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A multi-stage stochastic programming for condition-based maintenance with proportional hazards model

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Abstract

Condition-Based Maintenance (CBM) optimization using Proportional Hazards Model (PHM) is a kind of maintenance optimization problem in which inspections of a system relevant to its failure rate depending on the age and value of covariates are performed in time intervals. The general approach for constructing a CBM based on PHM for a system is to minimize a long run average cost per unit of time as an objective function in which the model is considered for an infinite span of time. In this paper, a CBM model is presented based on two types of maintenance actions (minimal repair and replacement) to determine control limits to cope with the class of real-life problems in which a system would be planned for a specified planning horizon. An effective multi-stage stochastic programming approach is used to come up with the minimum expected cost given the state scenarios of the system in periods over a planning horizon. An extensive computational study is presented to demonstrate the efficiency of the proposed model through numerical instances solved by a novel hybrid meta-heuristic algorithm. A sensitivity is also performed on cost parameters to designate the effects of minimal repair cost and replacement cost in the proposed model.

Keywords: Condition Based Maintenance, proportional hazards model, multi-stage stochastic programming.

1-Introduction

Significant development in technology has led to more economical vent the destructive failures and some costs of traditional preventive maintenance.

Maintenance planning can be classified into time-based maintenance and more precise maintenance strategies in the competitive market. CBM rose as a class of maintenance planning in which the health of equipment would be continuously monitored or discretely inspected to pre (TBM) and CBM. Against the planned periodic maintenance and corrective maintenance as TBM, CBM can predictively give effective information to preventively do appropriate maintenance action prior to failure. A novel investigation between TBM and CBM has been done by Jonge et al. (2017).

Over CBM process, the collected inspection data based on the age and effect of covariates forming the condition of the system can provide efficient information about the future behavior of the system, subsequently the appropriate maintenance actions would be planned to prevent dangerous or destructive failures (Jardine et al ., 2006).

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In a sense that age of the equipment can be under control through measuring the operating condition by various parameters such as temperature, state of the oil, vibration, noise, etc. These parameters through inspections can play an important role as signs to motivate maintenance planner set some maintenance plans before occurring any serious failure.

The process of CBM constitutes of two main steps: first one is condition monitoring and the second one is maintenance decision making (Jonge et al., 2017). The core of CBM is the condition monitoring step wherein the mentioned parameters are monitored either continuously using specific types of sensors associated to the technology, the nature and the structure of an equipment or in discrete using periodic inspections (Campos, 2009), consequently after observing the parameters, an optimal decision including replacement or repair would be made to reduce unnecessary maintenance leading to cost reduction and to prevent the destructive failures leading to a better safety and more availability of the equipment.

Discrepant models for CBM have been proposed (Kasier and Gebraeel, 2009; Tian et al., 2009; Koc and Lee, 2001; Fu et al., 2004; Yan et al., 2004; Bansal and Evans, 2004; Ahmad and Kamarruddin, 2012) and many survey studies have been done on CBM (Sharma and Yadava, 2011; Shin and Jun, 2015; Prajapati et al., 2012; Alaswad and Xiang, 2017). Proportional Hazards Model (PHM) was introduced by Cox (1972) and used widely in various fields; it was also used to model the hazard rate function in maintenance optimization using covariates as parameters being monitored. A CBM approach for single unit has been presented using PHM to determine an optimal control limit for hazard rate by which a maintainer can replace the equipment economically (Makis and Jardine, 1992; Banjevic et al., 2001). Ghasemi et al. (2007) presented an optimal replacement policy using Partially Observed Markov Decision Process (POMDP) and PHM in which equipment's state is unknown and solved the model using Dynamic Programming (DP) method. A multi-objective model for CBM optimization based on PHM has been proposed by Tian et al. (2012). They proposed a model in which reliability maximization and cost minimization are objectives and two constraints for cost and reliability are contemplated. The multi-objective model is transformed into single objective using physical programming approach to solve the problem more convenient in their study. Golmakani and Fattahipour (2011) developed the model proposed by Makis et al. (1992) in which a new cost parameter called the cost of inspection is considered and inspection interval is a new variable in their model. A development created by Golmakani and Fattahipour (2011) is a scheme in which an interval inspection is a variable with non-same interval times. Interval times directly depend on the age of the equipment. A model with two control limits for multi-component system has been proposed by (Tian and Liao, 2011). A new consideration has been done on PHM that repair policy with its cost is a new parameter in the model and both control limit and repair policy are simultaneously determined in a optimization problem in (Mousavi et al., 2014). They studied on the repair control limit policy, in a sense that the repair can refer the system state not just to "as-good-as-new". A model has been proposed by Golmakani and Pouresmaeeli (2014) to develop the CBM optimization model with PHM in which failure replacement cost depends on the equipment's degradation state and inspection interval is also considered as a variable with its cost. Naderkhani and Malik (2015) considered the problem of optimal sampling determination in CBM policy in which the stochastic deterioration has been constructed by PHM. The core of their study is to take a longer sampling interval for inspection and after estimating the hazard function of the system, if the estimated one exceeds a warning level, a decision would be made to inspect the system shorter and preventive maintenance is performed based on a maintenance control limit. Another study in CBM modeled by PHM refers to a study done by Caballea et al. (2015), they proposed a CBM strategy for a system with two types of failures (degradation and sudden failures). Degradation process follows non-homogeneous Poisson process and sudden failure follows doubly stochastic Poisson process. A threshold value is determined for degradation level and it is measured at inspection times. If the system degradation level exceeds threshold value, the system would be replaced at inspection; otherwise it continues to work until the next inspection time. The objective is to minimize the total expected cost. Lam and Banjevic (2015) presented a scheme based on a myopic approach to optimize the inspection scheduling in CBM modeled by PHM. They proposed a decision policy by which inspections are scheduled with regard to the current health of the system and optimized myopically over the next inspection interval. Jafari et al (2015) studied on the optimal control limit policy for a two-unit system in which two units are economically dependent, the first one is based CBM that its stochastic deterioration is measured by PHM and the second one is based age maintenance. In their work, the objective function is to seek an optimal opportunistic maintenance solution to minimize the long-run average cost per unit time. Their proposed problem is formulated through semi-Markov decision process approach.

Two common ways exist to model a maintenance optimization problem: infinite time and finite time (i.e., planning horizon) (Nakagawa, 2005). The expected cost per unit of time is usually adopted for infinite time interval and the expected cost for a planning horizon is usually adopted for finite interval time. All above-reported CBM optimization models based on PHM applied for single-unit or multi-unit system are constructed based on the long-run average cost per unit of time (i.e., infinite time). Practically, the working time of a system may be finite and it would be useful to model CBM optimization for equipment being set for a planning horizon. There is not any study for CBM using PHM over a planning horizon. For a finite planning horizon, there are a specified number of inspection points to monitor states of the system and total expected cost would be contemplated as an objective function being minimized based on an optimal control limit at each inspection.

When we are dealing with uncertainty, to make a constant decision for all the time is not efficient, all mentioned studies set a fixed control limit while the state of the system may change at any instant of time. This study aims to present a multi-stage model to cover a finite-time-interval feature for CBM optimization of a single unit system in which each inspection point can be contemplated as a stage. In each stage, the value of covariate for next stage is unknown, subsequently hazard rate of the system cannot certainly determined and a decision have to be made for the control limit of next stage, thus we would have a control limit for each stage that already has been determined. What motivates us to model a CBM system through a multi-stage stochastic programming is the value of covariate that behaves stochastically.

The main contributions of this study are to take into account existing gaps and to propose a proper solution approach:

- The first one is to present a model for finite interval of time applied in real life scenarios.
- To propose various optimal control limits (i.e., flexible control limit) in proportion to the current state of the system rather than fixed control limit for all inspection periods.
- To use multi-stage stochastic programming approach to model the condition-based maintenance with PHM as a novel approach to cover uncertainty issue in sequential problems.
- To apply an efficient hybrid meta-heuristic constructed by GA and IWO to cope with large scale issue and to use lower bound (LB) technique generated by DP to ensure the efficiency of the proposed meta-heuristic algorithm.

The outline of the paper is organized as follows. Section 2 elaborates the traditional CBM using PHM. Section 3 is assigned to describe a multi-stage stochastic programming for CBM and section 4 presents solution method based meta-heuristic algorithm so called hybrid GA/IWO algorithm and DP for obtaining a global optimum, also a LB technique is used to make a comparison with the solution by GA/IWO extensively. Section 5 gives some numerical examples and a sensitivity analysis on cost parameters. Finally, section 6 consists of summarization of the major findings of the study and some directions for future research in this area.

