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Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing and backorder

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Abstract

In a supplier-retailer-buyer supply chain, the supplier frequently offers the retailer a trade credit of S periods, and the retailer in turn provides a trade credit of R periods to her/his buyer to stimulate sales and reduce inventory. From the seller's perspective, granting trade credit increases sales and revenue but also increases opportunity cost (i.e., the capital opportunity loss during credit period) and default risk (i.e., the percentage that the buyer will not be able to pay off her/his debt obligations). Hence, how to determine credit period is increasingly recognized as an important strategy to increase seller's profitability. Also, many products such as fruits, vegetables, high-tech products, pharmaceuticals, and volatile liquids not only deteriorate continuously due to evaporation, obsolescence and spoilage but also have their expiration dates. In this paper along with deterioration and expiration date, we consider shortages that are very rarely investigated by researches. Therefore, this paper proposes an economic order quantity model for the retailer where: (a) the supplier provides an up-stream trade credit and the retailer also offers a down-stream trade credit, (b) the retailer's down-stream trade credit to the buyer not only increases sales and revenue but also opportunity cost and default risk, (c) deteriorating items not only deteriorate continuously but also have their expiration dates and (d) there is a shortage allowed in each time period. We then show that the retailer's optimal credit period and cycle time not only exist but also are unique. Furthermore, we discuss several special cases including for non-deteriorating items. Finally, we run some numerical examples to illustrate the problem and provide managerial insights.

Keywords: Supply chain management, deteriorating items, expiration dates, trade credit, Backorder

1-Introduction

In practice, the seller usually provides to his buyer a permissible delay in payments to give interest to the buyer and reduce inventory. During the credit period, the buyer can accumulate the revenue and earn interest on the accumulative revenue. However, if the buyer cannot pay off the purchase amount during the credit period then the seller charges to the buyer interest on the unpaid balance. One of the first studies

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on this issue is Goyal (1985). This paper established the retailer's optimal economic order quantity (EOQ) when the supplier offers a permissible delay in payments. On the other hand, Shah (1993) then considered a stochastic inventory model for deteriorating items when delays in payments are permissible. Later, Aggarwal and Jaggi (1995) extended the EOQ model from non-deteriorating items to deteriorating items. Jamal et al. (1997) further generalized the EOQ model with trade credit financing to allow shortages. After, Teng (2002) provided an easy analytical closed-form solution to this type of problem.

Afterwards, Huang (2003) extended the trade credit problem to the case in which a supplier offers its retailer a credit period, and the retailer in turn provides another credit period to its customers. Furthermore, Liao (2008) extended Huang's model to an economic production quantity (EPQ) model for deteriorating items. Subsequently, Teng (2009) provided the optimal ordering policies for a retailer to deal with bad credit customers as well as good credit customers. Conversely, Min et al. (2010) proposed an EPQ model under stock-dependent demand and two-level trade credit. Later, Kreng and Tan (2011) obtained the optimal replenishment decision in an EPQ model with defective items under trade credit policy. After, Teng et al. (2011) obtained the optimal ordering policy for stock-dependent demand under progressive payment scheme. Further, Teng et al. (2012) extended the demand pattern from constant to increasing in time. Recently, Ouyang and Chang (2013) built up an EPQ model with imperfect production process and complete backlogging. Concurrently, Chen et al. (2013) established the retailer's optimal EOQ when the supplier offers conditionally permissible delay in payments link to order quantity. In all articles described above, the EOQ/EPQ models with trade credit financing were studied only from the perspective of the buyer. How to determine the optimal credit period for the seller has received only a few attentions by the researchers such as Chern et al. (2013), and Teng and Lou (2012). Currently, Seifert et al. (2013) organized the trade credit literature and derived a detailed agenda for future research in trade credit area.

It well knows that many products such as vegetables, fruits, volatile liquids, blood banks, fashion merchandises and high-tech products deteriorate continuously due to several reasons such as evaporation, spoilage, obsolescence among others. In this course, Ghare and Schrader (1963) proposed an EOQ model by assuming an exponentially decaying inventory. Then Covert and Philip (1973) generalized the constant exponential deterioration rate to a two-parameter Weibull distribution. Later, Dave and Patel (1981) established an EOQ model for deteriorating items with linearly increasing demand and no shortages. Then Sachan (1984) further extended the EOQ model to allow for shortages. Conversely, Goswami and Chaudhuri (1991) generalized an EOQ model for deteriorating items from a constant demand pattern to a linear trend in demand. Concurrently, Raafat (1991) provided a survey of literature on continuously deteriorating inventory model. On the other hand, Hariga (1996) studied optimal EOQ models for deteriorating items with time-varying demand. Afterwards, Teng et al. (1999) generalized EOQ models with shortages and fluctuating demand. Later, Goyal and Giri (2001) wrote a survey on the recent trends in modeling of deteriorating inventory. Teng et al. (2002) further extended the model to allow for partial backlogging. Skouri et al. (2009) established inventory EOQ models with ramp-type demand rate and Weibull deterioration rate. In a subsequent paper, Skouri et al. (2011) further generalized the model for deteriorating items with ramp-type demand and permissible delay in payments. Mahata (2012) proposed an EPQ model for deteriorating items under retailer partial trade credit policy. Recently, Dye (2013) studied the effect of technology investment on deteriorating items. Wee and Widyadana (2013) developed a production model for deteriorating items with stochastic preventive maintenance time and rework. Although a deteriorating item has its own expiration date (a.k.a., maximum lifetime), none of the above mentioned papers take the maximum lifetime into consideration. Currently, Bakker et al. (2012) wrote a review of inventory systems with deterioration since 2001. In this paper, we propose an EOQ model for the retailer to obtain her/his optimal credit period and cycle time when: (a) the supplier grants to the retailer an up-stream trade credit of S years while the retailer offers a down-stream trade credit of R years to the buyer, (b) the retailer's down-stream trade credit to the buyer not only increases sales and revenue but also opportunity cost and default risk, and (c) a deteriorating item not only deteriorates continuously but also has its maximum lifetime. We then formulate the retailer's objective functions under different possible cases. In fact, the proposed inventory model forms a general framework that includes many previous models as special cases such as Goyal (1985), Teng (2002), Teng and Goyal (2007), Teng and Lou (2012), Lou and Wang (2013), Wang et al. (2014), and others. By applying concave fractional programming, we prove that there exists a unique global optimal solution to the retailer's replenishment cycle time. Similarly, using Calculus we show that the retailer's optimal down-stream credit period not only exists but also is unique. Furthermore, we discuss a special case for non-deteriorating items. Finally, we run several numerical examples to illustrate the problem and provide some managerial insights.

