A joint pricing-network design model for a resilient closed-loop supply chain under quantity discount

Monireh Shoaraye-Nejati¹, M.Saeed Jabalameli¹*, Mir Saman Pishvae1

¹School of Industrial Engineering, Iran University of Science & Technology, Tehran, Iran
monireh_shoaraye@ind.iust.ac.ir, jabal@iust.ac.ir, pishvae@iust.ac.ir

Abstract
In this paper, a novel resilient multi-echelon closed-loop location-allocation-inventory problem (RMCLIP) is addressed that optimizes strategic and tactical decisions simultaneously. In order to represent the purchasing cost of raw material from the supplier, a pricing model under quantity discounts is employed in the closed-loop supply chain (CLSC). Considering the capability of returning the reworked products to the forward logistics that can affect the ordering patterns of distribution centers (DCs) is another significant difference between this study and similar related researches. Furthermore, resilient capacity approach is used to provide a flexible SC toward the uncertainty of reworking centers (RCs) and suppliers' capacity. As this point, based on some facilities' capacity uncertainty, the robust model is formulated. The computational results and sensitivity analyses are presented using GAMS software to reveal the applicability of the proposed model. The results are analyzed in depth to provide some managerial insights.

Keywords: Closed-loop supply chain, resilient capacity, robust, pricing, quantity discount, returned products

1-Introduction
Recently, green supply chains tend to invest on integrating some business operations to minimize the side effects such as natural source reduction, water and air pollution, etc. (Abdallah et al., 2012a). In practice, product recovery is one of the most prevalent methods for making a green supply chain, which is one of the main requirements of making a closed-loop supply chain. In other words, implementing a reverse logistics in a specific channel is one of the vital needs to achieve a CLSC. In particular, the reverse logistics can be defined as some efforts done to return or properly dispose the unsold, damaged, and End-Of-Life (EOL) products. Such efforts include reworking, repairing, disposing, recycling, and remanufacturing. In a broader sense, CLSC management is a combination of forward and reverse logistics as traditional and modern processes, respectively (Karimi et al., 2015). It is noteworthy that the prevalent assumption in the related literature, that indicates the creation of new spare parts by combining the returned products with subassemblies to transfer to the secondary market, is not an acceptable one in real world (Diabat et al., 2015). Keeping this in mind, we decide to address a new approach in which the returned products from the customers can go back to the remanufacturing centers with the aim of re-entering the forward logistics. Regarding this point, DCs (Distribution Centers) may change their ordering patterns and, on the other side, it is expected that the retailers' demands be influenced by applying this strategy.

*Corresponding author
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On the other hand, robustness and resilience are two major abilities of channels to encounter the errors with the aim of returning to their initial conditions or shifting to the new states. The mentioned abilities of SC networks that make favorable state after being disturbed are called "flexibility" and "adaptability" (Christopher and Peck, 2004). Indeed, one of the effective issues must be addressed in SC (Supply Chain) network designs is resiliency which can be tracked in their own challenging natures and in this regard holds them more competitive. In a broader sense, measuring the ability of quickly returning to the normal performance is everything that the resilience discusses in the companies. In particular, one of the most important streams of the resiliency is the additional surplus capacity, which causes more flexibility. For instance, while a disruption occurs, the additional surplus capacity enables new delivery sites for new SC members. Regarding this point, the necessary changes in production capacity must be done to properly respond the new demand level(Kristianto et al., 2014). Tang (2006) presents another definition of risks related to the resilient SCs including two types of risk: (1) operational risks and (2) disruption risks. The main difference between the two named risks arises from the causative agents. For example, operational risks may be caused by uncertain demands, supply capacity and procurement costs. On the opposite, the main factors of disruption risks include some natural or man-made disasters e.g., floods, earthquakes, terrorist activities, etc (Jabbarzadeh et al., 2016).

Additionally, in most supply networks, the procurement of components and raw materials from the suppliers has incurred the greatest expenses on the downstream partners. For instance, 40–60% of production costs in most US manufacturers are related to the process of purchasing raw materials from the supplier (Wadhwa and Ravindran, 2007). In this regard, an effective way to properly handling the purchasing costs is to reduce the operational costs. With respect to these points, we address a discount method with the aim of realizing the optimal purchasing decisions. In this regard, a pricing model under quantity discounts is utilized in the proposed CLSC to represent the cost of purchasing raw materials from the supplier.

