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# A heuristic light robust approach to increase the quality of robust solutions

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## Abstract

In this paper, the optimizations problems to seek robust solutions under uncertainty are considered. The light robust approach is one of the strong and new methods to achieve robust solutions under conditions of uncertainty. In this paper, we tried to improve the quality of the solutions obtained from the Light Robust method by introducing a revised approach. Considering the problem concerned, an algorithm was also developed to properly choose the weight parameter in the proposed approach presented as much as possible. In addition, the data obtained from the proposed approach were investigated using the regression analysis. The results indicate that increased ratio of the number of constraints to the number of variables is directly correlated with increased likelihood of improving the quality of the solution. In conditions where the proposed approach has provided a solution better than the solution presented by the simple Light Robust approach, the mean value of the improvement accounts for about 9%.

**Keywords:** Robust Optimization, Light Robust, Uncertainty

## 1-Introduction

The basic premise of classical mathematical programming is to develop a model, which input data are certain values (Bertsimas and Sim, 2004). However, this assumption is often violated in real-world problems. This problem can be either attributed to the fact that the parameters used in the model are only the estimates of the real parameters or in a more general state due to the effect of uncertainty on some parameters.

The optimization issues affected by non-deterministic or uncertain parameters have been in the focus of attention from a long time ago. Two important factors in problems involving uncertainty are quality and feasibility of the solution. The optimization models occurring in uncertainty conditions may produce solutions so far from the optimal solution or even infeasible. Therefore, it seems natural to look for designing of solution methods capable of securing the planning models against uncertainty, or in other words, the solutions that are "robust" (Bertsimas and Sim, 2004).

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In this paper, we focus mainly on the linear programming problem in the following form:

$$\min \sum_{j \in N} c_j x_j \tag{1}$$

$$\sum_{j \in N} a_{ij} x_j \geq b_i \qquad i \in M \tag{2}$$

$$x_j \geq 0 \qquad j \in N \tag{3}$$

Some values of the coefficients matrix $A$ can vary in the range $\left[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}\right]$. The $a_{ij}$ and $\hat{a}_{ij}$ are the relevant nominal values and the input noise data, respectively. The actual values of the coefficients are also independent of each other. Since the uncertainty in vectors $b$ and $c$ can be eliminated directly, they are considered in the model. In this model, we assumed that $n = |N|$ and $m = |M|$ are respectively the number of variables and the number of constraints.

The purpose of this paper is to display and improve the solution's robustness. In fact, we tried to improve the quality of the solutions obtained from the Light Robust method by introducing a revised approach. Considering the problem concerned, an algorithm was also developed to properly choose the weight parameter in the proposed approach presented as much as possible. The related literature is presented in Section 2. In Section 3, the proposed Light Robust approach will be described. An innovative algorithm is presented in Section 4. The computational results on random data are presented in Section 5 and finally, Section 6 draws some conclusions and suggestions for future studies.

## 2- Literature Review

The ***sensitivity analysis*** approaches and ***stochastic programming*** were previously used in the classical methods to consider the uncertainty of parameters (Birge and Louveaux (2011)). In the first method, the effect of uncertainty on data was initially ignored, and subsequently, the sensitivity analysis was used to confirm the solutions obtained. However, the sensitivity analysis is only a tool for analyzing the solution's goodness and cannot be used to generate robust solutions. In addition, performing sensitivity analysis on the parameters simultaneously in models with a lot of unreliable data appears to be impossible.

The ***stochastic programming*** considers and uses scenario with different probabilities to scale the parameters (See Ruszczynski and Shapiro (2003); Linderoth et al (2006) and Birge and Louveaux (2011)). However, there are three main problems in the case of this approach:

- Identifying the distribution of data, and thus, numerating the scenarios using these distributions.
- The chance constraints eliminate the convexity characteristic of the main problem and significantly adds to its complexity
- The dimensions of the optimization model obtained will astronomically increase with increasing scenarios, which raises many computational challenges.

