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Leader-follower competitive facility Location and Design problem in an uncertain environment

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Abstract

This paper aims at providing a new approach to optimize location and design (quality) decision for new facilities as a leader-follower competitive configuration under the condition that competitor's reaction is unknown. A chain is considered as a leader in the first level and tends to open a new facility in a specific market where similar competitor facilities as follower already exist. In the second level, the follower decides on locating and designing some facilities through the market subject to the location and design of leader's facilities to keep or capture more market share. The market share captured by each facility depends on its distance to customers and its quality based on probabilistic Huff-like model. In facts, the leader decides on location and quality of its own new facility based on the follower reaction strategies to maximize its profit**.** Since the number of the follower's new facilities are unknown for the leader," robust optimization" is used for modeling this problem. A case from two chain stores in the city of Tehran, Iran, is studied and the proposed model is implemented. The computational results display the robustness and effectiveness of the model and highlight the importance of using robust optimization approach in uncertain competitive environments.

Keywords: Competitive location, location design, leader-follower, uncertain environment, robust optimization.

1-Introduction

In the location science, the best location of one or more facilities is determined with the purpose of optimizing a certain objective such as minimization of transportation costs, minimization of social costs, maximization of the market share, etc. One of the important factors to achieve this target is associated with existing/not existing competitors in the market that offer the same goods or services. The models of competitive facility location are proposed when there are some competitors in facility location. If there are some other facilities offering the same goods or it is likely to see a new competitor in near future, then the new facility will have to compete to gain more market share.

A review of this type of location problems can be seen in different papers (Plastria, 2001; Drezner, 2014). Hotelling (1929) introduced the first facility location model under competition in a linear market that all customers use the closest facility.

How to attract customers to the facility can be deterministic or probable, and knowing it is essential for estimating the market share captured by each facility.

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The probabilistic model for estimating the market share among the competing facilities was proposed by Huff (1964). It is assumed that the probability that a customer patronizes a certain facility is proportional to its area and inversely associated with a power of its distance from the costumer in Huff approach. Nakanishi and Cooper (1974) developed the mentioned approach by several attraction factors.

The competition may be static, which means that the competitors are already in the market and do not react to a new facility.it may also be with foresight, in which the competitors react after the entry of a new facility. If the competitors can change their decisions, then we have a dynamic model, in which the existence of equilibrium situations is of major concern.

The wide range of competitive location models is associated with leader-follower models. Consider a chain where the leader wants to open new facility/facilities in a market while similar facilities of a competitor, the follower, are already present (or will enter the market in the near future). The objective of the leader is to find the location of his facility/facilities that maximize his market share, following the location of the facility of the follower. These types of problems are known as Stackelberg problems in economic literature and as Simpson's problems in voting theory (Stackelberg, 1952). This type of problems in location literature was first introduced by Hakimi (1983). He introduced the terms *medianoid* for the follower problem, and *centroid* for the leader problem. An *(r|Xp)-medianoid* problem refers to the follower's problem of locating *r* new facilities in the presence of *p* leader's facilities located at a set of points *Xp*. In addition a *(r|p)-centroid* problem refers to the leader's problem of locating *p* new facilities, knowing that the follower will react by opening *r* new facilities by solving an *(r|Xp)-medianoid* problem (see for instance (Simpson,1969; Santos-Peñate 2007)). Drezner (1982) analyzed the problem in the plane and solved this problem heuristically by applying the gravity model. Such models are very difficult to solve. The value of the leader's objective function can be calculated for a given location if the follower's best location can be calculated. If the follower's optimal location cannot be guaranteed, the objective function cannot well-defined. Drezner and Drezner (1998) proposed three heuristic algorithms for the solving of the single-facility location problem (for both the leader and the follower) in the plane.

Ghosh and Craig (1983) have solved a problem similar to Drezner's one by making all variables discrete and also defining a set of predetermined potential locations for the leader-follower problem. They used integer programming for modeling the respective problem and their solution is only limited to relatively small-scale problems. Drezner (1998) discussed a model for the competitive location with a limited budget in which both the optimal allocation of the budget among the new facilities and the best locations for them were found in a continuous space.