In this paper, a resilient multi-echelon closed-loop location-allocation-inventory problem (RMCLIP) is presented in which some strategic decisions including facility location are investigated at the same time with the tactical ones such as allocation and inventory decisions. Moreover, we survey the environmental impacts of employing different production technologies at manufacturing centers in the form of costs. Additionally, a pricing model based on quantity discount is proposed to illustrate the cost of procurement of components and raw materials from the supplier. The confrontational approach with uncertainty of the facilities' capacity is considered in the form of resilient capacity. Thereafter, based on some facilities' capacity uncertainty, the robust model is formulated. According to the aforementioned importance of both location-allocation-inventory closed-loop supply chains and discount models, it seems that our attempt to consider the discount, production technology, and returned products all together in the closed-loop resilient SC would be a significant step to solve the problem, having a special position in the literature.

Major contributions of the present study distinguishing it from other similar works are:

- Considering the resilient capacity of RCs and suppliers with in a CLSC in order to neutralize the impacts of occurring disruption
- Considering both the incremental price breaks (i.e. price levels) for the raw material delivered by the supplier and failure rate of the raw material
- Investigating the effects of fraction of the products returned from the reverse logistics on channel's ordering pattern and chain's demand quantity in the forward logistics
- Considering the environmental impacts of using different technologies in the production process (i.e. CO₂ emission from the production process)

The remaining of this paper is organized as follows. Section 2 addresses a brief literature review of the previous related researches. Section 3 states the problem description and the mathematical formulation of the robust model. Section 4 deals with the solution of the model via GAMS software which additionally includes some model evaluations and sensitivity analyses. Finally, conclusions and future research suggestions are discussed in Section 5.
2-Literature Review

Closely related to the recent studies, the joint location-inventory problem is becoming an increasingly important issue, which was first introduced by Baumol and Wolfe (1958). After a while, Teo et al. (2001) used an analytical modeling approach to study the impact of consolidated DCs into a central DC on the facility investments and inventory costs and illustrated that for stochastic demand, the total investment costs of the facility and an integrated system can be worth a decentralized system. Nozick and Turnquist (2001) considered a more complete model for individual products in a multi-product and two-echelon inventory system that presents a method for optimizing the trade-off among customer service and cost together with developing concerns about dynamic environments. Freling et al. (2003) surveyed a single sourcing model with transportation and inventory costs in a dynamic environment. Shu et al. (2005) presented the stochastic inventory-transportation network design problem consisting of one supplier and multiple retailers in an uncertain environment. In addition, Miranda and Garrido (2006) simulated the inventory-location decisions by means of a non-linear mixed-integer model.

Recently, a new approach to the location-inventory problems was adopted by Daskin et al. (2002) and Shen et al. (2003) in which \( (Q, r) \) inventory policy was considered. Afterwards, Qi and Shen (2007) added the routing decision to the inventory-location problem and showed the effects uncertainty has on the supply chain decisions. Javid and Azad (2010) developed a novel model in a stochastic supply chain to optimize location, allocation, capacity, inventory, and routing decisions simultaneously. Berman et al. (2012) developed the literature by studying a coordinated location-inventory model where DCs follow a periodic review \( (R, S) \) inventory policy. To this end, they introduced two types of coordination mechanisms: (1) partial coordination, in which each DC may choose its own review interval from the menu, and (2) full coordination, where all the DCs have an identical review interval. To expand the related models, Tancrez et al. (2012) studied the integrated location-inventory problem for three levels supply chain networks, including suppliers, DCs, and retailers. Sadjadi et al. (2016) developed a three-level supply chain network with uncertain demands and lead time, which includes a single supplier, multiple DCs and retailers to simultaneously optimize the facility location-allocation, retailers’ demands, and inventory replenishment decisions. With the aim of focusing on the environmental considerations, Kumar et al. (2016) considered production and pollution routing problem with the time window in a vehicle routing model where the location and inventory decisions are integrated.