Another approach to consider the uncertainty is ***robust optimization*** (Bertsimas et al. (2011); Ben-Tal and Nemirovski (1999); Gabrel et al, (2014)). A robust solution is a solution that can remain feasible even when some of the input parameters change. In other words, it is desirable to provide a solution that is not necessarily optimal for a nominal problem but its feasibility and the cost generated by would not be grossly overwhelming by changing the coefficients (Mulvey, 1995). The first step in this direction was taken by Soyster (1973). He proposed a linear optimization model for generating a solution that will be feasible for all the data in a convex set. The provided model was highly conservative; meaning that, in order to ensure the feasibility, a relatively large proportion of the solution optimality of the nominal problem will be lost. In fact, the Soyster proposed approach was very strict and for the worst case. Ben-Tal and Nemirovski (2002) developed other robust approach with less conservatism compared to Soyster (1973). In their proposed model, the uncertainty was considered ellipsoidal. The problem with their method is the fact that it turns a linear programming problem into the form of quadratic or conical programming. Goerigk and Schöbel (2016) argue that the algorithm engineering methodology fits very well to the field of robust optimization and yields a rewarding new perspective on both the current state of research and open research directions.

Bertsimas and Sim (2004) considered a different viewpoint of robustness (See Section 2-1). Their approach is based on the fact that, in real conditions, it is unrealistic to assume that all coefficients take their worst-case value at the same time. This approach preserved the linearity of the model. In his proposed approach, a new Norm called D-Norm has been used instead of the Euclidean Norm. The classical robust counterpart of a problem requires the solution to be feasible for all uncertain parameter values in a so-called uncertainty set and offers no guarantees for parameter values outside this uncertainty set. The globalized robust counterpart (GRC) extends this idea by allowing controlled constraint violations in a larger uncertainty set (Ben-Tal, et al. 2017).

In addition to the mentioned approaches in the modeling of the uncertainty of input data, there is also another approach known as ***Light Robust*** that is presented by Fischetti and Monaci (2009) and is described in Section 2-2 of this paper. In general, the Light Robust approach has been developed to create balance between the optimality and feasibility of the solution. Ide and Schöbel (2016) extended the concepts of highly, and lightly robust efficiency and collected different types of min-max robust efficiency.

## 2-1- Bertsimas and Sim approach’s

The Bertsimas and Sim approach is based on the fact that in real-world problems, all uncertain parameters rarely happen to get their worst cases at the same time. Then, it seems logical to present a model in which the optimal solutions will still remain robust for any variation from the maximum $\Gamma_i$ parameters in the i[th] row. Here, the parameter $\Gamma_i$ is an input parameter that is determined based on our expectation from the robustness of the solution (Bertsimas and Sim (2004)). Therefore, in the robust counterpart model for the problem, the following constraint will replace the constraint associated with each row:

$$\sum_{j \in N} a_{ij} x_j + \beta(x, \Gamma_i) \le b_i \tag{4}$$

Where,

$\beta(x, \Gamma_i)$ is the protective level that is defined proportional to the uncertainty of each row, as follows.

$$\beta(x, \Gamma_i) = \max_{S \subset N : |S| \le \Gamma_i} \sum_{j \in S} \hat{a}_{ij} x_j \tag{5}$$

Therefore, $\beta(x, \Gamma_i)$ is the maximum amount of increase on the left side of the constraint, while the maximum $\Gamma_i$ parameters in the row $i$ gets their worst values.

As stated, $\Gamma_i$ provides the possibility to control the level of uncertainty: $\Gamma_i = 0$ means that we have failed to consider the uncertainty and chosen the nominal constraints, while $\Gamma_i = n$ means that all the uncertain parameters at the i[th] row can get the worst amount, which is the same situation considered by Soyster (1973). Thus, the robust model of the problem is formulated as follows (to find the detail description about the model and constrain please see Bertsimas and Sim (2004)):

$$\min \sum_{j \in N} c_j x_j \tag{6}$$

$$\sum_{j \in N} a_{ij} x_j + \Gamma_i z_i + \sum_{j \in N} p_{ij} \le b_i \quad i \in M \tag{7}$$

$$-\hat{a}_{ij} x_j + z_i + p_{ij} \ge 0 \quad i \in M \quad j \in N \tag{8}$$

$$z_i \ge 0 \quad i \in M \tag{9}$$

$$p_{ij} \ge 0 \quad i \in M \quad j \in N \tag{10}$$

$$x_j \ge 0 \quad j \in N \tag{11}$$

A remarkable point in the BS approach is that if the uncertain parameters under the assumptions change, the solutions will definitely be feasible. However, even if the number of coefficients that change in the i^th row would be more than $\Gamma_i$ , the solution provided by this approach will remain feasible with a high probability.

