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The optimal warehouse capacity: A queuing-based fuzzy programming approach

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Abstract

Among the various existing forms of warehousing management, the simultaneous use of private and public warehouses is a most well-known one. The purpose of this article is to develop a queuing theory-based model for determining the optimal capacity of private warehouse in order to minimize the total corresponding costs. In the proposed model, the available space and budget to create a private warehouse are assumed to be limited. Due to the vagueness, some parameters are simulated by expert-based triangular fuzzy numbers. Also two well-known methods are applied to solve the queuing-based fuzzy programming model and to optimize the private warehouse capacity. The numerical results confirm that our proposed method match well with various lines of manufacturing environments and conditions.

Keywords: Optimal warehouse capacity, Queuing theory, Fuzzy programming, Multi-objective programming.

1 - Introduction

Warehouses are one of the key components of a supply chain body which play an important role in handling the corresponding supplies and demands. The suitable application of warehouses may lead to satisfactory service levels with no interruption in the production and distribution flows. In fact, when random fluctuations and seasonal patterns occurs in supplies and demands, one can decrease the frequency and volume of stock out and increase the customer service level via effective management of inventories in warehouses. Warehouses also help to benefit from the purchase discount in

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recessions condition. Generally, the managers have two options for warehousing: (1) construction (private warehouse) and (2) renting for a limited time interval (public warehouse) (Gill, 2009) (Gu, Goetschalck and McGinnis, 2010).

Public warehouses do not need a lot of investment on the assets, properties, machineries and equipments as well as the hiring and training of employees. On the other hand, although a private warehouse usually requires all the above-mentioned investments, its related carrying costs are much lower than a public warehouse. Moreover, in the case of private warehouse, there is a tax saving due to the consideration of depreciation of buildings and equipments.

So, to make more effective decisions on warehousing problems, we need a model which can establish a compromise between the advantages and disadvantages of both types of warehouses. The products in a warehouse can be regarded as excess inventories waiting for the arrival of market demands. So, it seems that the use of queuing models to formulate the products in warehouses might lead to the promising results. Furthermore, one can present various simple and/or comprehensive queue models for the different conditions of a warehouse system. In this article, using a new queuing approach for the inventory and warehouse problem and analysis of construction costs for different capacities, a novel model is proposed for optimizing the private warehouse capacity.

The paper is organized as follows. In section 2, we discuss the supportive body of relevant literature. In section 3, a queuing model is presented and the solution algorithm is developed for various types of the model. In section 4, the solution method is proposed. Three numerical examples are solved and analyzed in section 5. Finally, section 6 is devoted to the conclusion and suggestions of future studies.

2 - Literature review

Several researches have been done in order to determine the optimal capacity of warehouses. For example, Ashayeri and Gelders (1985), discussed the topic in detail and suggested that for determining the optimal warehousing system, it is desirable to combine the analytic and simulation methods. Cormier and Gunn (1992), provided a comprehensive literature review on the warehousing and related optimization models. The proposed models for determining the optimal capacity of warehouses were basically aiming at optimizing the total costs as well as the customer service levels during the planning time horizon. In such models, the location of warehouse is assumed to be pre-specified but the company must make a decision for the optimal capacity in the case of a completely seasonal demand. If the capacity of considered private warehouse is not enough, the company can rent a public warehouse to supply the additional required capacity.

The dynamic version of optimal warehouse capacity (OWC) models as an extension of the plant capacity models was considered in Manne and Veinott (1967). Luss (1982), presented a review on the capacity expansion models of warehouses. Ballou (1985) developed a method to simultaneously use of the public and private warehouses with maximal savings. Hung and Fisk (1984), formulated a novel model for the OWC problem under both the static and dynamic demands. In the static case, the capacity may not be expanded in the future but in the dynamic one, it is expansible if the required personnel and equipments are available. By reviewing the OWC models, Cormier and Gunn (1996), proposed a model for using both private and public warehouses under the fixed demands. They concluded that when the private warehouse capacity is very limited, the public warehouse is useful.

