Applying a CVaR measure for a stochastic competitive closed-loop supply chain network under disruption

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Abstract
This paper addresses a closed-loop supply chain network design problem, in which two different supply chains compete on retail prices by defining a price-dependent demand function. So, the model is formulated in a bi-level stochastic form to demonstrate the Stackelberg competition and associated uncertainties more precisely. Moreover, it is capable of considering random disruptions in the leader supply chain while incorporating the inventory, pricing, location and allocation decisions. Afterwards, having a contract with reliable suppliers is examined to resist the consequent results of disruption in the supply process. Additionally, the sharing strategy with new resilient distribution centers is used for tackling disruption risks at distribution centers. Furthermore, after integrating the proposed bi-level model into an integrated equivalent form by using the Karush–Kuhn–Tucker (KKT) transformation method, the conditional value at risk (CVaR) measure is used to handle the considered uncertainties. Finally, a real industrial case of a filter company is applied to obtain numerical results and the performance of the stochastic model is investigated by several test problems to arrive in helpful managerial insights.

Keywords: Closed-loop supply chain; Competition; Conditional value at risk; Disruption.

1- Introduction
The supply chain network design (SCND) is of great importance and can simply impact a company’s effectiveness and efficiency. It includes strategic decisions on the number, location, capacity and commission of the production–distribution facilities of a firm (Drezner, 1987). The suitable SCND causes an optimum structure that makes it easy to manage the chain efficiently. An integrated forward and reverse supply chain network is one of the main fields of the logistics network design. Based on the environmental, legal, social and economic factors, the reverse logistics and closed-loop supply chain network design (CLSCND) have received great attention among researchers (Khosrojerdi et al., 2016).

During recent years, different kinds of unpredictable events (e.g., acts of terrorism and natural disasters) have taken place showing that the world is increasingly becoming uncertain and vulnerable.
Moreover, it seems that supply chains are more fragile due to the plurality of industries, decentered production, reduction in the number of suppliers and focus on deduction of inventory. Although different industries have decreased supply chain costs, they make them open to risks and disruptions simultaneously (Li et al., 2010). Failures in a supply chain are unplanned events that disrupt the normal flow of products and materials. Consequently, the companies inside the supply chain become more susceptible to financial and operational risks (Li, Wang et al., 2010). While the CLSCND has gained great attention from researchers and practitioners during the last decades, most of the existing models in the literature ignore disruption risks when configuring the CLSCND.

Generally, most supply chain failures can be categorized in three groups in relation with supply, demand and other risks. Supply chain resilience is concerned with the system’s ability to return to its original state or to a new and more desirable one after experiencing a disturbance and avoiding failure occurrence. In other words, it is not only the ability to maintain the system control over performance variability when encountering disturbance, but also a property of being adaptive and capable of sustained response to sudden and significant shifts of the environment in the form of uncertain demands. Finally, it develops the researches by introducing a multi-period CLSCND model under both demand and supply uncertainty while incorporating pricing, inventory, location and allocation decisions in a competitive environment where two different supply chains compete on retail prices.

The remainder of this work is organized as follows. The first section includes an introduction to the CLSCND. Then in the next part, the related literature is reviewed. After that, the mathematical modeling of the problem is presented in the third section. The fourth section deals with the application of the model on a real filter industrial case by carrying out some sensitivity analyses. Then, the conclusion and future directions are examined in the last section.

2- Literature review

In this section, the related literature about the CLSCND problem and proposed models for mitigating the uncertainty and disruption risks is reviewed. The first attempt in the context of disruption risks in the SCND was made by Drezner (1987) in facility location problems. After that Fleischmann et al. (2001) were among the pioneer practitioners who focused on the integrated design of the logistics network and showed that the traditional approaches might bring cost saving (Fleischmann, Beullens et al., 2001). Then, Salema et al. (2007) contributed to the study considered the previous study and proposed a mixed-integer linear programming (MILP) model. However, the demand uncertainty and capacity limitations and variations were left for future research.

Sayarshad et al. (2010) presented a novel multi-objective model to optimize the fleet planning problem, which was examined through a numerical case example for representing the efficient solution procedure. Then, Pishvaee et al. (2010) proposed a probabilistic bi-objective MIP model for tackling uncertainties. They integrated strategic decisions of both the reverse and forward supply chain networks to prevent sub-optimal solutions.

Chen et al. (2011) proposed a location-routing network design model considering disruption with pre-defined probabilities. Then, for the first time, Javid et al. (2010) presented a new model for optimizing the strategic and tactical decisions in a stochastic supply chain. They assumed the demand to be uncertain for each customer and follow a normal distribution. O’Hanley et al. (2012) considered two models to design a reliable system for the network of facilities: maximal expected covering and unreliable p-center problem. It is assumed in both models that p facilities should be located to serve a set of customers and each facility’s failure is known through location-based probabilities. After that, Wang (2013) presented a novel model for considering the disruption risk and uncertainties. Moreover, a scenario-relaxation method was applied to solve a model. Proposing a multi-objective model for the CLSCND was another attempt in this scope done by Amin et al. (2013). They examined the CLSCND including factories, collection centers, demand nodes and products. Then, they proposed an MILP model for minimizing the associated total costs.

Ramezani et al. (2014) showed an application of fuzzy sets in order to design a multi-period multi-product CLSCND problem and considered three objective functions to maximize the total profit, minimize the delivery time and maximize the product quality, respectively. Their model was carried out by implementing the reliability theory. Qi et al. (2010) presented an integrated supply chain network model for optimizing the location of retailers and the customer allocations. They assumed that the single-period single-product supply chain might disrupt the supplier section or retailer levels.
Yadegari et al. (2015) proposed an integrated forward/reverse logistic model by considering three transportation modes, which were solved by applying a memetic algorithm.

