - (\alpha_1+\alpha_2)\theta_s^2 + b_p\Bigg(2\eta_2\alpha_1 + c\Bigg(-3\eta_2\theta_p + (b_s+\theta_s)(b_s+2\theta_s)\Bigg)\Bigg)\Bigg))$$

$$A_4 = ((b_s+\theta_s)\Bigg(-2b_p^2 c\eta_1 - b_s^2\alpha_2 + \eta_1(\alpha_1+2\alpha_2)\theta_p - b_s(\alpha_1+2\alpha_2)\theta_s - (\alpha_1+\alpha_2)\theta_s^2 + b_p\Bigg(2\eta_1\alpha_2 + c\Bigg(-3\eta_1\theta_p + (b_s+\theta_s)(b_s+2\theta_s)\Bigg)\Bigg)\Bigg))$$

$$A_5 = \frac{1}{(2b_p+\theta_p)(2b_p+3\theta_p)} \quad , A_6 = 2b_s(b_p+\theta_p) + \theta_s(2b_p+\theta_p) \quad , A_7 = b_s\theta_p - \theta_s(2b_p+\theta_p)$$

$$A_8 = 2b_p(b_p c_R + \alpha_1) + (5b_p c_R + 2\alpha_1 + \alpha_2)\theta_p + 3c_R\theta_p^2 \quad , A_9 = 2b_p(b_p c_R + \alpha_2) + (5b_p c_R + \alpha_1 + 2\alpha_2)\theta_p + 3c_R\theta_p^2$$

$$A_{10} = \frac{1}{(4b_p^2 + 7b_p\theta_p + \theta_p^2)(4b_p^2 + 9b_p\theta_p + 3\theta_p^2)} \quad , A_{11} = 2(2b_p^2 + 4b_p\theta_p + \theta_p^2)^2 \quad , A_{12} = \theta_p(b_p+\theta_p)(2b_p^2 + 4b_p\theta_p + \theta_p^2)$$

$$A_{14} = 2b_s\theta_p(3b_p^2 + 6b_p\theta_p + 2\theta_p^2) - \theta_s(2b_p+\theta_p)(4b_p^2 + 7b_p\theta_p + \theta_p^2)$$

$$A_{13} = b_s(b_p+\theta_p)(8b_p^2 + 16b_p\theta_p + 5\theta_p^2) + \theta_s(2b_p+\theta_p)(4b_p^2 + 7b_p\theta_p + \theta_p^2)$$

$$A_{15} = 8b_p^4(c_D - c_R) + \theta_p^3(5\alpha_1 + 4\alpha_2 + 3c_D\theta_p) + b_p^2\theta_p(24\alpha_1 + 6\alpha_2 + 46c_D\theta_p - 33c_R\theta_p) + b_p^3(8\alpha_1 + 34c_D\theta_p - 30c_R\theta_p) + 3b_p\theta_p^2(7\alpha_1 + 4\alpha_2 + 7c_D\theta_p - 3c_R\theta_p)$$

$$A_{16} = 8b_p^4(c_D - c_R) + \theta_p^3(5\alpha_2 + 4\alpha_1 + 3c_D\theta_p) + b_p^2\theta_p(24\alpha_2 + 6\alpha_1 + 46c_D\theta_p - 33c_R\theta_p) + b_p^3(8\alpha_2 + 34c_D\theta_p - 30c_R\theta_p) + 3b_p\theta_p^2(7\alpha_2 + 4\alpha_1 + 7c_D\theta_p - 3c_R\theta_p)$$

$A_{17} = (\eta_1(2b_pc - \alpha_1 - \alpha_2)(2b_p + \theta_p)(2b_p + 3\theta_p)(4b_p^2 + 7b_p\theta_p + \theta_p^2)(4b_p^2 + 9b_p\theta_p + 3\theta_p^2))$

$A_{18} = \Bigg(2\Big(128b_p^7\eta_1 + 2b_p^4\theta_p^2(-113b_s^2 + 1027\eta_1\theta_p - 83b_s\theta_s) + 3b_p^2\theta_p^4(-53b_s^2 + 99\eta_1\theta_p - 23b_s\theta_s) + 2b_p\theta_p^5(-23b_s^2 + 16\eta_1\theta_p - 7b_s\theta_s) + 12b_p^5\theta_p(-8b_s^2 + 153\eta_1\theta_p - 7b_s\theta_s) - 16b_p^6\left(-49\eta_1\theta_p + b_s(b_s+\theta_s)\right) + \theta_p^6\left(\eta_1\theta_p - b_s(5b_s+\theta_s)\right) + 2b_p^3\theta_p^3\left(566\eta_1\theta_p - 11b_s(12b_s+7\theta_s)\right)\Big)\Bigg)$