Constructing facilities when attractiveness of facilities is a variable with a cost function depending on the attractiveness is termed ''design'', and the problem is called location and design model. The model was first proposed by Plastria and Carrizosa (2004) suggesting a multitude of approaches for calculating the market share attracted by facilities.

Ferna´ndez (2007) developed two solution methods which the location and the quality (design) of the new facilities were determined with the goal of maximization of the profit obtained for the chain. Redondo and Fernández (2010) solved the facility location and design (1|1)-centroid problem on the plane through heuristics approach. Four heuristics have been proposed for this hard-to-solve global optimization problem, namely, a grid search procedure, an alternating method and two evolutionary algorithms. Nasreddine Saidani (2012) developed a two-stage method which takes into account in the quality decision stage, the competitive decision process occurring among facilities is modelled as a game, the solution of which is given by its Nash equilibrium, In the location decision stage, an interval based global optimization method is used to determine the best location for the new facility. Usually, the demand is assumed to be fixed or constant in the literature regardless the conditions of the market. Redondo and Arrondo (2010) proposed a two-level evolutionary algorithm for solving the facility location and design (1|1)-centroid problem on the plane with variable demand where demand varies depending on the attraction for the facilities. Rafael Blanquero et al. (2016) studied the p-facility Huff location problem on networks formulated as a Mixed Integer Nonlinear Programming problem solved by a branch-and-bound algorithm and proposed and compared two approaches for the initialization and division of sub problems.

Alekseeva et al. (2009) and Kononov et al. (2010) proposed a model for the leader-follower problems in a discrete space considering the rule of the closest facility to the customers and maximizing the leader's and the follower's profit. In mentioned studies, the customer behavior is considered deterministic and on the basis of the least distance. On the basis of the probabilistic behavior of the customers and Huff rule, Gorji et al. (2011) solved the leader-follower model in a discrete space.

There are literally several studies in competitive location, which consider fixed reaction from a competitor which is not very practical in the real world. When one of the competitors increases his/her market share, of course, he/she can be faced with the reaction of other competitors, immediately. On the other hand, he cannot predict the exact reaction of competitors. Gorji et al. (2013) first considered the mentioned fact and proposed a robust model to determine the optimal locations for the leader's new facilities under the assumption that the number of the follower's new facilities is unknown for the leader.

In this paper, the presented model of Gorji et al. (2013) is extended to a location and design problem. The proposed model has two decision variables which influence each other. In other words, a leader-follower competitive facility location and design problem has been proposed where the numbers of the followers' new facilities are unknown in a discrete space.

The remaining structure of this article is organized as follows: Section 2 devotes to introducing of huff-like competitive location and design. Since Modeling is assumed with uncertainty, in section 3 the concept of robust optimization proposed by Mulvey et al. (1995) is described and in Section 4, the proposed model of this paper is explained. Section 5 presents the case study for obtaining solutions. The authors provide the conclusion of their findings and suggestions for future research in Section 6.

2-Huff-like competitive location and design problem

A chain, the *leader*, wants to locate a single new facility in a discrete space, where *m* facilities offering the same goods or product already exist. The first *t* of those *m* facilities belong to the chain, and the other (*m–t)* facilities belong to a competitor chain, the *follower*. The leader knows that the follower, as a reaction, will subsequently position a new facility too. The demand, supposed to be inelastic, is concentrated at *n* demand points, whose purchasing power (w_j) are known.

To estimate the market share of each facility, the model proposed by Huff (1964) is adopted. In the model, the patronizing behavior of customers is assumed to be probabilistic, that is each customer splits his buying power proportionally among the facilities in the market. The attraction of each facility to a given demand point is proportional to the quality of the facility and inversely proportional to the distance between the demand point and the facility.