## 2-2- Light robust approach

As noted in the previous sections, the Light Robust approach provides a compromise between the robustness of the solution of one hand and the quality of the solution on the other hand (Fischetti and Monaci (2009)). In other words, in this approach, we are looking for the best robust solution that does not have a very large distance from the optimal solution of the nominal problem. To explain this issue, considering the robust optimization model (BS) proposed by Bertsimas and Sim (2004), which mentioned in the previous section, the Light Robust counterpart will be as follows:

$$\min \sum_{i\in M} w_i \gamma_i \tag{12}$$

$$\sum_{j\in N} a_{ij} x_j + \beta(x, \Gamma_i) - \gamma_i \le b_i \quad i \in M \tag{13}$$

$$\sum_{j\in N} a_{ij} x_j \le b_i \quad i \in M \tag{14}$$

$$\sum_{j\in N} c_j x_j \le (1+\delta) z^* \tag{15}$$

$$x_j \ge 0 \quad j \in N \tag{16}$$

$$\gamma_i \ge 0 \quad i \in M \tag{17}$$

The auxiliary variables of $\gamma_i$ act as the second stage source variables and are used to avoid the possible infeasibility and their linear weight combination is minimized in the target function. Each of these variables specifies the robustness level of the solution in connection with the constraint associated with this auxiliary variable (constraint 12). More precisely, each of these variables gets a very positive amount, if and only if, the relevant constraint is violated. The constraint (15) also ensures that the worst valued obtained would not be less than a multiple of the optimal value of the nominal problem. The role of parameter $\delta$ is to create equilibrium between the robustness and the quality of the solution provided. The weight $\omega_i$ used in the target function actually creates a kind of penalty on different values of the auxiliary variables.

It should be noted that the Light Robust approach significantly depends on the robustness definition of the BS. Therefore, this approach can only be used only when the existing uncertainty can be formulated and described linearly. However, various forms of Light Robust approach can be used for a single problem. There is also another approach of the LR, which is not dependent on the BS, but is directly related to the auxiliary variables associated with the nominal problem constraint. The basic idea used here is that the robustness degree of a solution, in some way, is expressed by the value of auxiliary variables with uncertainty. Assume that $x^*$ is the optimal solution of the nominal problem. Then, we suppose:

$$L_i^* = \sum_{j\in N} (a_{ij} + \hat{a}_{ij}) x_j^* - b_i \tag{18}$$

The maximum amount of violation of $i$ constraint related to the optimal solution is equal to $x^*$. We also show the set of all the constraints, for which the above is positive, as follows:

$$U = \{ i \in M : L_i^* > 0 \} \tag{19}$$

In other words, $U$ is the set of the constraints experiencing the problem of uncertainty that must be resolved. Without diminishing the totality, we can assume that $|U| \geq 1$, since otherwise, the optimal solution of the $x^*$ nominal problem will be always feasible. We first solve the following LP problem:

$$\max \quad \sigma \tag{20}$$

$$\sum_{j \in N} a_{ij} x_j + s_i = b_i \quad i \in M \tag{21}$$

$$\sigma \leq \frac{s_i}{L_i^*} \quad i \in U \tag{22}$$

$$\sum_{j \in N} c_j x_j \leq (1+\delta) z^* \tag{23}$$

$$x_j \geq 0 \quad j \in N \tag{24}$$

$$s_i \geq 0 \quad i \in M \tag{25}$$

That maximizes the minimum of the auxiliary variables assigned to all the constraints with uncertainty. Also, innovatively, the, $s_i$ auxiliary variables are normalized by dividing by $L_i^*$. Considering the nature and form of the problem, which is of Max-Min type, the objective (target) function only considers the constraint that has the least amount of the normalized auxiliary variable, and therefore, it has no incentive to give larger auxiliary variables to the rest of the constraints; while this is important for the robustness improvement. Therefore, another LP will be solved to resolve this imbalance. If we assume that the optimal solution of the above LP problem is as $\left(x^*, s^*, \sigma^*\right)$, we can define the following parameters:

$$s_{avg} = \frac{\sum_{i \in U} s_i^* / L_i^*}{|U|} \qquad s_{min} = min\left\{ s_i^* / L_i^* : i \in U \right\} \left(= \sigma^*\right) \tag{26}$$

We solve the secondary LP problem as follows:

$$\min \sum_{i \in U} t_i \tag{27}$$

$$\sum_{j \in N} a_{ij} x_j + s_i = b_i \quad i \in M \tag{28}$$

$$\sum_{j \in N} c_j x_j \leq (1+\delta) z^* \tag{29}$$

$$\frac{s_i}{L_i^*} + t_i \geq s_{avg} \quad i \in U \tag{30}$$

$$x_j \geq 0 \quad j \in N \tag{31}$$

$$\frac{s_i}{L_i^*} \geq s_{\min} \quad i \in U \tag{32}$$

$$s_i \geq 0 \quad t_i \geq 0 \quad i \in U \tag{33}$$

In this model, we define a $t_i$ variable for each constraint belonging to the set U. The $t_i$ will take a positive value, if and only if, the auxiliary variable associated with that variable is smaller than $s_{avg}$ .The objective function of this LP minimizes the sum of these variables to create a balance.

## 3- Heuristic Weighted Light Robust approach (HWLR)

As mentioned above, the LR approach has been proposed to create a balance between the quality and robustness of the solution. Several attempts have been made to introduce new versions of this approach, some of which were mentioned in previous sections. In this section, by providing a model, we are to improve the quality of the solution presented by the light robust approach. In the proposed approach, providing a solution with a better quality would have a higher time cost. The general form of the proposed model is as follows:

$$\max \sum_i \vartheta_i s_i \qquad i \in U \tag{34}$$

$$\sum a_{ij} x_j + s_i = b_i \qquad \forall i \in N \tag{35}$$

$$(s_1 + \ldots + s_n) \le w \times z^* \tag{36}$$

$$\sum_j c_j x_j \le \delta \times z^* \tag{37}$$

$$S_i, x_j \ge 0 \qquad \forall i, j \tag{38}$$

The approach presented by Fischetti and Monaci (2009) (equations (27-33)), is based on solving two linear mathematical programming problems. In this paper, by providing a model, we tried to solve the problem of imbalance introduced in paper by Fischetti and Monaci (2009). Here, using the appropriate weight composition as the target function, we attempted to resolve the problem. The constraint (36) is to make a special linear combination of alternating variables bounded with a factor of the optimal value of the nominal problem. To complete this model, we need to obtain the $\vartheta_i$ coefficients, which are the cost of each additional $s_i$ unit. We will also determine the $V_i$ , which is the maximum amount that the problem quality is allowed to be reduced. We increase the $s_i$ s as much as the feasibility of the problem will be observed in the worst case as much as possible. For this reason, the model objective function will be maximization.

We first notice that if the $s_i$ is increased a single unit, the value of the objective function of the model will increase as $\vartheta_i$ . On the other hand, with increasing the $s_i$ as one unit, the total uncertainty would decrease. This coefficient is the same portion of the i$^{th}$ constraint in the total uncertainty. We define the total uncertainty of the model as $\sum_{i \in U} l^*_i$ . If this value is zero, the nominal model always obtains the optimal solution. Therefore, the uncertainty share of each $s_i$ will be equal to $\frac{l^*_i}{\sum_{i \in U} l^*_i}$ , which is the same $\vartheta_i$ .