$A_{19} = (\eta_1(\alpha_1 - \alpha_2)(2b_p + \theta_p)(2b_p + 3\theta_p)(4b_p^2 + 7b_p\theta_p + \theta_p^2)(4b_p^2 + 9b_p\theta_p + 3\theta_p^2))$

$A_{20} = (2\Big(128b_p^7\eta_1 - 16b_p^6\left(-63\eta_1\theta_p + (b_s+\theta_s)(b_s+2\theta_s)\right) + \theta_p^6\left(27\eta_1\theta_p - (5b_s+\theta_s)(b_s+2\theta_s)\right) - 12b_p^5\theta_p\left(-265\eta_1\theta_p + (b_s+2\theta_s)(8b_s+7\theta_s)\right) + 2b_p^3\theta_p^3\left(2190\eta_1\theta_p - 11(b_s+2\theta_s)(12b_s+7\theta_s)\right) + 2b_p\theta_p^5\left(198\eta_1\theta_p - (b_s+2\theta_s)(23b_s+7\theta_s)\right) - 3b_p^2\theta_p^4\left(-645\eta_1\theta_p + (b_s+2\theta_s)(53b_s+23\theta_s)\right) + 2b_p^4\theta_p^2\left(2553\eta_1\theta_p - (b_s+2\theta_s)(113b_s+83\theta_s)\right)\Big))$

$A_{21} = -1/((128b_p^7\eta_1 + 2b_p^4\theta_p^2(-113b_s^2 + 1027\eta_1\theta_p - 83b_s\theta_s) + 3b_p^2\theta_p^4(-53b_s^2 + 99\eta_1\theta_p - 23b_s\theta_s) + 2b_p\theta_p^5(-23b_s^2 + 16\eta_1\theta_p - 7b_s\theta_s) + 12b_p^5\theta_p(-8b_s^2 + 153\eta_1\theta_p - 7b_s\theta_s) - 16b_p^6\left(-49\eta_1\theta_p + b_s(b_s + \theta_s)\right) + \theta_p^6\left(\eta_1\theta_p - b_s(5b_s + \theta_s)\right) + 2b_p^3\theta_p^3(566\eta_1\theta_p - 11b_s(12b_s + 7\theta_s)))(128b_p^7\eta_1 - 16\eta_1\theta_p\left(-63\eta_1\theta_p + (b_s + \theta_s)(b_s + 2\theta_s)\right) + \theta_p^6\left(27\eta_1\theta_p - (5b_s + \theta_s)(b_s + 2\theta_s)\right) - 12b_p^5\theta_p\left(-265\eta_1\theta_p + (b_s + 2\theta_s)(8b_s + 7\theta_s)\right) + 2b_p^3\theta_p^3\left(2190\eta_1\theta_p - 11(b_s + 2\theta_s)(12b_s + 7\theta_s)\right) + 2b_p\theta_p^5\left(198\eta_1\theta_p - (b_s + 2\theta_s)(23b_s + 7\theta_s)\right) - 3b_p^2\theta_p^4\left(-645\eta_1\theta_p + (b_s + 2\theta_s)(53b_s + 23\theta_s)\right) + 2b_p^4\theta_p^2(2553\eta_1\theta_p - (b_s + 2\theta_s)(113b_s + 83\theta_s))))$