2-PHM for single unit

Makis and Jardine (1992) introduced a CBM policy based on an optimal control limit which directly depends on cost. They worked on deteriorating single unit with random failures to find an optimal replacement. PHM was used to model the replacement policy for equipment that its failure rate depends on the age and the value of stochastic covariates detected at discrete time points of operations. PHM refers to Cox's study (Cox, 1972) that failure rate of the equipment is the product of a classical failure rate indicating the age of the equipment and a positive function denoting the value of covariates. An optimal replacement rule which minimizes the long-run average cost per unit time is derived through a fixed-point iteration procedure. The equipment would be inspected at periodic intervals. Three events might be occurred: the first one is that the equipment has possibly failed between the previous inspection time and the current inspection time, in this situation, a corrective action would be performed, the second one might be a condition that the equipment's failure risk exceeds the optimal control limit determined by the fixed-point iteration procedure minimizing the objective function, i.e., long-run average cost, in this situation a preventive maintenance action should be performed and the third condition is that the equipment failure risk is lower than the optimal control limit in which it does not need to perform any action.

Three main assumptions must be contemplated in the model: the failure rate of the equipment is non-decreasing in time, the equipment behavior follows Markovian models and the cost of failure is larger than the cost of preventive maintenance.

In their model, T is time to failure of the system and $Z(t)$ is stochastic covariate at time t that is a diagnostic process denoting the effect of the operating environment on the system (e.g, it can be the vibration level of the equipment based on vibration analysis). Discrete time intervals are determined such as $\Delta, 2\Delta, 3\Delta, \ldots$ to perform inspections in which stochastic covariate is estimated by the beginning of the interval, i.e., $k\Delta \leq t < (k+1)\Delta$ interval is estimated by $Z(k\Delta)$. Hazard rate function in the model is shown by the product of a baseline failure rate, e.g., for Weibull distribution , $h_0(t) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}$ and a positive function to show the effect of covariate values denoting $\psi(Z(t)) = \exp\{\sum_{i=1}^{n} \gamma_i z_i(t)\}$, where γ_i is constant value and $z_i(t)$ is observed value of covariate at time t for i-th parameter.

Based on Makis and Jardine (1972) for Weibull hazard function and the stochastic covariate, the hazard function of the system is given by:

$$h(t, Z(t)) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1} \exp\{\gamma Z(t)\} \qquad t = 0, \Delta, 2\Delta, 3\Delta, \ldots \, and \, \Delta \geq 0, \tag{1}$$

Where the parameters of hazard function (i.e., β, η and γ) are estimated using maximum likelihood method in the presence of system's failure histories (Banjevic et al., 2001). The method maximum likelihood is also used to estimate the transition probabilities of the covariate process. The result is a $m \times m$ matrix, say $p(k)$, where the elements of $p(k)$ includes $p_{ij}(k)$ representing the conditional probability of the process $\{Z(k\Delta)\}$, i.e., the probability that in the next inspection , the state of the system is j given that the current state is i and failure may happen after and m denotes the number of covariate condition, e.g., the level of vibration, it constitutes a set $S = \{0, 1, \ldots, m\}$ that $Z(k\Delta) \in S$.

The replacement policy based on PHM is to perform a preventive maintenance if the failure hazard function constructed based on PHM is larger than the constant predetermined value denoted by d as a control limit and the time when the failure risk reaches the control limit is denoted by T_d. The average cost per unit time consists of two components; the total expected cost, i.e., C the cost of preventive maintenance (i.e., replacement) with probability that the failure does not happen before time T_d and $C + K$ the cost of replacement and failure the probability that the failure does happen before time T_d denoted by $Q(d)$, and the expected time interval, i.e., the expected time interval of minimum time to failure and T_d denoted by $W(d)$. This function is given by:

$$\varphi(d) = \frac{C(1 - Q(d)) + (C + K)(Q(d))}{W(d)} = \frac{C + KQ(d)}{W(d)}, \tag{2}$$

Where $Q(d) = \Pr\{T \leq T_d\}$ and $W(d) = \mathrm{E}(\min\{T, T_d\})$.

The aim of the model is to find the optimal control limit, d^*, that minimize the objective function, φ. Based on iteration procedure developed by Makis and Jardine (1972), d^* can be obtained through $d^{(n)} = \varphi(d^{(n-1)}), n = 1, 2, \ldots$ where $d^{(0)}$ as an initial value can be set an arbitrary value. After some iteration, a unique value for d is found called d^*that is satisfied in equation (3) as below.

$$\varphi(d^*) = d^* \tag{3}$$

So $\varphi(d)$ is the long run average cost per unit of time being minimized is constructed for infinite time in the model. We would construct a model based on finite time for systems in real world that work for a specific planning horizon. In the next section, a multi-stage model is proposed to cover the finite time interval for a system.

3-Multi-stage stochastic programming for condition-based maintenance

3-1-Problem description

Consider a repairable system that its failure follows Weibull distribution. PHM is contemplated to model the effect of covariates embedded in the hazard rate of the system under CBM as mentioned in section 2. The system is inspected every Δ unit of time and planned to be operated over a planning horizon$[0, T]$. At each inspection point (i.e., Δ, 2Δ, 3Δ,…), we should make an efficient decision for the control limit of the next inspection point due to the uncertain status of the system. Between two inspection points, if the system fails, a minimal repair is performed to proceed the task of the system that its cost as minimal repair cost is considered in the model. At each inspection point, if the proportional hazards rate of the system is greater than the control limit determined at the beginning of inspection point, the system must be replaced with a new one leading to a replacement cost in the model.

Over the planning horizon, n inspection points, i.e., $n = \left(\left\lceil\frac{T}{\Delta}\right\rceil - 1\right)$, could be followed as stages in which the status of the system (i.e., covariate value) may be different. Suppose that a system must be planned to perform CBM over T months, the system is inspected every Δ months. At the initial step, the status of the system is 0, after Δ months, at the first inspection point stated as the first stage, the status of the system may be one of the values in the set $S = \{0,1,\dots,m\}$ with a specific probability, consequently the hazard function of the system at the first inspection point will be a specific value. At the next inspection point, after 2Δ months, status of the system again may be one of the values in the set $S = \{0,1,\dots,m\}$ with a specific probability, so the hazard function of the system at the second inspection point may be a different value given action performed in the first inspection point. In this way, we can continue to the last inspection point.

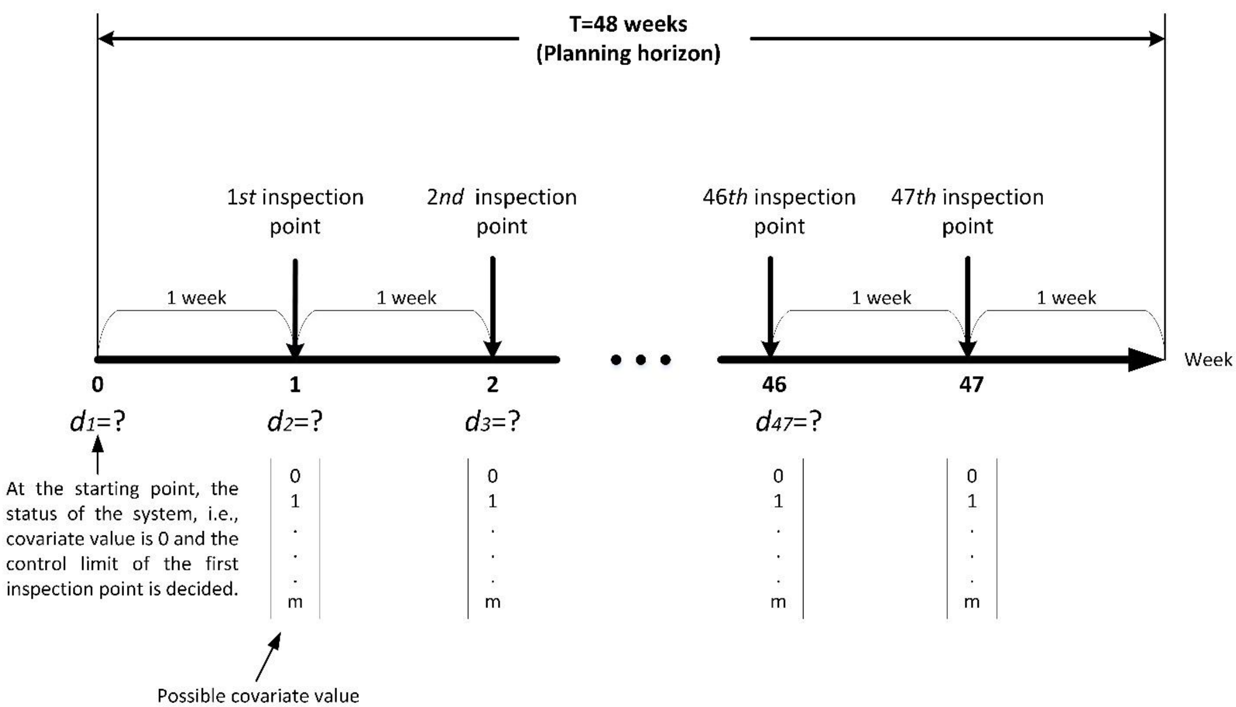

Fig.1. An instance of CBM planning

As an instance, in figure 1, a system is planned to be operated for 48 weeks and inspected every 1 week, consequently we have 48 stages. At the starting point, status of the system is 0 and based on the possible value of the system status at the next inspection point, a control limit, d_1, must be determined in advance. In this way, we must make a decision on the control limit of each stage except the last one (i.e., we suppose that because the mission of the system will be over at the last stage, so it doesn't need to determine any control limit).