CLSC is the most important stream of related research efforts to our work. As a preliminary investigation, Chung et al. (2008) investigated an inventory system for traditional forward-oriented material flow as well as a reverse material flow and analyzed a remanufacturing capability in a multi-echelon closed-loop model. Most recently, Abdallah et al. (2012b), Kannan et al. (2012), and Diabat et al. (2013) studied the effects of forward and reverse logistics on carbon emissions in the closed-loop supply chains. In order to investigate a wide spectrum of decisions totally, Nekooghadirli et al. (2014) studied a new bi-objective location-routing-inventory closed-loop problem which has been solved by four multi-objective meta-heuristic algorithms. Saffari et al. (2015) developed a novel sustainable CLSC including total cost, environmental factors, and social factors as three different objective functions. They applied an efficient non-dominated sorting genetic (NSGA) algorithm to solve the robust optimization model. In a broader case, Asl-Najafi et al. (2015) added the disruption problem to the dynamic location-allocation problem with two cost and time minimization objective functions solved by a new hybrid meta-heuristic algorithm based on MOPSO and NSGA-II. Kaya and Urek (2016) developed the location, inventory, and pricing decisions by MINLP models, solved by meta-heuristic algorithms. Al-Salem et al. (2016) formulated a closed-loop inventory-location problem as an MINLP. They transformed the existing problem to an MIP by using the reformulated problem and piecewise linearization, which is precisely solvable using CPLEX. In a new stream, Zhang and Unnikrishnan (2016) solved a coordinated inventory-location problem in a closed-loop supply chain. It is noteworthy that some recent researches have widely studied the pricing, advertising, and coordination problems using the game theory approach in CLSC i.e. Gao et al. (2016), Heydari and Asl-Najafi (2016), Bazan et al. (2017), Xie et al. (2017), and (Sahraeian and Mohagheghian, 2017).
In what follows, we provide a brief literature of the most relevant issues to our study called SC risk management. Different sources of SC risks are addressed in this stream. According to (Rezapour et al., 2016), fluctuation and disruption are two major groups of uncertainty nature. One appropriate example for fluctuation is market demand that has small and frequent variations. It should be noted that the other cases would have the same features e.g., price of materials and labor, delays in delivering the products, amount of damaged products, etc. The other kind of uncertainty called disruption includes unexpected and infrequent variations occurred by floods, earthquakes, fire, etc (Chopra et al., 2007). Recently, some risk mitigation methods with the aim of reducing the impacts of disruptions are proposed in the literature. These strategies discussed in both strategic and operational levels, would be able to create flexibility in SC’s performance. Among them, backup facilities are the most practical ways to compensate the damages happened during the disruption (Chopra et al., 2007). However, another practical tool which can be used in disruptions is to provide redundant reserve capacity in some of the facilities (Chopra and Sodhi, 2004). Additionally, in order to neutralize the risk of before-mentioned uncertainties, some other methods exist at the operational level such as keeping safety stock, production postponement, using multiple suppliers, etc (Rezapour et al., 2016). Finally, Zarrinpoor et al. (2016) investigated a reliable location-allocation problem under disruptions in which accelerated Benders decomposition method has been used to solve the model. In the following, Table 1 illustrates the main differences of present study compared to the other similar researches.

## Table 1. Findings of the literature survey

<table>
<thead>
<tr>
<th>Reference</th>
<th>Closed-loop</th>
<th>Location</th>
<th>Inventory</th>
<th>Resilient capacity</th>
<th>Price discounts</th>
<th>Returned products</th>
<th>Production technology</th>
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<td>Miranda and Garrido (2006)</td>
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<td>Chung et al. (2008)</td>
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<td>Nekooghadirli et al. (2014)</td>
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<td>Diabat et al. (2015)</td>
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<td>Pasandideh et al. (2015)</td>
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As can be seen in table 1, there are few studies that examine the main issues of this research all together. To be specific, studying a resilient CLSC taking into account the environmental impacts of different production technologies under a quantity discount model would be an interesting topic in the existing literature that have not been investigated up to now.

### 3- Problem description and mathematical formulation

Figure 1 displays a detailed schematic view of the proposed resilient CLSC. As depicted, first, the forward logistics supplier provides the required raw material for the manufacturers. Second, different kinds of products would be produced in the manufacturing centers using different production technologies. In the next step, DCs, retailers, and customers register their demands from the manufacturers, DCs, and retailers, respectively. We assume that the location of the retailers and both forward and reverse suppliers are known. Furthermore, the supplier and DCs are assumed un-capacitated. After shipping the products to the retailers, they separate the fraction of products that is not qualified for customer’s usage and send back a percentage of $\delta$ to the RCs so that $(1 - \delta)\%$ of products can be used by customers. In the meantime, the RCs purchase raw materials from a second
supplier in the reverse channel with incremental price breaks (i.e., price level) while considering failure rate of raw materials. With the aid of inspecting the returned products, they separate a percentage of $\eta$ which is unrepairable and dispose it with a specific disposal cost. Therefore, the rest of returned products including $(1 - \eta)\%$ will be repaired by the RCs spending reworking cost to prepare them for the forward logistics usage.