Table 1. A summary on important papers related to this paper

	Parameters				Decision variable					
Paper	Expiration date	Deterioration rate (time-varying)	S	Risk	Shortage	T	R	Two-level trade credit financials	Methodology	Objective function
Ouyang et al, 2014	✓	✓	✓	✓	-	✓	✓	✓	EOQ	Total annual profit
Wang et al, 2014	✓	✓	-	✓	-	✓	✓	-	EOQ	Total annual profit
Teng & Lou, 2012	-	-	✓	✓	-	✓	✓	✓	EOQ	Total annual profit
Liao, 2008	-	✓	✓	-	-	✓	-	✓	EOQ	Total cost
Min et al, 2010	-	✓	✓	-	-	✓	-	✓	EOQ	Average profit per unit of time
Chern et al, 2013	-	-	-	✓	-	T,Q	R,S	✓	EPQ	Total annual profit
Ouyang and Chang, 2013	✓	-	-	-	✓	T,Q	Fixed	-	EPQ	Inventory total cost per T
Chakrabarty & Chaudhuri, 1997	-	Fixed	-	-	✓	N,I(t)=0	Fixed	-	EOQ	Inventory total cost
Chakrabarty et al, 1998	-	✓	-	-	Stochastic	T,Q	-	-	EOQ	System total cost per T
Chandra Mahata, 2012	-	Stochastic	Fixed	-	-	T,Q	Fixed	✓	EPQ	Total annual cost
Kreng & Tan, 2011	-	Stochastic	Fixed	-	-	T,Q	Fixed	✓	EPQ	Total annual profit
Min et al, 2010	-	Fixed	Fixed	-	-	✓	Fixed	✓	A lot-sizing model	Average profit per unit of time
Haung, 2003	-	-	Fixed	-	-	✓	Fixed	✓	EOQ	Total annual cost
Teng et al, 2011	-	Fixed	Fixed	-	-	T,Q	Fixed	✓	EOQ	Total annual profit
Papachristosa & Skourib, 2000	-	✓	-	-	✓	n	-	-	A lot-sizing model	System total cost per T
Aggawarl & Jaggi, 1995	-	✓	-	-	-	T,Q	Fixed	-	EOQ	System total cost per T
This Paper	✓	✓	✓	✓	✓	✓	✓	✓	EOQ	Total annual profit

Based on the literature review, there are few researches considering shortages when presenting an EOQ/EPQ inventory model under two-level trade credit financing for deteriorating items. This study considers backorders (and cost of it) in an EOQ model with expiration dates under two-level trade credit financing for deteriorating items. In the rest of the paper the notations, assumptions and mathematical programming is presented.

2-Mathematical modeling

Notations

For the retailer:

o Ordering cost per order in dollars.
c Purchase cost per unit in dollars.
p Selling price per unit (dollars), with $p > c$
h Unit holding cost per year in dollars excluding interest charge.
r Annual compound interest paid per dollar per year.
I_e Interest earned per dollar per year.
I_c Interest charged per dollar per year.
t The time in years
$I(t)$ Inventory level in units at time t
$(\forall I(t) \geq 0, I(t) = I(t)^{+}, \forall I(t) < 0, I(t) = I(t)^{-})$

$\theta(t)$ The time-varying deterioration rate at time t, where $0 \leq \theta(t) < 1$
m The expiration date or maximum lifetime in years of the deteriorating item
S Up-stream credit period in years offered by the supplier
R Down-stream trade credit period in years offered by the retailer (a decision variable)
$D = D(R)$ The market annual demand rate in units which is a concave and increasing function of R
T Replenishment cycle time in years (a decision variable).
Q Order quantity
$TP(K, T)$ Total annual profit, which is a function of R and T.

K^* Optimal fraction of T before backorder occurs in years.
T^* Optimal replenishment cycle time in years.
TP^* Optimal annual total profit in dollars
K Fraction of T before backorder occurs.

b Backorder quantity

π Variable cost for backorder, which is a function of t

$\hat{\pi}$ Fixed cost for backorder.

Assumptions:

Next, the following assumptions are made to establish the mathematical inventory model:

1. All deteriorating items have their expiration dates (m). Hence, the deterioration rate must be closed to 1 when time is approaching to the expiration date m. We may assume that the deteriorating rate is $\theta(t) = \lambda/(\lambda + m - t)$, or $\theta(t) = e^{\lambda(t-m)}$, where λ is a constant. However, to make the problem tractable, we assume that the deterioration rate is the same as that in Sarkar (2012) and Wang et al. (2014) as follow:

$$\theta(t) = \frac{l}{l + m - t} \qquad 0 \leq t \leq T \leq m \tag{1}$$

Note that it is clear from (1) that the replenishment cycle time T must be less than or equal to$m$, and the proposed deterioration rate is a general case for non-deteriorating items, in which $m \to \infty$ and $\theta(t) \to 0$.

2. Similar to the assumption in Chern et al. (2013) and Teng and Lou (2012), we assume that the demand rate $D(R)$ is a positive exponential function of the retailer's down-stream credit period R as:

$$D(R) = Ke^{aR} \tag{2}$$

Where K and a are positive constants with$0 < a < 1$. For convenience, $D(R)$ and D will be used interchangeably.

3. The longer the retailer's down-stream credit period, the higher the default risk to the retailer. For simplicity, we may assume that the rate of default risk giving the retailer's down-stream credit period R is assumed as

$$F(R) = 1 - e^{-bR} \tag{3}$$

Where b is the coefficient of the default risk, which is a positive constant value.

4. If the annual compound interest rate is$r$, then a dollar received at time t is equivalent to e^{-rt} dollars received now. The retailer offers the buyer a credit period of$R$. Hence, the retailer's net revenue received after default risk and opportunity cost is:

$$pD(R)[1 - F(R)]e^{-rR} = pKe^{aR}e^{-bR}e^{-rR} = pKe^{[a-(b+r)]R}. \tag{4}$$

This value will be considered in the total profit as a positive income.