New and novel approaches for making a flexible supply chain facing operational risks were examined by Esmaeili-Kia et al. (2016). After that Azad et al. (2013) considered the partial failures at distribution centers and considered the failure risks in both distribution centers and transportation paths. Ahmadi-Javid et al. (2013)) proposed a vehicle routing problem (VRP) for a supply chain including producer-distributors for delivering a type of product to customers. They assumed that the capacity of each set could change based on stochastic disruptions. Azad et al. (2014) designed a reliable supply chain network and considered the disruption risk at distribution centers and transportation modes. They applied the conditional value at risk (CVaR) measure to control the associated risk of the considered problem and solved it by utilizing a hybrid algorithm. Babazadeh et al. (2012) applied the CVaR risk measure for designing an integrated forward-reverse logistics network in the presence of uncertainties. Moreover, they proved the power of a stochastic model with the CVaR criteria in mitigating data uncertainties and managing the risk levels. Considering that competition on an integrated pricing-inventory model was another attempt made by (Rashid et al., 2015). They applied the queuing theory to tackle the uncertainty of delivering time and customer’s demand. In addition, Hatefi et al. (2015) proposed an integrated supply chain network design model to implement the reliability concept for examining the facilities failures. Their model was formulated in a multi-level single-product form. After that, Hasani and Khosrojerdi (2016) considered six resilience strategies for a global supply chain network under uncertainty formulated in a mixed-integer non-linear form. A new model for the closed-loop supply chain network design problem considering supply disruptions was also proposed by (Ghomi-Avili et al., 2017). They applied two resilience strategies for mitigating supply disruption, (1) using extra inventory and, (2) having a contract with reliable suppliers in the earlier periods. More recently, Jabbarzadeh et al. (2017) studied demand and supply uncertainties in a realistic production-distribution problem, which was dealt with an enhanced robustness approach.

The disruption and uncertainty effect on the performance of a supply chain, in which Schmitt et al. (2015) compared through two different centralized and decentralized bi-level models. On the other hand, Khosrojerdi et al. (2016) applied a robust optimization approach to consider the stochastic failures in a supply network. Ghomi-Avili et al. (2018) proposed a fuzzy bi-objective bi-level model with a price-dependent demand to design a closed-loop supply chain network in the presence of random disruptions at suppliers. Moreover, they considered the environmental issues by applying two strategies; adding a reverse flow and controlling the amount of CO2 emissions, respectively. Afterwards, Dehghani et al. (2018) considered a solar photovoltaic supply chain for designing a robust supply chain considering associated uncertainties by applying a set of technical, social and geographical criteria.

In addition, Naderi et al. (2017) proposed a bi-level model for designing a water supply network under stochastic environment, in which both the water and wastewater networks were integrated to derive better solutions. They solved the model by applying an accelerated Benders' decomposition method. Additionally, Jabbarzadeh et al. (2017) proposed another model for designing a green and resilient supply chain network and presented a new multi-objective optimization method for electricity supply chain networks considering economic, environmental and resilience issues. Due to the importance of unexpected disruptions in supply chain management, Ghavamifar et al. (2018) proposed a bi-objective model to design a resilient supply chain including suppliers, distribution centers, and retailers considering disruption risks. Then, (Jabbarzadeh et al., 2018) applied a Lagrangian relaxation method to solve their proposed stochastic robust optimization model. They studied lateral transshipment strategy to mitigate operational risks and probable disruptions.

This paper develops the literature by presenting a multi-period CLSCND model under demand and supply uncertainty while incorporating pricing, inventory, location and allocation decisions in a competitive environment, in which two different SCs compete on retail prices. Additionally, the presented model can consider both disruption and operational risks to design a CLSCN. Unlike common studies on the CLSC subject, our paper tries to find strategic and tactical decisions in an integrated form in order to prevent the probable sub-optimality. Here, two types of disruption (i.e., total and partial) are considered in supplier and distribution centers, respectively. Thus, having a contract with reliable suppliers is examined through this paper to resist the consequent results of
disruption in the supply process. Also, the sharing strategy with new resilient distribution centers is used for tackling disruption risks at distribution centers. Finally, this paper attempts to contribute to the existing literature by introducing a new stochastic bi-level model for modeling the CLSCND problem under the disruption risk in a competitive environment. To tackle the resulting complexities in bi-level modeling, the Karush–Kuhn–Tucker (K-K-T) optimality conditions are applied to integrate the inner problem with the master one. The impacts of considering competition and disruption in the context of the CLSCND under demand uncertainties and consequent improvements can be known as the primary goals of this paper as investigated in the following sections.

3- Problem statement

This paper considers a market, in which two competing SCs involve producing and delivering the same substitutable products. These two SCs do not have the same authority in the market and one of them is the leader referred to as SC1 and the other one known as SC2 is the follower. For considering environmental issues, SC1 has a reverse flow, in which the used products will be bought back and transferred to the collection centers to be examined and disassembled for reuse in the forward flow. Fig 1 depicts both the forward and reverse structure of SC1. Moreover, as it can be seen in the customers are divided into two different categories, including the new-product and second-hand customers, respectively.

The competition between the two SCs is considered uncooperatively, which means that each SC aims to maximize its own profit and market share given the competitor's actions. The Stackelberg game is applied to form the competition among SCs in a bi-level form.

Hence, the problem involves two upper and lower optimization levels. The upper level known as the master problem involves the optimal structure of SC1 and the lower level deals with the optimal decisions of SC2 (as the follower). In addition, the demand function is considered to reflect the customer's reaction to the proposed final product retail prices by SC1 and SC2, respectively. Moreover, in this paper, disruption is considered along with competition in the CLSCND context. Therefore, the SC’s mechanism is first defined, and then the structure and type of disruption will be examined precisely.

Here, disruption will be examined by knowing the structure of the CLSCN. Two types of disruption occur in both supplier section and distribution level of the CLSCN. Suppliers will face total disruption and they will not be accessible after a disruption. Thus, having a contract with a reliable supplier is considered through this paper to resist the consequent results. Moreover, distribution centers will face partial disruption in which they will not be inaccessible but will lose some part of their capacity.

