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Pricing and warranty for hi-tech products considering failures due to unknown faults and human errors (A game theory approach)

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Abstract

Hi-tech warranty reports show that a considerable percent of product's failures are related to the unknown faults and human errors, respectively. In this paper, two joint price and warranty models in a supply chain are presented for Hi-tech products. The supply chain includes a retailer and a customer, regarding failures due to unknown faults and human errors, beside the production failures. In the first model, failures due to production defects and human errors are considered separable, and in the second model they are considered inseparable. For both models, the interaction between the retailer and the customer is considered by retailer-Stackelberg game. The optimal purchase time, price, and warranty length are determined by maximizing the profits of customer and the retailer. Finally, the numerical example and sensitivity analysis are presented to illustrate the working logic of the models.

Keywords: Game theory, hi-tech products, human errors, pricing, unknown failure, warranty cost

1- Introduction

Nowadays the effects of failures related to unknown faults and human errors in pricing and warranty length would not be neglected; specially, for Hi-tech electronic products due to rapid advancement, and complexity of technology (Wu, 2011). Unknown failure or No Fault Found (NFF) implies that a failure occurs at the time of using products, but nothing could be found with testing or analyzing by warranty agents. A common example for this type of failure is when a computer hangs up. It is clear that a failure has occurred, but often works again when it is restarted (Qi, Ganesan and Pecht, 2008). In the case of electronic products, almost half of the evidence claims that the faults are related to the NFF (Jones and Hayes, 2001). Failures due to human errors are mostly caused by misusing. Brombachar (1999) showed that 17% and 38% of products failures are related to the human errors and NFF phenomenon, respectively (Wu, 2014).

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Identification and separation of these failures will affect warranty, pricing policies, and consequently, the profit. The related literature on joint pricing and warranty policies are briefly summarized to compare the proposed models.

Over the years, the warranty issue has been studied from different views; however, in this paper, joint warranty and pricing models have been focused precisely, which aims to determine optimal warranty length and selling price, simultaneously. Zhou, Li and Tang (2009) determined optimal warranty and pricing policy for repairable Hi-tech products with fixed lifetime due to free repair warranty policy. Chen, Li and Zhou (2012) considered various pricing strategies in a two echelon supply chain with one manufacturer and two retailers. The model is presented as a non-cooperative game. Furthermore, for the first time a three-level warranty service contract among manufacturer, agent, and customers were presented under risk parameter by Shamsi et al. (2013). They also considered the penalty cost incurred due to waiting repair time. Later, Esmaeili et al., (2014) developed their model based on semi or non-cooperative games. Yeh and Fang (2015) determined optimal decision for warranty with consideration of marketing and production capacity. Moreover, Wei, Zhao and Li (2015) proposed price and warranty period decisions for complementary products with horizontal firms' cooperation/noncooperation strategies. Taleizadeh et al. (2017) proposed an optimization model for warranty and price in a competitive duopoly supply chain with parallel importation. Anand et al. (2017) determined optimal price and warranty length for profit determination (an evaluation based on preventive maintenance). Moreover, Cao and He (2017) considerd price and warranty competition in a supply chain with a common retailer.
The mentioned research focused on the failures due to production defects; while failures due to unknown faults and human errors could not be neglected. In the literature, the difference between the failures due to production defects, human errors, or NFF concept has not been highlighted, while, separating these failures would have significant effects on warranty and pricing policies.

Practically, warranty claims could be due to production defects, unknown faults (NFF), or human errors; which have significant effects on warranty costs (Wu, 2011). To reduce the cost of NFF phenomenon, coordination policy between a retailer and a supplier in a supply chain was proposed by Huang et al. (2011). Wu (2014) proposed three warranty return policies by considering human errors, and unknown failures to determine the warranty cost of its provider by assuming the infinite time horizon. Furthermore, Raza and Ulansky (2015) developed a mathematical reliability model of continuously tested LRU subject to permanent and intermittent failures. Erkoyuncu et al. (2016) proposed a framework to estimate maintenance costs attributed to the NFF phenomenon. Khan, Farnsworth and Erkoyuncu (2017) focused on outlining current industrial attitudes regarding the No Fault Found (NFF) phenomena and identifies the drivers that influence the NFF decision-making process. Moreover, Eskandari Dorabati, Zeinal Hamadani and Fazlollahtabar (2018) investigated the effects of operators on warranty cost under sales delay conditions. The proposed model is validated using a numerical example for a two types of intermittent and fatal failures occur under a non-renewing warranty policy.

