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Introducing a mathematical model in supply chain by adding trust flow

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Abstract

These day, supply chains (SCs) have become more and more complicated and have extensively expanded and due to these complexities, the supply chain management (SCM) has encountered several uncertainties, and, as a result, trust and assurance between members in SCs has become essential for a successful SCM. Although trust is an inevitable component in nearly all fields in SCs, like cooperation, coordination and management. As Trust increases the sense of security among members and cuts back on the losses, this research attempts to introduce a mathematical model that is able to utilize trust as a main element in a two-echelon SC. Defining trust is difficult since it is analyzed from different perspectives, and it is used in a wide range of situations. Therefore, the aim of this study is to propose an appropriate definition for this concept according to SCs, and to present a two-echelon SC, including a retailer and a supplier. The supplier and the retailer play Stackelberg game in newsvendor framework. The order quantity and stock, as the best sections for proposing the definition of trust, is developed for retailer and supplier. In addition, Beta model is presented for calculating trust and finally in order to verify the quality and efficiency of the proposed model, a numerical example is also offered.

Keywords: Newsvendor problem, Computational Trust model, Stackelberg game, Trust, Supply chain.

1. Introduction

In the 1980s, a host of companies were seeking for techniques and strategies to decrease their costs. One of the most important areas which offer a lot of potential opportunities for reducing costs is SCs which include goods, money, and information flows. SCM is one of the most popular management concepts that manage flows between stages to maximize profit. Even though boosting

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the profit is usually the most significant purpose in SCs, some cases in the real world don't follow this law.

For instance, some retailers always protest against the imposed policies of suppliers, but they also insist on keeping their cooperation. In other words, despite having considerably less profit or even in some cases losing huge sums, they keep their cooperation. What is the reason why they continue their relations?

This research aims to investigate this gap from a different perspective. Trust is as an umbrella term covering all the three flows of supply chain, goods, money and information, because if members do not trust each other, the interrelation between these three flows will certainly be lost.

Trust models are currently divided into two categories: methodological and mathematical methods. In the methodological models, based on cognitive perspectives, trust is built by normative aids and affected by the beliefs (Esfandiari and Chandrasekharan, 2001). The mathematical models not only depend on beliefs but also they are the result of a pragmatic game with practical functions, probabilities and evaluations of previous interactions. Methodological models attempt to reproduce human reasoning mechanisms, and they present that winning trust is as important as being able to trust. Nevertheless, the mathematical models do not utilize this method for winning trust (Gambeta, 1990). During the last 15 years, a lot of computational trust models have been proposed, each of which uses various techniques for finding trust. Some of these models will be briefly discussed as follow:

The first trust model is the marsh model provided by Marsh in 1994 in which only the direct interactions are used and trust is classified into three types: basic, general and position trust (Marsh 1994). However, the most important computational trust model is Eigen which includes basic trust, distributed trust and saving trust (Kamvar, Schlosser and Garcia-Molina, 2003). Some other important computational trust models are Peer, Fire (Huynh, Jenning and Shadbolt, 2006) and Beta (Jøsang and Sanderud. 2003). Beta model is used in this study as a computational trust model which will be explained in the following sections.

1.2. Definitions of trust

There are several definitions for trust in literature. Trust can be defined by reliability, utility, availability, reputation, risk, confidence, quality of services and other concepts. Nevertheless, none of these concepts can accurately propose the definition of trust. Because the trust is an abstract concept, which combines many complicated factors (Mcknight and Chervany, 1996). Trust has gotten attention in several fields: psychology, sociology, economics, political science, anthropology and recently in wireless networks, Computer Science and e-Commence Communication (Hassan, Strisena and Landfeldt, 2008; Nguyen, Lamont and Mason, 2009). Each field approaches the problem with its own disciplinary lens and filters. For example, while sociologists tend to consider trust as relationship in nature, some psychologists consider it as a personal view/attribute (Lewies and Wigert, 1985). Social psychologists are more likely to propose trust as an interpersonal phenomenon whereas Economists are more inclined to consider trust as a rational choice mechanism to increase its own utility (Williamson, 1993).

Some basic definitions of trust are presented below:

"Trust is the quantity of probability that a person by having information about another one estimates the honesty of another one's operations without knowing about the results of the operation" (Gambetta 1990).

Oxford Dictionary: "confidence or rely on some features or characteristics of a person or organization, accept or give credit to a person or organization without considering evidences, honesty, integrity and loyalty".

"Having safe confidence and reliable dependence on people" (Staples and Webster, 2008).

The rest of the paper is organized as follows: next section provides an overview of the related literature; Section 3 focuses on definitions of the concept of trust; and Section 4 describes how the computational trust model is defined; Section 5 formulates the model and provides the results; Section 6 proposes a numerical study to investigate the model. Finally, managerial conclusions and some directions for further research are provided in Section 7.

2. Literature review

To the best of the authors' knowledge, no research has been conducted in the context of trust as a main factor in SCs to investigate the mathematical trust models in SCs with respect to Stackelberg game in newsvendor framework. Based on this gap in the literature, the following four main steps will be elaborated on. First, some papers that have developed and used mathematical trust models in other different context will be explored. Second, the literature on Newsvendor will be scrutinized. Then the Stackelberg game is reviewed, and finally, recent studies about Stackelberg game in newsvendor framework are explained.

2.1. Trust literature

Trust models are very heterogeneous. This heterogeneity depends on many factors such as the trust definitions or application domains.
In some of the literature on SCs, the impact and role of trust have been examined in terms of collaboration at the strategic level (Akkermans, Borgerd and Van Doremalen, 2004; Panayides and Lun, 2009). However, few studies consider trust at the operational or managerial level and as a mathematical model. Kwon and Suh (2004) investigated the relationship between trust and commitment using empirical testing. Moore (2006) studied the role of trust in logistics alliances, and empirically tested it by using a sample of logistics alliances. Hung and Wang (2011) provided a model on Taiwan industry and indicated that sharing information and coordination based on trust led to remarkable reduction in unreliability and increase in performance of the SC.