White and Francis (1971), studied the OWC problem with both the deterministic and probabilistic demand conditions and proposed a solution method based on the linear programming, duality theory and network flow problem. They considered the warehouse construction costs, transportation costs and storage costs in the public warehouse. Rao and Rao (1998), developed an OWC model under the seasonal demand considering the time varying costs, economics of scale in the capital expenditure, and concave costs. By presenting a structure for the optimal solution, they claimed that the static OWC problem and its extensions can simply be solved with no need for the linear programming procedures.

Furthermore, they solved the dynamic OWC problem with concave costs using the network flow and dynamic programming methods. Goh et al (2001) presented a model to minimize the total cost of ordering, inventory holding and warehousing. Petinis et al (2005) solved a multi-product warehouse capacity problem in order to minimize the ordering and inventory costs using the quadratic programming. Gill (2009), presented a linear programming model for determining the amount of products in the public and private warehouses.

Note worthily, there have been few researches which address the OWC problem so that Ashayeri and Gelders (1985), Rowley (2000), and Rouwenhorst et al. (2000) confirmed the lack of sufficient researches in this area. Baker and Canessa (2009) reviewed the studies from 1973 and concluded that warehouses and their optimal capacity have not been studied sufficiently and the lack of scientific researches in this field is recognizable.

There have been many works applied the queuing models for the production environments; but, most of them address the inventory control, determination of the optimal inventory, production line balancing and operation sequencing. For example, Ke and Lin (2006) and Kumar (2012) studied the queuing system with unreliable server whose arrival rate, service rate of customers, breakdown rate and repair rate of server were considered to be fuzzy. Barak and Fallahnezhad (2012) studied two fuzzy queuing models of M/M/1 and M/E2/1 under the assumption that the arrival and service rates as well as the system costs were fuzzy parameters. They compared two practical systems to study the different conditions of operator's allocation in the queuing systems.

To the best of our knowledge, the queuing theory has not been used to model the OWC problem till now. Accordingly, this article aims at presenting a fuzzy queuing model for the OWC problem to determine the optimal private warehouse capacity. At first, a queuing model is proposed for the warehouse and its related components. Then, the total cost function for the different capacities of private and public warehouses is calculated. Due to the involved uncertainty, some of the cost parameters are assumed to be triangular fuzzy numbers; therefore, the total cost function will be characterized by the fuzzy mathematics. Afterward, using the fuzzy programming approach, the minimum of total fuzzy cost function is calculated to determine the corresponding optimal capacities. Finally, three numerical examples of warehouses in various production environments are solved and analyzed.
For warehouse capacity and management see Ross (2015) and Friemann, Rippel, and Schönsleben (2014).

3 - Proposed model

In this section, the queuing model for the OWC problem is presented. After completing the production process, final products get out the production site and enter into the warehouse. The customers' demands are then supplied from the warehouse inventory. To propose a queue model for the final warehouse, it is necessary to adapt the warehouse system with a queuing system. So we have:

- Queuing system is the warehouse system.
- Queuing system's customer are the final products.
- Customers' arrival rate to queue is the product departure rate from production site, denoted as λ.
- Service rate of queuing system to the customers is the products' demand rate denoted as μ.

3–1- Assumptions and notations

- The manufacturer produces only a single product.
- If the inventory of final product is less than the private warehouse capacity, all the products will be stored in the private warehouse with the storage cost of h_1 units per cycle.
- If the inventory of final product is more than the private warehouse capacity, the excess products should be stored in one or more external (public) warehouses with the storage cost more than h_1.

S_{max}: Maximum space allocated for constructing the private warehouse.
B_{max}: Maximum budget allocated for constructing the private warehouse.

a: Required space to store one unit of the product.
b: Per unit capital investment for constructing the private warehouse.
k: Capacity of the private warehouse.
h_1: Storage cost in private warehouse per unit per cycle time.
h_2: Storage cost in external (public) warehouse per unit per cycle time.
n: Number of customers (final product) in the queuing system (warehouse).
π_n: Probability of existing n customers (final product) in the queuing system (warehouse).
λ: Customers' arrival rate at queuing system (output rate of final product into warehouse system).
μ: Service rate of queuing system to customers (demand rate of final product from warehouse system).