$A_{22} = (128b_p^8c\eta_1 + \theta_p^6(5b_s^2\alpha_1 - \eta_1(14\alpha_1 + 13\alpha_2)\theta_p + b_s(6\alpha_1 + 5\alpha_2)\theta_s + (\alpha_1 + \alpha_2)\theta_s^2) + 2b_p^5\theta_p(b_s^2(48\alpha_1 - 113c\theta_p) + 3\eta_1\theta_p(-418\alpha_1 - 112\alpha_2 + 851c\theta_p) + 3b_s(30\alpha_1 + 16\alpha_2 - 103c\theta_p)\theta_s + 2(21(\alpha_1 + \alpha_2) - 83c\theta_p)\theta_s^2) + b_p^3\theta_p^3(3b_s^2(88\alpha_1 - 53c\theta_p) + \eta_1\theta_p(-4(689\alpha_1 + 406\alpha_2) + 1935c\theta_p) + b_s(418\alpha_1 + 264\alpha_2 - 387c\theta_p)\theta_s + 2(77(\alpha_1 + \alpha_2) - 69c\theta_p)\theta_s^2) + b_p^2\theta_p^4\left(b_s^2(159\alpha_1 - 46c\theta_p) + 9\eta_1\theta_p(-124\alpha_1 - 91\alpha_2 + 44c\theta_p) + b_s\left(228\alpha_1 + 53(3\alpha_2 - 2c\theta_p)\right)\theta_s + (69(\alpha_1 + \alpha_2) - 28c\theta_p)\theta_s^2\right) + 4b_p^6(4b_s^2(\alpha_1 - 6c\theta_p) + \eta_1\theta_p(-28(8\alpha_1 + \alpha_2) + 795c\theta_p) + b_s(8\alpha_1 + 4\alpha_2 - 69c\theta_p)\theta_s + 2(2(\alpha_1 + \alpha_2) - 21c\theta_p)\theta_s^2) + b_p\theta_p^5(b_s^2(46\alpha_1 - 5c\theta_p) + \eta_1\theta_p(-214\alpha_1 - 182\alpha_2 + 27c\theta_p) + b_s(60\alpha_1 + 46\alpha_2 - 11c\theta_p)\theta_s - 2(-7(\alpha_1 + \alpha_2) + c\theta_p)\theta_s^2) - 2b_p^4\theta_p^2(\eta_1\theta_p(1790\alpha_1 + 763\alpha_2 - 2190c\theta_p) + b_s^2(-113\alpha_1 + 132c\theta_p) + b_s(-196\alpha_1 - 113\alpha_2 + 341c\theta_p)\theta_s + (-83(\alpha_1 + \alpha_2) + 154c\theta_p)\theta_s^2) + 16b_p^7\left(-8\eta_1\alpha_1 - c\left(-63\eta_1\theta_p + (b_s + \theta_s)(b_s + 2\theta_s)\right)\right))$

$A_{23} = (128b_p^8c\eta_1 + \theta_p^6(5b_s^2\alpha_2 - \eta_1(13\alpha_1 + 14\alpha_2)\theta_p + b_s(5\alpha_1 + 6\alpha_2)\theta_s + (\alpha_1 + \alpha_2)\theta_s^2) + 2b_p^5\theta_p(b_s^2(48\alpha_2 - 113c\theta_p) + 3\eta_1\theta_p(-112\alpha_1 - 418\alpha_2 + 851c\theta_p) + 3b_s(16\alpha_1 + 30\alpha_2 - 103c\theta_p)\theta_s + 2(21(\alpha_1 + \alpha_2) - 83c\theta_p)\theta_s^2) + b_p^3\theta_p^3(3b_s^2(88\alpha_2 - 53c\theta_p) + \eta_1\theta_p(-4(406\alpha_1 + 689\alpha_2) + 1935c\theta_p) + b_s(264\alpha_1 + 418\alpha_2 - 387c\theta_p)\theta_s + 2(77(\alpha_1 + \alpha_2) - 69c\theta_p)\theta_s^2) + b_p^2\theta_p^4(b_s^2(159\alpha_2 - 46c\theta_p) + 9\eta_1\theta_p(-91\alpha_1 - 124\alpha_2 + 44c\theta_p) + b_s(159\alpha_1 + 228\alpha_2 - 106c\theta_p)\theta_s + (69(\alpha_1 + \alpha_2) - 28c\theta_p)\theta_s^2) + 4b_p^6(4b_s^2(\alpha_2 - 6c\theta_p) + \eta_1\theta_p(-28(\alpha_1 + 8\alpha_2) + 795c\theta_p) + b_s(4\alpha_1 + 8\alpha_2 - 69c\theta_p)\theta_s + 2(2(\alpha_1 + \alpha_2) - 21c\theta_p)\theta_s^2) + b_p\theta_p^5(b_s^2(46\alpha_2 - 5c\theta_p) + \eta_1\theta_p(-182\alpha_1 - 214\alpha_2 + 27c\theta_p) + b_s(46\alpha_1 + 60\alpha_2 - 11c\theta_p)\theta_s - 2(-7(\alpha_1 + \alpha_2) + c\theta_p)\theta_s^2) - 2b_p^4\theta_p^2(\eta_1\theta_p(763\alpha_1 + 1790\alpha_2 - 2190c\theta_p) + b_s^2(-113\alpha_2 + 132c\theta_p) + b_s(-113\alpha_1 - 196\alpha_2 + 341c\theta_p)\theta_s + (-83(\alpha_1 + \alpha_2) + 154c\theta_p)\theta_s^2) + 16b_p^7\left(-8\eta_1\alpha_2 - c\left(-63\eta_1\theta_p + (b_s + \theta_s)(b_s + 2\theta_s)\right)\right))$