Building the buyer-seller relationship should be grounded on trust. A meta-analysis of research on trust in a business context found that the concept of trust involves reliability as well as willingness and intention to act (Castaldo, Premazzi and Zerbini, 2010). Trust is a formidable element between buyers and sellers in a long time. In this regard two trust models were developed over the time by

Day et al. (2013). As a result of these models the trust within members depends on unreasonable allocation of sources, upgrade skills, information sharing empathy and reliability.

The trust in Computer Science derives from the concept in sociological, psychological and economical environments. The definition of trust is not unique and it may vary depending on the context and the purpose. Nguyen, Zhao and Yang (2010) integrated objective and mental trust in services of an on line network. This model proposed limitations based on trust. As a result, according to the numerical example, customers can choose the best option in service network. Vivekananth. (2011) considered an advanced form of calculating trust for the security of on line networks. He developed a trust model based on appropriate selections under reputation. Therefore, the nodes were selected based on the level of trust. The summaries of trust literature are shown in table 1.

Table 1. Summary of the models are reviewed in trust field

paper	*Information*		*Model*		*Context*		
			Computational				
	Asymmetric	Symmetric	Simulating and Stochastic	Game Theory	Business Relations	e-Commerce Communication	Computer Science
Akkermans, Bogerd and Van Doremalen (2004)		✓				✓	✓
Day et al. (2013)	✓			✓	✓		
Castaldo, Premazzi and Zerbini (2010)	✓				✓		
Hung and Wnag (2011)		✓	✓		✓		
Kwon and Suh (2006)		✓			✓		
Moore (2006)	✓				✓		
Neguen, Lamont and Mason (2010)	✓		✓				✓
Panayides (2009)		✓				✓	✓
Vivekananth. (2011)	✓		✓			✓	✓

2.2. Newsvendor problem

The newsvendor problem is one of the classical problems in the literature on inventory management. Key insights stemming from an analysis of this problem have wide ranging implications for managing inventory decisions for organizations in, for example, the hospitality, airline, and fashion goods industries. In the classic model, preparing products is done once at the beginning of a single period and the retailer determines the optimal order quantity and the supplier specifies the optimal wholesale price. (Khouja, 1999).

2.3. Stackelberg game

When we confront a lot of economic, social, political and military issues, we often have to analyze opposite situations. In fact, game theory is a mathematical theory for conflicting or competitive situations. This theory provides rational solutions to optimize decision making (Shakuri and Menhaj, 2008).

Stackelberg game is a kind of game theory which analyzes interactions among leaders and followers which was proposed by Von Stackelberg in 1934. In classic problems, the follower determines his optimal strategy, then the leader determines his optimal strategy based on the best optimal strategy of the follower (Romp, 1997). If the optimal strategies are accepted by the leader and the follower, the game will be over. Otherwise, the game continues until the sides are satisfied (Petruzzi, 1999). Although this game seems to be easy, the payoff matrix can be considerably more difficult to analyze mathematically (Binmore,1998 and Shubik, 2006).

2.4. Newsvendor literature

Most typically, subjects anchor their order quantities to the expected demand in cases where maximal profits could be achieved by ordering much more than the expected demand (if product has a high profit margin, i.e., shortage is relatively more costly than excess) or less than the expected demand (if product has a low profit margin). Qin et al. (2011) call for more studies of the newsvendor problem under both stochastic demand and stochastic supply. In general, uncertain supply either related to procurement (order) or production (production batch order) has been discussed in the inventory management literature. Stochastic yield has been considered in the newsvendor setting for both single and multi-supplier case. For example, Keren and Piliskin (2006) presents how stochastic yield impacts supply chain coordination. Lau, Hasija and Bearden (2014) analyze two newsvendor experiments and find that less than half of subjects suffer from pull-to-center bias. They conclude that there can be significant differences between the subjects within one experiment and urge researchers not to make too strong conclusions from averages across experiments.

In the SCs, which don’t follow the classic law in SCM, members seem to be more leader-follower. It means these SCs face a Stackelberg game. Trust intensifies the effect of leader-follower

relationships because the follower insists on having cooperation under any conditions (advantage or disadvantage).
Li, Li and Cai (2012) offered a SC where the retailer faces a stochastic demand and orders from the supplier, while the supplier manufactures new products and also remanufactures return to meet the order. They used Stackelberg game, where the manufacturing quantity is determined by the supplier after realizing the order quantity from the retailer. They attained that in the Stakelberg game the order and manufacturing quantities are larger than in the inaccessible return information case, and the profits for the supplier and retailer are also higher. Computational results are reported to show the effects of system parameters.

The newsvendor model is probably the most celebrated model in inventory literature and is applied extensively to investigate inventory centralization, cooperation and coordination.

Lin et al. (2010) considered the newsvendor SC problem which was formulated as a Stackelberg game: the manufacturer is the leader who designs a contract or a trade term, and the retailer was the follower who determines the order quantity and selling price. In addition, the perishable retailing problem was formulated as a two-period inventory system. They developed profit of maximization models by taking into account the return policy and quantity discount that are offered by the manufacturer to the retailer. With properly designed contracts, the inefficiency caused by double marginalization can be completely eliminated and, as a result, the SC is coordinated. Devangan et al. (2012) considered a SC coordination problem when demand faced by a retailer is influenced by the amount of inventory displayed on the retail shelf. The goal in this research was to design individually rational contracts that coordinated the SC when the retailer faced inventory-level-dependent demand. They presented a buyback contract where any leftover inventory at the retailer could be returned to the supplier at a pre-specified terms of the buyback contract. They provided managerial insights into the design of the contracts and analyzed the impact of shelf space inventory on the contract parameters. The summeries of newsvendor literature are shown in table 2.

The authors try to to investigate theses aims as follow: First, we try to study a reasonable trust model (Beta model) to calculate amount of trust that is adaptable in SCs. Second, some criteria which to win trust are proposed and for both of the retailer and the supplier a criterion is considered. Finally, a two-echelon SC including a supplier and a retailer with stochastic demand in newsvendor framework is presented. The members play Stackelberg game in which the supplier is the leader and the retailer is the follower. In addition, trust affects the stock level of supplier and the order quantity of retailer and decreases both of them. What’s more, the optimal order quantity of retailer is the demand of supplier.