Due to the uncertain demands of the retailers, DCs keep safety stock to overcome demands fluctuations. In this paper, single sourcing allocation has been utilized in which a retailer can be linked to a single DC in the forward logistics as well as the retailers that would be able to transfer the returned products to a single RC. Moreover, a customer, a supplier, and an RC can be served by a single retailer, manufacturer, and supplier, respectively. In order to bring down the transportation costs, we presume that each RC can be located only near DCs. The demands and returns for product $p$ at retailer $i$ are assumed to be independent and normally distributed, i.e. $N(\mu_{ip}, \sigma_{ip}^2)$ and $N(\lambda_{ip}, \rho_{ip}^2)$ where $\lambda_{ip} = \delta_p \mu_{ip}$ and $\delta \in [0,1]$. Furthermore, another uncertainty of the presented problem is considered in the capacity of RCs and suppliers. In this regard, a practical encountering tool, which is usable in disruptions, has been applied called redundant reserve capacity. Indeed, in order to neutralize the risk of capacity uncertainties, resilient capacity problem is provided.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Graphical model of investigation}
\end{figure}

3-1- Notations
In this section, in order to formulate the proposed problem, we first define the notations as follows:

**Sets**

$I$ set of retailers, indexed by $i$

$J$ set of potential regional distribution center sites, indexed by $j$

$K$ set of potential regional reworking center sites, indexed by $k$ ($j$ and $k$ are aliases)

$P$ set of products, indexed by $p$

$L$ set of customers, indexed by $l$

$M$ set of potential regional disposal center sites, indexed by $m$

$O$ set of potential supplier sites in the forward logistics, indexed by $o$

$R$ set of potential supplier sites in the reverse logistics, indexed by $r$

$S$ set of required raw material for reworking operations, indexed by $s$
\( U \) set of potential regional manufacturing center sites, indexed by \( u \)
\( T \) set of different production technologies, indexed by \( t \)
\( V \) set of incremental price breaks, indexed by \( v \)

**Parameters**
- \( F_j \) fixed cost of locating a DC at site \( j \)
- \( F'_k \) fixed cost of locating an RC at site \( k \)
- \( G'_m \) fixed cost of locating a disposal center at site \( m \)
- \( G_u \) fixed cost of locating a manufacturing center at site \( u \)
- \( d_{ijp} \) shipping cost per unit of product \( p \) from DC \( j \) to retailer \( i \)
- \( d_{skp} \) shipping cost per unit of product \( p \) from retailer \( i \) to RC \( k \)
- \( d'_{ip} \) shipping cost per unit of product \( p \) from retailer \( i \) to customer \( l \)
- \( d'_{mp} \) shipping cost per unit of product \( p \) from RC \( k \) to disposal center \( m \)
- \( dd_{kip} \) shipping cost per unit of product \( p \) from RC \( k \) to DC \( j \)
- \( b_{tp} \) unit reworking cost of product \( p \) at RC \( k \)
- \( c_{mp} \) unit disposing cost of product \( p \) at disposal center \( m \)
- \( \beta \) weight factor associated with transportation costs
- \( \theta \) weight factor associated with inventory costs
- \( \gamma \) weight factor associated with reworking costs
- \( \delta_p \) fraction of returned product \( p \) from forward logistics
- \( \eta_p \) fraction of unrepairable product \( p \)
- \( A_{jup} \) fixed ordering cost of product \( p \) from DC \( j \) to the manufacturing center \( u \)
- \( \lambda_r \) failure rate of raw material provided by supplier \( r \)
- \( Z_{\alpha} \) normal standard deviation with \( P(Z \leq Z_{\alpha}) \)
- \( g_{ujp} \) fixed shipping cost of product \( p \) from manufacturing center \( u \) to DC \( j \)
- \( g'_{skp} \) fixed shipping cost of product \( p \) from retailer \( i \) to RC \( k \)
- \( a_{ujp} \) variable shipping cost of product \( p \) from manufacturing center \( u \) to DC \( j \)
- \( a'_{skp} \) variable shipping cost of product \( p \) from retailer \( i \) to RC \( k \)
- \( LT_p \) lead time for product \( p \) from manufacturing center to DC \( j \)
- \( h_p \) unit holding cost of product \( p \) at DCs and RCs
- \( q_{lp} \) demand quantity of customer \( l \) for product \( p \)
- \( c'_{rs} \) all shipping and displacement costs per unit of raw material \( s \) from supplier \( r \) to RC \( k \)
- \( c'_{oup} \) all shipping and displacement costs per unit of product \( p \) from supplier \( o \) to manufacturing center \( u \)
- \( \tau_t \) cost of environmental impacts for the production process using technology \( t \)
- \( Cap_{ks} \) capacity of RC \( k \) for raw material \( s \)
- \( Cap_{rs} \) capacity of supplier \( r \) for raw material \( s \)
- \( a_{ks} \) percentage of total capacity lose when a disruption occurs in RC \( k \)
- \( a'_{rs} \) percentage of total capacity lose when a disruption occurs in supplier \( r \)
- \( b_{ks} \) is equal to \( (1-a_{ks}) \); the existence capacity of RC \( k \) for raw material \( s \)
- \( b'_{rs} \) is equal to \( (1-a'_{rs}) \); the existence capacity of supplier \( r \) for raw material \( s \)
demand quantity of RC k for raw material s to repair the product p