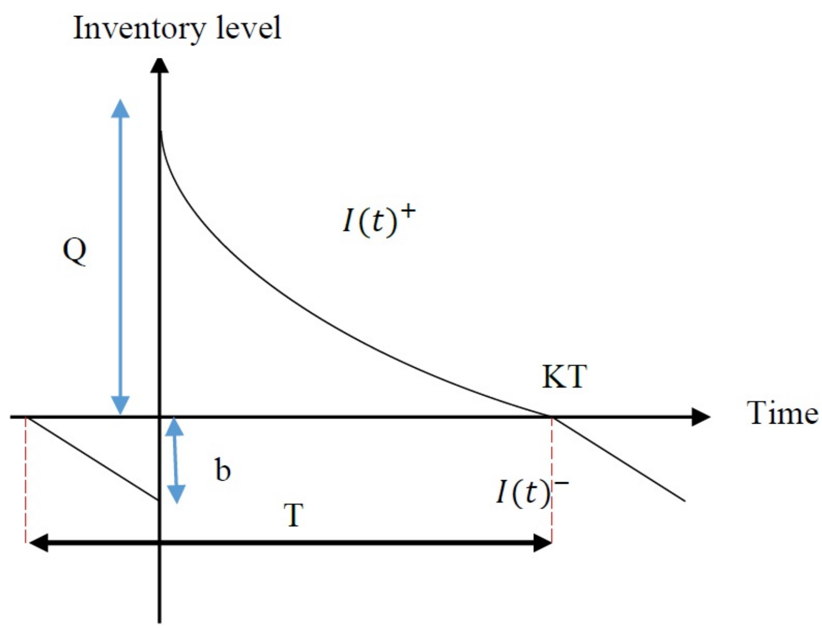


Fig 1. Inventory level in one period

Inventory equations:

During the replenishment cycle [0, T], the inventory level is depleted by demand and deterioration, and hence governed by the following differential equation:

$$\frac{dI(t)^+}{dt} = -D - \theta(t)I(t)^+; 0 \le t \le KT$$

$$\frac{dI(t)^-}{dt} = -D; KT \leq t \leq T$$

The $I(t)^+$ and $I(t)^-$ values are depicted in figure1.

The value of backorder quantity according to the inventory diagram (Figure1) is as follow:

$$b = (1-k)TD$$

Therefore, in all equations the value of backorder (b) will be replaced by $(1-k)TD$.

As a result, the retailer's order quantity is calculated by determining $I(t)^+$ when t equals to zero (at the beginning of the period when the order comes to the stock). Then we have:

$$Q = I(0)^+ = D(1+m)Ln(\frac{1+m}{1+m-KT})$$

Therefore, the retailer's holding cost excluding interest cost per cycle is:

$$h\int_0^{KT} I(t)^+ dt = hD\int_0^{KT}(1+m-t)Ln\left(\frac{1+m-t}{1+m-KT}\right)dt$$
$$= hD\left[\frac{(1+m)^2}{2}Ln\left(\frac{1+m}{1+m-KT}\right) + \frac{(KT)^2}{4} - \frac{(1+m)KT}{2}\right]$$

Another cost considered in this paper is the backorder cost, which occurs when the inventory level is zero or negative but there is a demand for the product. The backorder quantity (b) is depicted in figure1. In this situation the seller waits until the next replenishment and sells all b at once and gets the money at the retailers down-stream trade credit period (R).

$$\hat{\pi}b + \pi b \times \frac{(1-K)T}{2}$$

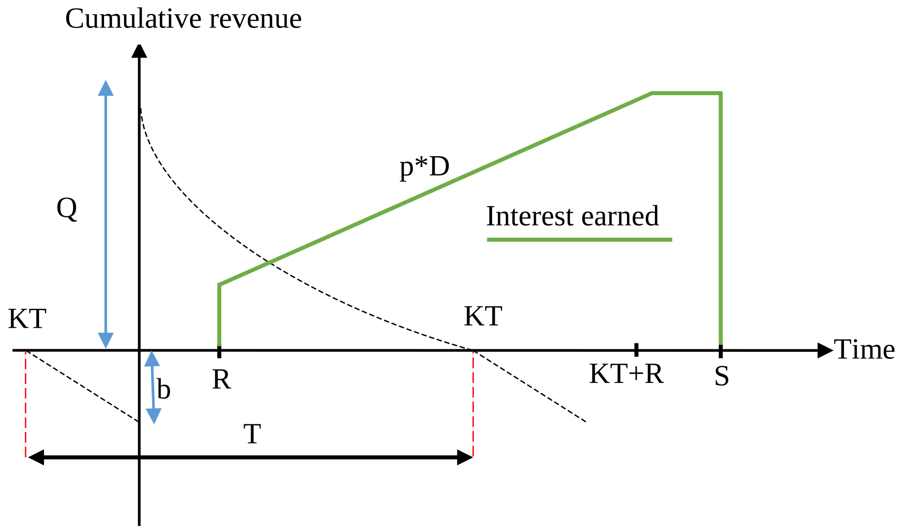
As it is obvious in the above equation, backorder cost is constructed by two parts (fixed and variable). The fixed cost ($\hat{\pi}b$) is paid just when the backorder occurs, but the variable cost ($\pi b \times \frac{(1-K)T}{2}$) is paid according to time.

From the values of R and S and K, we have two potential cases: (1) $R \leq S$, and (2) $S \geq R$. The first potential case has two sub cases. Let us discuss them separately.

1) $R \leq S$

In this case, the down-stream trade credit period is lower than the up-stream trade credit period. There can be two sub cases that are discussed in the nest section. Based on the values of S (i.e., the time at which the retailer must pay off the purchase amount to the supplier to avoid interest charge) and $KT + R$ (i.e., the time at which the retailer receives the payment from the last customer before backorder), we have two possible sub-cases. If $R \leq S - KT$ (i.e., there is an interest earned), then the retailer sells all units by $S - KT$ and at time S, pays back the supplier the money for ordering and buying $Q + b$ products, which is shown in figure2.

If $R \geq S - KT$, then the retailer sells some units by S (i.e., there is an interest earned) but at time S the retailer should pay for the supplier, then the retailer starts paying for the interest charges on the items sold after $S - KT$, which is shown in Figure 1.The graphical representation of this case is shown in figure 2. Now, let us discuss the detailed formulation in each sub-case.