The following notation is used to describe the facility location and design model under study:

m	the number of existing facilities
p	the number of leader's new facilities
Nf	the number of follower's new facilities
n	the number of demand points
n_{pot}^{l}	the number of potential locations for the leader
n_{pot}^{f}	the number of potential locations for the follower
i	index of existing facility; the range for leader's existing facilities is i = 1,2,…,t and the range for follower's existing facilities is i = t + 1, t + 2,…,m
j	index of demand points; j = 1,2, . . .,n

r	index of quality level
f	Index of potential for the follower f= 1,2,…, n_{pot}^{f}
l	Index of potential for the leader l= 1,2,…, n_{pot}^{l}
W_j	the buying power of demand point j
a_{ir}	quality of existing facility i with the quality level r
d_{ij}	the distance between existing facility i and demand point j
x_{fr}	location of follower's new facility with the quality level r
x_{lr}	location of leader's new facility with the quality level r
$d_{x_{fr}j}$	the distance between the follower’s new facility and demand point j
$d_{x_{lr}j}$	the distance between the leader’s new facility and demand point j
A_{ij}	the attractiveness level of existing facility i for demand point j
A_{Lj}	the attractiveness level of leader’s new facility for demand point j
A_{Fj}	the attractiveness level of follower’s new facility for demand point j
λ_j	the quality sensitivity of demand point j
β	the distance sensitivity parameter, β> 0

And the model variables are as follows:

xp_{fr}	a binary variable that is equal to 1 if the follower opens his new facility in potential location f with the quality level r
xp_{lr}	a binary variable that is equal to 1 if the leader opens his new facility in potential location l with the quality level r
q_{lr}	quality of leader's new facility with the quality level r
q_{fr}	quality of follower's new facility with the quality level r
M_L	the leader’s market share
M_F	the follower’s market share

According to the gravity model, the formulation of the attractiveness level for a customer j for facility j is as follows (see Drezner (1994), Fernández (2007)):

$$A_{ij} = \frac{q_i}{g(d_{ij})} \tag{1}$$

The use of a general non-negative and non-decreasing function g_{ij}, in the attraction functions generalizes the proposals found in literature, such as $g(d_{ij}) = d_{ij}^{\lambda_j}$ (see (Huff, D. L. (1964), see Drezner (1994)) or $g(d_{ij}) = e^{\lambda_j d_{ij}}$ (see Hodgson (1981)) with $\lambda_j > 0$, a given distance sensitivity parameter.

Therefore, in this paper the following formulation has been considered for of the attractiveness level for a customer j for facility i:

$$A_{ij} = a_{ir} / (\varepsilon + \mathrm{d}_{ij})^{\beta} \tag{2}$$

Where is $g(d_{ij}) = d_{ij}^{\lambda_j}$ has been considered as distance function, and β has been used instead of λ_j which is a given distance sensitivity parameter that is not different between. If the distance between the facility and the customer is zero, the denominator becomes also zero and consequently makes the fraction undefined. Therefore ε is added to d_{ij} avoid denominator becoming zero. Similarly, the attractiveness levels of the leader's and the follower's new facilities for customer j are respectively as follows:

$$A_{lj} = q_{lr} / (\varepsilon + d_{x_{lr}j})^{\beta} \tag{3}$$

$$A_{fj} = q_{fr} / (\varepsilon + d_{x_{fr}j})^{\beta} \tag{4}$$

Using these assumptions, the market share attracted by the leader's chain after the location of the leader and the follower's new facilities are determined is:

$$M_L = \sum_{j=1}^{n} w_j \frac{\sum_{i=1}^{t} \frac{\lambda_j a_{ir}}{(d_{ij} + \varepsilon)^{\beta}} + \sum_{r \in R} \sum_{L=1}^{N_{pot}^{l}} \frac{\lambda_j q_{lr}}{(d_{x_{lr}j} + \varepsilon)^{\beta}} xp_{lr}}{\sum_{i=1}^{m} \frac{\lambda_j a_{ir}}{(\varepsilon + d_{ij})^{\beta}} + \sum_{r \in R} \sum_{F=1}^{N_{pot}^{f}} \frac{\lambda_j q_{fr}}{(d_{x_{fr}j} + \varepsilon)^{\beta}} xp^{*}_{Fr} + \sum_{r \in R} \sum_{L=1}^{N_{pot}^{l}} \frac{\lambda_j q_{lr}}{(d_{x_{lr}j} + \varepsilon)^{\beta}} xp_{lr}} \tag{5}$$