The relation $\sum_i w_i s_i$ shows the reduced quality of the solution. In other words, its maximum can be as much as a percentage of $z^*$. Thus, $V = \delta z^*$ is defined. The value of $\delta$ is determined by DM. To specify the $s$ coefficients, we do as this: If the vector $(s_1, s_2, \ldots, s_n)$ changes, what a change will occur in the matrix $A$ ? To do this, we must first define the change in the vector. $S$ .

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} X + \begin{pmatrix} S_1 \\ \vdots \\ S_n \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} \tag{39}$$

Since the purpose of this calculation is to find out what change will occur in the following relation by changing the vector $(s_1, s_2, \ldots, s_n)$, we will have:

$$Z = (C_1, C_2, \ldots, C_n) \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \tag{40}$$

$$A^{-1}AX + A^{-1}S = A^{-1}b \to X + A^{-1}S = A^{-1}b \to CX + CA^{-1}S = CA^{-1}b \tag{41}$$

$$Z + CA^{-1}S = CA^{-1}b \tag{42}$$

It is clear from the above equation that if the $s_i$ increases by one unit, the $z$ will decrease as much as the i^th^ component of vector $CA^{-1}$, i.e., the vector $C$ in the i^th^ column of vector $A^{-1}$. Therefore, the factor of variable $s_i$ multiplied by the defined constraint will be considered equals to the same value; i.e., $w_i = \left(CA^{-1}\right)_i$. Therefore, the mentioned constraint will be as follows:

$$\sum_i \left(CA^{-1}\right)_i s_i \le \delta z^* \tag{43}$$

If $A^{-1}$ does not exist, this approach is useless. To solve this problem, the following calculations are useful. In fact, $z$ is the trace of the following matrix:

$$\begin{pmatrix} z_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & z_n \end{pmatrix} = \begin{pmatrix} c_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_n \end{pmatrix} \begin{pmatrix} x_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_n \end{pmatrix} \tag{44}$$

$$\begin{pmatrix} c_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_n \end{pmatrix}^{-1} \begin{pmatrix} z_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & z_n \end{pmatrix} = \begin{pmatrix} x_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_n \end{pmatrix} \tag{45}$$

The sum of the elements of the $i^{th}$ row of $\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_n \end{pmatrix}$ is equal to the $i^{th}$ element of $\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} X$. Then, on the two sides of the above equation, the matrix A is multiplied:

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} c_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_n \end{pmatrix}^{-1} \begin{pmatrix} z_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & z_n \end{pmatrix} = A \begin{pmatrix} \frac{z_1}{c_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{z_n}{c_n} \end{pmatrix} \tag{46}$$

$$= \begin{pmatrix} a_{11}\frac{z_1}{c_1} & \cdots & a_{1n}\frac{z_n}{c_n} \\ \vdots & \ddots & \vdots \\ a_{n1}\frac{z_1}{c_1} & \cdots & a_{nn}\frac{z_n}{c_n} \end{pmatrix} \tag{47}$$

If we show the sum of the elements of the i^th^ row of the above matrix with $K_i$ , with one unit increase in the $s_i$ , the $K_i$ must be reduced by one unit. Thus, if we have $\sum_i \Delta s_i = 1$, one can conclude that the reduction in the sum of the elements of the above matrix is equal to one. The sum of the above elements is equal to:

$$TE = \sum_i \left( \sum_j a_{ij} \frac{z_j}{c_j} \right) \tag{48}$$

Now, assume that in the i^th^ row, the largest values of $\frac{a_{ij}}{c_j}$ are related to index $j_k$ and the least values are related to index $j_l$ . In this case, we will have:

$$TE = \sum_i \left( \sum_j z_j \frac{a_{ij}}{c_j} \right) \le \sum_i \left( \sum_j z_j \left( \max \frac{a_{ij}}{c_j} \right) \right) \le \sum_i \frac{a_{ij_k}}{c_{j_k}} \sum_j z_j \le z \sum_i \frac{a_{ij_k}}{c_{j_k}} \tag{49}$$

If we put $M = \sum_i \frac{a_{ij_k}}{c_{j_k}}$ , we will have $TE \le Mz$ . Similarly, if we put $m = \sum_i \frac{a_{ij_l}}{c_{j_l}}$, we will have $TE \ge mz$ . On one hand, $TE$ is equal to $\sum_i s_i$ . Therefore, the constraint $mz \le \sum_i s_i \le Mz$ is always established.