Fig 1. Structure of two competing SCs
It is assumed that the new distribution centers are reliable and resilient enough. Conversely, the existing distribution centers are not as strong as the new centers and they may disrupt. Then, when an existing distribution center faces disruption, the sharing strategy with reliable distribution centers can compensate for the lost capacity for delivering products to the customers. Therefore, the model will be formulated in a stochastic bi-level form. The other assumptions considered in this paper are as follows:

- All customer demands must be fulfilled and all of the returned products should be examined.
- Suppliers will face only total disruption.
- The existing distribution centers may face partial disruption.
- The lost capacity in the existing distribution centers follows a normal random distribution function with parameters $\mu_k$ (mean) and $\sigma_k^2$ (variance).
- The Stackelberg game is used for modelling the competition among SC1 and SC2.

Before formulating the competitive CLSCND model under random disruption, let us introduce the following notations used in this paper.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Index sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>$P_1$ Set of unreliable suppliers</td>
</tr>
<tr>
<td>$l'$</td>
<td>$P_2$ Set of recyclable units</td>
</tr>
<tr>
<td>$I$</td>
<td>$P_2$ Set of unrecyclable units</td>
</tr>
<tr>
<td>$K_1$</td>
<td>$L$ Set of collection/inspection centers</td>
</tr>
<tr>
<td>$K_2$</td>
<td>$O$ Set of recycling centers (outsourcing)</td>
</tr>
<tr>
<td>$K$</td>
<td>$G$ Set of disposal centers</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$T$ Time periods</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$S$ Set of scenarios</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>$\bar{tr}_{ij}$</td>
</tr>
<tr>
<td>$trp_{k_2k_i}$</td>
</tr>
<tr>
<td>$trd_{j,k}$</td>
</tr>
<tr>
<td>$trc_{\alpha,i}$</td>
</tr>
<tr>
<td>$cad_k$</td>
</tr>
<tr>
<td>$cap^F$</td>
</tr>
<tr>
<td>$\lambda_{s_{is}}$</td>
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<tr>
<td>$\pi_s$</td>
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<tr>
<td>$\pi_k$</td>
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<tr>
<td>$\alpha$</td>
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<tr>
<td>Parameters</td>
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<tr>
<td>------------</td>
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<tr>
<td>$ca_{i}'$</td>
</tr>
<tr>
<td>$B_p$</td>
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<tr>
<td>$cah_j$</td>
</tr>
<tr>
<td>$cap_j$</td>
</tr>
<tr>
<td>$pr_{c_2s}$</td>
</tr>
</tbody>
</table>

### Notes
- $\beta$: The sensitivity of each SC's demand with respect to its own retail price
- $\alpha_k$: Return fraction of used products from second-hand customers to inspection centers
- $\phi_2^k$: Fraction of raw material indirectly gained from recyclable units
- $L^L$: Lower bound for a price in SC1
- $U^L$: Upper bound for a price in SC1
- $S^L$: Upper bound for the quantity of final product shipped SC1 to new product customer
- $S^F$: Upper bound for the quantity of final product shipped SC2 to new product customer
- $D_t$: Total demand at time period $t$
<table>
<thead>
<tr>
<th>Decision variables</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{l,t}^L$</td>
<td>1, if a distribution center $k$ is located at candidate place by the Leader; 0, otherwise</td>
</tr>
<tr>
<td>$y_{l,t}^L$</td>
<td>1, if an inspection center $l$ is located at candidate place by the Leader; 0, otherwise</td>
</tr>
<tr>
<td>$W_{l,t}^L$</td>
<td>1, if the Leader SC contracts with a reliable supplier $l'$; 0, otherwise</td>
</tr>
<tr>
<td>$S_{j,i,ts}^L$</td>
<td>1, if the production center $j$ is allocated to unreliable supplier $i$ at time period $t$ under scenario $s$ for the Leader SC; 0, otherwise</td>
</tr>
<tr>
<td>$S_{j,i,ts}^{L-}$</td>
<td>1, if the production center $j$ is allocated to a reliable supplier $i'$ at time period $t$ under scenario $s$ for the Leader SC; 0, otherwise</td>
</tr>
<tr>
<td>$S_{k,i,ts}^L$</td>
<td>1, if the distribution center $k$ is allocated to a customer $c_1$ at time period $t$ under scenario $s$ for the Leader SC; 0, otherwise</td>
</tr>
<tr>
<td>$F_{k,ts}^L$</td>
<td>Quantity of final products shipped from distribution center $k$ ($K = K_1 \cup K_2$) to new-product customer $c_1$ at time period $t$ under scenario $s$ (Leader)</td>
</tr>
<tr>
<td>$F_{c,ts}^F$</td>
<td>Quantity of final products shipped from the follower supply chain to new-product customer $c_1$ at time period $t$ under scenario $s$</td>
</tr>
<tr>
<td>$F_{c,ts}^{LP}$</td>
<td>Quantity of raw material type $p$ shipped from unreliable supplier $i$ to production center $j$ at time period $t$ under scenario $s$ (Leader)</td>
</tr>
<tr>
<td>$F_{c,ts}^{LP}$</td>
<td>Quantity of raw material type $p$ shipped from reliable supplier $i'$ to production center $j$ at time period $t$ under scenario $s$ (Leader)</td>
</tr>
<tr>
<td>$F_{k,ts}^{L}$</td>
<td>Quantity of final products shipped from a new distribution center $k_2$ to existing distribution center $k_1$ at time period $t$ under scenario $s$</td>
</tr>
<tr>
<td>$F_{c,ts}^R$</td>
<td>Quantity of returned products shipped from second-hand customer $c_2$ to inspection center $l$ at time period $t$ under scenario $s$</td>
</tr>
<tr>
<td>$F_{k,ts}^{RP}$</td>
<td>Quantity of unrecyclable units shipped from inspection center $l$ to disposal center $g$ at time period $t$ under scenario $s$</td>
</tr>
<tr>
<td>$F_{c,ts}^{RP}$</td>
<td>Quantity of directly recyclable units shipped from inspection center $l$ to production center $j$ at time period $t$ under scenario $s$</td>
</tr>
<tr>
<td>$F_{c,ts}^{RP}$</td>
<td>Quantity of indirectly recyclable units shipped from inspection center $l$ to recycling center $o$ at time period $t$ under scenario $s$</td>
</tr>
<tr>
<td>$F_{k,ts}^{RP}$</td>
<td>Quantity of recycled units shipped from recycling center $o$ to production center $j$ at time period $t$ under scenario $s$</td>
</tr>
<tr>
<td>$F_{k,ts}^L$</td>
<td>Quantity of final products shipped from the distribution center $k_1$ to new-product customer $c_1$ at time period $t$ under scenario $s$ (Leader)</td>
</tr>
<tr>
<td>$D_{c,ts}^F$</td>
<td>Demand of customer $c_1$ satisfied by the follower at time period $t$ under scenario $s$</td>
</tr>
<tr>
<td>$D_{c,ts}^L$</td>
<td>Demand of customer $c_1$ satisfied by the leader at time period $t$ under scenario $s$</td>
</tr>
<tr>
<td>$P_{c,ts}^L$</td>
<td>Retail price for customer $c_1$ offered by the leader at time period $t$ under scenario $s$</td>
</tr>
<tr>
<td>$P_{c,ts}^F$</td>
<td>Retail price for customer $c_1$ offered by the follower at time period $t$ under scenario $s$</td>
</tr>
<tr>
<td>$PR_{j,ts}$</td>
<td>Quantity of production online $b$ at production center $j$ at time period $t$ under scenario $s$</td>
</tr>
</tbody>
</table>
### Decision variables