Sub-case 1-1) $R \leq KT, S \geq R + KT$

Fig 2. $R \leq KT$ and $S \geq R + KT$

The retailer's ordering cost per cycle is O dollars, and the purchase cost per cycle is $c * I(t)$ dollars. Hence, the retailer's annual total profit can be expressed as follows:

TP=Net annual revenue- Annual purchase cost- Annual ordering cost- Annual holding cost- Annual backorder cost -Interest charged +Interest earned

Net annual revenue = $pKe^{[a-(B+r)R]}$

Annual purchase cost = $c * Q = c * I(t) = \frac{c}{T}\left[D(1+m)Ln\left(\frac{1+m}{1+m-KT}\right) + b\right]$

In which $Q = D(1+m)Ln\left(\frac{1+m}{1+m-KT}\right)$ and $b = (1-K)TD$

Annual ordering cost = $\frac{O}{T}$

Annual holding cost = $\frac{hD}{T}\left[\left[\frac{(1+m)^2}{2}Ln\left(\frac{1+m}{1+m-KT}\right) + \frac{(KT)^2}{4} - \frac{(1+m)KT}{2}\right]\right]$

Annual backorder cost = $\frac{\hat{\pi} b}{T} + \pi b \times \frac{(1-K)T}{2T} = \hat{\pi}(1-K)D + \pi(1-K)D \times \frac{(1-K)T}{2} = \hat{\pi}(1-K)D + \pi\frac{(1-K)^2TD}{2}$

As can be seen in figure2, the retailer sells deteriorating items at time 0, but receives the money at time R. Thus, the retailer accumulates revenue in an account that earns I_e per dollar per year from R through KT+R. Therefore, the interest earned per cycle is I_e multiplied by the area of the trapezoidal from R through KT+R as shown in figure 2. At time S the retailer must pay off the cost to bank and therefore the rectangular from KT+R through S is calculated equal to the previous parts and is considered as earned interest. Hence, the interest earned per year is as follows. Notice that the vertical axis in Figures 1-4 represents the cumulative revenue, not cumulative sale volume, and the slope of the increasing line in Figures 2 – 4 is P*D.

Interest charged = 0

Interest earned = $\frac{T*pDT*I_e}{2T} + \frac{T*pb*I_e}{T} + \frac{(pb+pDT)(S-(R+KT))*I_e}{T}$

Therefore, the total profit gained by the first case will be calculated as follows:

$$TP_1(K,T) = pK1e^{[a-(B+r)R]} - \frac{c}{T}\left[D(1+m)Ln\left(\frac{1+m}{1+m-KT}\right)+b\right] - \frac{O}{T} - \frac{hD}{T}\left[\left[\frac{(1+m)^2}{2}Ln\left(\frac{1+m}{1+m-KT}\right)+\frac{(KT)^2}{4} - \frac{(1+m)KT}{2}\right]\right] - \left[\hat{\pi}(1-K)D + \pi\frac{(1-K)^2TD}{2}\right] + \frac{T*pDT*I_e}{2T} + \frac{T*pb*I_e}{T} + \frac{(pb+pDT)(S-(R+KT))*I_e}{T}$$

Next, we discuss the other sub-case in which $R \leq KT$ and $S \geq R + KT$.

Sub-case 1-2) $R \leq KT$ and $S \geq R + KT$

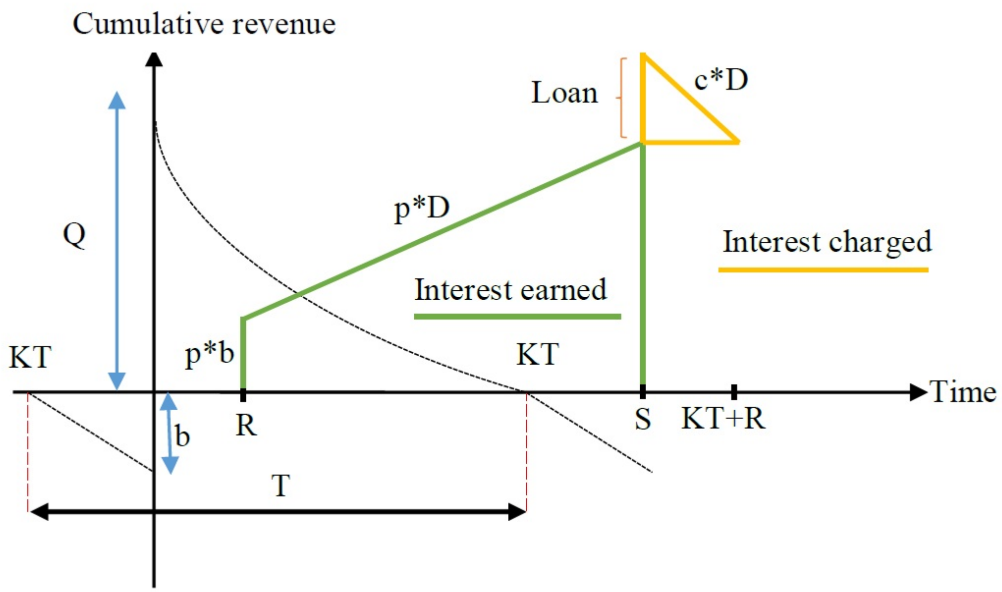


Fig 3. Subcase 1-2: $R \leq KT$ and $S \geq R + KT$

In this sub-case, the retailer receives some of revenue at time R through S. A part of the purchase cost at time S along with a loan that is gained at time S. Hence, there is no interest charge while the interest earned per cycle is I_e multiplied by the area of the trapezoid on the interval [R,S] as shown in Figure3. On the other hand, the retailer sells deteriorating items at time 0, but receives the money at time R. Thus, the retailer accumulates revenue in an account that earns I_e per dollar per year from R through S. Therefore, the interest earned per cycle is I_e multiplied by the area of the trapezoid as shown in Figure3. Consequently, the retailer's annual interest earned is

Interest charged = $\frac{(cD)(R+KT-S)^2*I_e}{2T}$

In this sub-case, the supplier's up-stream credit period S is shorter than or equal to the customer last payment time KT. Hence, the retailer cannot pay off the purchase amount at time S, and must finance some items sold after time S at an interest charged I_c per dollar per year. As a result, the interest charged per cycle is times the area of the above triangle as shown in figure3. Therefore, the interest charged per year is given by