\( q'_{kps} \)

purchasing cost per unit of raw materials from supplier r to RC k at price level v

\( \phi'_{skrv} \)

incremental price breaks occurred for raw material s shipped from supplier r to RC k at price level v

\( \phi'_{skrv} \)

the existing price level v for raw materials from supplier r to RC k

**Decision variables**

- \( X_j \): 1 if a DC is located at site j, otherwise 0
- \( Y_{ijp} \): 1 if product p is served to retailer i by DC j, otherwise 0
- \( W_k \): 1 if an RC is located at site k, otherwise 0
- \( z_{alp} \): 1 if the returned product p from retailer i is collected by RC k, otherwise 0
- \( V_{ljp} \): 1 if product p is served to customer l by retailer i, otherwise 0
- \( H_m \): 1 if a disposal center is located at site m, otherwise 0
- \( O_{sup} \): 1 if product p is served to reworking center k by disposal center located at m, otherwise 0
- \( W'_u \): 1 if a manufacturing center is located at site u, otherwise 0
- \( X'_{skv} \): 1 if price level v is used for raw materials shipped from supplier r to RC k, otherwise 0
- \( X'_{sup} \): 1 if product p is shipped from supplier o to manufacturing center u, otherwise 0
- \( U'_{kip} \): 1 if product p is shipped from RC k to DC j, otherwise 0
- \( X_P_{opt} \): 1 if product p is produced at manufacturing center u using production technology t, otherwise 0
- \( x'_{skv} \): quantity of raw material s from the supplier to RC k at price level v
- \( z_1, z_2 \): robust counterpart dual variables
- \( P_1, P_2 \): robust counterpart dual variables

3.2-Problem formulation

We first describe a comprehensive background of robust optimization to better express the formulation of the proposed problem. Then, the method of making the resilient capacity against disruptions is presented. Finally, the robust model is mentioned.

3.3- Background of robust optimization

The model by (Bertsimas and Sim, 2004) will be further described for the linear optimization problem while the objective function is minimization and there are coefficients of uncertainty in both the objective function and the constraints so that they will be more consistent with the main model of the study.

We consider the following optimization problem in general:

\[
\begin{align*}
\text{Min} & \quad c^T x \\
\text{st.} & \quad Ax \leq b \\
& \quad l \leq x \leq u
\end{align*}
\]

(1)

Each of the coefficients of constraints \( a_{ij}, j \in N = \{1, 2, \ldots, n\} \) is modeled as an independent random variable with the symmetric but unknown distribution \( \tilde{a}_{ij}, j \in N \) which takes a value in the interval \( [a_{ij} - \tilde{a}_{ij}, a_{ij} + \tilde{a}_{ij}] \), \( \tilde{a}_{ij} \) showing deviation from the nominal coefficient \( a_{ij} \). Each of the coefficients
of objective function $c_j, j \in N$ takes a value in the interval $[c_j - d_j, c_j + d_j]$, $d_j$ showing deviation from the nominal coefficient $c_j$. It is noteworthy that since the objective function is minimization and the objective of robust models is to obtain the maximum regret, only one side of the aforementioned interval is used, that is we assume that $c_j$ takes a value in the interval $[c_j, c_j + d_j]$.