- \( F_{jkts}^L \): Quantity of final products shipped from production center \( j \) to distribution center \( k \) at time period \( t \) under scenario \( s \) (Leader)
- \( I_{kts} \): Total inventory (of final products) held at distribution center \( k \) at time period \( t \) under scenario \( s \)
- \( RP_{jts}^c \): Quantity of returned products by customer \( c \) at time period \( t \) under scenario \( s \)
- \( I_{jts}^p \): Total inventory (of raw material type \( p \)) held at production center \( j \) at time period \( t \) under scenario \( s \)

### Model formulation

#### 3-1- Model formulation

#### 3-1-1- Upper-level model (leader)

The proposed model for optimizing the structure of the leader supply chain is as follows:

\[
Z_i = \text{Max} \quad \sum_{k,c_1,t,s} \pi_x F_{kc_1ts}^L \cdot p_{c_1ts} - \left[ \sum_{s} \pi_s \left( \sum_{k,c_1,t} F_{kc_1ts}^L \cdot FC_{k_2}^L + \sum_{i} \sum_{j} j_{i}^L \cdot FC_{i_1}^L + \sum_{l} W_{i}^L \cdot HC_{i}^L + \frac{\sum_{j} F_{jts}^{LP} \cdot tr_{jj} + \sum_{j,k,t} F_{jks}^{LP} \cdot tr_{jk} + \sum_{j,k,t} F_{jts}^{LP} \cdot tr_{jk}}{\sum_{k,c_1,t} F_{kc_1ts}^L \cdot tr_{kci} + \sum_{j,t} F_{jts}^L \cdot tr_{ijkl} + \sum_{j,k,t} F_{jts}^L \cdot tr_{lgt}} \right) \right] \]

s.t.

1. \( \sum_{j} F_{jts}^{LP} \leq cah_1 \) \hspace{1cm} (\( \forall i \in I, \forall t \in T, \forall p \in P, \forall s \in S \))
2. \( \sum_{j} F_{jts}^{LP} \leq cah_i' \) \hspace{1cm} (\( \forall i' \in I', \forall t \in T, \forall p \in P, \forall s \in S \))
3. \( \sum_{i,p} F_{jts}^{LP} + \sum_{i,p} F_{jts}^{LP} + \sum_{o,p} F_{jts}^{LP} \leq cah_j \) \hspace{1cm} (\( \forall j \in J, \forall t \in T, \forall s \in S \))
4. \( \sum_{k} F_{kts}^{LP} \leq cah_j \) \hspace{1cm} (\( \forall j \in J, \forall t \in T, \forall s \in S \))
5. \( \sum_{j} F_{jts}^{LP} \leq cad_k \) \hspace{1cm} (\( \forall k \in K_1, \forall t \in T, \forall s \in S \))
6. \( \sum_{j} F_{jts}^{LP} \leq cad_k \) \hspace{1cm} (\( \forall k \in K_2, \forall t \in T, \forall s \in S \))
7. \( \sum_{c} F_{kc_1ts}^L \leq x_{k_2}^L \cdot cad_k \) \hspace{1cm} (\( \forall k \in K_2, \forall t \in T, \forall s \in S \))
8. \( \sum_{c} F_{kc_1ts}^L \leq x_{k_2}^L \cdot cad_k \) \hspace{1cm} (\( \forall k \in K_2, \forall t \in T, \forall s \in S \))
\[
\sum_{k \in K_t} F_{k,c_{jts}}^L + \sum_{k \in K_t} F_{k,c_{jts}}^R = D_{c_{jts}}^L
\]  
\[\forall c_i \in C, \forall t \in T, \forall s \in S \quad (9)\]

\[
D_{c_{jts}}^L + D_{c_{jts}}^R = D_t
\]  
\[\forall c_i \in C, \forall t \in T, \forall s \in S \quad (10)\]

\[
D_{c_{jts}}^L = d^L - \alpha p_{c_{jts}}^L + \beta p_{c_{jts}}^F
\]  
\[\forall c_i \in C, \forall t \in T, \forall s \in S \quad (11)\]