Interest earned = $\frac{(S-R)^2 * pD * I_e}{2T} + \frac{(S-R) * pb * I_e}{T}$

Therefore, the total profit gained by the first case will be calculated as follows:

$$TP_2(K,T) = pK1e^{[a-(B+r)R]} - \frac{c}{T}\left[D(1+m)Ln\left(\frac{1+m}{1+m-KT}\right) + b\right] - \frac{O}{T} - \frac{hD}{T}\left[\left[\frac{(1+m)^2}{2} Ln\left(\frac{1+m}{1+m-KT}\right) + \frac{(KT)^2}{4} - \frac{(1+m)KT}{2}\right]\right] - \left[\hat{\pi}(1-K)D + \pi\frac{(1-K)^2 TD}{2}\right] + \frac{(S-R)^2 * pD * I_e}{2T} + \frac{(S-R) * pb * I_e}{T} - \frac{(cD)(R+KT-S)^2 * I_e}{2T}$$

Finally, we formulate the retailer's annual total profit for the case of $R \geq S$ below.

Case 2) $R \geq S$

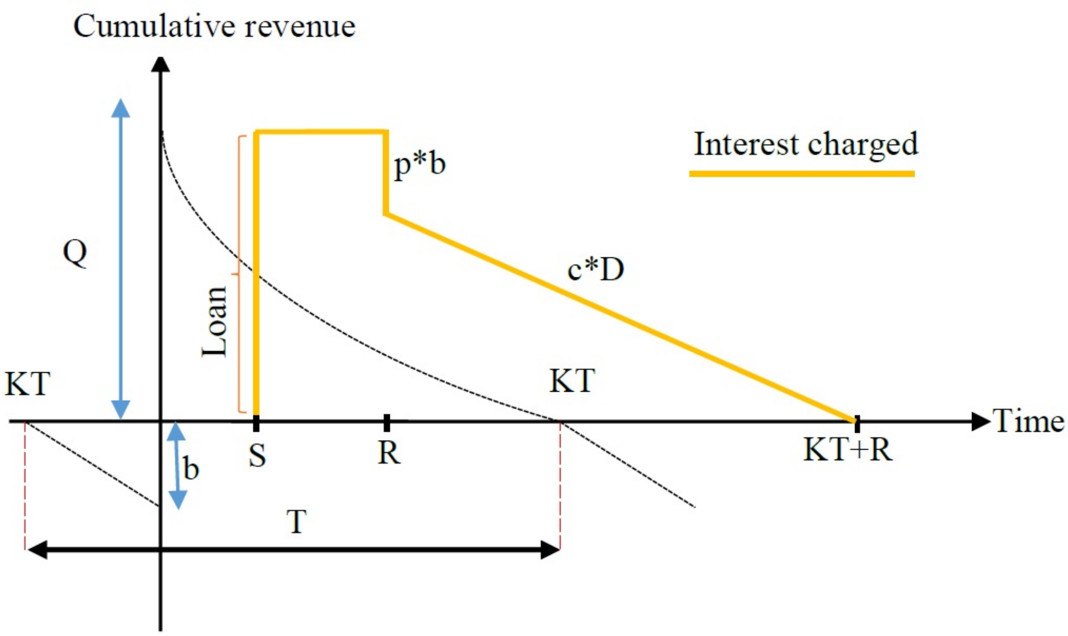


Fig 4. Case2:$R \geq S$

As can be seen in figure 4, when the up-stream credit period is lower than the down-stream credit period from the time S the retailer has to get loan to afford the D demand. Hence, the retailer cannot pay off the purchase amount at time S, and must finance all items sold after time R at an interest charged I_C per dollar per year. As a result, the interest charged per cycle is the area of the triangle and rectangular as shown in figure 4(the whole shape). Therefore, the interest charged per year is given by

Interest charged = $\frac{(R-S) * cDT * I_C}{T} + \frac{(cD)(KT)^2 * I_C}{2T}$

And the interest earned as depicted in Figure4 is equal to zero.

Interest earned = 0

Therefore, the total profit gained by the first case will be calculated as follows:

$$TP_3(K,T) = pK1e^{[a-(B+r)R]} - \frac{c}{T}\left[D(1+m)Ln\left(\frac{1+m}{1+m-KT}\right) + b\right] - \frac{O}{T} - \frac{hD}{T}\left[\frac{(1+m)^2}{2}Ln\left(\frac{1+m}{1+m-KT}\right) + \frac{(KT)^2}{4} - \frac{(1+m)KT}{2}\right] - \left[\hat{\pi}(1-K)D + \pi\frac{(1-K)^2TD}{2}\right] - \left[\frac{(R-S)*cDT*I_c}{T} + \frac{(cD)(KT)^2*I_C}{2T}\right]$$

Therefore, the retailer's objective is to determine the optimal fraction of T before backorder occurs $K^*$and cycle time T^* such that the annual total profit $TP_i(K,T)$ for (i=1,2 and 3) is maximized. In the next section, we characterize the retailer's optimal K and T in each case, and then obtain the conditions in which the optimal $T^*$is in either sub-cases.

3-Theoretical results and optimal solution

To solve the problem, we apply the existing theoretical results in concave fractional programming. We know from Cambini and Martein (2009) that the real-value function:

$$q(x) = \frac{f(x)}{g(x)}$$

is (strictly) pseudo-concave, if $f(x)$ is non-negative, differentiable and (strictly) concave, and $g(x)$ is positive, differentiable and convex. For any given K, by applying $q(x)$, we can prove that the retailer's annual total profit $TP_i(K,T)$ for (i=1,2 and 3) is strictly pseudo-concave in T. As a result, for any given$K$, there exists a unique global optimal solution T_i^*such that $TP_i(K,T)$ is maximized. Similar to the previous section we discuss the case of $R \leq S$ first, and then the case of $R \geq S$.

3-1-Optimal solution for the case of R $\leq$ S

By applying the concave fractional programming as in $q(x)$, we can prove that the retailer's annual total profit $TP_i(K,T)$ for (i=1 and 2) is strictly pseudo-concave in T. Consequently, we have the following theoretical results.