Now, we consider that $\sum_i s_i \le Mz$ is always feasible and $\sum_i s_i < mz$ is always infeasible. Hence, we will look for a "$w$ " that optimizes the constraint $\sum_i s_i \le wz$ of the model solution meanwhile becoming feasible as much as possible. To this end, we develop the following innovative algorithm:

## 4- Proposed innovative algorithm

**Step 0:** We put $w_f = M$ and $w_s = m$ . We solve the problem by the assumption of $w = w_s$ . If it is infeasible, we will go to step 1. If it becomes feasible, the algorithm will end and the solution obtained will not get better.

**Step 1:** We solve the problem by the assumption of $w = \frac{w_s + w_f}{2}$. If it is infeasible, we will go to step 2. If it becomes feasible, we will go to step 3.

**Step 2:** We put $w_s = w$ and go to step 1.

**Step 3:** If the quality of the problem is sufficient, the algorithm ends. To improve the quality of the problem solution, we put $w_f = w$ and go to step 1.

## 5- Computational results

In this part of the paper, the computational results of the proposed HWLR approach are presented. The input parameters of the problem were developed randomly for a minimizing linear programming problem. A comparison between the optimal value obtained from the simple LR approach (Fischetti and Monaci (2009)), the proposed HWLR approach as well as the solution resulted from the GAMS software for the nominal problem is provided. The HWLR approach described in Section 3 is seeking to improve the quality of the solution obtained from the LR method. To evaluate the performance of the new approach, the likelihood of improving the quality of the solution compared to simple LR approach was selected as the measurement criterion. The solution obtained from HWLR approach was compared with the solution obtained from the simple LR approach as the probability of improvement (column 4, Table 1). To estimate this probability, 40 test problems in large sizes of *i* and *j* (i.e, Numbers of variables and constrains) were considered. Then for each test problem, 100 times the parameters were randomly generated. Finally, the number of improvement events was used as an estimate of the likelihood of improvement. For example, for i = 447 and j = 245, out of 100 random generated problems, the new approach provided a definite better solution than the simple LR approach in 41 cases and in other 59 solutions provided the same results are obtained by simple LR approach. In fact, the proposed approach never provided a solution worse than the simple LR and the probability of improvement as a simulated measure shows the likelihood of providing better results. The results of these calculations are in accordance with Table 1. Also, to generate all data of the problem completely randomized as much as possible, the lower and upper bounds of the range for producing random numbers were generated randomly as well.

$$c_j = uniform[uniform[10,30], uniform[50,150]] \tag{50}$$

$$b_i = uniform[uniform[500,1500], uniform[2500,3500]] \tag{51}$$

$$a_{ij} = uniform[uniform[0,5], uniform[10,15]] \tag{52}$$

The calculations show that the improvement of the quality of the solution varies from 0 to 12% of the variable compared to the simple Light Robust approach for cases where improvements are made, and on average, is slightly more than 9%. As can be seen in Table 1, in the case where the ratio of the number of constraints to the number of variables is small, the proposed approach shows no significant improvement compared to the simple Light Robust approach. However, as this ratio increases, the improvement of the proposed approach is significant compared to the simple Light Robust approach. For example, in a case that the number of constraints and the number of variables are 328 and 35, respectively, in 100 different implementations with random data, the proposed approach provides a significantly better solution in 88 cases compared to the simple Light Robust approach, which is a remarkable result.