Most of these models have focused on the failures due to human errors, but the interaction between the customers and warranty service agents had not been considered. Moreover, the hi-tech characteristics, such as finite time horizon, and decreasing sale price during the time have not been considered in their models.

In this paper, two joint price and warranty models are presented in a supply chain for Hi-tech products by considering costs of failures due to unknown faults and human errors. Warranty costs include costs of failures due to production defects, unknown faults, and human errors. Unknown faults are irreparable, and production defects and human errors are repairable. In the first model, production defects and human errors are considered separable, and in the second model, they are considered inseparable. The supply chain includes a customer and a retailer that the interaction between them is being studied by retailer-Stackelberg. The optimal purchase time, price, and warranty length are determined by maximizing the profits of customer and the retailer. Finally, the numerical example and sensitivity analysis for both models are presented.

The reminder of this paper is organized as follows: In section 2, the notation and assumptions underlying the proposed models will be disclosed. In section 3, the problem is formulated. In section 4, the solution method is being described and in Section 5, the numerical example and sensitivity analysis is presented. Finally, in section 6, conclusions are presented with some managerial insight and suggestions for future research.

2- Notation and assumptions

This section describes the notations, decision variable, input parameters and assumptions of the model, which will be used to formulate the models.

2-1- Decision variables

t customer's purchase time
w warranty period length
p_0 initial selling price

2-2- Input parameters

k product value for the customer per unit of time
T product's lifetime in market
p_s price reduction percentage per unit of time
C_r repair cost
λ_1 failure rate due to production defects
λ_2 failure rate due to human errors
λ_3 failure rate due to unknown faults
C_m manufacturer's wholesale price for the retailer
P product value reduction percentage due to unknown failures (after warranty expiration)
S salvage value of the failed product due to unknown faults
Π_{ci} customer's profit in *i*th model; i=1,2
Π_{ri} retailer's profit in *i*th model; i=1,2

2-3- Assumptions

The proposed models are developed based on the following assumptions:

1. The planning horizon is finite [0, T] (Zhou, Li and Tang, 2009).
2. Warranty policies are nonrenewable.
3. Failure rate due to production defects, unknown faults and human errors are fixed and statistically independent (Wu, 2014).
4. Failures due to production defects and human errors are detectable and repairable.
5. Since the product is Hi-tech, the purchase time (t) will affect the product value of the customers;

Therefore, the product value will be k (T-t) for t Є [0, T] (Zhou, Li and Tang, 2009). Moreover, after warranty expiration, a reduction percentage is being considered as value reduction of product based on customer's dissatisfaction for unknown failures, such that:

$$Pk(T-t-w) \tag{1}$$

6. The prices of the manufacturer and the retailer for Hi-tech products are continuously decreasing function of time (Yang et al., 2011; Tsao et al., 2014):

$$p_0(1-p_s)^t \tag{2}$$

$$C_m(1-p_s)^t \tag{3}$$

3- The proposed models

Consider an item (hi-tech product) is received for warranty service that its failure could be detectable or undetectable. In warranty period, for unknown faults (undetectable failures), a new product will be replaced and failures due to production defects or human errors will be repaired. Since the failures caused by production defects and human errors sometimes are not distinguishable and could not be recognized from each other; separating production defects and human errors would be an efficient tool, therefore this assumption has been considered in the first model but not the second one.

3-1-The first model

In this model, the failures due to production defects and human errors are separated. For the failures due to human errors in the warranty period and after that, the customer pays the repair costs, and for the failures due to production defects, the retailer serves free repairing in the warranty period, and the customer pays the repair costs after warranty's expiration. Moreover, for the failures due to unknown faults in the warranty period, a new product will be replaced.