To find T_1^*, taking the first-order partial derivative of $TP_1(K,T)$, setting the result to zero, and re-arranging terms, we get

$$\frac{\partial TP_1(K,T)}{\partial T} = \frac{O}{T^2} - \left[cD(1+m)\left(\frac{-1}{T^2}\ln\frac{1+m}{1+m-KT} + \frac{K}{T(1+m-KT)}\right)\right] + \left[\frac{hD(1+m)^2}{2T^2}\ln\frac{1+m}{1+m-KT} + \frac{-hKD(1+m)^2}{2T(1+m-KT)}\right] - \frac{\pi D(1-K)^2}{2} + \left[\frac{pDI_e}{2} + p(1-K)DI_e + pDI_e(2K-K^2)\right] = 0$$

For any given T, taking the first-order partial derivative of $TP_1(K,T)$ with respect to K, setting the result to zero, and re-arranging terms, we have

$$\frac{\partial TP_1(K,T)}{\partial K} = 0 - \left[cD(1+m)\left(\frac{1}{1+m-KT}\right) - cD\right] + \left[\frac{-hD(1+m)^2}{2(1+m-KT)} - \frac{hDTK}{2} + \frac{hD(1+m)}{2}\right] - [-D\hat{\pi} - \pi DT(1-K)] + [-pTDI_e + pDI_e(-S+R-2T+2KT]$$

Taking the second-order partial derivative of$TP_1(K,T)$ with respect to K, and re-arranging terms, we obtain

$$\frac{\partial^2 TP_1(K,T)}{\partial K^2} = 0 - \left[\frac{-cDT(1+m)}{(1+m-KT)^2}\right] - \left[\frac{hDT(1+m)^2}{2(1+m-KT)^2} + \frac{hDT}{2}\right] - [\pi DT] + 2TpDI_e$$

Theorem 1. For any given T the second-order partial derivative of$TP_1(K,T)$ with respect to K is negative (the objective function is maximization) so that the objective is convex and concave fractional programming can be applied.

Therefore,

1- $TP_1(K,T)$ is a strictly pseudo-concave function in T, and hence exists a unique maximum solution at T_1^*.

2- If $S \geq R + KT$ then $TP_1(K,T)$ subject to $S \geq R + KT$ is maximized atT_1^*
3- If $S \leq R + KT$ then $TP_1(K,T)$ subject to $S \geq R + KT$ is maximized at $R + KT$

Proof. Let's use concave fractional programming to define

$$f_1(T)= T(pK1e^{[a-(B+r)R]} - \frac{c}{T}\left[D(1+m)Ln\left(\frac{1+m}{1+m-KT}\right) + b\right] - \frac{O}{T} - \frac{hD}{T}\left[\left[\frac{(1+m)^2}{2}Ln\left(\frac{1+m}{1+m-KT}\right) + \frac{(KT)^2}{4} - \frac{(1+m)KT}{2}\right]\right] - \left[\hat{\pi}(1-K)D + \pi\frac{(1-K)^2TD}{2}\right] + \frac{T*pDT*I_e}{2T} + \frac{T*pb*I_e}{T} + \frac{(pb+pDT)\left(S-(R+KT)\right)*I_e}{T})$$

And

$$g_1(T) = T$$

Taking the first-order and second-order derivatives of $f_1(T)$ we have

$$f_1{}'(T)= \frac{O}{T} - \left[cD(1+m)\left(\frac{-1}{T}\ln\frac{1+m}{1+m-KT} + \frac{K}{T(1+m-KT)}\right)\right] + \left[\frac{hD(1+m)^2}{2T}\ln\frac{1+m}{1+m-KT} + \frac{-hKD(1+m)^2}{2(1+m-KT)}\right] - \frac{\pi D(1-K)^2}{2} + \left[\frac{pDI_e}{2} + p(1-K)DI_e + pDI_e(2K-K^2)\right]$$

And

$$f_1{}''(T)= \frac{O}{T^2} - \left[\frac{-1}{T}\ln\frac{1+m}{1+m-KT} + \frac{K}{T(1+m-KT)}\right] - \left[\frac{hD(1+m)^2}{2T}\ln\frac{1+m}{1+m-KT}\right] - \frac{\pi D(1-K)^2}{2} \leq 0$$

Therefore $TP_1(K,T) = \frac{f_1(T)}{g_1(T)}$ is a strictly pseudo-concave function at T which completes the proof of Theorem 1.

3-2-Optimal solution for the case of R $\leq$ KT and S $\geq$ R + KT

To find T_1^*, taking the first-order partial derivative of $TP_2(K,T)$, setting the result to zero, and re-arranging terms, we get

$$\frac{\partial TP_2(K,T)}{\partial T} = \frac{O}{T^2} - \left[cD(1+m)\left(\frac{-1}{T^2}\ln\frac{1+m}{1+m-KT} + \frac{K}{T(1+m-KT)}\right)\right] + \left[\frac{hD(1+m)^2}{2T^2}\ln\frac{1+m}{1+m-KT} + \frac{-hKD(1+m)^2}{2T(1+m-KT)}\right] - \frac{\pi D(1-K)^2}{2} - \left[\frac{(S-R)^2pDI_e}{2T^2} + \frac{4(R+KT-S)KcDI_c - 2cDI_c(R+KT-S)^2}{4T^2}\right] = 0$$

For any given T, taking the first-order partial derivative of $TP_2(K,T)$ with respect to K, setting the result to zero, and re-arranging terms, we have

$$\frac{\partial TP_2(K,T)}{\partial K} = 0 - \left[cD(1+m)\left(\frac{1}{1+m-KT}\right) - cD\right] + \left[\frac{-hD(1+m)^2}{2(1+m-KT)} - \frac{hDTK}{2} + \frac{hD(1+m)}{2}\right] - [-D\hat{\pi} - \pi DT(1-K)] - [(S-R)pDI_e + cDI_c(R+KT-S)]$$

Taking the second-order partial derivative of$TP_2(K,T)$ with respect to K, and re-arranging terms, we obtain

$$\frac{\partial^2 TP_2(K,T)}{\partial K^2} = 0 - \left[\frac{-cDT(1+m)}{(1+m-KT)^2}\right] - \left[\frac{hDT(1+m)^2}{2(1+m-KT)^2} + \frac{hDT}{2}\right] - [\pi DT] - cDTI_c$$

To identify whether K^* is 0 or positive, lets define the discrimination term below:

Theorem2. For any given T the second-order partial derivative of$TP_2(K,T)$ with respect to K is negative (the objective function is maximization) so that the objective is convex and concave fractional programming can be applied. Therefore:

1- $TP_2(K,T)$ is a strictly pseudo-concave function in T, and hence exists a unique maximum solution at T_2^*.
2- If $S \geq R + KT$ then $TP_2(K,T)$ subject to $S \geq R + KT$ is maximized atT_2^*
3- If $S \leq R + KT$ then $TP_2(K,T)$ subject to $S \geq R + KT$ is maximized at $R + KT$

Proof. Let's use concave fractional programming to define

$$f_2(T)= T(pK1e^{[a-(B+r)R]} - \frac{c}{T}\left[D(1+m)Ln\left(\frac{1+m}{1+m-KT}\right)+b\right] - \frac{O}{T} - \frac{hD}{T}\left[\left[\frac{(1+m)^2}{2}Ln\left(\frac{1+m}{1+m-KT}\right)+\frac{(KT)^2}{4} - \frac{(1+m)KT}{2}\right]\right] - \left[\hat{\pi}(1-K)D + \pi\frac{(1-K)^2TD}{2}\right] + \frac{(S-R)^2*pD*I_e}{2T} + \frac{(S-R)*pb*I_e}{T} - \frac{(cD)(R+KT-S)^2*I_e}{2T})$$

And

$$g_2(T) = T$$

Taking the first-order and second-order derivatives of $f_2(T)$ we have

$$f_2{}'(T)= \frac{O}{T} - \left[cD(1+m)\left(\frac{-1}{T}\ln\frac{1+m}{1+m-KT}+\frac{K}{T(1+m-KT)}\right)\right] + \left[\frac{hD(1+m)^2}{2T}\ln\frac{1+m}{1+m-KT}+\frac{-hKD(1+m)^2}{2T(1+m-KT)}\right] - \frac{\pi D(1-K)^2}{2} + \left[\frac{pDI_e}{2} + p(1-K)DI_e + pDI_e(2K-K^2)\right]$$

And

$$f_2{}''(T)= -\left[cD(1+m)\left(\frac{-1}{T^2}\ln\frac{1+m}{1+m-KT}+\frac{K}{T(1+m-KT)}\right)\right] - \left[\frac{hD(1+m)^2}{2T^2}\ln\frac{1+m}{1+m-KT}+\frac{-hKD(1+m)^2}{2T(1+m-KT)}\right] - \frac{\pi D(1-K)^2}{2} + \left[\frac{pDI_e}{2} + p(1-K)DI_e\right] \leq 0$$

Therefore $TP_2(K,T) = \frac{f_2(T)}{g_2(T)}$ is a strictly pseudo-concave function at T which completes the proof of Theorem 2.

$$\Delta K_1 = -\left[\frac{-cDT(1+m)}{(1+m-KT)^2}\right] - \left[\frac{hDT(1+m)^2}{2(1+m-KT)^2}+\frac{hDT}{2}\right] - [\pi DT] + 2TpDI_e - \left(\left[\frac{-cDT(1+m)}{(1+m-KT)^2}\right] - \left[\frac{hDT(1+m)^2}{2(1+m-KT)^2}+\frac{hDT}{2}\right] - [\pi DT] - cDTI_c\right)$$

Theorem2.

For any $T > 0$, if ΔK_1 has an acceptable answer, then we have:

1- $TP_2(K,T)$ is a strictly concave function in K, and hence exists a unique maximum solution K_1T^*+$R_1{}^*$
2- If $\Delta K_1 \leq 0$, then $TP_2(K,T)$ is maximized at $K_1{}^*$.
3- If $\Delta K_1 \geq 0$, then a unique $K_1{}^*$exists and $TP_2(K,T)$ is maximized.

3-3-Optimal solution for the case: R ≥ S

To find T_1^*, taking the first-order partial derivative of $TP_3(K,T)$, setting the result to zero, and re-arranging terms, we get

$$\frac{\partial TP_3(K,T)}{\partial T} = \frac{O}{T^2} - \left[cD(1+m)\left(\frac{-1}{T^2}\ln\frac{1+m}{1+m-KT} + \frac{K}{T(1+m-KT)}\right)\right] + \left[\frac{hD(1+m)^2}{2T^2}\ln\frac{1+m}{1+m-KT} + \frac{-hKD(1+m)^2}{2T(1+m-KT)}\right] - \frac{\pi D(1-K)^2}{2} + \left[\frac{K^2 cDI_c}{2}\right] = 0$$

For any given T, taking the first-order partial derivative of $TP_3(K,T)$ with respect to K, setting the result to zero, and re-arranging terms, we have

$$\frac{\partial TP_3(K,T)}{\partial K} = 0 - \left[cD(1+m)\left(\frac{1}{1+m-KT}\right) - cD\right] + \left[\frac{-hD(1+m)^2}{2(1+m-KT)} - \frac{hDTK}{2} + \frac{hD(1+m)}{2}\right] - [-D\hat{\pi} - \pi DT(1-K)] + [cDTI_cK]$$

Taking the second-order partial derivative of$TP_3(K,T)$ with respect to K, and re-arranging terms, we obtain

$$\frac{\partial^2 TP_3(K,T)}{\partial K^2} = 0 - \left[\frac{-cDT(1+m)}{(1+m-KT)^2}\right] - \left[\frac{hDT(1+m)^2}{2(1+m-KT)^2} + \frac{hDT}{2}\right] - [\pi DT] + cDTI_c$$

Theorem 3. For any given T the second-order partial derivative of$TP_3(K,T)$ with respect to K is negative (the objective function is maximization) so that the objective is convex and concave fractional programming can be applied.