Based on the concepts of finite capacity queuing models, products enter in the private and public warehouses by the rate of $\lambda(1-\pi_k)$ and $\lambda\pi_k$, respectively. As denoted before, π_k is the probability of existing k units in the private warehouse. Fig. 1 shows the above descriptions schematically. The aim is to optimize the private warehouse capacity (k^*).

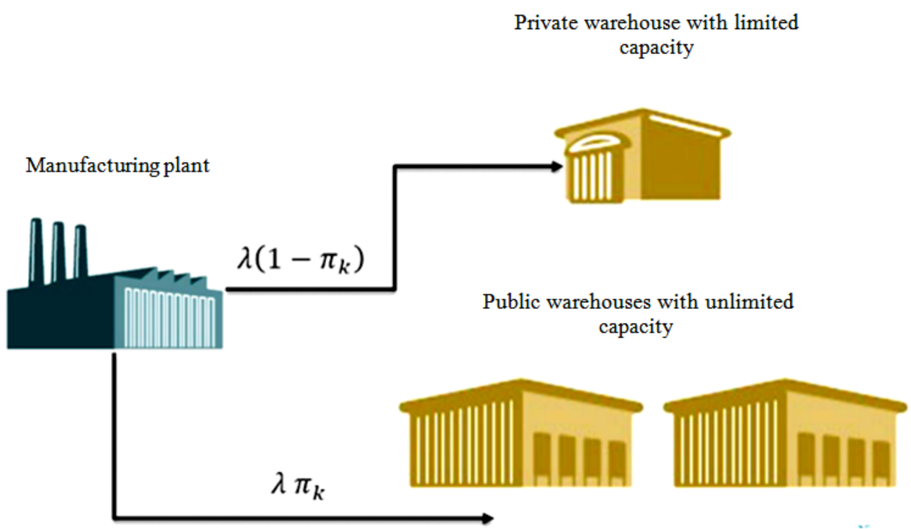


Fig. 1. Arrival rate of final products to public and private warehouses.

3-2- Mathematical model

Objective function (1) aims at minimizing the total costs including two types of cost due to the violation of private warehouse capacity from its optimal value. In fact, the total costs of constructing a warehouse with capacity of k are equal to sum of those two types of cost.

$$\mathrm{MIN}\,C_T = \left[\frac{i.(1+i)^N}{(1+i)^N-1}\right].\sum_{n=0}^{k}(k-n).\pi_n.b + \sum_{n=k+1}^{\infty}(n-k).\pi_n.(h_2-h_1) \quad (1)$$

s.t.

$$a.k \le S_{max} \quad (2)$$

$$b.k \le B_{max} \quad (3)$$

Constraints (2) and (3) ensure no violation of the space and budget limitations.

The first term in the objective function (1) is the cost of unused capacity when constructing a private warehouse with an excess capacity compared to the optimal size (i.e., $k > k^*$).The cost is referred to as *cost of excess capacity* (C_{EC}) as shown in the following equation:

$$C_{EC} = \sum_{n=0}^{k}(k-n).\pi_n.b \quad (4)$$

The second term in the objective function (1) is the cost of capacity lacks when constructing a private warehouse with an inadequate compared to the optimal size (i.e., $k^* > k$). In fact, if a warehouse with less than optimal capacity is constructed, the company will be forced to turn to the public warehouses and pay more money (i.e., h_2-h_1) per unit of final products not stored in the private warehouse. The cost is referred to as the c*ost of the lack of capacity* (C_{LC}) as shown in the following equation:

$$C_{LC} = \sum_{n=k+1}^{\infty}(n-k).\pi_n.(h_2-h_1) \quad (5)$$

The two types of cost behave in an opposite manner; by increasing the private warehouse capacity, C_{EC} is increased while C_{LC} is decreased and vice versa. C_{EC} is imposed once at the beginning of planning horizon if the company constructs an excess warehouse capacity. On the other hands, C_{LC}, if exists, is paid in each cycle during the planning horizon. To be able to combine the two costs, C_{EC} should be prorated over the cycles of planning horizon considering the time value of money. Therefore, it is multiplied by the *capital recovery factor* (*CRF*) denoted as follows:

$$CRF=(\frac{A}{P},i,N)=[\frac{i.(1+i)^N}{(1+i)^N-1}] \quad (6)$$

In which, i and N are the interest rate and the number of cycles during the planning horizon, respectively. Notably, objective function (1) is the sum of C_{LC} and C_{EC} multiplied by *CRF*.