$A_{24} = \{1/\theta_p(-4b_p^2\eta_2 + 2b_p(-4\eta_2\theta_p + (b_s + \theta_s)^2) + \theta_p\left(-3\eta_2\theta_p + (b_s + \theta_s)(2b_s + \theta_s)\right))\}$

$A_{25} = -\theta_p(b_p + \theta_p)(2\eta_2(b_p + \theta_p) - (b_s + \theta_s)^2)$, $A_{26} = \theta_s + (b_s\theta_p - (2b_p + \theta_p)\theta_s)(-2\eta_2(b_p + \theta_p) + (b_s + \theta_s)^2)$

$A_{27} = 4b_p^3c\eta_2 - \alpha_2 + 2b_p^2(-b_s^2c[-2\eta_2\alpha_2 + (6(c_D + c_M) + 5c_R)\eta_2\theta_p - 2b_sc\theta_s - c\theta_s^2) + b_p\Big(c_M + c_R + 2b_s^2\alpha_2 - b_s^2(4c_M + 3c_R)\theta_p - 2\eta_2\theta_p(\alpha_1 + 4\alpha_2 - 5c_M\theta_p - 3c_R\theta_p) + 2b_s(2\alpha_2 - 4c_M\theta_p - 3c_R\theta_p)\theta_s + (2\alpha_2 - 4c_M\theta_p - 3c_R\theta_p)\theta_s^2 + c_D\left(1 + 2\theta_p(5\eta_2\theta_p - 2(b_s + \theta_s)^2)\right)\Big) + \theta_p(c_R + (\alpha_1 + 2\alpha_2)(-2\eta_2\theta_p + (b_s + \theta_s)^2) - c_D\left(-1 + \theta_p(-2\eta_2\theta_p + (b_s + \theta_s)^2)\right) - c_M\left(-1 + \theta_p(-2\eta_2\theta_p + (b_s + \theta_s)^2)\right))$

$A_{28} = c - 2b_p^2\eta_2c + 2b_p\eta_2\alpha_2 + \eta_2(-4b_p(c_D + c_M) - 3b_pc_R + \alpha_1 + 2\alpha_2)\theta_p - (c_D + c_M)\eta_2\theta_p^2$

$A_{29} = 2b_p(-b_pc + \alpha_2) + (-4b_p(c_D + c_M) - 3b_pc_R + \alpha_1 + 2\alpha_2)\theta_p - (c_D + c_M)\theta_p^2$

$A_{30} = 1/\{4\,b_p^2\eta_2 + 2\theta_p(\eta_2\theta_p - b_s(b_s + \theta_s)) - 2b_p(-4\eta_2\theta_p + (b_s + \theta_s)^2)\}$

$A_{31} = -b_s^3 + 2b_s\eta_2(b_p + \theta_p) - 3b_s^2\theta_s + \eta_2(2b_p + \theta_p)\theta_s - 2b_s\theta_s^2$

$A_{32} = 2b_p^2(c_D - c_R)\eta_2 - b_s^2(\alpha_1 + 2c_D\theta_p + c_M\theta_p) + \eta_2\theta_p(2\alpha_1 + \alpha_2 + 2c_D\theta_p + c_M\theta_p) - b_s(2\alpha_1 + \alpha_2 + 2c_D\theta_p + c_M\theta_p)\theta_s - (\alpha_1 + \alpha_2)\theta_s^2 + b_p(b_s^2(-c_D + c_R) + 2\eta_2\alpha_1 + (5c_D + c_M - 3c_R)\eta_2\theta_p + b_s(-c_D + c_M + 3c_R)\theta_s + (c_M + 2c_R)\theta_s^2)$

$A_{33} = (2\eta_1 \left(4b_p^2\eta_2 - 2b_p\left(-4\eta_2\theta_p + (b_s + \theta_s)^2\right) + \theta_p \left(3\eta_2\theta_p - (b_s + \theta_s)(2b_s + \theta_s)\right)\right)\left(2b_p^2 c\eta_2 + b_s^2\alpha_1 - \eta_2(2\alpha_1 + \alpha_2)\theta_p + b_s(2\alpha_1 + \alpha_2)\theta_s + (\alpha_1 + \alpha_2)\theta_s^2 + b_p\left(-2\eta_2\alpha_1 - c\left(-3\eta_2\theta_p + (b_s + \theta_s)(b_s + 2\theta_s)\right)\right)\right))$