Therefore,

1- $TP_3(K,T)$ is a strictly pseudo-concave function in T, and hence exists a unique maximum solution at T_2^*.
2- If $S \geq R + KT$ then $TP_3(K,T)$ subject to $S \geq R + KT$ is maximized atT_3^*
3- If $S \leq R + KT$ then $TP_3(K,T)$ subject to $S \geq R + KT$ is maximized at $R + KT$

Proof. Let's use concave fractional programming to define

$$f_3(T)=T(pK1e^{[a-(B+r)R]} - \frac{c}{T}\left[D(1+m)Ln\left(\frac{1+m}{1+m-KT}\right) + b\right] - \frac{O}{T} - \frac{hD}{T}\left[\frac{(1+m)^2}{2}Ln\left(\frac{1+m}{1+m-KT}\right) + \frac{(KT)^2}{4} - \frac{(1+m)KT}{2}\right] - \left[\hat{\pi}(1-K)D + \pi\frac{(1-K)^2TD}{2}\right] - \left[\frac{(R-S)*cDT*I_c}{T} + \frac{(cD)(KT)^2*I_C}{2T}\right])$$

And

$$g_3(T) = T$$

Taking the first-order and second-order derivatives of $f_3(T)$ we have

$$f_3{}'(T)= \frac{O}{T} - \left[cD(1+m)\left(\frac{-1}{T}\ln\frac{1+m}{1+m-KT} + \frac{K}{T(1+m-KT)}\right)\right] + \left[\frac{hD(1+m)^2}{2T^2}\ln\frac{1+m}{1+m-KT} + \frac{-hKD(1+m)^2}{2(1+m-KT)}\right] - \frac{\pi D(1-K)^2}{2} + \left[\frac{K^2 cDI_c}{2}\right]$$

And

$$f_3{}''(T)=- \left[cD(1+m)\left(\frac{-1}{T}\ln\frac{1+m}{1+m-KT} + \frac{K}{T(1+m-KT)}\right)\right] - \left[\frac{hD(1+m)^2}{2T^2}\ln\frac{1+m}{1+m-KT} + \frac{-hKD(1+m)^2}{2(1+m-KT)}\right] - \frac{\pi D(1-K)^2}{2} \leq 0$$

Therefore $TP_3(K,T) = \frac{f_3(T)}{g_3(T)}$ is a strictly pseudo-concave function at T which completes the proof of Theorem 3.

4-Numerical example

In this section, we use numerical example in order to illustrate theoretical results as well as to gain some managerial insights. Using the data as those given below, we study the sensitivity analysis on the optimal solution with respect to each parameter in appropriate unit. The computational results are shown in table 2.

Table 1. Information related to numerical example

Data	Value	Data	Value
O	20$	p	15 $ per unit
r	0.05 per year	c	10 $ per unit
$K1$	1000 units/year	h	2 $ per unit per year
S	0.25 year	R	0.75 year
m	1 year	I_e	0.03 per year
I_c	0.06 per year		

Table 2. Computational results related to sensitivity analysis

parameter	K^*	T^*	$TP^*(K^*,T^*)$
K_1 =1000 units/year	0.5345	0.0534	29,907.00
K_1 =2000 units/year	0.7245	0.0324	29,933.00
K_1 =3000 units/year	0.8531	0.0253	29,996.00
p =20 dollars per unit	0.1679	0.0167	26,441.00
p =25 dollars per unit	0.1779	0.0157	26,968.00
p =30 dollars per unit	0.2678	0.0067	27,316.00
c =10 dollars per unit	0.1679	0.0167	22,298.00
c =12 dollars per unit	0.1999	0.0159	23,435.00
c =14 dollars per unit	0.2679	0.0117	25,516.00
o =20dollars	0.5345	0.0534	17,126.00
o =15dollars	0.6245	0.0324	21,779.00
o =10dollars	0.6531	0.0253	29,458.00
h =2 dollars per unit per year	0.1679	0.0167	23,067.00
h =4 dollars per unit per year	0.1779	0.0147	26,610.00
h =8 dollars per unit per year	0.2678	0.0122	28,124.00
m =1year	0.1679	0.0167	19,601.00
m =1.5year	0.1999	0.0199	21,456.00
m =2year	0.2679	0.0267	22,552.00
S =0.25 years	0.5345	0.0534	26,618.00
S =0.5 years	0.3245	0.0324	19,244.00
S =0.75 years	0.2531	0.0253	18,220.00
R =0.75 years	0.1679	0.0167	29,625.00
R =1 years	0.1379	0.0117	22,651.00
R =1.25 years	0.0678	0.0017	20,453.00

The sensitivity analysis reveals that: if the value of h, K_1, p, or o increases then the values of K^* and $TP^*\left(K^*, T^*\right)$ increase while the value of T^* decreases; and by contrast, if the value of S and R increases then the values of K^* and $TP^*\left(K^*,\ T^*\right)$ decrease while the value of T^* increases. Hence, a higher value of c and h causes lower values of K^*, T^*, and $TP^*\left(K^*,\ T^*\right)$; and conversely, a higher value of m causes higher values of K^*, T^*, and $TP^*\left(K^*,\ T^*\right)$. A simple economic interpretation is as follows: if h, K_1, p, or o are higher, then the effect of backorder fraction K to demand (as well as annual profit) gets higher. Hence, higher value of h, K_1, p, or o causes higher values of backorder fraction K^* and annual total profit $TP^*\left(K^*,\ T^*\right)$ while a lower value of T^* to reduce holding cost. Similarly, a simple economic interpretation is as follows: if the expiration date of the deteriorating item m is longer, then it is worth to increase the backorder fraction K^* as well as the cycle time T^* in order to increase the sales and the annual total profit $TP^*\left(K^*,\ T^*\right)$. Likewise, one can easily interpret the rest of the managerial insights by using the analogous argument.

5-Conclusions

Taking care of both up-stream and down-stream trade credits simultaneously for deteriorating items with expiration dates along with shortages has received relatively little attention from the researchers. In this paper, we have built an EOQ model for the retailer to obtain its optimal credit period and cycle time in a supplier-retailer-buyer supply chain in which (a) the retailer receives an up-stream trade credit from the supplier while offer a down-stream trade credit to the buyer, (b) deteriorating items not only deteriorate continuously but also have their expiration dates, (c) down-stream credit period increases not only demand but also opportunity cost and default risk, and in contrast with most previous researches there is a shortage allowed. Then we have proved that the optimal trade credit and cycle time exist uniquely. Moreover, we have shown that the proposed model is a generalized case for non-deteriorating items and several previous EOQ models. Finally, we have used numerical example to study the optimal solution with respect to each parameter to illustrate the inventory model and provide some managerial insights.

For future research, we can extend the mathematical inventory model in several ways. For example, one immediate possible extension could be variable up and down-stream credit periods, cash discounts, etc. Also, one may generalize a single player local optimal solution to an integrated cooperative solution for both players. Finally, one can extend the fully trade credit policy to the partial trade credit policy in which a seller requests its credit-risk customers to pay a fraction of the purchase amount at the time of placing an order as a collateral deposit, and then grants a permissible delay on the rest of the purchase amount.

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