\[
R_{p_{c_{jts}}}^L = \alpha_R \times D_{c_{jts}}^L
\]  
\[\forall c_i = c_2, \forall t \in T, \forall s \in S \quad (12)\]

\[
\sum_{l \in L} F_{c_{jts}}^L \leq R_{p_{c_{jts}}}^L
\]  
\[\forall c_2 \in C_2, \forall t \in T, \forall s \in S \quad (13)\]

\[
\sum_{c_i} F_{c_{jts}}^R \leq c_{acj}
\]  
\[\forall i \in L, \forall t \in T, \forall s \in S \quad (14)\]

\[
\sum_{j \in J} F_{j_{kts}}^{RP} = \theta_p^0 \times \sum_{c_i} F_{j_{kts}}^{R}
\]  
\[\forall p_i \in P_i, \forall l \in L \quad (15)\]

\[
\sum_{c_i} F_{j_{kts}}^{RP} = \theta_p^0 \times \sum_{l \in L} F_{j_{kts}}^{R}
\]  
\[\forall p_i \in P_i, \forall o \in O \quad (16)\]

\[
\sum_{c_i} F_{j_{kts}}^{R} = (1 - \sum_{p_i} \theta_p^0 - \theta_p^0) \times \sum_{l \in L} F_{j_{kts}}^{R}
\]  
\[\forall g \in G, \forall t \in T, \forall s \in S \quad (17)\]

\[
\sum_{c_i} F_{j_{kts}}^{R} = \sum_{l \in L} F_{j_{kts}}^{RP} + \sum_{l \in L} F_{j_{kts}}^{RP} + \sum_{l \in L} F_{j_{kts}}^{R}
\]  
\[\forall j \in J, \forall p \in P \quad (18)\]

\[
I_{kts} = I_{k-1s} + \sum_{j \in J} F_{j_{kts}}^{L} - \sum_{j \in J} F_{j_{kts}}^{L}
\]  
\[\forall k \in K, \forall t \in T, \forall s \in S \quad (20)\]

\[
\sum_{j \in J} I_{j_{kts}} + F_{j_{kts}}^{LP} + F_{j_{kts}}^{LP} + F_{j_{kts}}^{RP} + F_{j_{kts}}^{RP} \leq PR_{j_{kts}}
\]  
\[\forall j \in J, \forall p \in P, \forall t \in T, \forall s \in S \quad (21)\]

\[
\sum_{j \in J} I_{j_{kts}} + F_{j_{kts}}^{LP} + F_{j_{kts}}^{LP} \leq PR_{j_{kts}}
\]  
\[\forall j \in J, \forall p \in P, \forall t \in T, \forall s \in S \quad (22)\]

\[
\sum_{k \in K_t} F_{k,j_{kts}}^{L} + (1 - \pi_k)S_{k,c_{jts}}^{L} \times cad_k \geq \sum_{c_i} F_{k,c_{jts}}^{L}
\]  
\[\forall k_i \in K_t, \forall w_i \in C, \forall t \in T, \forall s \in S \quad (23)\]

\[
W_i^{L} \leq \lambda S_{k_{c_{jts}}}^{L}
\]  
\[\forall i \in I, \forall i' \in I, \forall j \in J, \forall t \in T, \forall s \in S \quad (24)\]

\[
S_{i_{jts}}^{L} \leq W_i^{L}
\]  
\[\forall i' \in I', \forall j \in J, \forall t \in T, \forall s \in S \quad (25)\]

\[
F_{j_{kts}}^{LP} \leq M \times S_{j_{kts}}^{L}
\]  
\[\forall i \in I, \forall p \in P, \forall j \in J, \forall t \in T, \forall s \in S \quad (26)\]

\[
F_{j_{kts}}^{LP} \leq M \times S_{j_{kts}}^{L}
\]  
\[\forall i' \in I', \forall p \in P, \forall j \in J \quad (27)\]

\[
(x_{k_{c_{jts}}}, y_{k_{c_{jts}}}, S_{k_{c_{jts}}}, S_{k_{c_{jts}}}^{L}, W_{j_{c_{jts}}}^{L}) \in \{0, 1\}
\]  
\[(F_{j_{kts}}^{LP}, F_{j_{kts}}^{LP}, F_{j_{kts}}^{LP}, F_{j_{kts}}^{LP}, F_{j_{kts}}^{LP}, F_{j_{kts}}^{LP}, F_{j_{kts}}^{LP}, F_{j_{kts}}^{LP}, F_{j_{kts}}^{LP}) \in R^{+} \quad (28)\]

\[
F_{j_{kts}}^{LP}, F_{j_{kts}}^{LP}, F_{j_{kts}}^{LP}, F_{j_{kts}}^{LP}, I_{kts}^{L}, I_{j_{kts}}^{L}, R_{j_{kts}}^{LP}, R_{j_{kts}}^{LP}, R_{j_{kts}}^{LP}, R_{j_{kts}}^{LP}, R_{j_{kts}}^{LP}
\]  
\[\forall t \in T, \forall s \in S \quad (27)\]
The first objective function (1) maximizes the leader profit in the proposed CLSCND model. It consists of the total income subtracted by other associated costs. Constraints (2) and (3) state that all amounts of raw materials sent from suppliers must be equal to the received amount by distribution centers. Constraints (4) show that the total quantity of delivered products from suppliers, recycling and inspection centers should be equal or less than production centers’ capacity. Constraints (5) and (6) state the maximum capacity of production centers and admission capacity of distribution centers, respectively. Constraints (7) and (8) state the capacity limitations in each facility. Constraints (9) assure that all the new-product customer demands must be fulfilled through the leader’s distribution channels.