In order to formulate the robust counterpart of the problem, $\Gamma_i$ is defined as follows. Let the $i$-th constraint be $a_i^T x \leq b_i$. $\Gamma_i$ is defined as the set of uncertain coefficients in the row $i$. We define for each row $i a_i \Gamma_i$, which is not necessarily an integer, so that we have $\Gamma_i \in [0, |\Gamma_i|]$. In fact, the role the $\Gamma_i$ plays in the constraints is to adjust the robustness of the proposed method against the level of conservatism of the solution. Bertsimas and Sim (2004) showed there is a low probability that all coefficients will be undergone uncertainty at the same time. Hence, we assume a maximum number of $|\Gamma_i|$ coefficients are allowed to change and a $a_{it}$ coefficient can change to a maximum value of $(\Gamma_i - |\Gamma_i|) \hat{a}_{ij}$. Then, the solution still remains justified. In other words, we assume that only one subset of the coefficients will be allowed to adversely affect our solution. This assumption assures that if this happens in a real case, our optimal robust solution will be definitely justified. In addition, considering the symmetric distribution of the variables, even if the number of changing variables exceeds $|\Gamma_i|$, the optimal solution remains justified with a high probability. Thus, we call $\Gamma_i$ the protection level for the $i$-th constraint. The parameter $\Gamma_0$ controls the level of robustness in the objective function. Therefore, we tend to find the optimal solution value in cases where it undergoes a change equal to $\Gamma_0$ from the objective function coefficients, having the greatest effect on the solution. In general, higher values of $\Gamma_0$ raise the level of conservatism versus the higher costs we have to pay for it in the objective function. $\Gamma_0$ is necessarily an integer but other $\Gamma_i$’s can be integers or not.

Accordingly, the robust counterpart of the nominal linear optimization mentioned can be obtained as follows (Bertsimas and Sim, 2003):

$$\min \ c^T x + \max_{\{\alpha_i \mid \alpha_i \leq \Gamma_0, \alpha_i \in \mathbb{R}\}} \left\{ \sum_{j \in \mathbb{R}} d_j \left| x_j \right| \right\}$$

s.t.

$$\sum_{j} a_{ij} x_j + \max_{\{\alpha_i \mid \alpha_i \leq \chi, \chi \leq \Gamma, i \in \mathbb{R}, \gamma_i \leq \mathbb{R}\}} \left\{ \sum_{j \in \mathbb{R}} \hat{a}_{ij} \left| x_j \right| + (\Gamma_i - \Gamma) \hat{a}_{ij} \left| x_j \right| \right\} \leq b_i$$

$$l \leq x \leq u$$

If we want to change the aforementioned model into a linear optimization model, the following theorem is needed.

**Theorem.** For every vector $x^*$, the protection function of the $i$-th constraint is obtained from the equation below:

$$\beta_i (x^*, \Gamma_i) = \max_{\{\alpha_i \mid \alpha_i \leq \chi, \chi \leq \Gamma, i \in \mathbb{R}, \gamma_i \leq \mathbb{R}\}} \left\{ \sum_{j \in \mathbb{R}} \hat{a}_{ij} \left| x^*_j \right| + (\Gamma_i - \Gamma) \hat{a}_{ij} \left| x^*_j \right| \right\}$$

Equation (3) shows the optimal value of the objective function, which is linear.
\[ \beta_i \left( x^*, \Gamma_i \right) = \max \sum_{j \in J_i} \hat{a}_{ij} \left| x^*_j \right| z_{ij} \]

s.t.
\[ \sum_{j \in J_i} z_{ij} \leq \Gamma_i \]
\[ 0 \leq z_{ij} \leq 1 \]

By replacing the dual variables of equation (4) in the original robust counterpart, it can be formulated as follows in which \( r \) and \( z \) are the vectors of the dual variables provided for linearization of nonlinear formula.

\[ \begin{align*}
\min \quad & c^T x + z_0 \Gamma_0 + \sum_{j \in J_0} r_{0j} \\
\text{s.t.} \quad & \sum_j a_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} r_{ij} \leq b_i \quad \forall i \\
& z_0 + r_{0j} \geq d_j y_j \quad \forall i \in J_0 \]
\[ r_{ij} \geq 0 \quad \forall i, j \in J_i \]
\[ y_j \geq 0 \quad \forall j \]
\[ z_i \geq 0 \quad \forall i \]
\[ l_i \leq x_i \leq u_i \quad \forall j \]
\end{align*} \]

3-4. The robust model

In this paper, uncertainty is to consider the resiliency of capacity of RCs and suppliers. Thus, in addition to the parameter for capacity of these facilities, a parameter is defined for percentage of loss of part of capacity for each facility and each raw material \( s \) (i.e., \( b_{ks} \) and \( b'_{rs} \)), being greater than or equal to zero. In fact, it can both have a value for each facility per raw material and have a value of zero. Since this parameter plays a major role in determining the exact available capacity of the facilities and is of an uncertain nature, it will be brought into the model as interval uncertainty defined in the intervals \[ \left[ b_{ks} - b'_{ks}, b_{ks} + b'_{ks} \right] \text{ and } \left[ b'_{rs} - b'_{rs}, b'_{rs} + b'_{rs} \right]. \]