$A_{34} = 32b_p^4\eta_1\eta_2^2 - 4b_p^3\eta_2\left(b_s^2(8\eta_1 + \eta_2) - 32\eta_1\eta_2\theta_p + 2b_s(8\eta_1 + \eta_2)\theta_s + (8\eta_1 + \eta_2)\theta_s^2\right) + 4b_p^2\left(b_s^4(2\eta_1 + \eta_2) + 42\eta_1\eta_2^2\theta_p^2 + 4b_s^3(2\eta_1 + \eta_2)\theta_s - b_s\eta_2(42\eta_1 + 5\eta_2)\theta_p\theta_s - 2\eta_2(9\eta_1 + \eta_2)\theta_p\theta_s^2 + 2b_s(4\eta_1 + \eta_2)\theta_s^3 + 2\eta_1\theta_s^4 + b_s^2\left(-3\eta_2(8\eta_1 + \eta_2)\theta_p + (12\eta_1 + 5\eta_2)\theta_s^2\right)\right) + \theta_p\left(-b_s^6 - 6b_s^5\theta_s + b_s^4\left(8\eta_1\theta_p + 4\eta_2\theta_p - 13\theta_s^2\right) + 2b_s^3\theta_s\left(10\eta_1\theta_p + 7\eta_2\theta_p - 6\theta_s^2\right) + 4b_s\theta_p\theta_s\left(-\eta_2(6\eta_1 + \eta_2)\theta_p + (\eta_1 + \eta_2)\theta_s^2\right) + \eta_2\theta_p^2\left(12\eta_1\eta_2\theta_p - (4\eta_1 + \eta_2)\theta_s^2\right) + 2b_s^2\left(-2\eta_2(5\eta_1 + \eta_2)\theta_p^2 + (8\eta_1 + 7\eta_2)\theta_p\theta_s^2 - 2\theta_s^4\right)\right) + b_p(-b_s^6 - 6b_s^5\theta_s + b_s^4\left(8(2\eta_1 + \eta_2)\theta_p - 13\theta_s^2\right) + 2b_s^3\theta_s\left(26\eta_1\theta_p + 15\eta_2\theta_p - 6\theta_s^2\right) + 4b_s\theta_p\theta_s\left(-2\eta_2(15\eta_1 + 2\eta_2)\theta_p + (7\eta_1 + 3\eta_2)\theta_s^2\right) + 2b_s^2\left(-6\eta_2(7\eta_1 + \eta_2)\theta_p^2 + (30\eta_1 + 17\eta_2)\theta_p\theta_s^2 - 2\theta_s^4\right) + \theta_p\left(80\eta_1\eta_2^2\theta_p^2 - \eta_2(36\eta_1 + 5\eta_2)\theta_p\theta_s^2 + 4\eta_1\theta_s^4\right))$

$A_{35} = (b_p + \theta_p)(-b_s^3 + 2b_s\eta_2(b_p + \theta_p) - 3b_s^2\theta_s + \eta_2(2b_p + \theta_p)\theta_s - 2b_s\theta_s^2)(2b_p^2 c\eta_2 + b_s^2\alpha_1 - \eta_2(2\alpha_1 + \alpha_2)\theta_p + b_s(2\alpha_1 + \alpha_2)\theta_s + (\alpha_1 + \alpha_2)\theta_s^2 + b_p\left(-2\eta_2\alpha_1 - c\left(-3\eta_2\theta_p + (b_s + \theta_s)(b_s + 2\theta_s)\right)\right))$

$A_{36} = \frac{4}{(A_{24})^2\theta_p^2}\{A_{24}(-\theta_p)\left(\eta_1\left(-2b_p^2\eta_2 + \theta_p\left(-\eta_2\theta_p + b_s(b_s + \theta_s)\right) + b_p\left(-4\eta_2\theta_p + (b_s + \theta_s)^2\right)\right)\right) + \frac{(A_{24})^2\theta_p^2}{4}\left(\left(b_p + \theta_p\right)\left(b_s^3 + 3b_s^2\theta_s - \eta_2\left(2b_p + \theta_p\right)\theta_s + 2b_s\left(-\eta_2\left(b_p + \theta_p\right) + \theta_s^2\right)\right)^2\right)$