3-3- Fuzzy model

As known, in the long run, parameters i, h_1 and h_2 are vague and imprecise; therefore, their exact estimates cannot be provided. Accordingly, those parameters are introduced into the model through the triangular fuzzy estimates, based on the experts' opinion and experiences. Note worthily, the subscripts *L*, *M* and *R* indicate the pessimistic, the possible and the optimistic prominent values of the corresponding fuzzy triangular estimate, respectively.

$\tilde{h}_1 = (h_1^L, h_1^M, h_1^R)$
$\tilde{h}_2 = (h_2^L, h_2^M, h_2^R)$
$\tilde{\imath}=(i^L, i^M, i^R)$

Therefore, the fuzzy objective function (1) is as follows:

$$\text{MIN}\tilde{C}_T = \left[\frac{\tilde{\imath}.(1+\tilde{\imath})^N}{(1+\tilde{\imath})^N-1}\right].\sum_{n=0}^{k}(k-n).\pi_n.b + \sum_{n=k+1}^{\infty}(n-k).\pi_n.(\tilde{h}_2-\tilde{h}_1) \quad (7)$$

Notably, the proposed model is very general; it may be used to determine the optimal private warehouse capacity in different companies with different conditions of production and distribution systems. At first, an appropriate queuing system must be established in accordance with the specific conditions (e.g., rate and function of demand and production, single or group based ordering and delivery of products) of each company. Afterwards, π_k should be calculated for the queuing system using the Markov chains and equilibrium equations. Finally, the optimal warehouse capacity is determined by minimizing the total cost function ($\tilde{C}_T$) for various warehouse capacities (k).

4. Proposed solution method

We apply Lai and Hwang (1992) to convert objective function (7) to three crisp equivalent objective functions. To do so, consider the following model with its fuzzy objective function:

$$\begin{aligned} &\text{Max } \tilde{Z} = \tilde{C}X \\ &\text{s.t. } AX \le b; \\ &X \ge 0 \end{aligned} \quad (8)$$

If coefficients of the above objective function are triangular fuzzy estimates such as $\tilde{C}=(C^L, C^M, C^R)$, we have:

$$\begin{aligned}&\text{Max } \tilde{Z} = \textstyle\sum_{j=1}^{n} (c_j^L, c_j^M, c_j^R).x_j \\ &\text{s.t.} \textstyle\sum_{j=1}^{n} a_{ij} \,.\, x_j \leq b i = 1,2,\ldots,m; \\ &x_j \geq 0\end{aligned} \tag{9}$$

Therefore, $\tilde{Z}$ would be a triangular fuzzy variable such as $\tilde{Z} = (C^L, C^M, C^R)X$. According to the method of Li and Huang [20], in order to maximize objective function (9), one must simultaneously maximize $(C^M)X$, minimize $[(C^M - C^L)X]$ and maximize $[(C^R - C^M)X]$. So, mathematical model (9) is transformed to the following equivalent three-objective model:

$$\begin{aligned}&\text{Min } (Z_1) = (C^M - C^L)X \\ &\text{Max } (Z_2) = C^M X \\ &\text{Max } (Z_3) = (C^R - C^M)X \\ &\text{s.t. } AX \leq b; \quad X \geq 0\end{aligned} \tag{10}$$

To obtain the efficient solution, we employ the aggregate function proposed by Torabi and Hassini (2008) which establish a reasonable trade-off between the maximization of minimum achievement degrees of three objectives and the maximization of a weighted sum of the achievement degrees of three objectives. To determine a fuzzy achievement function for objective function Z_i, at first, the corresponding best and worst objective function value is obtained by optimizing the following models:

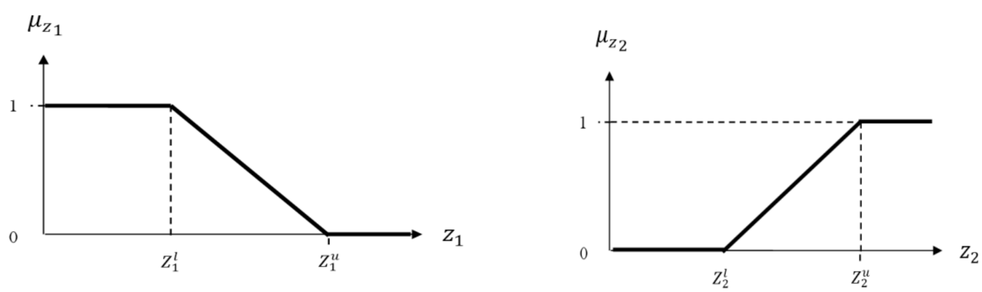
$$\begin{aligned}&Z_i^l = \text{Min } Z_i \\ &\text{s.t. } AX \leq b; \quad X \geq 0\end{aligned} \tag{11}$$

$$\begin{aligned}&Z_i^u = \text{Max } Z_i \\ &\text{s.t. } AX \leq b; \quad X \geq 0\end{aligned} \tag{12}$$

Afterward, the linear achievement degree for minimizing Z_1 is calculated as equation (13) and shown in Fig. 2(a). Also, the linear achievement degree for maximizing Z_i (i=2,3) is calculated as equation (14) and shown in Fig. 2(b).

$$\mu_{Z_1}(Z_1) = \begin{cases} 1 & Z_1 \leq Z_1^l \\ \frac{Z_1^u - Z_1}{Z_1^u - Z_1^l}; & Z_1^l \leq Z_1 \leq Z_1^u \\ 0; & Z_1 \geq Z_1^u \end{cases} \tag{13}$$

$$\mu_{Z_i}(Z_i) = \begin{cases} 0 & Z_i \leq Z_i^l \\ \frac{Z_i - Z_i^l}{Z_i^u - Z_i^l}; & Z_i^l \leq Z_i \leq Z_i^u \,;\, i=2,3 \\ 1; & Z_i \geq Z_i^u \end{cases} \tag{14}$$



2(a). Z_1 2(b). Z_2 (Z_3)

Fig. 2. Linear achievement functions.

Accordingly, the model (10) might be converted to the following single objective problem using the following aggregate function.

$$\text{Max} \quad Z = \gamma\mu + (1-\gamma)\sum_i \theta_i \mu_{z_i}(z_i)$$
$$\text{s.t.} \quad \mu \le \mu_{z_i}(z_i); \quad i = 1,2,3 \tag{15}$$
$$\gamma \text{ and } \theta_i \in [0,1],$$

In which, $\mu_{z_i}(z_i)$ and $\mu = \min_i\{\mu_{z_i}(z_i)\}$ denote the achievement degree of Z_i and the minimum achievement degrees of the objective functions, respectively. Moreover, γ and θ_i indicate the coefficient of compensation and the relative importance of Z_i, respectively. Parameter θ_i is determined by the decision maker based on her/his preferences such that $\sum_i \theta_i = 1$, $\theta_i > 0$. The above aggregate function thus provides a convex combination of the minimum and weighted sum of $\mu_{z_i}(z_i)$ values to ensure yielding an adjustably balanced compromise solution. A higher value for γ means that a more attention is paid to obtain a higher minimum for the achievement degrees of objectives (μ) and accordingly more balanced compromise solutions. On the contrary, a lower value for γ means that a more attention is paid to obtain a solution with high achievement degree for some objectives with higher relative importance yielding unbalanced compromise solutions.