Table 2 . Summary of the models are reviewed in supply chains field

Paper	*Stochastic Supply Chain*		*Model*			*Supply chain dimensions*		*Newsvendor problem*	
				Steckelberg game					
	Other Component	*Demand*	*Others*	*Retailer Leader*	*Supplier Leader*	*Multi-echelon*	*Two-echelon*	*Number of retailer*	*Number of supplier*
Devangan et al. (2012)		✓			✓		✓	1	1
Keren (2009)	✓	✓	✓				✓	1	1
Lau, Hasija and Bearden (2014)	✓	✓	✓			✓		1	1
Li, Li and Wang (2012)	✓				✓		✓	1	1
Lin, Chen and Chiang (2010)	✓				✓		✓	1	1
Qin et al. (2011)	✓	✓	✓				✓	1	1
Wang and Chen (2013)	✓				✓		✓	1	1
Xu, Wei and Jun (2012)	✓	✓			✓		✓	1	1

3. Beta model

The most important component of a probabilistic trust model is the treatment of model to estimate probabilistic future of outputs. Beta model was provided by Jøsang ans Sanderud (2003). According to this model, the trustee's behavior is a random variable having specific probability distribution. Outputs are successes and failures

3.1. Parameters of Beta model

α_{pr} Validation parameter of trustee having prior mixed beta distribution.

β_{pr} Validation parameter of trustee having prior mixed beta distribution.

α_{post} Parameter of post beta distribution.

β_{post} Parameter of post beta distribution.

h Bernoulli sequence from specific number of interactions including cooperation and none-cooperation for trustee.

$N_{SR}(h)$ Number of cooperation for trustee in specific transactions.

$N_{fR}(h)$ Number of none-cooperation for trustee in specific transactions.

$N_R(h)$ Number of total interacts.

Θ_{te} Bayesian parameter estimation.

ϑ_{te} Probability of success in a transition.

o_i Output including cooperation or defect $i = 1 \dots n$.

k Number of successes (cooperation).

We have a sequence of successes and failures in which the sequence is estimated by Bayesian parameter estimationΘ_{te}. The sequence can be suggested by a probabilistic distribution ϑ_{te}that is the probability of success in a transition. Under the assumption, $h = o_1 \dots o_n$ is a sequence of outputs considered as a Bernoulli trial, and successes follow binomial distribution. (Gelman and Buchwald, 2003).

$$P(k\ successes) = \binom{n}{k} \vartheta_{te} (1-\vartheta_{te})^{n-k} \tag{1}$$

Taking $p(\vartheta_{te})$ from the beta family, a two-parameter class of distributions can be expressed as follow: (Casella et al. 2003)

$$f\,(\vartheta_{te} \left| \alpha_{pr}, \beta_{pr} \right.) = \frac{\Gamma(\alpha_{pr} + \beta_{pr})}{\Gamma(\alpha_{pr})\Gamma(\beta_{pr})} \vartheta_{te}{}^{\alpha_{pr}-1} (1-\vartheta_{te})^{\beta_{pr}-1} \tag{2}$$

If $f(\vartheta_{te}|\alpha_{pr}, \beta_{pr})$ is selected as a prior distribution, post distribution of ϑ_{te}will be a binomial distribution with $(\alpha_{post}, \beta_{post})$ parameters. Thus prior parameters and post parameters are related to each other as follow:

$$\begin{aligned} \alpha_{post} &= N_{sR}(h) + \alpha_{pr} \\ \beta_{post} &= N_{fR}(h) + \beta_{pr} \end{aligned} \tag{3}$$

$N_{sR}(h), N_{fR}(h)$ are the cooperation and the defect in h sequence respectively. Trust is the expected value of the post distribution (Jøsang and Sandrud, 2012).

$$TRUST = E(\vartheta_{te}) = \frac{\alpha_{post}}{\alpha_{post} + \beta_{post}} \tag{4}$$

4. The proposed model

A relationship based on trust between two members of SCs includes the reliability of the two members and the ability of each member to create trust on the opposite side.

4.1. The assumptions

Assumption 1. Utility functions and order quantities are none-negative.
Assumption 2. The model uses exact optimization approach so it is limited to classic techniques.
Assumption 3. Trust is an exogenous parameter for the SC.
Assumption 4. Trust as a variable is multiplied in stock level for supplier and order quantitylevel for retailer. We assume that the trust decline the stock of the supplier and the order quantity of the retailer.
Assumption 5. $c < w < p$

c	Unit production cost of supplier
p	Retailer price
w	Unit wholesale price of supplier

When members of the SC deal, the rated parameters of deal should be specified. If these parameters are not exactly determined, every member will decide based on their opinion. Therefore, there are some criteria which create trust in a SC for both retailer and supplier in a transact. For instance, price, quality, delivery time, payment, unreasonable criticizing, public charity performances and customer services, etc. can be applied as a trust factor.

In this study, payment is a factor which creates trust for the supplier. In other words, the supplier relies on the retailer under specified conditions that is proposed in the contract. Delivery time is a factor the retailer relies on the supplier. Simply put, the retailer trusts the supplier when he delivers services on time.

4.2. Definitions of variables

D	Stochastic demand with normal distribution with mean μ, vaiance δ^2.
p	The retail price of retailer.
$\sqrt{Tr}$	Trust function.
Tr_{RS}	Trust of retailer to supplier.
Tr_{SR}	Trust of supplier to retailer.

h_R^+ Unit inventory cost (mitigated by salvage cost) is incurred for units left over at the end of period for retailer.

h_R^- Shortage cost per unit of unsatisfied demand is incurred for retailer.

w The unit wholesale price of supplier.

a Initial demand (fixed).

π_R Utility function of retailer.

$f(\xi)$ Distribution function of demand.

$F(\xi)$ Cumulative function of demand.

c The unit Production cost of supplier.

b Initial stock (fixed).

q_S Order quantity of supplier.

h_S^+ Unit inventory cost (mitigated by salvage cost) is incurred for units left over at the end of period for supplier.

h_S^- Shortage cost per unit of unsatisfied demand for supplier.