Consequently, the corresponding market share attracted by the follower's chain is:

$$M_F = \sum_{j=1}^{n} w_j \frac{\sum_{i=t+1}^{m} \frac{\lambda_j a_{ir}}{(d_{ij} + \varepsilon)^{\beta}} + \sum_{r \in R} \sum_{F=1}^{N_{pot}^{f}} \frac{\lambda_j q_{fr}}{(d_{x_{fr}j} + \varepsilon)^{\beta}} xp_{fr}}{\sum_{i=1}^{m} \frac{\lambda_j a_{ir}}{(\varepsilon + d_{ij})^{\beta}} + \sum_{r \in R} \sum_{F=1}^{N_{pot}^{f}} \frac{\lambda_j q_{fr}}{(d_{x_{fr}j} + \varepsilon)^{\beta}} xp_{fr} + \sum_{r \in R} \sum_{L=1}^{N_{pot}^{l}} \frac{\lambda_j q_{lr}}{(d_{x_{lr}j} + \varepsilon)^{\beta}} xp_{lr}} \tag{6}$$

3-Background of robust optimization

First, the framework of the robust optimization used to obtain a set of solutions that are robust against the conversion of parameters is described briefly. It's introduced by Mulvey et al. (1995).The optimization model has the following structure:

$$Min\ c^T x + d^T y \tag{7}$$

subject to:

$$Ax = b \tag{8}$$

$$Bx + Cy = e \tag{9}$$

$$x, y \geq 0 \tag{10}$$

x is the vector of "design" decision variable. It should be noted that the optimal value of x does not depend on the uncertain parameters. $y \in R^{n^2}$ is the vector of "control" variables; the optimal values of which value depend both on the realization of uncertain parameters and on the optimal value of the design variables. Constraint (8) is the structural constraints in which coefficients are fixed and free of noise. Constraint (9) denotes the control constraints. The coefficients of this constraint set are uncertain. A set of scenarios Ω={1,2,3,…,S} is introduced. p^s is the probability of scenario $(\sum_{s=1}^{S} p^s = 1)$ and a set $\{B^s, C^s, e^s, d^s\}$ is the set of uncertain parameters under each scenario. In each scenario control variable is determined then set $\{z_1, z_2, \ldots, z_s\}$ are error vectors to measure the infeasibility allowed in the control constraints under scenario s. the scenario based robust optimization approach is as follow:

$$min\ \sigma(x, y_1 y_2, \ldots, y_s) + \omega\rho(z_1, z_2, \ldots, z_3) \tag{11}$$

subject to

$$Ax = b, \tag{12}$$

$$B_s x + C_s y + z_s = e_s, \qquad \forall s \in Q \tag{13}$$

$$X, y_s \geq 0\ , \qquad \forall s \in Q \tag{14}$$

The function σ can be considered expected value i.e. $\sigma(x, y_1, y_2, ..., y_s) = \sum_{s=1}^{S} p_s \varepsilon_s$ .The value ε_s in each scenario is the objective function $\varepsilon_s = C^T + d^T y_s$ with probability p_s .The second term in the objective function (11) is a feasibility penalty function used for errors of control constraints under various scenarios. Appropriate function Mulevey et al. (1995) and Mulvey and Ruszczynski (1995) considered is $\lambda \sum_{s \in Q} (\varepsilon_s - \sum_{s' \in Q} p_{s'} \varepsilon_{s'})^2$ in which λ denotes the weighting scale to measure the tradeoff between feasibility and cost. In this paper, the RO used in the leader's problem is as follows:

$$Min \sum_{s \in Q} p_s \varepsilon_s + \lambda \sum_{s \in Q} (\varepsilon_s - \sum_{s' \in Q} p_{s'} \varepsilon_{s'})^2 \tag{15}$$

subject to :

$$Ax = b\ , \tag{16}$$

$$B_{s+} C_s Y_s + Z_s = e_s\ , \quad \forall s \in Q \tag{17}$$

$$X, Y_S \geq 0\ , \qquad \forall s \in Q \tag{18}$$

4-Robust model for competitive location-design problem

In the real world competition, with the reaction of competitors, we face uncertainty. Therefore in this study, we have eliminated the assumption that the number of the follower's new facilities is definite and the leader's problem has been solved in a condition that the leader does not know that after locating her new facilities, how many facilities are going to be opened by the follower. Mulvey et al. (1995) and Mulvey and Ruszczynski (1995) presented an improved stochastic programming approach called the robust programming, capable of tackling the decision-makers' favored risk aversion or service-level function and yielding a series of solutions that are progressively less sensitive to realizations of the data in a scenario set. In this section, we present the robust model for competitive facility location and design.