In accordance with the interval uncertainty, each uncertain \( b'_{rs} \) and \( b_{ks} \) is in the form of a symmetrical, narrow interval with the center of \( b'_{rs} \) and \( b_{ks} \) like \( \bar{b}_{ks} = \rho \bar{b}_{ks} \) and \( \bar{b}_{rs} = \rho \bar{b}_{rs} \). \( b_{ks} \) and \( b'_{rs} \) are estimated values of percentage of loss of RC capacity for the raw material \( s \). Percentage of loss of the supplier’s capacity \( r \) for the raw material \( s \), \( b_{ks} \) and \( b'_{rs} \) are variations of these parameters, respectively, and \( \rho > 0 \) shows the level of uncertainty.

Now, the proposed robust problem can be formulated as:
\[
\begin{align*}
\min & \sum_{j \in J} f_j x_j + \beta \sum_{j \in J} \sum_{i \in I} \mu_x d_{ij}(y_{ij} - \sum_{k \in K} (1 - \eta_{ij})y_{ij}(U'_{ij} + \sum_{i \in I} \mu_x d_{ij} y_{ij} - \sum_{k \in K} \lambda_{ij}(1 - \eta_{ij})y_{ij}(U'_{ij}) + \beta \sum_{k \in K} \sum_{i \in I} \mu_x d_{ij} y_{ij}) \\
+ & \theta \sum_{k \in K} \sum_{i \in I} \sum_{j \in J} h_j(1 - \eta_{ij})y_{ij}(U'_{ij} + \sum_{i \in I} \lambda_{ij}(1 - \eta_{ij})y_{ij}(U'_{ij}) + \beta \sum_{k \in K} \sum_{i \in I} \mu_x d_{ij} y_{ij} + \theta \sum_{k \in K} \sum_{i \in I} \mu_x d_{ij} y_{ij} + \lambda_{ij}(1 - \eta_{ij})y_{ij}(U'_{ij}) \\
\text{s.t.} & \sum_{j \in J} y_{ij} = 1 \forall i \in I, p \\
y_{ij} - x_j \leq 0 \forall i \in I, j \in J, p \\
\sum_{k \in K} z_{ij} = 1 \forall i \in I, p \\
z_{ij} - w_k \leq 0 \forall i \in I, k \in K, p \\
w_k - \sum_{i \in I} y_{ij} \geq 0 \forall k \in K, p \\
\sum_{k \in K} v_{ij} = 1 \forall p, l \\
\sum_{i \in I} (1 - \delta_p) \mu_x y_{ij} \geq \sum_{i \in I} q \delta \mu_x v_{ij} \forall i \in I, p \\
\sum_{m \in M} o_{ij} = 1 \forall k \in K, p \\
o_{ij} - h_m \leq 0 \forall k \in K, m, p \\
\sum_{i \in I} q \delta \mu_x y_{ij} - (1 - \delta_p) \mu_x y_{ij} \leq \phi(\alpha)(1 - \delta_p) \sum_{j \in J} y_{ij} \sigma_{ij} \forall i \in I, p \\
\sum_{p \in P} X^1 = 1 \forall p \in u \\
X^1 - w^1 \leq 0 \forall p, o, u \\
\sum_{j \in J} \sum_{i \in I} y_{ij} = w_k \forall p, k \in K \\
z_{ij} - \sum_{j \in J} u_{ij} = 0 \forall i \in I, p, k \in K \\
\sum_{z \in V} x_{zv} \geq \sum_{i \in I} \sum_{j \in J} \gamma_{ij} \sum_{s \in S} p_{js} \leq \sum_{s \in S} \gamma_{js} \sum_{k \in K} c_{ps} w_k \forall k \in K \\
z^1_{zv} + p_{ks}^1 \geq \sum_{k \in K} c_{ps} w_k \forall k, s \\
z^1_{zv} \geq 0 \forall k, s \\
\sum_{k \in K} \sum_{v \in V} x_{zv} ^{k} \geq \sum_{j \in J} \sum_{i \in I} \gamma_{ij} \sum_{s \in S} p_{js} \leq \sum_{s \in S} \gamma_{js} \sum_{k \in K} c_{ps} w_k \forall r, s \\
z_2^2 \geq \sum_{k \in K} \sum_{v \in V} x_{zv} ^{k} \forall r, s \\
z_2^2 \geq 0 \forall r, s
\end{align*}
\]
The objective function (6) minimizes the total cost of the CLSC. The first term of objective function (6) indicates the fixed location cost of the DCs. The second term represents the delivery cost to the retailers from the assigned DCs. The third term implies three main costs including holding cost at DCs, ordering cost of the DCs from manufacturers, shipping cost from manufacturers to DCs. The fourth term shows the delivery cost to the DCs from the assigned RCs. The fifth term is the holding cost of the reworked products, which are kept at RCs and would be transferred to the DCs as soon as needed. The sixth term represents the total expected safety stock inventory cost based on risk pooling of the uncertainty in demand. The seventh term represents the fixed location cost of the remanufacturing centers. The eighth term symbolizes the delivery cost from the retailers to the assigned RC. The Ninth term represents the total expected working inventory at the RC relative to the assigned returns. The Tenth term depicts the total expected safety stock inventory cost. The Eleventh term represents the delivery cost from the Remanufacturing Centers. The Twelfth term represents the delivery cost from the RC to the assigned disposal center. The Thirteenth term indicates disposing cost of product at disposal center. The Fourteenth term denotes reworking cost of product at RC. The Fifteenth term represents the fixed location cost of the manufacturing centers. The Sixteenth term represents environmental impact of production at the manufacturing center by using a special technology. The Seventeenth term represents the fixed cost of sending products from supplier to the manufacturing centers. The Eighteenth term represents the fixed cost of sending raw material from supplier to RC. The Nineteenth term represents the fixed cost of buying raw material from supplier and the failure rate of raw material provided by suppliers.