π_S Utility function of supplier.

4.3. decision variables

q_R Order quantity for retailer.

I_S Inventory quantity for supplier $I_s = \left(b - \sqrt{Tr_{SR}}\right)SS + q_r$.

SS Stock of supplier.

4.4. The retailer model

Normally, the customer is assumed to be independent of the supplier. It means that the customer does not have any information about the supplier. However, in some special positions, the customer may plan based on the supplier's inventory. Therefore, we assume the order quantity of retailer decreases by the stock of supplier and the trust.

The retailer model considers the following problem

$$D = a + \frac{m}{\sqrt{Tr_{RS}}SS} q_R + \xi \qquad 0 < \frac{m}{\sqrt{Tr_{RS}}SS} < 1 \qquad \xi \sim N(\mu, \delta^2) \tag{6}$$

Trust is assumed to decline the order quantity of the retailer. It seems to be correct because a lot of retailers order more than their needs in each ordering in the real cases.
Retailer utility function is:

$$Max\pi_R = Max\left\{E[\min(D,q_R) - h_R^+(q_R - D) - h_R^-(D - q_R)] - wq_R\right\} \tag{7}$$

$$Max\,\pi_R = (p-w)(a + \frac{m}{\sqrt{Tr_{RS}}SS} q_R + \mu) - (w + h_R^+) \int_0^{q_R - \left[a + \frac{m}{\sqrt{T_{RS}}SS} q_R\right]} F(\xi)d\xi$$

$$-(p + h_R^- - w) \int_{q_R - \left[a + \frac{m}{\sqrt{Tr_{RS}}SS}\right]}^{\infty} [1 - F(\xi)]d\xi$$

Since the retailer seeks maximum profit, the utility function should be concave. Therefore, at first, we calculate derivative of the utility function:

$$\frac{\partial \pi_R}{\partial q_R} = \frac{m}{\sqrt{Tr_{RS}}SS}(p-w) - [(w+h_R^+)(1-\frac{m}{\sqrt{Tr_{RS}}SS})F(q_R - [a + \frac{m}{\sqrt{Tr_{RS}}SS} q_R])]$$
$$+[(p + h_R^- - w)(1-\frac{m}{\sqrt{Tr_{RS}}SS})[1 - F(q_R - [a + \frac{m}{\sqrt{Tr_{RS}}SS}])] \tag{8}$$

Then the second derivative of the utility function is obtained as the follow:

$$\frac{\partial^2 \pi_R}{\partial q_R^2} = -[(w+h_R^+)(1-\frac{m}{\sqrt{Tr_{RS}}SS})^2 f(q_R - [a + \frac{m}{\sqrt{Tr_{RS}}SS} q_R])]$$
$$-[(p + h_R^- - w)(1-\frac{m}{\sqrt{Tr_{RS}}SS})^2 f(q_R - [a + \frac{m}{\sqrt{Tr_{RS}}SS} q_R])] < 0 \qquad concave \tag{9}$$

Finally, the optimal order quantity is acquired as bellow:

$$\frac{\partial \pi_R}{\partial q_R} = 0$$

$$-[(w+h_R^+)(1-\frac{m}{\sqrt{Tr_{RS}}SS})F(q_R - [a + \frac{m}{\sqrt{Tr_{RS}}SS} q_R])] -$$

$$[(p+h_R^- - w)(1-\frac{m}{\sqrt{Tr_{RS}}SS})F(q_R - [a + \frac{m}{\sqrt{Tr_{RS}}SS} q_R])] = -\frac{m}{\sqrt{Tr_{RS}}SS}(p-w) - (p+h_R^- - w)(1-\frac{m}{\sqrt{Tr_{RS}}SS})$$

$$F(q_R - [a + \frac{m}{\sqrt{Tr_{RS}}SS} q_R])[(p + h_R^- - w)(1 - \frac{m}{\sqrt{Tr_{RS}}SS}) + (w + h_R^+)(1 - \frac{m}{\sqrt{Tr_{RS}}SS})] =$$

$$\frac{m}{\sqrt{Tr_{RS}}SS}(p - w) + (p + h_R^- - w)(1 - \frac{m}{\sqrt{Tr_{RS}}SS})$$

$$F(q_R - \left[a + \frac{m}{\sqrt{Tr_{RS}}SS} q_R \right]) = \frac{\frac{m}{\sqrt{Tr_{RS}}SS}(p - w) + (p + h_R^- - w)(1 - \frac{m}{\sqrt{Tr_{RS}}SS})}{(p + h_R^- - w)(1 - \frac{m}{\sqrt{Tr_{RS}}SS}) + (w + h_R^+)(1 - \frac{m}{\sqrt{Tr_{RS}}SS})}$$

$$q_R^* = \frac{F^{-1}(\frac{\frac{m}{\sqrt{Tr_{RS}}SS}(p - w) + (p + h_R^- - w)(1 - \frac{m}{\sqrt{Tr_{RS}}SS})}{(p + h_R^- - w)(1 - \frac{m}{\sqrt{Tr_{RS}}SS}) + (w + h_R^+)(1 - \frac{m}{\sqrt{Tr_{RS}}SS})})}{(1 - \frac{m}{\sqrt{Tr_{RS}}SS})} \tag{10}$$

4.5. The supplier model

In fact the order quantity of the retailer is the demand of the supplier. The remarkable thing is that the demand is stochastic, but in the current models, the order quantity is considered deterministic. This assumption seems to be incorrect in the real world. Demand is stochastic; hence, the order quantity also should be practically random.