In this model, the leader knows that after opening the p new facilities, the follower will surely respond to this and will open its own new facilities but the leader is not certain about is the number and quality of the facilities that the followers are going to open. In this model, it is assumed that the maximum number of follower's new facilities and also the probability of opening a different number of follower's new facilities are known for the leader. The leader's problem has been modeled by RO in an uncertain condition in which the number and quality of competitor's new facilities are unknown. Each number of the follower's new facilities here is defined as a scenario and RO is consequently applied. In fact, it is assumed that the follower may locate 1, 2,... r new facilities with the quality of level 1,2, 3. Therefore these different scenarios are obtained. The follower's objective in each scenario is to maximize his profit share and the leader's one is also to maximize his own profit after the follower’s new facilities entry. At first, the leader opens his new facility in potential point xp_{lr} .The follower’s problem in scenario *s* with respect to his knowledge about xp_{lr}^{s} (the location and design of the leader’s new facilities which were opened in scenario*s*) is as follows:

$$M_F^s = \sum_{j=1}^{n} w_j \frac{\sum_{i=t+1}^{m} \frac{\lambda_j a_{ir}}{(d_{ij}+\varepsilon)^{\beta}} + \sum_{r\in R}\sum_{F=1}^{N_{pot}^{f}} \frac{\lambda_j q_{fr}}{(d_{x_{fr}j}+\varepsilon)^{\beta}} xp_{Fr}^{s}}{\sum_{i=1}^{m} \frac{\lambda_j a_{ir}}{(\varepsilon+d_{ij})^{\beta}} + \sum_{r\in R}\sum_{F=1}^{N_{pot}^{f}} \frac{\lambda_j q_{fr}}{(d_{x_{fr}j}+\varepsilon)^{\beta}} xp_{Fr}^{s} + \sum_{r\in R}\sum_{L=1}^{N_{pot}^{l}} \frac{\lambda_j q_{lr}}{(d_{x_{lr}j}+\varepsilon)^{\beta}} xp_{Lr}^{s}} \quad (19)$$

$$max\ B_F^S = c.(M_F^S)-G_1(x)-G_2(r) \quad (20)$$

subject to:

$$\sum_{r\in R} xp_{fr}^{s} \leq 1 \quad , \qquad f=1,2,\ldots, n_{pot}^{f} \quad (21)$$

$$\sum_{r\in R}\sum_{F=1}^{N_{pot}^{f}} xp_{fr}^{s} = nf_s \quad (22)$$

$$xp_{fr}^{s} \in \{0,1\}$$

Equation (20) is the objective function that maximizes the follower’s profit, where C is the income per unit of good sold and M_F^s is the market share (described in the previous section as equation (19)), and multiply M_F^s by C results expected sales. To calculate the profit, the costs must be deducted from the amount of sales. G1(x) and G_2(r) are functions which give the operating and design cost of a facility located at x with quality level r. G_1(x) and G_2(r) are as follows:

$$G_1(X) = \sum_{J} \frac{W_J}{(\varphi_{i1} + (d_{jx})^{\varphi_{i0}})}$$
$$\varphi_{i1}, \varphi_{i0} \succ 0 \quad (23)$$
$$G_2(r) = \exp(\frac{r}{\beta_0} + \beta_1) - \exp(\beta_1)$$
$$\beta_0, \beta_1 \succ 0$$

According to the formulation of the function G1(x), Operating costs are a function of the buying power of customers and their distance from the facility. It means that the operation cost of locating a new facility in places where customers have more buying power and less distance is higher. About the formulation of the G2(r), it is clear that the design cost of a facility is an increasing function of quality level r. A more detailed explanation of these functions can be found in Fernández et al. (2007).

Constraint (21) ensures that each follower's new facility is opened in only one of the design level. The number of the follower's new facilities for each scenario is ensured by constrains (22).