Based on the minimal repair cost and the replacement cost, we should seek to find an optimal control limit for each inspection point with respect to uncertain states of the system in order to minimize the total expected cost over the planning horizon. As we know the probability transition distribution of the covariate value, the multi-stage stochastic programming would be suitable approach to cover the uncertainty in the system.

3-2-Multi-stage programming approach for CBM

In multi-stage stochastic programming, most focus is on the decision to be made today, future uncertainty and the possible action in the future. Thus, for our problem, the decision that must be made today is the control limit of the system for the next inspection point, future uncertainty is the value of covariate at the next inspection point and the possible action is to replace or do nothing. The objective function is to minimize the total expected cost of the system over planning horizon. Two types of cost would be considered in the model, the cost of minimal repair denoted by C_{mr} and the cost of replacement denoted byC_{rep}.

Uncertain status of the system behaves as a discrete-time stochastic process with finite probability space, so the uncertainty in this problem can be shown through a scenario tree as illustrated in the figure 2. A scenario tree is a usual way to describe the dynamic stochastic data over time (Birge and Louveaux 2010).

Figure 2 illustrates an example that each stage of the tree expresses the inspection point. There are some nodes indicating possible realization of covariate value (i.e., the system state) in each stage. For each node, the node number and the corresponding realization of the covariate value are depicted in the circle. For instance, node 4 corresponds to the scenario in which state of the system is 0 at stage 2 . The probability of each node depends on the decision relevant to the previous stage if the system was replaced or not.

In our problem as a model of sequential decision problem under uncertainty, a decision maker have to take a decision on the first control limit (i.e.,d_1) based on the first observation of the uncertain parameter, i.e., state of the system or covariate value being one of the values in the set $S = \{0,1, \dots, m\}$ at the first inspection point, the decision d_1 can specify the age of the system at the initial point and at the end of the second stage denoted by $x_2(\xi_1)$ and $x_2'(\xi_1)$, repectively. In a sense that if the proportional hazards rate of the system is greater than the first control limit i.e., $I_{\xi_1}\big((h_0(x_1')\psi(\xi_1) - d_1)\big) = 1$, the system must be replaced with new one, so the age of the new system at the initial of the second stage will be $x_2(\xi_1)$=0 and the age of the system at the end of the second stage will be $x_2'(\xi_1) = x_2(\xi_1) + \Delta= \Delta$, otherwise if the proportional hazards rate of the system is smaller than the first control limit i.e., $I_{\xi_1}\big((h_0(x_1')\psi(\xi_1) - d_1)\big) = 0$, the system must continue to work without any replacement, so $x_2(\xi_1) = \Delta$ and $x_2'(\xi_1) = x_2(\xi_1) + \Delta= 2\Delta$. Therefore, the decision on the age of the system at the beginning of the second stage and the end of the second stage are a function of the realized covariate value (i.e., ξ_1). After which, the decision maker must make a decision on the control limit for the end of the second stage or the second inspection point. Based on the determined second control limit (i.e., d_2) and the realized covariate value at the end of the second stage, the indicator function, $I_{(\xi_1,\xi_2)}\big((h_0(x_2'(\xi_1,\xi_2))\psi(\xi_2) - d_2)\big)$, takes 0 or 1 and consequently the age of the system at the beginning and the end of the third stage are determined accordingly. This sequence of uncertain covariates value leading to alternating proportional hazards rate of the system and determination of control limit value at each stage continue to the end of the planning horizon.

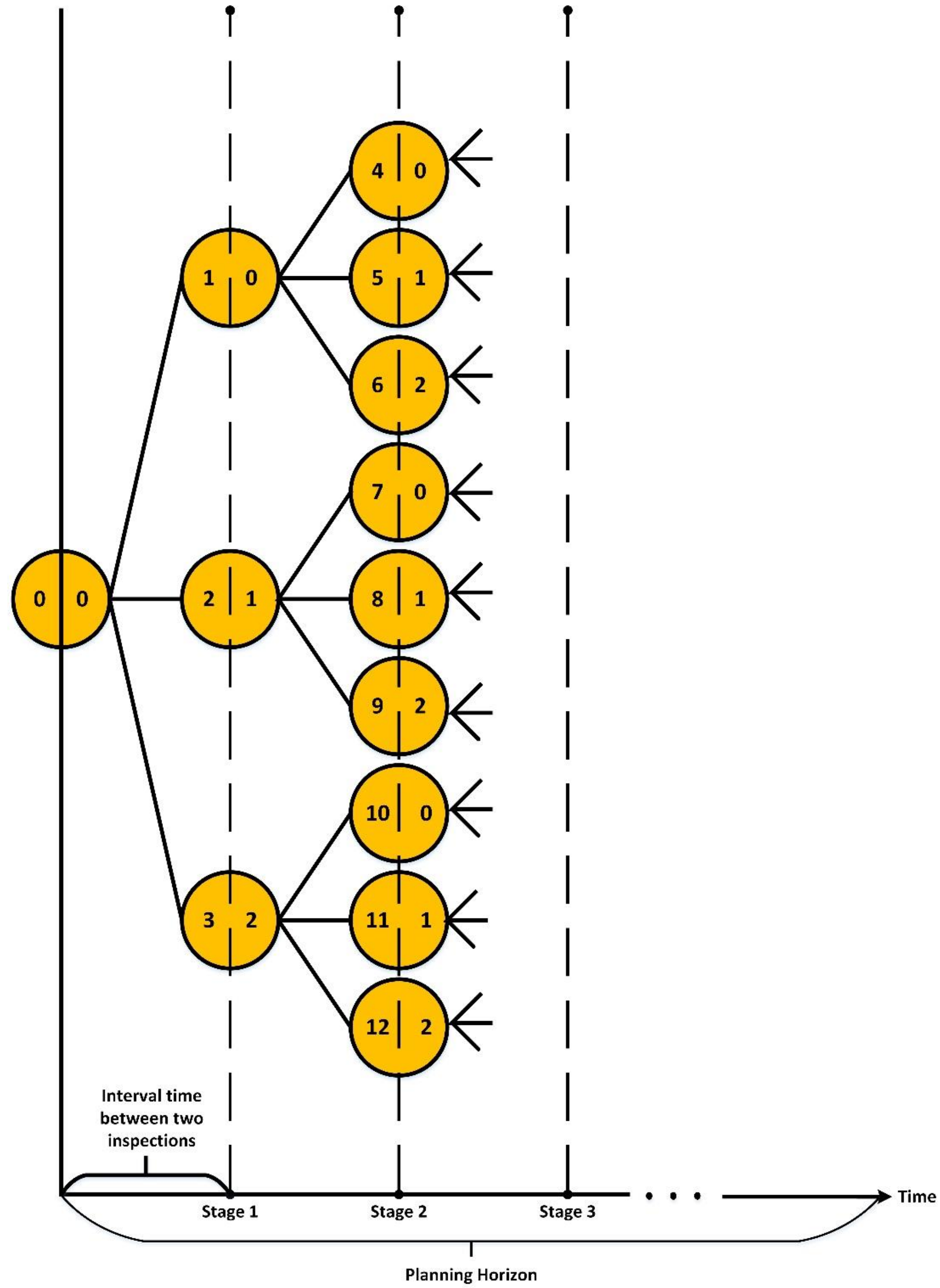

Fig.2. A part of a scenario tree for a system with $S = \{0,1,2\}$

Following the notation in chapter 2 of Birge and Louveaux (2010), a multi-stage nonlinear stochastic model for CBM of a single unit would be presented as sequence below:

$$Min\ Total\ Expected\ \text{Cost} = E_{\xi_1}[\min(C_{mr}\int_{x_1}^{x_1^{'}} h_0(t)\psi(\xi_1)dt + C_{rep}I_0(h_0(x_1^{'})\psi(\xi_1)-d_1))$$

$$+E_{(\xi_2|\xi_1(1-I_{\xi_1}))}[\min(C_{mr}\int_{x_2(\xi_1)}^{x_2^{'}(\xi_1)} h_0(t)\psi(\xi_2)dt + C_{rep}I_{(\xi_1)}(h_0(x_2^{'}(\xi_1))\psi(\xi_2)-d_2))$$

$$+\cdots+E_{\left(\xi_n\left|(\xi_1,\xi_2,...,\xi_{n-1})(1-I_{(\xi_1,\xi_2,...,\xi_{n-1})})\right.\right)}[\min(C_{mr}\int_{x_n(\xi_1,\xi_2,...,\xi_{n-1})}^{x'_n(\xi_1,\xi_2,...,\xi_{n-1})} h_0(t)\psi(\xi_n)dt$$

$$+C_{rep}I_{(\xi_1,\xi_2,...,\xi_{n-1})}(h_0(x'_n(\xi_1,\xi_2,...,\xi_{n-1}))\psi(\xi_n)-d_n))]\cdots]] \quad (4)$$