3-1-1-The customer's model

In the customer's model, the optimal purchase time is determined by maximizing the customer's profit. The customer's profit is given by:

Customer's profit = product value at purchase time - product value reduction after the warranty period due to claims with unknown faults - selling price- repair costs of failures due to production defects after the warranty period- repair costs of failures due to human errors in the warranty period and then

$$\pi_{c1}(t) = k(T-t) - Pk(T-t-w) - p_0(1-p_s)^t - C_r \int_{t+w}^{T} \lambda_1 \, dx - C_r \int_{t}^{T} \lambda_2 \, dx \qquad (4)$$

The customer purchase time (t) is determined by maximizing equation (4).

3-1-2- The retailer's model

The retailer obtains the optimal selling price and warranty length by maximizing the profit. The retailer's profit includes:

Retailer's profit = selling price - purchase cost - repair costs of failures due to production defects in the warranty period - product's replacement cost in the warranty period for claims with unknown faults - salvage value.

$$\pi_{r1}(p_0, w) = p_0(1-p_s)^t - C_m(1-p_s)^t - C_r \int_{t}^{t+w} \lambda_1 \, dx - \int_{t}^{t+w} \lambda_3 \, [C_m - S](1-p_s)^x dx \qquad (5)$$

Subject to $w \le T$ (6)

$$\pi_{c1}(t) = k(T-t) - Pk(T-t-w) - p_0(1-p_s)^t - C_r \lambda_1 (T-t-w) - C_r \lambda_2 (T-t) > 0 \qquad (7)$$

Equation (6) refers that the retailer doesn't provide warranty service after the end of product lifetime in the market. To prevent the warranty length of being zero, equation (7) has been considered.

3-2- The second model

In contrast to the first model, here, production defects and human errors are not recognized from each other. Therefore, the retailer repairs both failures due to production defects and failures due to human errors for free, in the warranty period. However, after the warranty expiration, the customer pays the repair costs for both failures. Moreover, for the failures due to unknown faults in the warranty period, a new product will be replaced.

3-2-1- The customer's model

The customer's profit includes:

Customer's profit = product value at purchase time- product value reduction after the warranty period due to claims with unknown faults- selling price- repair costs of production defects and human errors after warranty period.

$$\pi_{c2}(t) = k(T-t) - Pk(T-t-w) - p_0(1-p_s)^t - C_r \int_{t+w}^{T} \lambda_2\, dx - C_r \int_{t+w}^{T} \lambda_1\, dx \quad (8)$$

The customer's purchase time (t) is determined by maximizing equation (8).

3-2-2- The retailer's model

The retailer's profit includes:

Retailer's profit = selling price – purchase cost- repair costs of failures due to production defects and human errors in the warranty period– product's replacement cost in the warranty period for claims with unknown faults -salvage value

$$\pi_{r2}(p_0, w) = p_0(1-p_s)^t - C_m(1-p_s)^t - C_r \int_{t}^{t+w} \lambda_1\, dx - C_r \int_{t}^{t+w} \lambda_2\, dx - \int_{t}^{t+w} \lambda_3\, [C_m - S](1-p_s)^x dx \quad (9)$$

subject to $t+w \leq T$ (10)

$$(11)\ \pi_{c2}(t) = k(T-t) - Pk(T-t-w) - p_0(1-p_s)^t - C_r\lambda_1(T-t-w) - C_r\lambda_2(T-t-w) > 0$$

Equation (10) shows that the retailer doesn't provide warranty service after the end of product lifetime in the market. To prevent the warranty length of being zero, equation (11) has been considered.

4- Retailer- Stackelberg game

The interaction between the retailer and the customer in both models is considered as retailer-Stackelberg game, because the retailer first sets the selling price and warranty policy, and then the customer decides to purchase the product. Regarding to backward induction method, for a given initial price, p_0, and warranty length, w, the customer obtains the best purchase time, according to the customer's model, which is given by equation (13) and equation (15),then the retailer obtains his own initial price and warranty length by maximizing the profit , $\pi_r(p_0, w)$, based on the best response of the customer. The optimal solutions of the first and second models based on the retailer-Stackelberg game will be obtained as follows.