Constraints (10) ensure that the amount of new-product customer demand fulfilled by the leader and follower should be equal to the total market demand. Constraints (11) state that total demand of the new-product customer is directly and reversely sensitive to its rival and own retail prices, respectively. Constraints (12) represent the balance of returned products by second-hand customers. Constraints (13) assure that the total quantity of used product transferred from second-hand customers to inspection centers should be equal or less than the total returned products. Constraints (14) represent capacity limitation on inspection centers and Constraints (15)-(17) state that all returned products should be transferred to production, recycling and disposal centers. Therefore, constraints (18) show the balance relation among them. Constraints (19) show the flow balance for the inventory of unit type p at each time period.

Constraints (20) represent the flow balance for the inventory of final products in each time period. Constraints (21) and (22) ensure that the total inventory balance and capacity limitations in production centers. Constraints (23) ensure that the total amount of received final products from reliable distribution centers with the safe capacity of an unreliable distribution center should be equal or greater than the amount of delivered final products to the customer. Constraints (24) show the supply contracts facing disruption. Constraints (25) state that the suppliers can allocate to the production centers if a contract is made with reliable suppliers in advance. Constraints (26) and (27) ensure that raw materials can be sent from either the reliable or unreliable suppliers to the production centers if they are allocated at that time period. Constraints (28) state the binary conditions and non-negativity of decision variables.

3-1-2- Lower level model (follower)

The proposed model for optimizing the following decisions in the competitive market is as follows:

\[
Z_3 = \text{Max} \sum_{c_t, t, s} p_F^{c_t, t} \times D_{c_t, t}^{F} - \sum_{c_t, t, s} r_F^{c_t, t} \times TC^F
\]

s.t.

\[
\sum_{c_t, t} F^{c_t, t} \leq \text{cap}_F^t \quad (\forall t \in T, \forall s \in S)
\]

\[
F_{c_t, t}^F = D_{c_t, t}^F \quad (\forall c_t \in C, \forall t \in T, \forall s \in S)
\]

\[
D_{c_t, t}^F = d^F - \beta p_{c_t, t}^F + \alpha p_{c_t, t}^L \quad (\forall c_t \in C, \forall t \in T, \forall s \in S)
\]

\[
(F_{c_t, t}^F, p_{c_t, t}^F, D_{c_t, t}^F) \in R^+
\]

The objective function (29) maximizes the profit of the follower in the same market. Constraints (30) represent the capacity limitation in the follower supply chain. Constraints (31) assure that all the new-product customer’s demand should be fulfilled through the follower distribution channels. Constraints (32) state that the total demand of the new-product customer in the follower supply chain. Constraints (33) state the non-negativity of decision variables.
\section*{3-2- Linearization of the model}

The proposed MIP model formulated in a bi-level form incorporates non-linear objective functions. The nonlinearity of the objective functions is made by multiplying two continuous variables. Thus, the McCormick Envelopes method is applied to linearize the proposed model (McCormick, 1976; Vidal et al., 2001, Kolodziej et al., 2013). Here, the SC1 (as the market leader) will determine a special range for the selling price of his final products. Consequently, the follower will have another range for the selling price which should not violate the leader’s proposed price in the market. Then, the following constraints will be used for linearizing the non-linear terms of the objective functions:

\begin{align}
L^L & \leq p^L_{c,ts} \leq U^L & \quad (\forall c_i \in C_1, \forall t \in T, \forall s \in S) \\
L^F & \leq p^F_{c,ts} \leq U^F & \quad (\forall c_i \in C_1, \forall t \in T, \forall s \in S) \\
0 & \leq F^L_{k,ts} \leq SL & \quad (\forall k \in K, \forall c_i \in C_1, \forall t \in T, \forall s \in S) \\
0 & \leq D^F_{c,ts} \leq SF & \quad (\forall c_i \in C_1, \forall t \in T, \forall s \in S)
\end{align}

Finally, the following constraints should be added to both the leader and followers’ problems for linearizing them by means of the McCormick Envelopes method.

\begin{align}
U^L F^L_{k,ts} + S^L p^L_{c,ts} - S^L U^L & \leq M^L_{k,ts} \leq S^F p^F_{c,ts} + L^F F^L_{k,ts} & \quad (\forall k \in K, \forall c_i \in C_1, \forall t \in T, \forall s \in S) \\
L^F F^L_{k,ts} & \leq M^L_{k,ts} \leq U^F F^L_{k,ts} & \quad (\forall k \in K, \forall c_i \in C_1, \forall t \in T, \forall s \in S) \\
U^F D^F_{c,ts} + S^F p^F_{c,ts} - S^F U^F & \leq M^F_{c,ts} \leq S^F p^F_{c,ts} + L^F D^F_{c,ts} & \quad (\forall c_i \in C_1, \forall t \in T, \forall s \in S) \\
L^F D^F_{c,ts} & \leq M^F_{c,ts} \leq U^F D^F_{c,ts} & \quad (\forall c_i \in C_1, \forall t \in T, \forall s \in S)
\end{align}

\section*{4- Solution procedure}

The proposed bi-level stochastic model is hard to be solved. So, in this paper, the Karush–Kuhn–Tucker (KKT) optimality conditions are implemented to change the bi-level model into a single-level form. In should be stated that the KKT optimality conditions are applicable since the bi-level model follows convex programming principles. The associated non-linear constraints added by implementing the K-K-T conditions are then linearized by a suitable approach. After integrating the proposed model, some stochastic constraints exist which are mitigated by the application of the CVaR risk measure.

\section*{4-1- KKT transformation method}

As stated before, a bi-level stochastic MINLP model is applied to formulate the competition among the two SCs. In bi-level optimization problems, there are two optimization levels, which are known as leader and follower optimization levels, respectively. The bi-level programming problem is NP-hardness, and bi-level models are also possible to be non-convex problems while the upper and lower levels are convex. Thus, it is not usually easy to solve them (Sun et al., 2008). Therefore, here the K-K-T reformulation method is applied to represent the proposed model as an equivalent integrated one (Sinha et al. (2002)).