For applying the above method, we present the minimization of the objective function (7) as a maximization objective function (16).

$$\text{Max } (-\tilde{Z}) = -\left[\frac{\tilde{\imath}.(1+\tilde{\imath})^N}{(1+\tilde{\imath})^N-1}\right].b.\sum_{n=0}^{k}(k-n).\pi_n - (\tilde{h}_2-\tilde{h}_1).\sum_{n=k+1}^{\infty}(n-k).\pi_n \tag{16}$$

Then, the following three-objective model is provided as the equivalent crisp version of our fuzzy model:

$$\text{Min } (-Z_1) = -\left[\frac{(i^M-i^L).(1+(i^M-i^L))^N}{(1+(i^M-i^L))^N-1}\right].b.\sum_{n=0}^{k}(k-n).\pi_n - ((h_2^M-h_2^L)-(h_1^M-h_1^L)).\sum_{n=k+1}^{\infty}(n-k).\pi_n$$

$$\text{Max } (-Z_2) = -\left[\frac{i^M.(1+i^M)^N}{(1+i^M)^N-1}\right].b.\sum_{n=0}^{k}(k-n).\pi_n - (h_2^M-h_1^M).\sum_{n=k+1}^{\infty}(n-k).\pi_n$$

$$\text{Max } (-Z_3) = -\left[\frac{(i^R-i^M).(1+(i^R-i^M))^N}{(1+(i^R-i^M))^N-1}\right].b.\sum_{n=0}^{k}(k-n).\pi_n - ((h_2^R-h_2^M)-(h_1^R-h_1^M)).\sum_{n=k+1}^{\infty}(n-k).\pi_n$$

s.t.

$$a.k \le S_{max}$$
$$b.k \le B_{max}$$

After determining $Z_i^l; (i = 1,2,3)$ and $Z_i^u; (\text{i} = 1,2,3)$ according to equations (11) and (12); also, calculating $\mu_{Z_i}(Z_i); (i = 1,2,3)$ based on equations (13) and (14), we present the following equivalent single-objective model of the multi-objective model (17):

$$\text{MaxZ} = \gamma\mu + (1-\gamma)\sum_i \theta_i \mu_{z_i}(z_i)$$

s.t.

$$\mu \le \frac{Z_1^u + \left[\frac{(i^M-i^L).\left(1+(i^M-i^L)\right)^N}{\left(1+(i^M-i^L)\right)^N-1}\right].b.\sum_{n=0}^{k}(k-n).\pi_n + \left((h_2^M-h_2^L)-(h_1^M-h_1^L)\right).\sum_{n=k+1}^{\infty}(n-k).\pi_n}{Z_1^u - Z_1^l}$$

$$\mu \leq \frac{-\left[\frac{i^M.\left(1+i^M\right)^N}{\left(1+i^M\right)^N-1}\right].b.\sum_{n=0}^{k}(k-n).\pi_n-\left(h_2^M-h_1^M\right).\sum_{n=k+1}^{\infty}(n-k).\pi_n-Z_2^l}{Z_2^u-Z_2^l}$$

$$\mu \leq \frac{-\left[\frac{(i^R-i^M).\left(1+(i^R-i^M)\right)^N}{\left(1+(i^R-i^M)\right)^N-1}\right].b.\sum_{n=0}^{k}(k-n).\pi_n-\left((h_2^R-h_2^M)-(h_1^R-h_1^M)\right).\sum_{n=k+1}^{\infty}(n-k).\pi_n-Z_3^l}{Z_3^u-Z_3^l} \quad (18)$$

γ and $\theta_i \in [0,1]$,

The above model was implemented in MATLAB to optimize the achievement degrees of the three objective functions. Fig. 3 shows the process of determining the optimal warehouse capacity using the proposed method.