4-1- Optimal solution of the first model

Since the customer's profit function is concave in t (Appendix A), the optimal customer's purchase time will be obtained by setting the first derivative to zero, such that:

$$\frac{\partial \pi_{c1}(t)}{\partial(t)} = -k + Pk - p_0(1-p_s)^t \ln(1-p_s) + C_r\,\lambda_2 + C_r\,\lambda_1 = 0 \quad (12)$$

$$t = \frac{\ln\left[\frac{k(1-P) - C_r\lambda_2 - C_r\lambda_1}{-p_0 \ln(1-p_s)}\right]}{\ln(1-p_s)} \quad (13)$$

Substituting equation (13) into equations (5), (6), and (7), the problem transforms into a non-constrained nonlinear function of two variables w and p_o, where the optimal solution will be found by using a grid search.

4-2- Optimal solution of the second model

The customer's profit function is also concave in t (Appendix B), and the optimal customer's purchase time will be obtained by setting the first derivative to zero, such that:

$$\frac{\partial \pi_{c2}(t)}{\partial (t)} = -k + Pk - p_0(1-p_s)^t \ln(1-p_s) + C_r\,\lambda_2 + C_r\,\lambda_1 = 0 \quad (14)$$

$$t = \frac{\ln[\frac{k(1-P)-C_r\,\lambda_2 - C_r\,\lambda_1}{-p_0\ln(1-p_s)}]}{\ln(1-p_s)} \quad (15)$$

Substituting equation (15) into equations (9), (10) and (11), the problem transforms into a non-constrained nonlinear function of two variables w and p_o, where the optimal solution would be found by using a grid search.

5- Computational results

In this section, a numerical example and sensitivity analysis are presented to illustrate some significant features of the models established in previous sections.

5-1- Numerical example

In this section a numerical example is presented to illustrate the model precisely. Consider a retailer who sells a product. The product lifetime (T) is 150 weeks, and the failure rates due to production defects, unknown faults and human errors are 0.013, 0.011 and 0.005 per week, respectively. The value of the product for the customer (k) is 12000 monetary units per unit of time during the product lifetime, and the manufacturer's wholesale price (C_m) is 700000 monetary units. Furthermore, product salvage value (S) is 380000 monetary units and repair cost (C_r) is 60000 monetary units. The price reduction percentage (p_s) is 0.01 per unit of time, and product value reduction percentage due to unknown failures, after warranty expiration (P) is 0.38. The obtained result is shown in table1:

Table 1. The optimal solutions of the first and second models

	p_0	w	t	Πc_i	Πr_i
First model	1659279	54.10	95.90	1.47	267648.89
Second model	1681982	52.70	97.30	2.50	258289.60

According to table 1, by separating the failures due to production defects and the failures due to human errors, retailer obtains more profit, but customer's profit decreases, which makes sense. Moreover, since the customer wants to benefit whole of the warranty advantage, the customer's purchase time will be obtained based on the warranty length in both models.

5- 2- Sensitivity analysis

The effects of parameters k, λ_2 and λ_3 on Π_c and Π_r are investigated according to the data in the numerical example.