Subject to:

$$x_l(\xi_1,\xi_2,\dots,\xi_{l-1}) = (l-1)\Delta - (l-1)I_{(\xi_1,\xi_2,\dots,\xi_{l-1})}\Delta \quad , \ l \in \{2,3,\dots,n\} \text{ and } \xi_l \in \{1,2,\dots,m\} \quad (5)$$

$$x'_l(\xi_1,\xi_2,\dots,\xi_{l-1}) = \Delta + x_l(\xi_1,\xi_2,\dots,\xi_{l-1}) \quad , \ l \in \{2,3,\dots,n\} \text{ and } \xi_l \in \{1,2,\dots,m\} \quad (6)$$

$$x_1 = 0 \quad (7)$$

$$x'_1 = \Delta \quad (8)$$

Where ξ_l denotes the uncertain parameter indicating the covariate value or the state of the system at the l-*th* inspection point or stage that is one of the values in the set $S = \{0,1,\dots,m\}$. $x_l(\xi_1,\xi_2,\dots,\xi_{l-1})$ and $x'_l(\xi_1,\xi_2,\dots,\xi_{l-1})$denote the age of the system at the beginning and at the end of stage l with respect to whole history past observation of covariate values , i.e., $\xi_1,\xi_2,\dots,\xi_{l-1}$. $h_0(x'_l(\xi_1,\xi_2,\dots,\xi_{l-1}))\psi(\xi_l)$ is the proportional hazards function at the age $x'_n(\xi_1,\xi_2,\dots,\xi_{l-1})$ and with covariate value ξ_l and $\psi(\xi_l)$ equals to $e^{\gamma\xi_l}$. As above explained, If the proportional hazards value at each stage is greater than the control limit, the replacement action must be performed leading to replacement cost, C_{rep}.
When the system fails between inspections, it is back to the hazard rate exactly before the failure through performing a minimal repair; cost C_{mr} is taken into account for each minimal repair. Thus, $\int_{x_n(\xi_1,\xi_2,\dots,\xi_{n-1})}^{x'_n(\xi_1,\xi_2,\dots,\xi_{n-1})} h_0(t)\psi(\xi_n)dt$ can express the mean number of failures between two inspection points.

The function $E_{\left(\xi_l\left|(\xi_1,\xi_2,\dots,\xi_{l-1})\left(1-I_{(\xi_1,\xi_2,\dots,\xi_{l-1})}\right)\right.\right)}[.]$ is mean function with respect to the realized value of ξ_{l-1} and the decision made in the $(l-1)$-*th* stage. Then, If the decision is a replacement, so $I_{(\xi_1,\xi_2,\dots,\xi_{l-1})} = 1$ and consequently $E_{\left(\xi_l\left|(\xi_1,\xi_2,\dots,\xi_{l-1})\left(1-I_{(\xi_1,\xi_2,\dots,\xi_{l-1})}\right)\right.\right)} = E_{(\xi_l|0)}$, If the decision is do nothing, so $I_{(\xi_1,\xi_2,\dots,\xi_{l-1})} = 0$ and consequently $E_{\left(\xi_l\left|(\xi_1,\xi_2,\dots,\xi_{l-1})\left(1-I_{(\xi_1,\xi_2,\dots,\xi_{l-1})}\right)\right.\right)} = E_{(\xi_l|(\xi_1,\xi_2,\dots,\xi_{l-1}))}$.

4-Solution approach

With the increasing number of values in the set S and the number of inspection points or stages, the dimension of the proposed multi-stage model increases exponentially, in a sense that we have m^n scenarios in a problem with m states in set S and n stages and due to the complex non-linear model presented in this study, to solve the problem through an exact method may takes very long time which might be inefficient. Thus, we present a meta-heuristic algorithm to minimize the total expected cost over a planning horizon. We also present a Dynamic Programming (DP) approach explained in **Appendix A** and run it on test problems for a rational time. Depending on the size of the problem, DP algorithm may achieve to the exact solutions and also can be a LB, however both can help us to show the efficiency of the proposed meta-heuristic algorithm to achieve optimal or near optimal solutions of these problems in a very short time. Thus, in the following section, we would propose an efficient hybrid heuristic algorithm constructed by hybrid GA and IWO to cope with the mentioned issues.

4-1- Hybrid GA/IWO

GA has been used widely to solve optimization problem as an efficient meta-heuristic algorithm. Their easy-to-use characteristic and being straightforward to cope with complex non-linear problems or large scale problems can be most important justification to use for this study. The most important challenge to use heuristic algorithms is to obtain a local optimum, so we use dispersion and reproduction features of IWO algorithm to prevent it from converging to a local optimum. The proposed algorithm is depicted through a flowchart in Fig.6 and for more explanation; all steps are described as follows.

4-1-1- Population Representation

Each chromosome represented by string in Fig.3 indicates information about the control limits of the inspection points over planning horizon. For instance, figure 3 shows a 6-stage problem with two-state covariate (as mentioned in section 3, note that we need 5 control limits) in which the stage 5 includes 32 nodes, consequently 32 proportional hazards rates can exist in this stage, the value in the last part of the chromosome, (e.g., 7) reveals that the control limit equals to the value of the seventh proportional hazard rate.

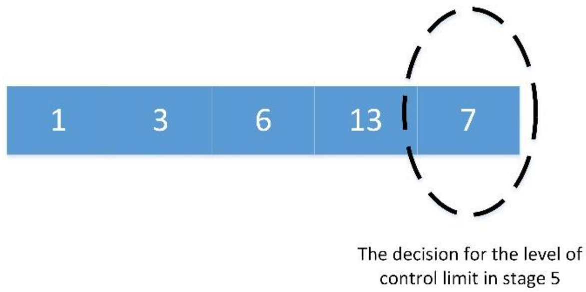

Fig.3. An instance of a chromosome, (a feasible solution).

The initial population consisting of chromosomes will be randomly generated according to the corresponding number of nodes in each period.

4-1-2- Fitness function

The fitness function as the objective function of the model mentioned in equation (4) is considered to assess the value of each solution represented by a chromosome.

4-1-3- Selection

This step in GA is to choose parents to generate new population, the usual way is to use the elitist strategy and roulette wheel method, but we propose a modification here to prevent from local optimum that is to use reproduction method in IWO. Each member of the population (i.e., chromosome) as parents can generate new children corresponding to its fitness value. The number of children generated by each member linearly varies from the lowest possible numbers of children for a chromosome with the worst fitness to the maximum number of children for a chromosome with the best fitness. The formula proposed by Mehrabian and Lucas (2006) is constructed for minimizing problems as presents below:

$$w_i = \frac{F-F_{min}}{F_{max}-F_{min}}(N_{min} - N_{max}) + N_{max} \tag{9}$$

Where w_O represents the number of children generated by chromosome i, F is the value of fitness function for chromosome i, F_{min} is the minimum fitness value among chromosome in the current iteration, F_{max} is the maximum value of fitness function among chromosome in the current iteration, N_{min} is the minimum number of children and N_{max} is the maximum number of children. Both N_{min} and N_{max} are the input parameters of the algorithm. Mehrabian and Lucas (2006) mentioned that usually in evolutionary algorithm for optimization problems, feasible solutions with better fitness would be thought to be the ones with having more chance to reproduce better solutions. We adopt this reproduction feature to enhance the efficiency of classical GA.

4-1-4- Crossover operator

Crossover is a well-applied operator to construct offspring by combining the gens of their parents. In the proposed GA, we used a two-point crossover for each chromosome with other chromosome in which two gens are selected and all gens between two selected gens are replaced with other chromosomes as depicted in figure 4.