For each arbitrary location for the leader's new facilities in each scenario, the follower problem has solved and optimal location and design of leader's new facility is obtained in point where profit was maximized. Supposing M^S_L as the optimal solution of the problem in scenario *s*, the RO for the leader's problem is as follows:

$$\max \sum_{s=1}^{S} p_s.\mathrm{B}_L^s - \lambda \sum_{s=1}^{S} p_s (\mathrm{B}_L^s - \sum_{s=1}^{S} p_s.\mathrm{B}_L^s)^2 \tag{24}$$

$$B_l^S = c.\left(M_l^S\right) - G_1\left(x\right) - G_2\left(r\right) \tag{25}$$

$$M_L^s = \sum_{j=1}^{n} w_j \frac{\sum_{i=1}^{t} \frac{\lambda_j a_{ir}}{(d_{ij}+\varepsilon)^{\beta}} + \sum_{r\in R}\sum_{L=1}^{N_{pot}^{l}} \frac{\lambda_j q_{ir}}{(d_{Lj}+\varepsilon)^{\beta}} xp_{Lr}^{s}}{\sum_{i=1}^{m} \frac{\lambda_j a_{ir}}{(\varepsilon + d_{ij})^{\beta}} + \sum_{r\in R}\sum_{F=1}^{N_{pot}^{f}} \frac{\lambda_j q_{fr}}{(d_{Fj}+\varepsilon)^{\beta}} xp^{*s}_{Fr} + \sum_{r\in R}\sum_{L=1}^{N_{pot}^{l}} \frac{\lambda_j q_{lr}}{(d_{lj}+\varepsilon)^{\beta}} xp_{Lr}^{s}} \tag{26}$$

subject to:

$$\sum_{r\in R} xp_{lr}^{s} \le 1 \quad , \quad \mathrm{l}= 1,2,\ldots,\ n_{pot}^{l} \tag{27}$$

$$\sum_{r\in R}\sum_{L=1}^{N_{pot}^{l}} xp_{lr}^{s} = P \tag{28}$$

$$xp_{lr}^{s} \in \left\{0,1\right\} \tag{29}$$

The objective function (24) gives a robust solution for the leader's problem. Equation (25) maximizes the expected value of the leader's profit for different scenarios. The difference between the expected value and the scenarios' optimal solutions is used in the penalty function. λ represents weighting penalty. When the difference between expected value and the optimal solution in each scenario is more important, λ gives more value and conversely. The constraints (27)–(29) are similar to (8)–(10).

5-Case study description

In this section, due to the novelty of aforementioned characteristics for the proposed model in the field of competitive facility location and design, a case study along with its solution is provided. According to the increasing trend of chain stores in urban life, some new chain stores are created which could be found a good place among the townspeople to meet their needs. In the city of Tehran, Iran, a large portion of people's needs of essential goods are supplied by such chain stores. In this section, we are going to address *Hyper*star and *Hyperme* as two new competing chain stores in *Tehran* as an application case of the developed model in this study. These stores offer a variety of products to customers and have the remarkable reputation for providing amenities. In the present study the senior store, *Hyper star*, aims to open a new branch in one of the regions of Tehran and knows that after opening a new branch, *Hyper me,* as the follower, will surely respond to it by opening its own new facilities. The leader is not certain about the number and quality of follower's new facilities that he is going to open. Although exact information on the number of new branches of *Hyperme* is not available, the probability of *Hyperme* opening different number of new facilities is known for the experts of *Hyper star*.

The proposed robust optimization model in this study is used to find the best location and design for the leader (*hyper star*) and maximize its profit with regard to the different scenarios for number of new facilities of the follower (*hyper me).* Tehran is divided into 22 regions and the central point of each region was considered as the demand point having a different buying power from the others. The buying power is given in range 1-10 In accordance with the financial situation of people living in every region.

Three Quality levels are also defined for new and existing branches (facilities) of chain stores as follow:

- Level 1: in this quality level, the facility *i* with a mixture of several facility attributes approximately is "not good" and value of the quality is 1.
- Level 2: in this quality level, the facility *i* with a mixture of several facility attributes approximately is "mediocre" and value of the quality is 3.
- Level 3: in this quality level, the facility *i* with a mixture of several facility attributes approximately is "very good" and value of the quality is 5.