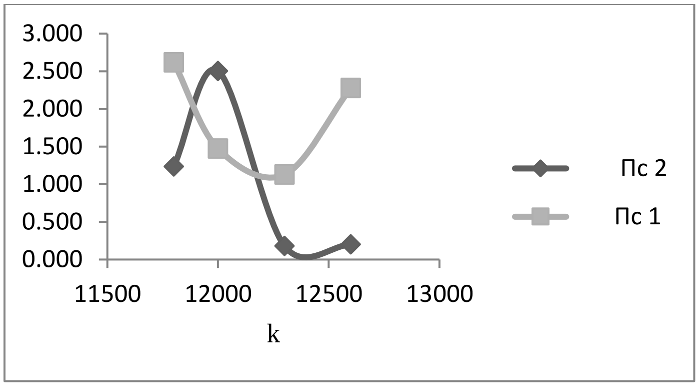

Fig.1. The effect of k on Π_c in the unit of time

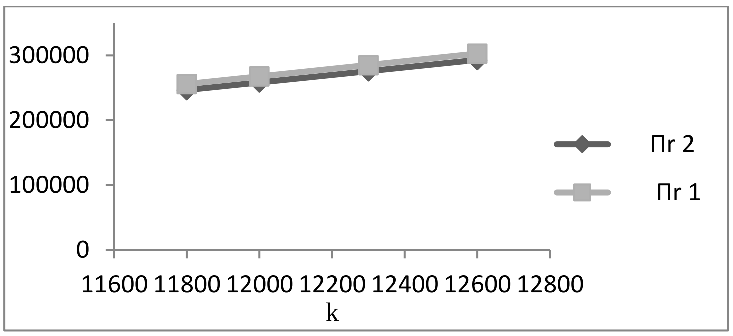

Fig.2. The effect of k on Π_r in the unit of time

As shown in figure1, for the high product values, since the customer is interested in finding the cause of failure (he/she uses the product carefully); therefore, the first model has more profit for the customer. But for the low product value, the second model will bring more profit.

Figure 2 shows that by raising the product value, k, the retailer's profit will increase in both models. In fact, by increasing k, the retailer will increase the warranty length and consequently the initial selling price. Moreover, the customer will purchase the product earlier to use the whole warranty length of product i.e. the purchase time will decrease based on the raise in warranty length.

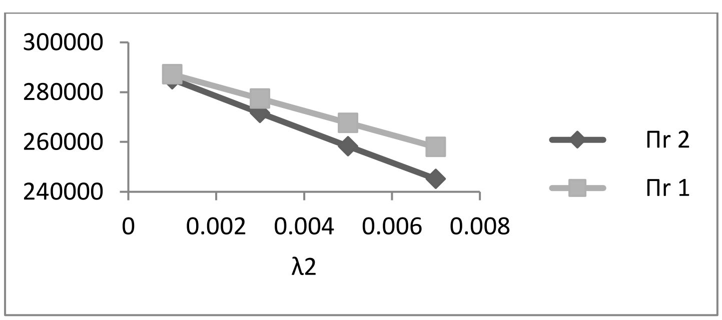

Fig.3. The effect of λ2 on Π_r in the unit of time

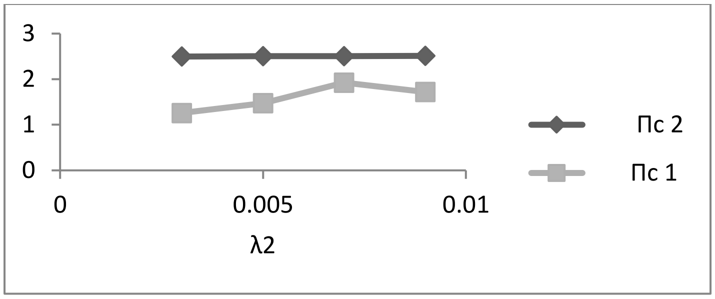

Fig.4. The effect of λ_2 on Π_c in the unit of time

As it is seen in figure 3 the retailer's profit will decreases by increasing the λ_2; because in the second model, λ_1 and λ_2 are inseparable and the retailer will repair the product free of charge in the warranty period, which will increase the retailer's warranty costs. In the first model, the customer will purchase the product later to decrease the repair cost; consequently, selling price will decrease and the retailer's profit will decreases too.

According to figure 4, the customer's profit is fixed in the second model but in the first model it has an increasing trend since the customer will purchase the product later to decrease the repair costs.