Considering the following maximization problem:

\begin{align}
\text{Max } Z & = f(x_1, \ldots, x_i) & \quad (42) \\
\text{s.t. } g_i(x_i) & \leq b_i & \quad (\forall i = 1, \ldots, m) & \quad (43)
\end{align}
The KKT optimality conditions are as follows:

1) \( \nabla f(x) = \sum_{i=1}^{m} \gamma_i \times \nabla g_i(x_i) \) \hspace{1cm} (44) 

2) \( \gamma_i \times (g_i(x_i) - b_i) = 0 \) \hspace{1cm} (\forall i = 1, \ldots, m) \hspace{1cm} (45) 

3) \( g_i(x_i) - b_i \leq 0 \) \hspace{1cm} (\forall i = 1, \ldots, m) \hspace{1cm} (46) 

4) \( \gamma_i \geq 0 \) \hspace{1cm} (\forall i = 1, \ldots, m) \hspace{1cm} (47) 

Then, the above constraints should be added to the upper level model.

**4.2- Linearization of the integrated model**

Some non-linear constraints are added to the upper-level problem by using the KKT optimality conditions. These constraints can be linearized by a suitable approach proposed by Grossmann et al. (1987). Therefore, a binary variable \( v_i \) and the following set of constraints are introduced by:

\[
\begin{align*}
\lambda_i - M v_i & \leq 0 \\
G_i(cv) - M (1 - v_i) & \leq 0 
\end{align*}
\] 

(48)

Afterwards, the non-linear constraints are replaced with the linearized form. Then, after solving the model by using the active constraints strategy, the active constraints should be written in an equality form.

**4.3- Applying risk measures for the proposed model**

As we know, decision making under uncertainty is usually involved with the expected value criterion. However, this criterion might not be suitable in situations with considerable variations in uncertain parameters. When the distribution functions of the uncertain parameters are known, different risk measures can be used in stochastic programming models (Govindan et al., 2017). In this paper, we examine a popular risk measure for the proposed stochastic CLSCND problem. Here, the CVaR measure is investigated, since it is suitably tractable. The well-known CVaR measure will be defined briefly. If the \( F_z(\cdot) \) shows the cumulative distribution function of a random variable \( z \), both VaR and CVaR at confidence level \( \alpha \) will be defined by (Rockafellar et al., 2000):

\[
\begin{align*}
\alpha - {\text{VaR}}(Z) &= \inf \{ t : \Pr( Z \leq t ) \geq \alpha \} \\
\alpha - {\text{CVaR}}(Z) &= \inf \{ t + \frac{1}{1-\alpha} E[ Z - t ] , \} : E[ Z - t ] , = M \Pr\{ Z - t, 0 \} 
\end{align*}
\] 

(49) \hspace{1cm} (50)

The CVaR measure has the following properties so it is a coherent convex risk measure and can be used later (Ahmadi-Javid and Seddighi, 2013). Moreover, we know that \( \alpha - {\text{VaR}}(Z) \) and \( C - {\text{VaR}}(Z) \) are defined for Normal distribution as follows (Rockafellar et al., 2002):

\[
\begin{align*}
\alpha - {\text{VaR}}(Z) &= E(Z) + \phi^{-1}(\alpha) \text{STD}(Z) \\
C - {\text{VaR}}(Z) &= E(Z) + \frac{\phi^{-1}(\alpha)}{\alpha} \text{STD}(Z) 
\end{align*}
\] 

(51)

With knowing the above formulation, stochastic constraints (23) can be rewritten by:
\[
\sqrt{\sigma_k^2 \left( S^L_{k,c,t} \right)} \leq \frac{\alpha}{\phi^{-1} (\alpha)} S^L_{k,c,t} [1 - \mu_k] + \frac{\alpha}{\phi^{-1} (\alpha)} \left\{ \sum_{k_2} F^L_{k_2,k,c,t} - \sum_{c_1} F^L_{k_2,k,c,t} \right\} \quad \forall k_i \in K_i, \forall c_l \in C_i, \forall t \in T, \forall s \in S
\] (52)

Constraints (52) conformed to a Second Order Cone Programming (SOCP) problem, which has the following general form with the standard Euclidean norm of the constraints:

\[
\text{Min } f^T x \\
s.t. \\
\|Ax + b\| \leq \delta_i^T x + B_i \\
i = 1, \ldots, m
\] (53)

When \((A, = 0, \forall i)\) the SOCP will change into a linear programming model and by setting \((\delta_i = 0, \forall i)\), it will change into a convex quadratic programming model. Finally, we will have the following constraint, which is tractable for solving each optimization software package:

\[
\sqrt{\sigma_k^2 \left( S^L_{k,c,t} \right)} \leq \frac{\alpha}{\phi^{-1} (\alpha)} S^L_{k,c,t} [1 - \mu_k] + \frac{\alpha}{\phi^{-1} (\alpha)} \left\{ \sum_{k_2} F^L_{k_2,k,c,t} - \sum_{c_1} F^L_{k_2,k,c,t} \right\}
\] (54)

5- Numerical example

The proposed model is applied to Sepanta Palayeh Pars Company which is an Iranian corporation involving the production and distribution of different filters. It is well-known as a pioneer company among his rivals in the market for some kind of products. Considering disruption and competition in an uncertain situation simultaneously is of high importance for the company. Therefore, implementing the proposed model on the real data leads to the following results.

5-1- Model validation

To validate the accuracy of the model, it is solved by the GAMS/ CIPLEX. The following results are achieved by solving the model. As it is obvious in Table 1, increasing the holding cost in each warehouse will reduce the total hold inventory in all periods, which assures the exactness of the proposed model and proves its accuracy.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Holding cost (per unit of product)</th>
<th>Total inventory hold in each period</th>
<th>Total hold inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>88.75</td>
<td>97.51</td>
</tr>
<tr>
<td>2</td>
<td>43</td>
<td>85.66</td>
<td>85.31</td>
</tr>
<tr>
<td>3</td>
<td>49</td>
<td>72.48</td>
<td>74.74</td>
</tr>
<tr>
<td>4</td>
<td>52</td>
<td>64.56</td>
<td>72.52</td>
</tr>
<tr>
<td>5</td>
<td>59</td>
<td>80.24</td>
<td>67.33</td>
</tr>
</tbody>
</table>
5-2- Computational results

To assess the performance of the proposed model, a computational study is carried out, and then the related results are reported in this section. As can be seen in Fig 2, by increasing the leader’s final product price coefficient, the total profit increases. So, the objective value improves gradually by increasing its own retail price coefficient to about 0.75. So, it is not profitable to increase the price coefficient more than 0.75 since the market share declines enormously.