In the proposed model, order quantity is provided based on a crisp measure of a stochastic variable. The amount has confidence level=$1/2$ because it is an expected value. In other words, fifty percent is the probability that the order quantity to be higher than the expected value and faces shortage, and also fifty percent is the possibility which I will bet higher than the expected value and faces some extra quantity. Thus, q_r^* is not the order quantity for the supplier. In fact, it is the average of the order quantity of the supplier.
The supplier model is suggested as follow:

$$q_S = N(q_R^*, \delta^2) \tag{11}$$

It is assumed that trust function reduces the stock, so according to this assumption, the inventory of supplier is obtained as follow:

$$I_S = (b - K(Tr_{SR}))ss + q_S \tag{12}$$

The utility function of the supplier is:

$$Max\pi_S = Max\left\{\begin{matrix} E(w-c)\min((b-\sqrt{Tr_{SR}})SS, q_S) \\ -h_S^+((b-\sqrt{Tr_{SR}})SS-q_S)-h_S^+((b-\sqrt{Tr_{SR}})SS-q_S)-h_S^-(q_S-(b-\sqrt{Tr_{SR}})SS) \end{matrix}\right\} \tag{13}$$

$$\pi_S = (w-c)\left[\begin{matrix} \int_0^{(b-\sqrt{Tr_{SR}})} q_S f(q_S)dq_S + \int_{(b-\sqrt{Tr_{SR}})}^{\infty} (b-\sqrt{Tr_{SR}}) f(q_S)dq_S \\ -h_S^+ \int_0^{(b-\sqrt{Tr_{SR}})} (q_S-(b-\sqrt{Tr_{SR}})SS) f(q_S)dq_S - h_S^- \int_0^{(b-\sqrt{Tr_{SR}})} ((b-\sqrt{Tr_{SR}})SS)-q_S) f(q_S)dq_S \end{matrix}\right]$$

Because the supplier will seeks maximum profit, the utility function should be concave. The derivative of the supplier utility function is:

$$\begin{aligned} \frac{\partial \pi_S}{\partial SS} &= (w-c)\left[\int_{(b-\sqrt{Tr_{SR}})SS}^{\infty} (b-\sqrt{Tr_{SR}}) f(q_S)dq_S\right] \\ &-h_S^+(b-\sqrt{Tr_{SR}}) \int_0^{(b-\sqrt{Tr_{SR}})SS} f(q_S)dq_S + h_S^-(b-\sqrt{Tr_{SR}}) \int_{(b-\sqrt{Tr_{SR}})SS}^{\infty} f(q_S)dq_S \end{aligned} \tag{14}$$

Second derivative of the supplier utility function is:

$$\begin{aligned} \frac{\partial^2 \pi_S}{\partial SS^2} &= -\left[(b-\sqrt{Tr_{SR}})^2 f((b-\sqrt{Tr_{SR}})SS)\right](w-C) \\ &-h_S^-\left[(b-\sqrt{Tr_{SR}})^2 f((b-\sqrt{Tr_{SR}})SS)\right] - h_S^+\left[(b-\sqrt{Tr_{SR}})^2 f((b-\sqrt{Tr_{SR}})SS)\right] \quad Concave \end{aligned} \tag{15}$$

Optimal inventory of the supplier is:

$$\begin{aligned} \frac{\partial \pi_S}{\partial SS} &= (w-c)\left[\int_{(b-\sqrt{Tr_{SR}})SS}^{\infty} (b-\sqrt{Tr_{SR}}) f(q_S)dq_S\right] \\ &-h_S^+(b-\sqrt{Tr_{SR}}) \int_0^{(b-\sqrt{Tr_{SR}})SS} f(q_S)dq_S + h_S^-(b-\sqrt{Tr_{SR}}) \int_{(b-\sqrt{Tr_{SR}})SS}^{\infty} f(q_S)dq_S \end{aligned}$$

$$SS^* = \frac{F^{-1}\left(\frac{w-c+(b-\sqrt{Tr_{SR}})h_S^-}{w-c+h_S^+(b-\sqrt{Tr_{SR}})+h_S^-(b-\sqrt{Tr_{SR}})}\right)}{(b-\sqrt{Tr_{SR}})} \tag{16}$$

$$I_S^* = F^{-1}\left(\frac{w-c+(b-\sqrt{Tr_{SR}})h_S^-}{w-c+h_S^+(b-\sqrt{Tr_{SR}})+h_S^-(b-\sqrt{Tr_{SR}})}\right) + q_S^* \tag{17}$$

If optimal amounts are accepted by two sides, the game will be over, and if the optimal amounts are not accepted, the game will be continued.

5. A numerical example for solving the model

In this section, the model is solved by a numerical example. The default values of parameters for solving the model are presented in table 3.Since trust is considered as a variable parameter, the decision variables and utility functions for both supplier and retailer are calculated by different values of trust. This model is solved by Matlab and is results are presented in table A4 (see appendix A).

5.1. Retailer

The surge in retailer's trust enhances order quantity. This rise is non-linear and ascending Figure 1. The utility function has irregular trend. First, it is ascending and then linear in a small interval. Overall, it has a downward trend Figure 2.