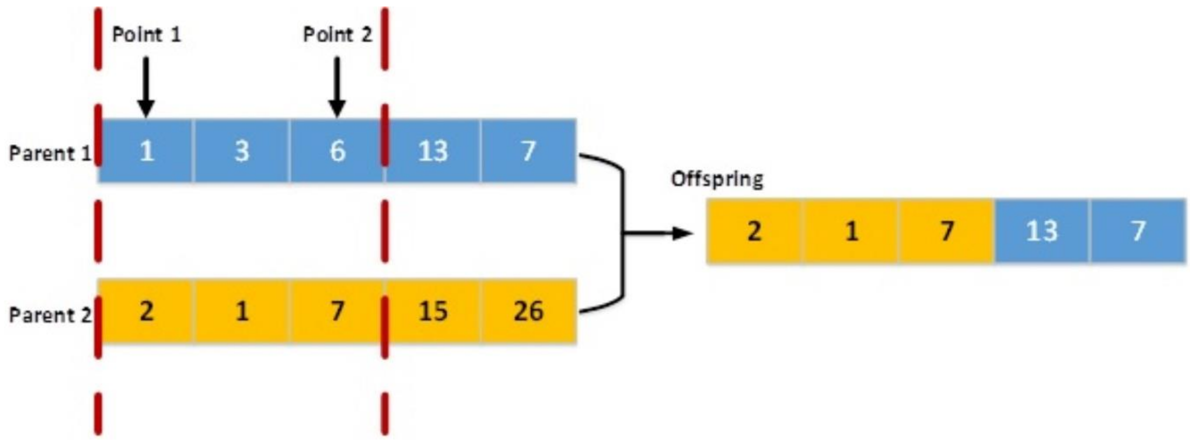

Fig.4. Two-point crossover operator for two parents

Based on the previous step, each chromosome is supposed to generate the number of offspring corresponding to its value calculated in equation (9), and a chromosome to be selected to do the two-point crossover with the main parent are selected randomly.

4-1-5- Mutation operator

Mutation operator is an efficient operator to help algorithm avoid premature convergence. It is performed on randomly selected gens and change its value to a new one with a special method in which we select an integer value between the possible lowest integer value and the possible greatest integer value through dispersion method in IWO. For instance, based on Fig.3, the first gene indicating the possible level of proportional hazards rate at the first stage can be selected between 1 and 2, the second gene indicating the possible level of proportional hazards rate at the second stage can be selected between 1 and 4 and so on. In this study we adopt here dispersion method of IWO to create a new chromosome. It is performed as follows:

$$\sigma_{cur} = \frac{(iter_{max} - iter)^{\pi}}{(iter_{max})^{\pi}} \left(\sigma_{initial} - \sigma_{final}\right) + \sigma_{final} \quad (10)$$

Where, standard deviation (SD), σ, of the random function will be reduced from a previously defined initial value, $\sigma_{initial}$, to a final value, σ_{final}, in each iteration ($iter$). $iter_{max}$ designates the maximum number of iterations, $\sigma_{cur}$is the standard deviation at the current step and π indicates the nonlinear modulation index.

Then, two gens are randomly selected and based on the possible values, a new value can be randomly created through a normal distribution with $\mu = \frac{\text{the possible greatest value} + \text{the possible lowest value}}{2}$ and $\sigma = \sigma_{cur}$. If the created value is between possible value, it will be adopted, otherwise again a new one is created until one is adopted. It is noted that if the new value is not integer, it have to be rounded.

For instance, based on Fig.3, supposed that the second gene is randomly selected. The possible level of proportional hazard rate can be selected between 1 and 4, and $\sigma_{cur} = 3$, so a new value would be randomly created through $N(\frac{4+1}{2} = 2.5, \sigma_{cur} = 3)$. An instance of the proposed mutation operator in which randomly two gens are changed is depicted in figure 5.

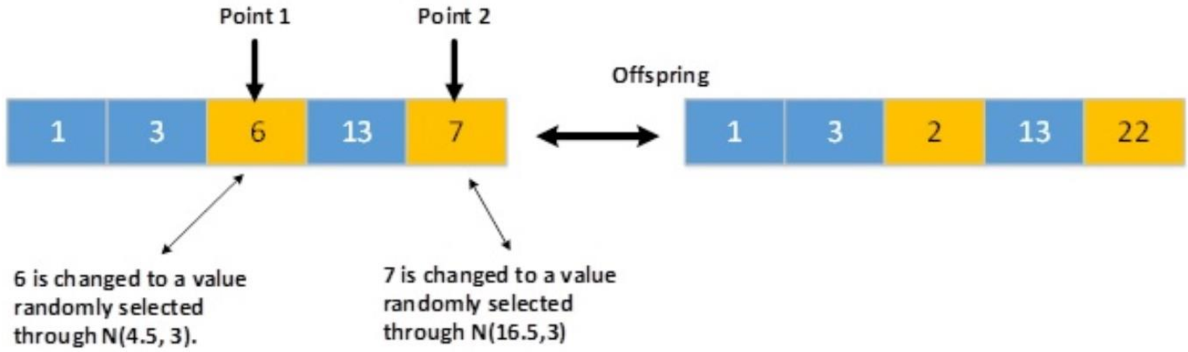

Fig.5. Mutation operator for one of the chromosomes

This step is also performed such as cross-over step that of each chromosome is supposed to generate the number of offspring corresponding to its value calculated in equation (9), and gens to be selected to perform two-point mutation are selected randomly.

Finally in this section, we illustrate the meta-heuristic algorithm through a flowchart in figure 6.

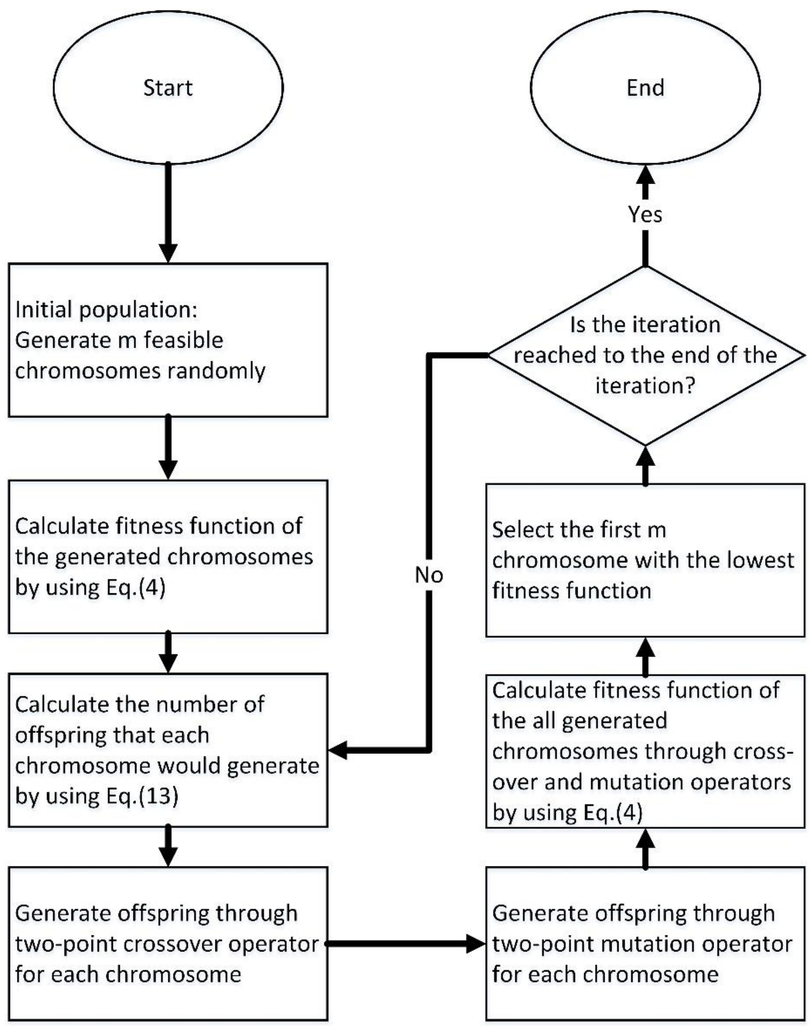

Fig.6. A flowchart to show the steps of the proposed GA algorithm

5-Computational study and results

The aim of the computational study section is two-fold. Firstly, ten various random test problems are to illustrate the performance of the proposed GA algorithm and secondly, a sensitivity analysis would be done on the cost-related parameters to validate the model. Before the experiments, the input parameters of the proposed meta-heuristic algorithm are tuned in the next section.

5-1-Parameter tuning

Study on finding the optimum values of the input parameters in the meta-heuristic algorithms makes algorithms more effective to find better solution. Response Surface Methodology (RSM) as a statistical tool is used to optimize functions and to estimate parameters in various areas (Box and Wilson, 1951). RSM proposed by Box and Wilson (1951) as an optimization method is used to tune the input parameters of the proposed GA. The steps of RSM would be usually described as follows:

- Selection of input parameters as factors to be tuned.
- Using an experimental design to reach minimum variances of outputs.
- Running simulation based on the factors of the experimental design.
- Construction of a stable regression model to find the optimum.