Fig 3 reveals the impacts of considering disruption at the planning phase in the CLSCND problem and after the planning time. It can be noticed that considering disruption on the planning time is much more efficient than a corporation fails to consider disruption besides the other planning decisions. It also depicts that by considering disruption risks, the supply chain profit increased the customer satisfaction will increase that represents the improvement of the proposed closed-loop supply chain.

![Fig 2](image1.png)

**Fig 2.** Effect of changing the final product price coefficient in the demand function (Leader)

![Fig 3](image2.png)

**Fig 3.** Effects of considering the disruption

Table 2 represents the sensitivity of the market demand to the distribution centers risk level. To deal with the stochastic constraints defined in Section 4.4, different scenarios are generated with respect to a specified risk level to evaluate the number of lost sales for the leader in the proposed market. By decreasing the risk level, the total percentage of lost demand declines consequently.
Table 2. Results of computational experiments on the risk level

<table>
<thead>
<tr>
<th>Retailer’s risk preference level (1-α)</th>
<th>Retailer’s risk aversion level (α)</th>
<th>Lost demand (%)</th>
<th>Retailer’s risk preference level (1-α)</th>
<th>Retailer’s risk aversion level (α)</th>
<th>Lost demand (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.99</td>
<td>0.36400</td>
<td>0.50</td>
<td>0.50</td>
<td>0.23938</td>
</tr>
<tr>
<td>0.02</td>
<td>0.98</td>
<td>0.34427</td>
<td>0.60</td>
<td>0.40</td>
<td>0.23005</td>
</tr>
<tr>
<td>0.03</td>
<td>0.97</td>
<td>0.33819</td>
<td>0.70</td>
<td>0.30</td>
<td>0.22669</td>
</tr>
<tr>
<td>0.04</td>
<td>0.96</td>
<td>0.33503</td>
<td>0.80</td>
<td>0.20</td>
<td>0.21217</td>
</tr>
<tr>
<td>0.05</td>
<td>0.95</td>
<td>0.32516</td>
<td>0.90</td>
<td>0.10</td>
<td>0.20044</td>
</tr>
<tr>
<td>0.06</td>
<td>0.94</td>
<td>0.3252</td>
<td>0.91</td>
<td>0.09</td>
<td>0.19936</td>
</tr>
<tr>
<td>0.07</td>
<td>0.93</td>
<td>0.32831</td>
<td>0.92</td>
<td>0.08</td>
<td>0.19251</td>
</tr>
<tr>
<td>0.08</td>
<td>0.92</td>
<td>0.32562</td>
<td>0.93</td>
<td>0.07</td>
<td>0.18865</td>
</tr>
<tr>
<td>0.09</td>
<td>0.91</td>
<td>0.32416</td>
<td>0.94</td>
<td>0.06</td>
<td>0.18102</td>
</tr>
<tr>
<td>0.10</td>
<td>0.90</td>
<td>0.31628</td>
<td>0.95</td>
<td>0.05</td>
<td>0.18754</td>
</tr>
<tr>
<td>0.15</td>
<td>0.85</td>
<td>0.28533</td>
<td>0.96</td>
<td>0.04</td>
<td>0.17542</td>
</tr>
<tr>
<td>0.20</td>
<td>0.80</td>
<td>0.27761</td>
<td>0.97</td>
<td>0.03</td>
<td>0.17320</td>
</tr>
<tr>
<td>0.30</td>
<td>0.70</td>
<td>0.24148</td>
<td>0.98</td>
<td>0.02</td>
<td>0.17022</td>
</tr>
<tr>
<td>0.40</td>
<td>0.60</td>
<td>0.23596</td>
<td>0.99</td>
<td>0.01</td>
<td>0.16847</td>
</tr>
</tbody>
</table>

6- Conclusion
Nowadays disruptions and uncertainties have great impacts on the supply chains performance. This paper dealt with the closed-loop supply chain network design considering disruption under competition. It was assumed that two supply chains referred to as SC1 and SC2 are competing in the same market. Moreover, the Stackelberg game was used to model the competition among them more clearly. For reflecting the reaction of customers to the final price of the product, the demand was assumed to be a function of each supply chain and his rival’s selling prices. In this paper, the uncertainty and disruption risks were presented on both supply and distribution risks by having a contract with reliable suppliers and sharing strategy in the distribution centers. Here, total and partial disruptions were considered in suppliers and distribution centers, respectively. So having contract with reliable suppliers was examined to resist the consequent results for disruption in the supply process. Using the sharing strategy with new resilient distribution centers was used for tackling the disruption risks at distribution centers. Finally, this paper attempted to contribute to the existing literature by introducing a new stochastic bi-level model for modelling the closed-loop supply chain network design under risk of disruption in the competitive environment.

In order to examine the application of the proposed model in a real-world industrial case, data of an actual company in a filter industry was applied. Then, the proposed stochastic bi-level model was converted to an equivalent single-level model by using the K-K-T transformation method. Then the CVaR measure was implemented to mitigate the stochastic constraints added to model after implementing sharing strategy. Finally, some sensitivity analyses were carried out on the proposed model to evaluate its efficiency and derive some managerial insights.

Adding inventory management concepts to the model can be an important direction for the future research study. Additionally, developing exact or heuristic solution methods seem to be useful when both the problem size and the number of disruption scenarios increases.
References