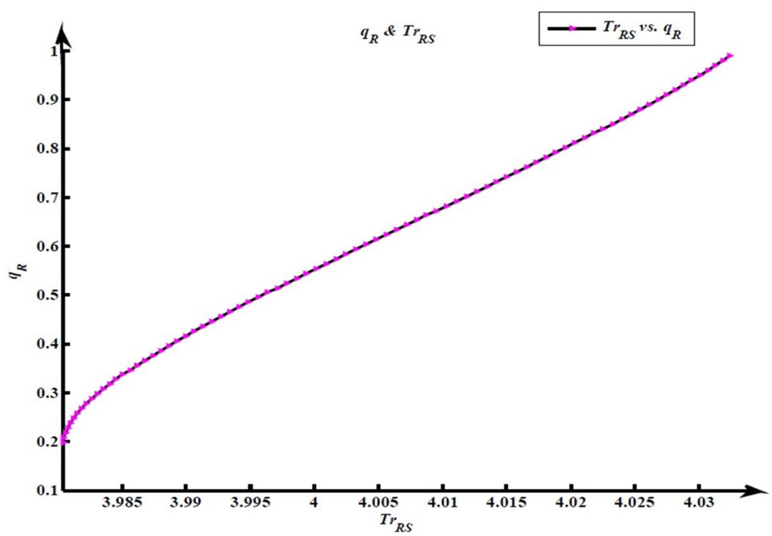

Figure 1. trust vs. Order quantity of retailer

Table 3.default value of parameters

P	h_R^+	h_R^-	w	a	C	h_S^+	h_S^-	μ	δ^2	$N_{CR}(h)$	$N_R(h)$	$N_{SR}(h)$	N_S	α_{pr}	β_{pr}
20	2	2	18	5	10	1	1	0	1	1	200	2	200	1	1
20	2	2	18	5	10	1	1	0	1	3	200	4	200	1	1
20	2	2	18	5	10	1	1	0	1	5	200	6	200	1	1
20	2	2	18	5	10	1	1	0	1	7	200	8	200	1	1
20	2	2	18	5	10	1	1	0	1	.	200	.	200	1	1
20	2	2	18	5	10	1	1	0	1	.	200	.	200	1	1
20	2	2	18	5	10	1	1	0	1	.	200	.	200	1	1
20	2	2	18	5	10	1	1	0	1	.	200	.	200	1	1
20	2	2	18	5	10	1	1	0	1	.	200	.	200	1	1
20	2	2	18	5	10	1	1	0	1	.	200	.	200	1	1
20	2	2	18	5	10	1	1	0	1	.	200	.	200	1	1
20	2	2	18	5	10	1	1	0	1	.	200	.	200	1	1
20	2	2	18	5	10	1	1	0	1	195	200	196	200	1	1
20	2	2	18	5	10	1	1	0	1	197	200	198	200	1	1
20	2	2	18	5	10	1	1	0	1	199	200	200	200	1	1

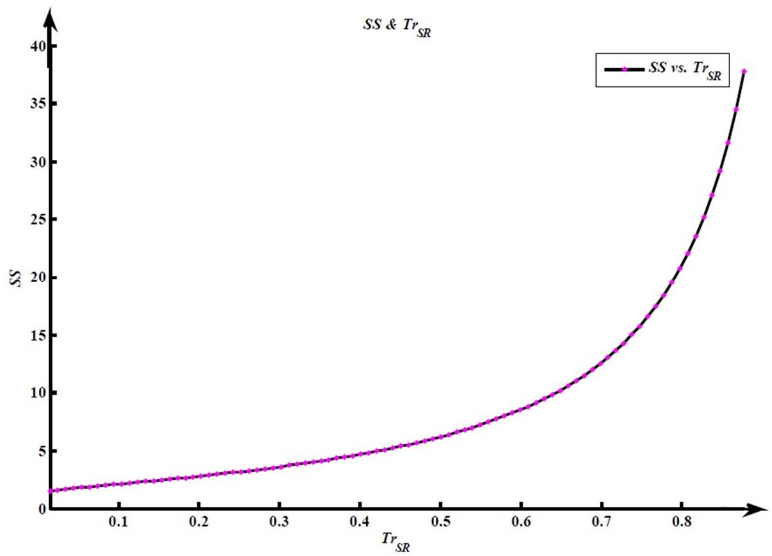

Figure 2. Retailer utility function vs. trust

5.2. Supplier

The Stock, inventory and utility functions of the supplier have raised by increasing trust ascend Figure 3, 4 and 5.