Input parameters of the proposed GA consists of the size of the population (n_{pop}), N_{max}, N_{min}, $\sigma_{initial}$, σ_{final} and π. Box-Behnken Design (BBD) as one of the best RSM designs is used to obtain a combination with minimum variance. In BBD, three levels are represented by coded values, -1 , 0 and 1 for low, zero and high levels of the factors (i.e., input parameters), respectively (Majumder et al., 2009). The factors can be coded according to the sequence following equation.

$$X_i = \frac{x_i - x_0}{\Delta x}, \qquad (11)$$

Where, X_i is codified value, x_i is actual value, x_0 is the value at the center point and Δx is the step change value. To run the simulation and to study the variation of response factors, the popular regression model is presented as below:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1}^{k} \beta_{ii} X_i^2 + \sum_{i<j} \beta_{ij} X_i X_j + \varepsilon \qquad (12)$$

Where y is the response value, β_0 denotes modification value, β_i denotes linear influence, β_{ij} denotes interaction influence.

Three levels of the input parameters as variables of the regression model is illustrated in table1.

Table 1. The experimental factors and their coded values

Input parameters	Symbol	Coded value		
		-1	0	1
n_{pop}	X_1	60	100	200
N_{max}	X_2	30	40	60
N_{min}	X_3	5	10	20
$\sigma_{initial}$	X_4	0.2	1	3
σ_{final}	X_5	0.001	0.01	0.1
π	X_6	2	2.5	3

All combinations of input parameters illustrated in Table 1 are run for five replications and for each run the best response is obtained. The significant input parameters and their interactions are determined and based on significant factors, the regression model in equation (12) is considered as an objective function and the range of the parameters can be constraints of the model, then the model is solved and the optimal parameters are determined. Table 2 shows the optimal parameters for an instance in the next section through BDD.

Table 2. The optimal value for input parameters

Input parameters	n_{pop}	N_{max}	N_{min}	$\sigma_{initial}$	σ_{final}	π
Optimal value	100	60	5	3	0.01	2

5-2- Numerical instances

In this section, to illustrate the performance of the proposed GA, various random test problems are solved through the proposed GA and DP explained in Appendix (A) in which two states may happen. The first state is relevant to the problems that the exact optimal solution could be found by both algorithms and the second one is associated to problems that due to being large, the problems are run by DP for 3600 seconds to reach a LB and a near optimal or good solution can be found by the proposed GA compared with LB.

10 test problems illustrated in Table 3 have run on a personal computer with an Intel core2, processor of 2.53 GH and 2 GB of RAM memory. The proposed GA and DP algorithm were coded by MATLAB.

Table 3. The test problems

Problem	PHM parameters			Time interval between two inspections	Planning horizon	Possible states of covariate value	Minimal repair cost	Replace ment cost
	α	β	γ	Δ	T	S	C_{mr}	C_{rep}
1	1	2	0.5	1	6	{0,1}	2.5	5
2	1	5	0.3	3	18	{0,1}	1	5
3	2	3	0.5	2	10	{0,1,2}	2	10
4	1	1.5	1	5	15	{0,1,2}	20	100
5	2	4.5	0.2	10	20	{0,1,2}	1	20
6	1	2	0.5	1	7	{0,1}	0.1	5
7	10	12	0.2	10	50	{0,1,2}	5	50
8	10	16	1.5	4	20	{0,1,2}	35	300
9	2	4	1	2	10	{0,1,2}	100	5000
10	3	8	0.5	20	100	{0,1,2}	10	3500

The first five problems can be categorized into the small scale problems because below 3600 second the optimal solution will be found and the second ones is categorized into the large scale problems because solving by DP will take a time much more than 3600 seconds.

The transition probability for test problems is assumed as below:

If $S \in \{0,1\} \rightarrow P = \begin{pmatrix} 0.9 & 0.1 \\ 0 & 1 \end{pmatrix}$, if $S \in \{0,1,2\} \rightarrow P = \begin{pmatrix} 0.2 & 0.6 & 0.2 \\ 0 & 0.4 & 0.6 \\ 0 & 0 & 1 \end{pmatrix}$.

The proposed GA and DP algorithm have run for each test problem and the results are reported in table 4.

Table 4. Results of running the proposed GA and DP algorithm

Test problem	The proposed GA			Dynamic programming		
	Total Expected Cost	CPU time (Second)	Solution	Total Expected Cost	CPU time (Second)	Solution
1	45.8385	7.5	$d_1 = 2$ $d_2 = 4$ $d_3 = 6$ $d_4 = 8$ $d_5 = 10$	45.8385	1678.45	$d_1 = 2$ $d_2 = 4$ $d_3 = 6$ $d_4 = 8$ $d_5 = 10$
2	1789.05	7.2	$d_1 = 405$ $d_2 = 6480$ $d_3 = 32805$ $d_4 = 103680$ $d_5 = 253125$	1789.05	1710.67	$d_1 = 405$ $d_2 = 6480$ $d_3 = 32805$ $d_4 = 103680$ $d_5 = 253125$
3	55.32	8.7	$d_1 = 2.47$ $d_2 = 6$ $d_3 = 13.5$ $d_4 = 24$	55.32	3120.4	$d_1 = 2.47$ $d_2 = 6$ $d_3 = 13.5$ $d_4 = 24$
4	2419.60	1.6	$d_1 = 3.35$ $d_2 = 4.74$ $d_3 = 5.80$	2419.60	3400.98	$d_1 = 3.35$ $d_2 = 4.74$ $d_3 = 5.80$
5	5202	1.8	$d_1 = 628.89$ $d_2 = 7115.10$ $d_3 = 29410.46$	5202	3460.03	$d_1 = 628.89$ $d_2 = 7115.10$ $d_3 = 29410.46$
6	6.4744	8.2	$d_1 = 3.3$ $d_2 = 6.6$ $d_3 = 9.9$ $d_4 = 13.19$ $d_5 = 16.49$ $d_6 = 19.79$	6.4751	3600	$d_1 = 3.3$ $d_2 = 6.6$ $d_3 = 9.9$ $d_4 = 13.19$ $d_5 = 16.49$ $d_6 = 12$
7	476.9388	8.4	$d_1 = 1.2$ $d_2 = 1.2$ $d_3 = 1.2$ $d_4 = 1.2$ $d_5 = 1.2$	504.0820	3600	$d_1 = 1.2$ $d_2 = 1.2$ $d_3 = 1.2$ $d_4 = 1.2$ $d_5 = 1.4657$
8	647.8825	7.5	$d_1 = 3.46 \times 10^{-5}$ $d_2 = 1.13$ $d_3 = 110.47$ $d_4 = 0.056$ $d_5 = 0.056$	713.5283	3600	$d_1 = 3.46 \times 10^{-5}$ $d_2 = 0.056$ $d_3 = 24.65$ $d_4 = 0.056$ $d_5 = 0.056$
9	26992.8208	6.9	$d_1 = 2$ $d_2 = 2$ $d_3 = 2$ $d_4 = 2$ $d_5 = 2$	30292.9582	3600	$d_1 = 5.43$ $d_2 = 2$ $d_3 = 14.77$ $d_4 = 2$ $d_5 = 2$
10	4.3479×10^8	7.2	$d_1 = 1.56 \times 10^6$ $d_2 = 1.56 \times 10^6$ $d_3 = 1.56 \times 10^6$ $d_4 = 1.56 \times 10^6$ $d_5 = 1.56 \times 10^6$	6.0202×10^8	3600	$d_1 = 4.24 \times 10^6$ $d_2 = 1.56 \times 10^6$ $d_3 = 4.24 \times 10^6$ $d_4 = 1.56 \times 10^6$ $d_5 = 1.56 \times 10^6$

As seen in table 4, the result shows that for the first five problems, the solution by the proposed GA and DP is the same and for other problems the solution by the proposed GA is much better than LB obtained by DP. To show the difference between two algorithms, all results are depicted in figure 7 and two algorithms are compared from total expected cost perspective.