Appendix C. Parameters for the Stackelberg game

$B = 1/(-2\eta_2(b_p + \theta_p) + (b_s + \theta_s)^2)$, $B_1 = b_s^2 c - \eta_2\left(\alpha_2 + c(b_p + \theta_p)\right) + 2b_s c\theta_s + c\theta_s^2$, $B_2 = (-\alpha_2 + c(b_p + \theta_p))(b_s + \theta_s)$

$B_3 = \eta_1(-2\eta_2(b_p + \theta_p) + (b_s + \theta_s)^2)(b_s^3 + 3b_s^2\theta_s - \eta_2(2b_p + \theta_p)\theta_s + 2b_s(-\eta_2(b_p + \theta_p) + \theta_s^2))(-2b_p^2 c\eta_2 - b_s^2\alpha_1 + \eta_2(2\alpha_1 + \alpha_2)\theta_p - b_s(2\alpha_1 + \alpha_2)\theta_s - (\alpha_1 + \alpha_2)\theta_s^2 + b_p\left(2\eta_2\alpha_1 + c\left(-3\eta_2\theta_p + (b_s + \theta_s)(b_s + 2\theta_s)\right)\right))$

$B_4 = (-b_s^3 + 2b_s\eta_2(b_p + \theta_p) - 3b_s^2\theta_s + \eta_2(2b_p + \theta_p)\theta_s - 2b_s\theta_s^2)(\left(b_s^3 + 3b_s^2\theta_s - \eta_2(2b_p + \theta_p)\theta_s + 2b_s(-\eta_2(b_p + \theta_p) + \theta_s^2)\right)^2 - 2\eta_1(-2\eta_2(b_p + \theta_p) + (b_s + \theta_s)^2)\left(-2b_p^2\eta_2 + \theta_p\left(-\eta_2\theta_p + b_s(b_s + \theta_s)\right) + b_p(-4\eta_2\theta_p + (b_s + \theta_s)^2)\right))$

$B_5 = (b_s^3 + 3b_s^2\theta_s - \eta_2(2b_p + \theta_p)\theta_s + 2b_s(-\eta_2(b_p + \theta_p) + \theta_s^2))(-2b_p^2 c\eta_2 - b_s^2\alpha_1 + \eta_2(2\alpha_1 + \alpha_2)\theta_p - b_s(2\alpha_1 + \alpha_2)\theta_s - (\alpha_1 + \alpha_2)\theta_s^2 + b_p\left(2\eta_2\alpha_1 + c\left(-3\eta_2\theta_p + (b_s + \theta_s)(b_s + 2\theta_s)\right)\right))$

$B_6 = \left(b_s^3 + 3b_s^2\theta_s - \eta_2(2b_p + \theta_p)\theta_s + 2b_s(-\eta_2(b_p + \theta_p) + \theta_s^2)\right)^2 - 2\eta_1(-2\eta_2(b_p + \theta_p) + (b_s + \theta_s)^2)\left(-2b_p^2\eta_2 + \theta_p\left(-\eta_2\theta_p + b_s(b_s + \theta_s)\right) + b_p(-4\eta_2\theta_p + (b_s + \theta_s)^2)\right)$

$B_7 = \eta_1(-8\eta_2(b_p + \theta_p) + (b_s + \theta_s)^2)(b_s^3 + 3b_s^2\theta_s - \eta_2(8b_p + \theta_p)\theta_s + 2b_s(-4\eta_2(b_p + \theta_p) + \theta_s^2))(-8b_p^2 c\eta_2 - b_s^2\alpha_1 + \eta_2(8\alpha_1 + 7\alpha_2)\theta_p - b_s(2\alpha_1 + \alpha_2)\theta_s - (\alpha_1 + \alpha_2)\theta_s^2 + b_p\left(8\eta_2\alpha_1 + c\left(-15\eta_2\theta_p + (b_s + \theta_s)(b_s + 2\theta_s)\right)\right))$

$B_8 = (-b_s^3 + 8b_s\eta_2(b_p + \theta_p) - 3b_s^2\theta_s + \eta_2(8b_p + \theta_p)\theta_s - 2b_s\theta_s^2)(\left(b_s^3 + 3b_s^2\theta_s - \eta_2(8b_p + \theta_p)\theta_s + 2b_s(-4\eta_2(b_p + \theta_p) + \theta_s^2)\right)^2 - 2\eta_1(-8\eta_2(b_p + \theta_p) + (b_s + \theta_s)^2)\left(-8b_p^2\eta_2 + \theta_p\left(-\eta_2\theta_p + b_s(b_s + \theta_s)\right) + b_p(-16\eta_2\theta_p + (b_s + \theta_s)^2)\right))$