It should be noted that people of every region with respect to their income and cultural situation prevailing in that region have different sensitivity to the quality level. Therefore, we considered parameter λj as the quality sensitivity of demand point j which can take values in the range [0.75-1.25]. Table 1 provides the location and the buying power of different demand points obtained by performing the survey with 89% of *Cronbach's alpha.*

Table 1. The locations and the buying power of different demand points

Demand points				
region No	buying power wj	quality sensitivity λj	location	
			x	y
1	10	1.25	51.462	35.801
2	10	1.25	51.359	35.749
3	6.63	1.25	51.425	35.767
4	2.87	1.1	51.521	35.752
5	4.54	1	51.304	35.746
6	4.32	0.9	51.401	35.722
7	4.55	1	51.443	35.720
8	4	1	51.489	35.724
9	2.77	0.9	51.320	35.683
10	2.14	0.9	51.365	35.682
11	3.59	0.7	51.394	35.679
12	1.89	0.6	51.428	35.678
13	3.35	0.5	51.504	35.707
14	2.03	0.5	51.485	35.671
15	2.14	0.8	51.474	35.634
16	1.26	0.8	51.412	35.638
17	1.98	0.5	51.361	35.654
18	1.81	0.5	51.298	35.654
19	1.8	0.5	51.367	35.625
20	2.54	0.5	51.436	35.599
21	2.81	1	51.209	35.708
22	3.75	1	51.211	35.742

The locations of demand points and the leader and follower existing facilities are depicted in figure 1.

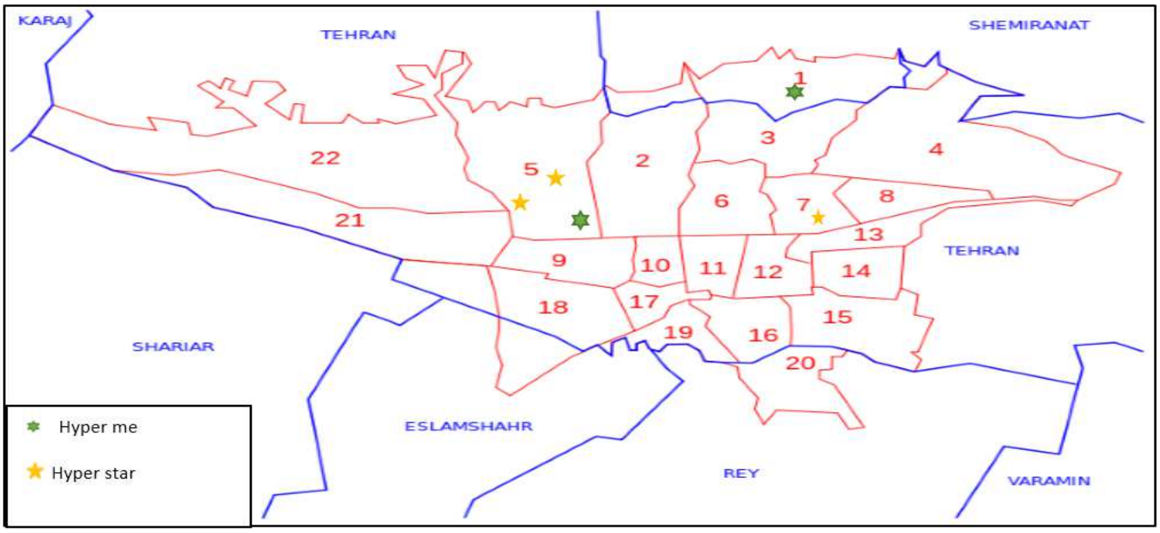

Fig.1 Schematic location of Hyper star and Hyper me branches in Tehran

As shown in figure 1, currently, there are three branches of *Hyper star* and two branches of *Hyper me* competing each other. In this case, 107 potential points for opening new branches are considered, in the central points of zones in each region. It is noteworthy that some regions are not suitable for this purpose, which has been removed. The coordinates of all points have been collected by using a map. The values of the primary input parameters are shown in table 2.