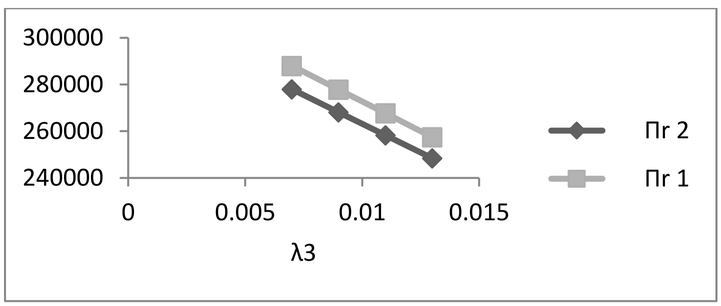

Fig.5. The effect of λ3 on Π_r in the unit of time

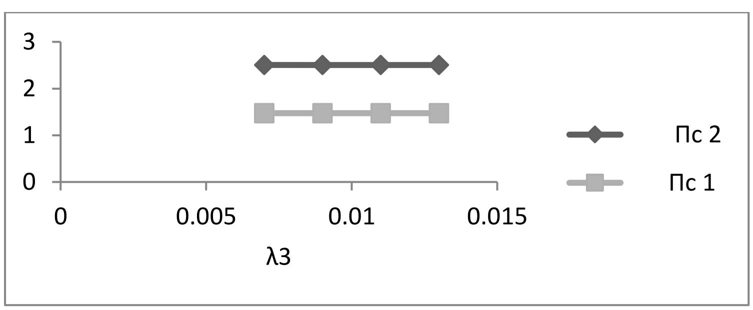

Fig.6. The effect of λ3 on Π_c in the unit of time

As it is seen in figures 5 and 6, by increasing the λ3, the retailer's profit decreases but the customer's profit is fixed in both models. This makes sense because the retailer will have more product replacing cost in the warranty period, also λ3 has not be used in the customer's profit equations, therefore, it doesn't have any effect on the customer's profit.

6- Conclusion

Rapid advancement of technology and expanding use of hi-tech products make the financial issues to surround the use of these products. Regarding to high warranty cost of Hi-tech products, recognizing the type of failures will cause an increase on the retailer's profits. Moreover, considering appropriate pricing and warranty policies simultaneously can increase customer's pleasure and satisfaction. In this paper, two models are presented for determining pricing and warranty length of Hi-tech products in supply chain containing a retailer and a customer. The warranty cost includes repair costs of failure due to production defects, human errors or unknown failures. In the first model, failures due to production defects and human errors are considered separable, and in the second model they have been considered to be inseparable. In both models, the interaction between the retailer and the customer is considered by a retailer- Stackelberg game. The optimal purchase time, price and warranty length are determined by maximizing the profits of customer and the retailer. Finally, the numerical example and sensitivity analysis are presented to illustrate the working logic of the models.

There is a broad scope to extend the present paper. For instance, the risk parameter can be considered for the customer or the retailer. In addition, incomplete information game can be used to model the interaction between customer and retailer. Finally, the competition between several manufacturers or retailers can be considered to make better solutions.

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Appendix A.

$$\pi_{c1}(t) = k(T-t) - Pk(T-t-w) - p_0(1-p_s)^t - C_r\,\lambda_1(T-t-w) - C_r\,\lambda_2(T-t) \quad \text{(A.1)}$$

$$\frac{\partial \pi_{c1}(t)}{\partial(t)} = -k + Pk - p_0(1-p_s)^t \ln(1-p_s) + C_r\lambda_2 + C_r\lambda_1 \quad \text{(A.2)}$$

$$\frac{\partial^2 \pi_{c1}(t)}{\partial(t)^2} = -p_0(1-p_s)^t\,[\ln(1-p_s)]^2 \quad \text{(A.3)}$$

When p_s<1, p_0>0 and the second derivative is negative, then the customer's profit function surely will have a maximum value.

Appendix B.

$$\pi_{c2}(t) = k(T-t) - Pk(T-t-w) - p_0(1-p_s)^t - C_r\,\lambda_1(T-t-w) - C_r\,\lambda_2(T-t-w) \quad \text{(B.1)}$$

$$\frac{\partial \pi_{c2}(t)}{\partial(t)} = -k + Pk - p_0(1-p_s)^t \ln(1-p_s) + C_r\,\lambda_2 + C_r\,\lambda_1 \quad \text{(B.2)}$$

$$\frac{\partial^2 \pi_{c2}(t)}{\partial(t)^2} = -p_0\,(1-p_s)^t\,[\ln(1-p_s)]^2 \quad \text{(B.3)}$$

When p_s<1, p_0>0 and the second derivative is negative, then the customer's profit function surely will have a maximum value.