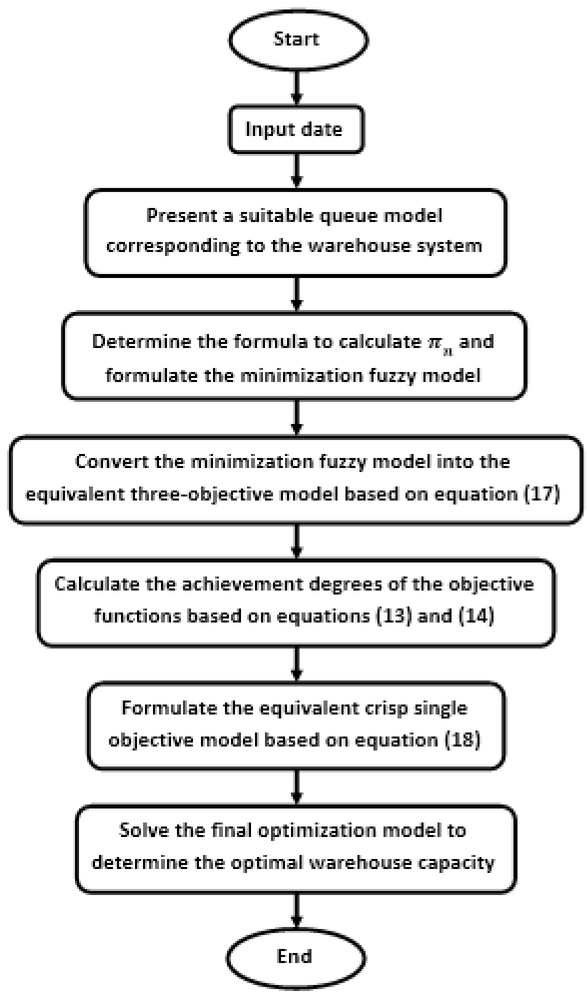


Fig. 3. The process of optimizing private warehouse capacity using the proposed method.

5- Analysis of computational results

In this section, some examples of production systems with various conditions are studied and analyzed using the proposed method. All the numerical examples were coded in MATLAB.

5-1- M/M/1 queuing model

First, consider an M/M/1 queue model. It is assumed that the average departure rate of final products to the private warehouse is λ. The time it takes to receive a single order from the customers is an exponential random variable with the rate μ. In fact, all the customers place the orders in a one-by-one manner. Consequently, π_n is calculated by the following equation:

$$\pi_n = P^n(1-P); \qquad P = \frac{\lambda}{\mu} \tag{19}$$

As a numerical instance, we assume that the departure of final products to private warehouse is a Poisson process with rate of 99 units per month. Also, the time it takes to receive a single order from the customers is supposed to be an exponential variable with rate of 0.01 per month. Other data is given in Table 1. Consequently, optimal capacity of private warehouse would be equal to 173. It is worth noting that in MATLAB code, infinite upper bound of second sigma in the numerator on the right-hind side of constraints in model (18) should be quantified so that the addition of probabilities equals almost one. We set upper bound to 1000 so that: $\sum_{n=0}^{1000} \pi_n = 0.999957$.

Table 1. Example data.

Parameters	Values
b	300
a	1.5
N	60
$\tilde{\boldsymbol{i}}$	$(1.5\%, 2\%, 4\%)$
$\tilde{\boldsymbol{h}}_{\mathbf{1}}$	(20,30,40)
$\tilde{\boldsymbol{h}}_{\mathbf{2}}$	(60,70,80)
S_{max}	400
B_{max}	75000

5-2- M/M/m queue model

This example is applied when the end consumers can satisfy their demands through m sale agents. Therefore, the sale agents are considered as servers. For simplicity, it is assumed that the average demand rates of those agents are the same; i.e., $\mu_1 = \mu_2 = \mu_3 = \cdots = \mu_m$.$\pi_n$ in such a model, as an M/M/m queue model, is calculated as follows:

$$\pi_n = \begin{cases} \left(\frac{\lambda}{\mu}\right)^n \frac{\pi_0}{n!} & ; \quad n < m \\ \left(\frac{\lambda}{\mu}\right)^n \frac{\pi_0 m^{m-n}}{m!} & ; \quad n \geq m \end{cases}$$

$$\pi_0 = \left[1 + \sum_{n=1}^{m-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \sum_{n=m}^{\infty} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{m!} * \frac{1}{m^{n-m}}\right]^{-1}$$

As a numerical experiment, the data of M/M/1 numerical example is used to except that the final products are delivered to sale agents and not to the consumers. It is assumed that the company has two sale agents and the time it takes to receive an order for products from each agent is an exponential random variable with the mean of 0.020 per month.