$B_9 = (b_s^3 + 3b_s^2\theta_s - \eta_2(8b_p + \theta_p)\theta_s + 2b_s(-4\eta_2(b_p + \theta_p) + \theta_s^2))(-8b_p^2 c\eta_2 - b_s^2\alpha_1 + \eta_2(8\alpha_1 + 7\alpha_2)\theta_p - b_s(2\alpha_1 + \alpha_2)\theta_s - (\alpha_1 + \alpha_2)\theta_s^2 + b_p\left(8\eta_2\alpha_1 + c\left(-15\eta_2\theta_p + (b_s + \theta_s)(b_s + 2\theta_s)\right)\right))$

$B_{10} = \left(b_s^3 + 3b_s^2\theta_s - \eta_2(8b_p + \theta_p)\theta_s + 2b_s(-4\eta_2(b_p + \theta_p) + \theta_s^2)\right)^2 - 2\eta_1(-8\eta_2(b_p + \theta_p) + (b_s + \theta_s)^2)\left(-8b_p^2\eta_2 + \theta_p\left(-\eta_2\theta_p + b_s(b_s + \theta_s)\right) + b_p(-16\eta_2\theta_p + (b_s + \theta_s)^2)\right)$

$B_{11} = 1/\{4b_p^2\eta_2 + 2\theta_p(\eta_2\theta_p - b_s(b_s + \theta_s)) - 2b_p(-4\eta_2\theta_p + (b_s + \theta_s)^2)\}$

$B_{12} = -b_s^3 + 2b_s\eta_2(b_p + \theta_p) - 3b_s^2\theta_s + \eta_2\theta_s(2b_p + \theta_p) - 2b_s\theta_s^2$

$B_{13} = 2b_p^2\eta_2 c_R - b_s^2(\alpha_1 + (c_D + c_M + 2c_R)\theta_p) + (\eta_2\theta_p - b_s\theta_s)(2\alpha_1 + \alpha_2 + (c_D + c_M + 2c_R)\theta_p) - (\alpha_1 + \alpha_2)\theta_s^2 + b_p(-b_s^2 c_R + 2\eta_2\alpha_1 + (c_D + c_M + 5c_R)\eta_2\theta_p + b_s\theta_s(c_D + c_M - c_R) + (c_D + c_M)\theta_s^2)$

$B_{14} = 2b_p^2\eta_2(c_D - c_R) - b_s^2(\alpha_1 + (2c_D + c_M)\theta_p) + (\eta_2\theta_p - b_s\theta_s)(2\alpha_1 + \alpha_2 + (2c_D + c_M)\theta_p) - (\alpha_1 + \alpha_2)\theta_s^2 + b_p(b_s^2(c_R - c_D) + 2\eta_2\alpha_1 + (5c_D + c_M - 3c_R)\eta_2\theta_p + b_s\theta_s(-c_D + c_M + 3c_R) + (2c_R + c_M)\theta_s^2)$

$B_{15} = 4\eta_1(-2\eta_2(b_p+\theta_p)+(b_s+\theta_s)^2)(-2b_p^2c\eta_2 - b_s^2\alpha_1 + \eta_2(2\alpha_1+\alpha_2)\theta_p - b_s(2\alpha_1+\alpha_2)\theta_s - (\alpha_1+\alpha_2)\theta_s^2 + b_p\left(2\eta_2\alpha_1 + c\left(-3\eta_2\theta_p + (b_s+\theta_s)(b_s+2\theta_s)\right)\right))$

$B_{16} = \left(b_s^3 + 3b_s^2\theta_s - \eta_2(2b_p+\theta_p)\theta_s + 2b_s(-\eta_2(b_p+\theta_p)+\theta_s^2)\right)^2 - 8\eta_1(-2\eta_2(b_p+\theta_p)+(b_s+\theta_s)^2)(-2b_p^2\eta_2 + \theta_p\left(-\eta_2\theta_p + b_s(b_s+\theta_s)\right) + b_p(-4\eta_2\theta_p + (b_s+\theta_s)^2))$

$B_{17} = \left(b_s^3 + 3b_s^2\theta_s - \eta_2(2b_p+\theta_p)\theta_s + 2b_s(-\eta_2(b_p+\theta_p)+\theta_s^2)\right)^2 (-2b_p^2c\eta_2 - b_s^2\alpha_1 + \eta_2(2\alpha_1+\alpha_2)\theta_p - b_s(2\alpha_1 + \alpha_2)\theta_s - (\alpha_1+\alpha_2)\theta_s^2 + b_p\left(2\eta_2\alpha_1 + c\left(-3\eta_2\theta_p + (b_s+\theta_s)(b_s+2\theta_s)\right)\right))$