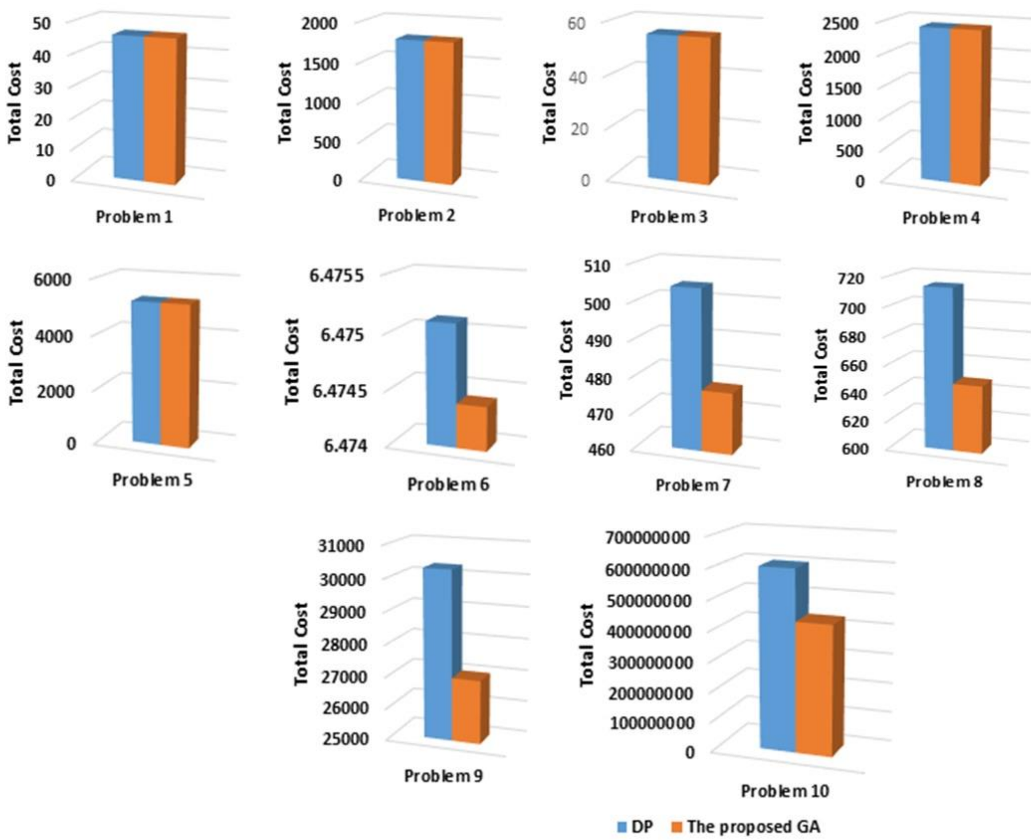

Fig.7. A comparison on total expected cost between the proposed GA and DP

Another comparison to illustrate the effective performance of the proposed GA is CPU time. As observed in table 4, all test problems by the proposed GA have run below 10 seconds and the results designate a significant difference between two algorithms. Relative Percentage Deviation (RPD) as a known performance measure is used to show the difference percentage of CPU time between two algorithms. RPD for CPU time can be computed by the formula as follows:

$$RPD = \frac{CPU\ time\ by\ DP - CPU\ time\ by\ the\ proposed\ GA}{CPU\ time\ by\ DP} \times 100 \quad (13)$$

The RPD of all problems is shown in figure 8. As observed, the difference percentage between two algorithms is more than 90 and it is clear that the performance of the proposed GA is much better than DP.

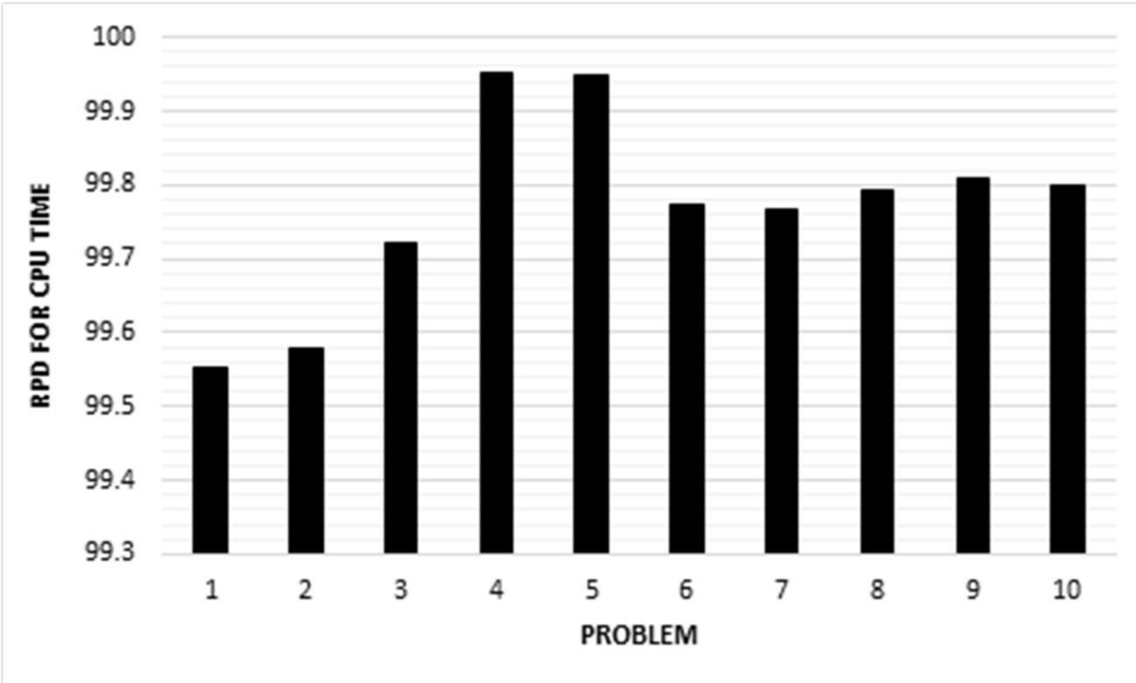

Fig.8. RPD for CPU time of all test problems

5-3-Sensitivity analysis

A sensitivity analysis would be performed to assess the efficiency of the model and to show the role of minimal repair and replacement cost in the model. We focus on the main two cost-related parameters of the model (i.e., C_{mr}, C_{rep}). We expect, in turn, when the minimal repair cost value (C_{mr}) increases, level of the control limits (d_i) decreases and when the replacement cost value (C_{rep}) increases, level of the control limits (d_i) increases. We have tested it on the first instance mentioned in section 5.2 with $C_{rep} = 4$ and $C_{mr} = 0.1, 1, 3$. The result is depicted in figure 9.

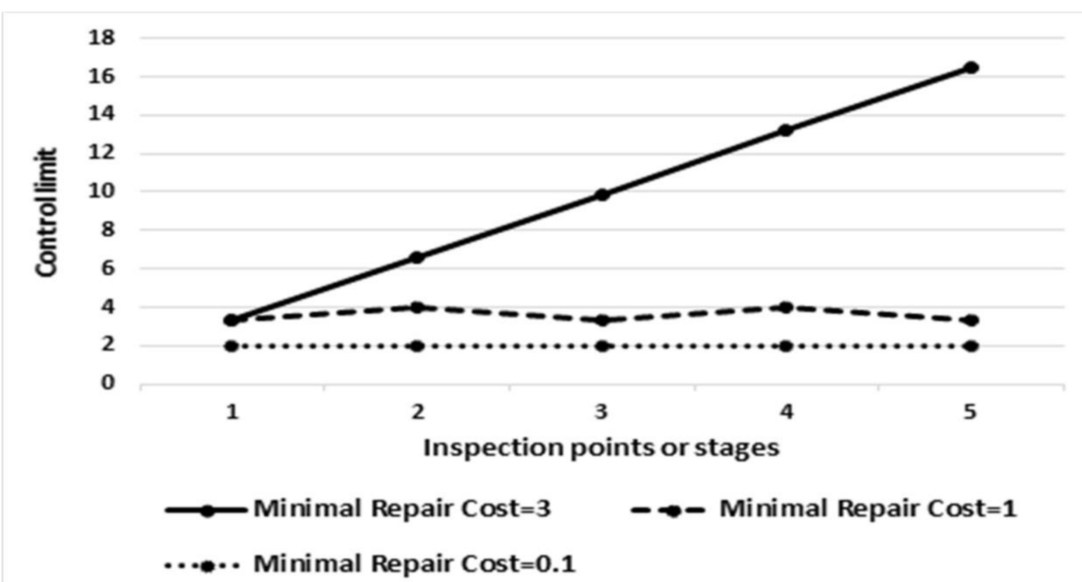

Fig.9. A comparison between various minimal repair costs on the control limit

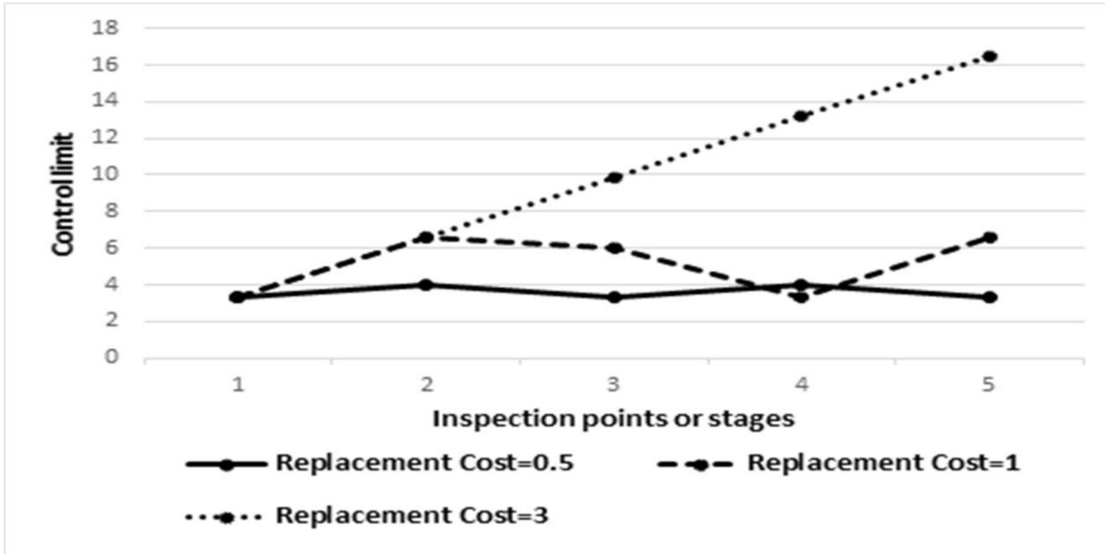

Fig.10. A comparison between various replacement repair costs on the control limit