Table 2. The location and design of leader and follower existing facilities

	branch NO	location		Quality
		x	y	
leader				
	1	51.293	35.729	5
	2	51.313	35.738	1
	3	51.452	35.709	1
follower				
	1	51.308	35.705	3
	2	51.444	35.787	3

5-1 Solution method and computational results

In this study, the follower's profit function in each of the potential locations and quality levels is calculated with regard to leader's potential location and various quality levels for all scenarios. Furthermore, the optimal locations and quality of new facility of the follower are obtained and leader's profit function (equation (25)) values are achieved correspondingly. The probability of scenario multiplied by leader's profit function value in each scenario is used to obtain the expected value of the leader's profit function in a specific potential location. The expected value minus the penalty value of solution robustness is calculated equation (11). This operation is done for all potential points and various quality levels of the leader and finally, the obtained maximum value of the objective function is considered as the optimal robust solution for the leader's problem. In this case, there are three scenarios for the number of follower's new facilities with determined probability. The performance of each competitor in each scenario is shown in table 3. The optimal location and quality of the leader, *Hyper star*, in different scenarios with a λ value of 0.2 and given the probability of each scenario are shown in the last row of table 3.

Table 3. Leader's optimal location for all scenarios

	number of new facilities of the follower	probability	the new facility of leader			the new facility of the follower		
			location (region-zone)	quality	profit	location (region-zone)	quality	profit
scenario1	1	0.4	2-4	5	287	3-3	3	320
scenario 2	2	0.4	2-8	5	319	21-2 , 3-3	5,3	266
scenario 3	3	0.2	11-1	1	387	2-3 ,21-2 ,22-2	3,1,1	277
robust solution		location (region-zone): 2-4		value of the quality:3				

As can be seen in scenario 1 that follower will open 1 new branch with a probability of 0.4, the optimal location of leader's new branch is in region 2 and zone 4 at the quality level 3 (i.e. mixture of several facility attributes is "very good" and value of the quality is 5) and then maximum profit for the follower is in location 3-3 and quality level 2 (i.e. mixture of several facility attributes is "mediocre" and value of the quality is 3). As it is observed, the optimal location and design (quality) for leader's new facilities are different in various scenarios. For asserting the superiority of the proposed robust optimization model compared to other solutions, we have compared the robust solution with the optimal solutions of scenarios in Table 4. It is shown that the robust solution has the minimum deviation compared to scenarios 1 and but it doesn't have maximum expected value. Although the solution of scenario 3 has the least deviation, due to its expected value, that is the lowest compared to the rest; after subtracting penalty function from the expected value, it not remains maximum.

Table 4. Comparison between the robust solution and different solutions

	Penalty function	**expected value**	**Subtracting penalty function from the expected value**
scenario 1	195	300	105
scenario2	40	308	268
scenario3	10	271	261
robust	31	300	269

The robust solution that makes a trade-off between profit deviation and the expected value of profit and considers both simultaneously is the best solution to an uncertain situation.

In table 5, the robust optimal locations for this case with different values of λ are depicted. λ represents weighting penalty. When the difference between expected value and the optimal solution in each scenario is more important, λ gives more value and conversely the robust optimal solutions in the range of 0.2-0.8

for λ are similar, at region–zone 20-1 and quality level of 2. For λ>0.8 and λ <0.2 optimal solutions are different. It is possible to provide various solutions to the decision maker and allow him to select the best.

Table 5. The leader's optimal robust solution for different values of λ

λ	0.06	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
optimal location (region–zone)	2-4	11-3	20-1	20-1	20-1	20-1	20-1	20-1	20-1	20-2	15-3
quality	3	1	3	3	3	3	3	3	3	3	1

6-Conclusion and future research

In this study, a robust competitive facility location problem model was proposed in a leader-follower configuration. Market share was estimated by using Huff-like models and costs devoted to both the design and the location of the new facilities were also taken into account. These two factors were considered as the variables of the problem. Computational results showed that how the proposed model can be used to determine location and quality (design) of new facilities in a case study. Different scenarios for numbers of follower's new facilities were generated and the optimal solution that maximizes the expected value of the leader's profit for different scenarios and minimizes the difference between the expected value and the scenarios' optimal solutions simultaneity was obtained.

As future research, continuous space for the same model and developing heuristic algorithms to solve are suggested. Also, considering an elastic demand function in the facility location problem model can be considered as another future work insight. .

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