$B_{18} = 1/(2(8b_p^2\eta_2 + \theta_p(\eta_2\theta_p - b_s(b_s+\theta_s)) - b_p(-16\eta_2\theta_p + (b_s+\theta_s)^2)))$

$B_{19} = -b_s^3 + 8b_s\eta_2(b_p+\theta_p) - 3b_s^2\theta_s + \eta_2\theta_s(8b_p+\theta_p) - 2b_s\theta_s^2$

$B_{20} = 8b_p^2\eta_2c_R - b_s^2(\alpha_1 + (c_D+c_M+2c_R)\theta_p) + \eta_2\theta_p(8\alpha_1 + 7\alpha_2 + (c_D+c_M+2c_R)\theta_p) - b_s\theta_s(2\alpha_1+\alpha_2+(c_D+c_M+2c_R)\theta_p) - (\alpha_1+\alpha_2)\theta_s^2 + b_p(-b_s^2c_R + 8\eta_2\alpha_1 + (c_D+c_M+17c_R)\eta_2\theta_p + b_s\theta_s(c_D+c_M-c_R) + (c_D+c_M)\theta_s^2)$

$B_{21} = 8b_p^2\eta_2(c_D-c_R) - b_s^2(\alpha_1 + (2c_D+c_M)\theta_p) + \eta_2\theta_p(8\alpha_1+7\alpha_2+(2c_D+c_M)\theta_p) - b_s\theta_s(2\alpha_1+\alpha_2+(2c_D+c_M)\theta_p) - (\alpha_1+\alpha_2)\theta_s^2 + b_p(b_s^2(c_R-c_D) + 8\eta_2\alpha_1 + (17c_D+c_M-15c_R)\eta_2\theta_p + b_s\theta_s(-c_D+c_M+3c_R) + (2c_R+c_M)\theta_s^2)$

$B_{22} = 4\eta_1(-8\eta_2(b_p+\theta_p)+(b_s+\theta_s)^2)(-8b_p^2c\eta_2 - b_s^2\alpha_1 + \eta_2(8\alpha_1+7\alpha_2)\theta_p - b_s(2\alpha_1+\alpha_2)\theta_s - (\alpha_1+\alpha_2)\theta_s^2 + b_p\left(8\eta_2\alpha_1 + c\left(-15\eta_2\theta_p + (b_s+\theta_s)(b_s+2\theta_s)\right)\right))$

$B_{23} = \left(b_s^2 - 8\eta_2(b_p+\theta_p)\right)\left(b_s^4 - 8b_s^2(\eta_1+\eta_2)(b_p+\theta_p) + 8\eta_1\eta_2(8b_p^2 + 16b_p\theta_p + \theta_p^2)\right) + 2b_s(3b_s^4 - b_s^2(16b_p(\eta_1 + 2\eta_2) + (12\eta_1+25\eta_2)\theta_p) + 8\eta_2(8b_p^2(2\eta_1+\eta_2) + b_p(28\eta_1+9\eta_2)\theta_p) + (5\eta_1+\eta_2)\theta_p^2))\theta_s + (13b_s^4 - 2b_s^2(24b_p\eta_1 + 40b_p\eta_2 + 12\eta_1\theta_p + 19\eta_2\theta_p) + \eta_2(64b_p^2(2\eta_1+\eta_2) + 16b_p(12\eta_1+\eta_2)\theta_p + (8\eta_1+\eta_2)\theta_p^2))\theta_s^2 - 4b_s(-3b_s^2 + 8b_p(\eta_1 + \eta_2) + (2\eta_1+\eta_2)\theta_p)\theta_s^3 + 4(b_s^2 - 2b_p\eta_1)\theta_s^4$

$B_{24} = \left(b_s^3 + 3b_s^2\theta_s - \eta_2(8b_p+\theta_p)\theta_s + 2b_s(-4\eta_2(b_p+\theta_p)+\theta_s^2)\right)^2 - 8\eta_1(-8\eta_2(b_p+\theta_p)+(b_s+\theta_s)^2)(-8b_p^2\eta_2 + \theta_p\left(-\eta_2\theta_p + b_s(b_s+\theta_s)\right) + b_p(-16\eta_2\theta_p + (b_s+\theta_s)^2))$