Figure 9 shows the level of minimal repair cost and the level of control limit in each stage. As presented, when the level of minimal repair cost increases, the level of control limit decreases, in a sense that the equipment should be replaced earlier. For the replacement cost, figure 10 also shows that when the replacement cost increases, the level of control limit increases.

Another aspect of the model presented in this paper is that increasing either minimal repair cost or replacement cost leads to increasing the total expected cost. We have tested the first instance again with various minimal repair costs and replacement costs, subsequently; the result is illustrated in figure 11. As observed, the result is in our favor.

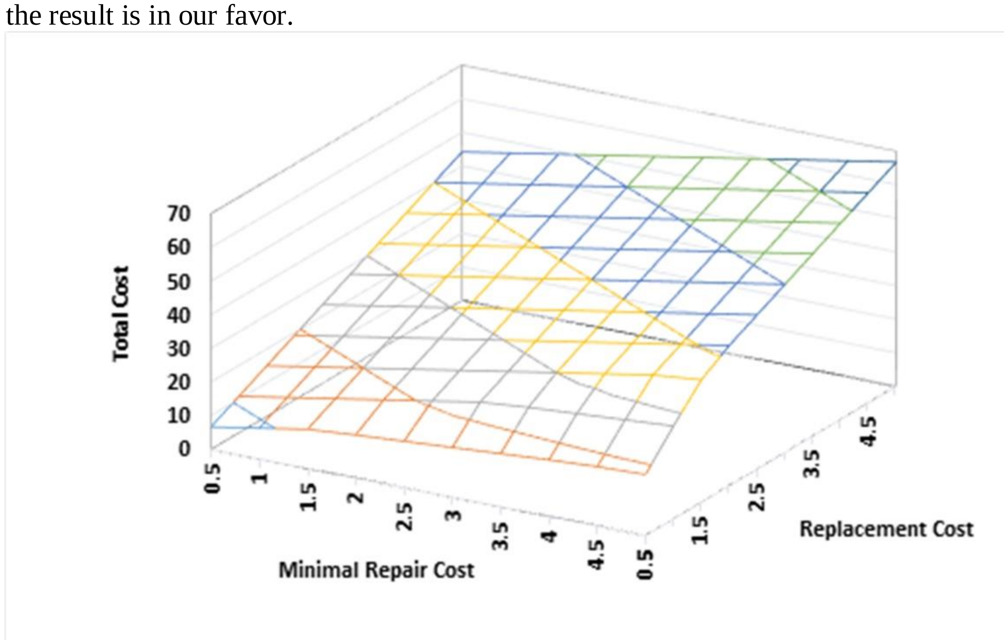

Fig.11. The result of total costs for various minimal repair cost and replacement cost

6-Conclusion and future research

In this paper, a non-linear multi-stage stochastic programming was applied to model CBM of a system for a finite planning horizon in which the state of the system or covariate value constituting PHM was an uncertain parameter at each inspection point or stage. Two types of repair were supposed in the model as a maintenance strategy, one is minimal repair to restore the system upon any failure between inspections to the state exactly before failure and the second one is perfect repair or replacement to restore the system to a new one at inspection point when the proportional hazards rate exceeds the control limit being decided before. Subsequently, two types of costs were contemplated in the model, minimal repair cost and replacement cost, respectively. Various random test problems were designed to illustrate the model numerically. In order to solve the problems, a hybrid GA/IWO was used to find good control limits so that the total expected cost was minimized. Furthermore, to show the efficiency of the proposed meta-heuristic algorithm, a DP was used to run the problems for a rational time as a LB. The result showed that the proposed GA outperformed in terms of both the quality of solution and CPU time. As future directions, the multi-stage stochastic programming approach could be developed for multi-component structures. Also, other relevant costs with discount can be considered in the model.

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Appendix A: DP approach to solve the proposed multi-stage stochastic model

Because of the sequential stages of the model and the non-linear programming structure, DP is applied to find the global solution. DP is usually proposed as a certain methodology to find the global solution for optimization of various complex problems. A problem solved by DP approach usually would be characterized by some features that described for our problem as below:

- When a problem is supposed to be solved by DP, it can be divided to multiple stages in which a decision making policy is contemplated at each stage. As we have to make sequential decisions, (i.e., control limit) over planning horizon, thus the n-stage problem is converted to n single stages in which, at each stage, an optimal control limit must be determined for next stage.

In each stage, there exist some states associated to that stage. The states provide the required information to map future actions according to the current decision. To specify the states of the system might be the most essential section of DP. In CBM problem, states could be the age of the system in the first of the stage and the end of the stage denoted by $x_l(\xi_1, \xi_2, \ldots, \xi_{l-1})$ and $x'_l(\xi_1, \xi_2, \ldots, \xi_{l-1})$, respectively considering the possible condition of the system (i.e., covariate value). Based on equations 5-8, each policy at each stage can be shown through a transition function as follows:

$$x_i(\xi_1, \xi_2, \ldots, \xi_{i-1}) = Y_{1,i,\xi_i}(x_{i-1}(\xi_1, \xi_2, \ldots, \xi_{i-2}), I_{(\xi_1,\xi_2,\ldots,\xi_{i-2})}) \quad \text{for } i = 1, \ldots \tag{A.1}$$

$$x'_{i,\xi_i} = Y_{2,i,\xi_i}(x_i(\xi_1, \xi_2, \ldots, \xi_{i-1})) \quad \text{for } i = 1, \ldots, \tag{A.2}$$

- A recursive procedure based on a backward process can be usually used to seek the optimal solution where algorithm initiates from the last stag and moving back stage by stage until all stage are analyzed and the recursive procedure can be based on a forward process, like backward process, but the algorithm initiate from the first and moves forward. We define TC_i as a separable function of minimal repair and replacement cost at stage i.

$$TC_i = \sum_{\xi_i} p_{(\xi_i|(\xi_1,\xi_2,\cdots,\xi_{i-1})(1-I_{(\xi_1,\xi_2,\ldots,\xi_{i-1})}))}[C_{mr}(((\frac{x'_i(\xi_1,\xi_2,\ldots,\xi_{i-1})}{\alpha})^{\beta}$$
$$-(\frac{x_i(\xi_1,\xi_2,\ldots,\xi_{i-1})}{\alpha})^{\beta})e^{\gamma\xi_i}) + C_{rep} I_{(\xi_1,\xi_2,\ldots,\xi_{i-1})}(\frac{\beta x'_i(\xi_1,\xi_2,\ldots,\xi_{i-1})^{\beta-1}}{\alpha^{\beta}} e^{\gamma\xi_i} - d_{i,\xi_i})] \tag{A.3}$$

The non-linear multi-stage stochastic programming model can be converted to DP model by using separable function represented in equation (A.3). We define f_i as recursive sequential function at stage i for the total cost, the DP formulation for our problem would be presented as below:

$$f_i\left(Y_{1,i,\xi_i}\big(Y_{2,i,\xi_i}\big(x_{i-1}(\xi_1,\xi_2,\ldots,\xi_{i-2})\big), I_{(\xi_1,\xi_2,\ldots,\xi_{i-2})}\big)\right) =$$
$$\min\left\{f_{i+1}\left(Y_{1,i,\xi_i}\big(Y_{2,i,\xi_i}\big(x_{i-1}(\xi_1,\xi_2,\ldots,\xi_{i-2})\big), I_{(\xi_1,\xi_2,\ldots,\xi_{i-2})}\big)\right) +\right.$$
$$\left. TC_i\left(Y_{1,i,\xi_i}\big(Y_{2,i,\xi_i}\big(x_{i-1}(\xi_1,\xi_2,\ldots,\xi_{i-2})\big), I_{(\xi_1,\xi_2,\ldots,\xi_{i-2})}\big)\right)\right\} \tag{A.4}$$