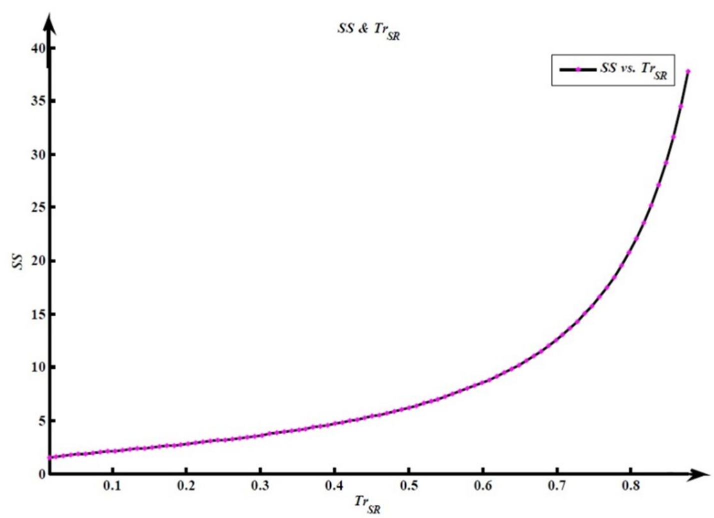

Figure 3. Supplier stock vs. trust

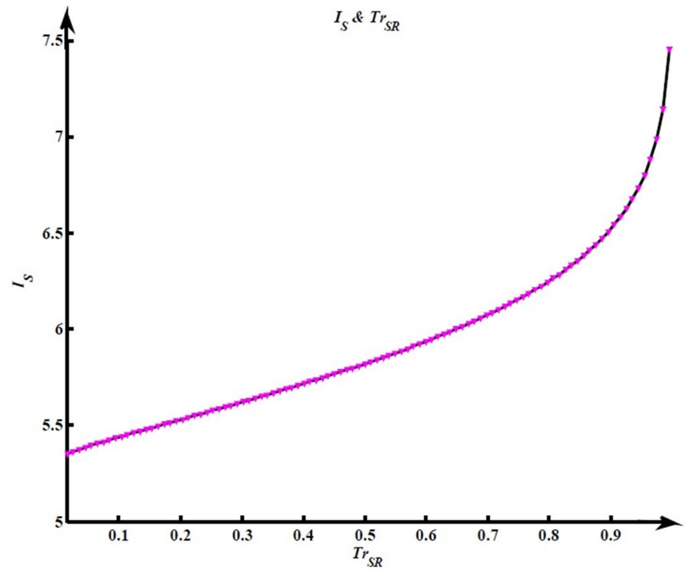

Figure 4. Supplier inventory vs. trust

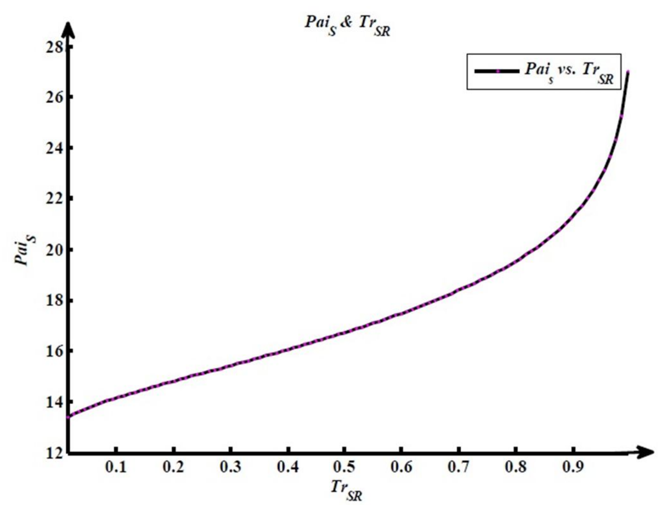

Figure 5. Supplier utility function vs. trust

6. CONCULSION

In this research, we proposed a mathematical trust model in the SC. We found several components such as price, quality, delivery time, payment, after sales services, unreasonable criticizes and public charity performance to create trust for both retailer and supplier. This research assumes that the supplier gains trust when the retailer pays financial bills according to their contract. The retailer also trusts when the supplier sends its orders without delay. We assumed trust as a variable parameter which was computed by Beta model. The Beta model was the most compatible with the computational trust model for calculating trust. Trust reduces stock and order quantity in demand function. Finally, a numerical example was solved to investigate the efficiency of the model. In conclusion, we found that more trust made the stock grew considerably and according to the model optimal order quantity depends on stock. In fact, non-linear and ascending approach of stock causes the optimal order quantity to increase as well. The retailer utility function had generally a downward trend. Simply put, the retailer did not achieve profit in this process. In the supplier model, where there is more trust, the utility function, inventory, and stock also enhance because these variables directly depend on each other. In other words, the supplier gains significantly more profit. To sum up, another flow was proposed to the SC that may have led to more profit and as a facilitator can cut extra costs and accelerate operations in SCs.

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Appendix A

Table A4. Results of solving example

Tr_{SR}	Tr_{RS}	q_R	SS	I_S	π_S	π_R
0.014851	0.009901	4.011542584	1.526774156	5.352253858	13.38953388	12.17819576
0.024752	0.019802	4.004249663	1.6132429	5.363682358	13.5126767	12.21626673
0.034653	0.029703	3.999317456	1.689779008	5.374536646	13.61772577	12.23450103
0.044554	0.039604	3.995612247	1.761025135	5.384921493	13.71217453	12.24448481
0.054455	0.049505	3.992691168	1.829102424	5.39495991	13.79952331	12.25022331
0.064356	0.059406	3.990325931	1.895196904	5.404738477	13.88173824	12.25354752
0.074257	0.069307	3.988380403	1.960057061	5.414317686	13.96005358	12.25542377
0.084158	0.079208	3.986765609	2.024196311	5.423741154	14.0353045	12.25640867
0.094059	0.089109	3.985419579	2.087989557	5.433041407	14.10808761	12.25684174
0.10396	0.09901	3.984297085	2.151724537	5.442243445	14.17884711	12.25693826
0.113861	0.108911	3.983363911	2.215631417	5.451367001	14.24792475	12.25683818
0.123762	0.118812	3.98259342	2.279900939	5.460428007	14.31559048	12.25663376
0.133663	0.128713	3.981964392	2.344696128	5.469439595	14.3820624	12.25638609
0.143564	0.138614	3.981459594	2.410160156	5.478412782	14.44752006	12.25613528
0.153465	0.148515	3.981064803	2.476421826	5.487356963	14.51211372	12.25590706
0.163366	0.158416	3.980768114	2.543599513	5.49628027	14.57597095	12.25571712
0.173267	0.168317	3.980559447	2.611804082	5.505189839	14.6392015	12.25557411
0.183168	0.178218	3.980430173	2.681141105	5.514092009	14.70190088	12.25548161
0.193069	0.188119	3.980372842	2.751712589	5.522992479	14.7641531	12.25543964
0.20297	0.19802	3.980380964	2.823618362	5.53189643	14.82603279	12.25544562
0.212871	0.207921	3.980448847	2.896957209	5.540808621	14.88760687	12.25549516
0.222772	0.217822	3.980571468	2.971827827	5.549733466	14.94893587	12.25558256
0.232673	0.227723	3.980744363	3.048329652	5.558675096	15.01007495	12.25570124
0.242574	0.237624	3.980963549	3.126563592	5.567637414	15.07107484	12.25584401
0.252475	0.247525	3.981225449	3.20663269	5.576624135	15.13198248	12.25600331
0.262376	0.257426	3.981526842	3.288642748	5.585638822	15.19284164	12.25617136
0.272277	0.267327	3.981864812	3.372702915	5.594684922	15.25369343	12.25634027
0.282178	0.277228	3.982236709	3.458926264	5.603765786	15.31457669	12.25650218