$\frac{1}{\mu_1} = \frac{1}{\mu_2} = 0.02; \quad \mu_1 = \mu_2 = \mu = 50$

λ=99; b= 300; P= $\frac{\lambda}{m\mu} = 0.99$; $\frac{\lambda}{\mu} = 1.98$; N=60

$\pi_0 = \frac{1-P}{1+P} = \frac{1-0.99}{1+0.99} = \frac{0.01}{1.99} = 0.005$

$$\pi_n = \begin{cases} (1.98)^n \frac{\pi_0}{n!} = (1.98)^n \frac{0.005}{n!} & n < 2 \\ (1.98)^n \frac{\pi_0 2^{2-n}}{2!} = (1.98)^n \frac{0.005 \times 2^{2-n}}{2!} & n \geq 2 \end{cases}$$

Using the developed MATLAB code, the optimal solution (i.e., the optimal capacity of private warehouse) is k^*=73.

A question which might be addressed is why the two examples with almost similar parameters have different results in the different optimal capacities for the private warehouse? This could be explained by the different input data precision. In fact, although the demand rates of final customer in both cases are equal but in the second example (i.e., sales agents), the more detailed and separated data are used instead of the aggregate data (as in case 1) with lots of error. In the other words, with more precise data, a better planning and more precise calculation of warehouse capacity is possible.

5-3- M/M$^{[r]}$/1 queue model

The main difference between the M/M/1 queuing model and the model described in this section is that the customer's demand is a constant amount such as r units, based on the concept of the economic order quantity considering ordering, transportation and shipment costs. In fact, the departure rate of final products from the private warehouse is r units. In this model, π_0 and π_n are obtained as follows:

$$\pi_n = \begin{cases} \frac{1-x_0^{n+1}}{r} & 1 \leq n < r \\ \pi_0 \frac{\lambda}{\mu} x_0^{n-r} & n \geq r \end{cases}$$

$\pi_0 = \frac{1-x_0}{r}$

In which $0 \leq x_0 \leq 1$ is a unique root of the following equation:

$\mu x^{r+1} - (\lambda + \mu)x + \lambda = 0$

The data are similar to the previous ones except that the customers each time place an order for two products (r=2); Also, the demand rate of customers is 50 times in a month. To determine π_0 and π_n, the following equation must be solved.

$\mu x^{r+1} - (\lambda + \mu)x + \lambda = 0$

$50x^3 - (149)x + 99 = 0$

$x = -1.993 \;;\; 0.993;1 \Rightarrow x_0 = 0.993$

$$\pi_n = \begin{cases} \frac{1-x_0^{n+1}}{r} = \frac{1-0.993^{n+1}}{2} & 1 \leq n < 2 \\ \pi_0 \frac{\lambda}{\mu} {x_0}^{n-r} = 0.0035 \times \frac{99}{50} \times 0.993^{n-2} & n \geq 2 \end{cases}$$

$\pi_0 = \frac{1-x_0}{r} = \frac{1-0.993}{2} = 0.0035$

The optimal capacity of private warehouse in this example is k^*=247 to minimize the warehouse-related costs. In this example, the demand rate is assumed to be 50 times per month and considering that each time 2 products are demanded, the demand rate is almost similar to the M/M/1 model (i.e., 100 units per month). However, why are the optimal warehouse capacities in the two cases different? The answer is that if there are less than r unit demands for a product, no products are delivered until the demand reaches r units. This means that we need more warehouse capacity in M/M$^{[r]}$/1 case.

6- Concluding remarks

In this paper, the well-known problem of determining the optimal warehouse capacity is addressed. At first, the departure of final products to the warehouse and the arrival of customer's orders were formulated as a queuing system. Then, considering the limitations on the available budget and space, a fuzzy programming model was developed to optimize the private warehouse capacity. The proposed fuzzy model was converted to an equivalent crisp version through two recent well-known methods in the literature. The resulted solution indicated that the proposed method, unlike the existing forecasting methods which are only suitable for some special cases, may easily be extended for the different conditions of production environments. Notably, there are some complex conditions in which the queuing models are not singly applicable. For such cases, it is suggested to use a combination of the queuing and simulation methods. The extension of the proposed model for the case of multiple product companies may be considered as a direction for the further work.

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