**Table 1.** Computational results of proposed approach

| Number | Number of constraints | Number of variables | Probability of improvement | Number | Number of constraints | Number of variables | Probability of improvement |
|---|---|---|---|---|---|---|---|
| 1 | 72 | 455 | 6 | 21 | 328 | 35 | 88 |
| 2 | 144 | 313 | 16 | 22 | 62 | 211 | 13 |
| 3 | 389 | 10 | 92 | 23 | 405 | 281 | 30 |
| 4 | 92 | 173 | 29 | 24 | 372 | 311 | 26 |
| 5 | 458 | 149 | 52 | 25 | 262 | 279 | 25 |
| 6 | 168 | 395 | 13 | 26 | 158 | 416 | 16 |
| 7 | 447 | 245 | 41 | 27 | 420 | 466 | 17 |
| 8 | 157 | 206 | 28 | 28 | 15 | 199 | 0 |
| 9 | 359 | 47 | 78 | 29 | 388 | 174 | 50 |
| 10 | 342 | 378 | 27 | 30 | 136 | 373 | 12 |
| 11 | 107 | 498 | 7 | 31 | 196 | 285 | 26 |
| 12 | 273 | 497 | 11 | 32 | 498 | 424 | 30 |
| 13 | 102 | 39 | 76 | 33 | 282 | 343 | 24 |
| 14 | 251 | 279 | 35 | 34 | 97 | 104 | 36 |
| 15 | 276 | 329 | 26 | 35 | 216 | 302 | 22 |
| 16 | 110 | 403 | 8 | 36 | 129 | 66 | 54 |
| 17 | 404 | 472 | 19 | 37 | 69 | 187 | 11 |
| 18 | 285 | 190 | 37 | 38 | 184 | 13 | 95 |
| 19 | 92 | 43 | 68 | 39 | 342 | 268 | 42 |
| 20 | 117 | 498 | 2 | 40 | 95 | 393 | 8 |

To further examine the effect of the number of constraints and the number of variables on the likelihood of improvement, we will go forward a little further. To do so, using the data presented in Table 1, we conducted a statistical survey using multiple regressions with the help of $EViews$ 9 software. From available models, using the model $R^2$ benchmark, Durbin-Watson criterion and the Schwarz criterion, we extracted the best model as shown below:

$$PROB = 91.6 - 23.3*LOG(VAR) + 12.2*LOG(CONS) \tag{53}$$

As predicted, with considering the number of constraints constant, increased number of variables reduces the chance of the solution improvement. Also, if the number of variables is considered constant, increased number of constraints will increase the improvement probability of the problem solution quality.

## 6- Conclusion, limitation and future research

The real-world problems, when expressed as optimization models, mostly confess to uncertainty. The uncertainty in optimization problems and the importance of achieving a robust solution have been the focus of attention in the recent literature. In this regard, several approaches have been proposed to solve this problem, some of which were mentioned in the section 2 of this paper. The feasibility of the solution and the quality of solution are two important factors in all of these approaches. In this paper, as noted in the previous sections, the authors' main focus was on the quality of the solution. Accordingly, a new constraint was introduced and an algorithm was also developed, which tries to generate the best possible value to estimate the parameter of this constraint. To review and implement the proposed approach, an indicator was introduced for the likelihood of improving the quality of the solution and its computational results were reported. The computational results obtained from the implementation of this approach indicate that whenever we use this approach in case of problems where the number of constraints is more the number of variables, the likelihood of improving the quality of the solution obtained significantly increases compared to the classic Light Robust approach, and will well cover the time cost of running the algorithm. Also, when this improvement occurs, the average solution's improvement rate would be of 9%, which is considerable. It should be noted that this improvement rate is only the average of values, and in some cases, this improvement rate is much better as well. A multiple regression model was performed to investigate the results more accurately and test the hypothesis obtained from the observations. Evaluation of the obtained model suggests that the probability of improving the solution quality of the proposed approach has an increasing trend in cases where the number of constraints of the problem is more the number of variables. The purpose of this paper was to provide an approach that can improve the quality of the solution of optimization problems while the obtained solution will remain feasible at the same time. To achieve this goal, many efforts have been made and this process will continue. As seen, a fairly intelligent addition of a constraint significantly improved the quality of the resulting solution in some cases.

Despite the above contributions, our research is not without limitations. The proposed model can set the stage for the incorporation of real parameters and constraints. As future research, to find the parameter used in this constraint, the meta-heuristic algorithms as well as other intelligent methods can be used. In the same vein, designing a properly designed DSS will have a significant impact on improving the quality of the solution and improving the timing of the problem solving simultaneously.

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