Appendix A

Table A4. Results of solving example

Tr_{SR}	Tr_{RS}	q_R	SS	I_S	π_S	π_R
0.292079	0.287129	3.982640117	3.547430365	5.612884697	15.37552835	12.25664926
0.30198	0.29703	3.983072829	3.638337857	5.62204489	15.43658375	12.25677382
0.311881	0.306931	3.983532816	3.731777042	5.631249567	15.49777693	12.25686832
0.321782	0.316832	3.984018214	3.827882496	5.64050192	15.55914082	12.25692539
0.331683	0.326733	3.984527302	3.926795712	5.649805139	15.62070749	12.25693788
0.341584	0.336634	3.985058489	4.028665775	5.659162432	15.6825083	12.25689884
0.351485	0.346535	3.9856103	4.133650092	5.668577037	15.74457413	12.25680155
0.361386	0.356436	3.986181363	4.241915159	5.678052232	15.80693546	12.25663952
0.371287	0.366337	3.986770403	4.353637405	5.687591353	15.86962258	12.2564065
0.381188	0.376238	3.987376229	4.469004096	5.697197802	15.93266571	12.25609646
0.391089	0.386139	3.987997727	4.58821432	5.706875058	15.9960951	12.25570362
0.40099	0.39604	3.988633855	4.711480066	5.716626695	16.05994117	12.25522241
0.410891	0.405941	3.989283633	4.839027402	5.726456389	16.12423468	12.25464753
0.420792	0.415842	3.989946144	4.971097773	5.736367933	16.18900674	12.25397386
0.430693	0.425743	3.99062052	5.107949428	5.746365251	16.25428904	12.25319654
0.440594	0.435644	3.991305947	5.249858998	5.756452409	16.32011391	12.25231092
0.450495	0.445545	3.992001654	5.397123239	5.766633632	16.38651443	12.25131257
0.460396	0.455446	3.992706911	5.550060977	5.77691332	16.45352458	12.25019727
0.470297	0.465347	3.993421029	5.709015258	5.78729606	16.52117938	12.24896101
0.480198	0.475248	3.994143353	5.874355756	5.797786648	16.58951498	12.2476
0.490099	0.485149	3.994873261	6.046481461	5.808390106	16.65856883	12.24611065
0.5	0.49505	3.995610162	6.225823683	5.8191117	16.72837982	12.24448956
0.509901	0.50495	3.996353492	6.412849442	5.829956968	16.79898844	12.24273356
0.519802	0.514851	3.997102714	6.608065265	5.840931737	16.87043695	12.24083966
0.529703	0.524752	3.997857315	6.812021481	5.852042154	16.94276954	12.23880505
0.539604	0.534653	3.998616803	7.025317083	5.863294711	17.01603257	12.23662714
0.549505	0.544554	3.99938071	7.248605231	5.874696282	17.09027476	12.23430353
0.559406	0.554455	4.000148583	7.482599519	5.886254151	17.16554742	12.231832

Appendix A

Table A4. Results of solving example

Tr_{SR}	Tr_{RS}	q_R	SS	I_S	π_S	π_R
0.569307	0.564356	4.00091999	7.728081125	5.897976056	17.24190473	12.22921053
0.579208	0.574257	4.001694514	7.985906981	5.909870228	17.31940401	12.22643729
0.589109	0.584158	4.002471754	8.257019156	5.921945442	17.39810603	12.22351064
0.59901	0.594059	4.003251321	8.542455654	5.934211068	17.47807539	12.22042913
0.608911	0.60396	4.004032842	8.843362879	5.946677132	17.5593809	12.2171915
0.618812	0.613861	4.004815954	9.161010092	5.959354381	17.64209598	12.21379669
0.628713	0.623762	4.005600304	9.496806218	5.972254361	17.72629924	12.21024383
0.638614	0.633663	4.006385553	9.852319481	5.985389501	17.81207495	12.20653224
0.648515	0.643564	4.007171365	10.22930041	5.998773211	17.89951372	12.20266146
0.658416	0.653465	4.007957418	10.62970894	6.012419993	17.98871321	12.19863121
0.668317	0.663366	4.008743393	11.05574645	6.026345567	18.07977892	12.19444143
0.678218	0.673267	4.009528979	11.50989387	6.040567016	18.17282517	12.19009226
0.688119	0.683168	4.01031387	11.99495713	6.055102954	18.26797614	12.18558407
0.69802	0.693069	4.011097766	12.51412179	6.069973719	18.36536713	12.18091742
0.707921	0.70297	4.011880368	13.07101894	6.085201604	18.46514601	12.17609315
0.717822	0.712871	4.012661383	13.66980532	6.100811112	18.56747487	12.17111227
0.727723	0.722772	4.013440517	14.31526121	6.116829275	18.67253205	12.1659761
0.737624	0.732673	4.014217478	15.01291096	6.133286016	18.78051442	12.16068616
0.747525	0.742574	4.014991975	15.76917237	6.150214584	18.89164014	12.15524427
0.757426	0.752475	4.015763712	16.59154345	6.167652081	19.00615198	12.14965251
0.767327	0.762376	4.016532395	17.48883778	6.185640084	19.12432128	12.14391327
0.777228	0.772277	4.017297722	18.47148378	6.204225411	19.24645271	12.13802923
0.787129	0.782178	4.018059388	19.55190921	6.223461053	19.37289011	12.13200342
0.79703	0.792079	4.01881708	20.74504035	6.243407325	19.50402373	12.12583923
0.806931	0.80198	4.019570475	22.06895773	6.264133301	19.6402991	12.11954041
0.816832	0.811881	4.02031924	23.54576863	6.285718615	19.78222823	12.11311115
0.826733	0.821782	4.021063028	25.20278451	6.308255764	19.93040388	12.10655608
0.836634	0.831683	4.021801472	27.07413476	6.331853066	20.08551771	12.09988035

Appendix A

Table A4. Results of solving example

Tr_{SR}	Tr_{RS}	q_R	SS	I_S	π_S	π_R
0.846535	0.841584	4.022534187	29.20301721	6.356638536	20.24838415	12.09308967
0.856436	0.851485	4.02326076	31.64489767	6.382765026	20.4199717	12.08619035
0.866337	0.861386	4.023980747	34.47215881	6.410417178	20.6014453	12.07918947
0.876238	0.871287	4.024693666	37.7810234	6.439821004	20.79422422	12.0720949
0.886139	0.881188	4.025398985	41.70215929	6.471257398	21.0000634	12.06491549
0.89604	0.891089	4.02609611	46.41746162	6.5050817	21.22117041	12.05766124
0.905941	0.90099	4.026784373	52.18764077	6.541752894	21.46037894	12.05034355
0.915842	0.910891	4.027462999	59.39966674	6.581878815	21.72141526	12.04297558
0.925743	0.920792	4.028131083	68.65292457	6.626289232	22.00932553	12.03557266
0.935644	0.930693	4.028787533	80.92650983	6.676160544	22.33119764	12.02815309
0.945545	0.940594	4.02943099	97.93282572	6.733243391	22.69746399	12.02073917
0.955446	0.950495	4.030059702	122.9528355	6.800316417	23.12446368	12.01335906
0.965347	0.960396	4.030671271	163.1359991	6.882206823	23.64010898	12.00605012
0.975248	0.970297	4.031262149	237.4593487	6.988530065	24.29876718	11.99886572
0.985149	0.980198	4.031826314	417.485281	7.143561229	25.23320907	11.99189273
0.99505	0.990099	4.032350057	1381.204018	7.455413605	26.99690237	11.98531952