Constraints (7) state that each retailer can only purchase from one and only one DC. Constraints (8) indicate that service provided by a DC is not possible unless the corresponding DC is opened. Constraints (9) indicate that each retailer can return products just to one RC. Constraints (10) state that returns can only be made to open RCs. Constraints (11) imply that an RC cannot be located unless a DC is opened at the same site and the retailers are assigned to this DC. Constraints (11) ensure that the proposed SC is closed-loop. Constraints (12) are similar to constraints (7). Constraints (13) ensure that all the customers' demands are satisfied. Constraints (14) indicate that each RC can deliver the products just to one disposal center. Constraints (15) denote that shipping to a disposal center is not possible unless the corresponding disposal center is opened. Constraints (16) are used for CCP implementation. Constraints (17) state that one and only the one supplier can only supply each manufacturer. Constraints (18) denote that service by a manufacturer is not possible unless the corresponding manufacturing center is opened. Constraints (19) state that if an RC is selected, the entered reverse products must be shipped to a DC, otherwise there should not be any assignment. Constraints (20) show that the possibility of having a reverse flow to the specific RC depends on the selection of that RC. The constraints (21)-(26) are obtained based on the robust model by (Bertsimas and Sim, 2004) described in previous subsection. Constraints (27) ensure that RCs' demand for raw materials is satisfied. Constraints (28) and (29) force quantities in the discount range for a vendor to be incremental. Because the "quantity" is incremental, if the order quantity lies in the discount interval \( V \), i.e. \( X_{v_{k_{sv}}^r} = 1 \), the quantities in interval 1 to \( V - 1 \) should lie in the maximum of those ranges (Wadhwa and Ravindran, 2007). Constraints (29) assure that a quantity in any range is no greater than the width of the range. Finally, constraints (30) are the standard integrality constraints.
4-Solution method

In this section, we discuss the application of the discussed model in a real case study provided by (Asl-Najafi et al., 2015). Based on this case study, the product flow would be started by Tehran as a supplier in the forward logistics and continued through the DCs to the customers. In this case study, 14 major zones have been considered in Iran as DCs. In order to solve the presented problem via GAMS in a short time, only one kind of product i.e. \( p = 1 \) and one kind of raw material i.e. \( s = 1 \) are considered. Some important parameter values of the case study addressed in (Asl-Najafi et al., 2015) beside the other values of parameters that are generated randomly have been illustrated in tables 2-4.

Table 2. Parameter values considering \( p = 1 \)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \beta )</th>
<th>( \theta )</th>
<th>( \gamma )</th>
<th>( \alpha )</th>
<th>( \eta_p )</th>
<th>( \delta_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>1</td>
<td>0.8</td>
<td>0.5</td>
<td>1.96</td>
<td>0.05</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 3. Retailer parameters considering \( p = 1 \)

<table>
<thead>
<tr>
<th>No.</th>
<th>Retailer</th>
<th>( \mu_{ip} )</th>
<th>( \rho_{ip} )</th>
<th>( \sigma_{ip}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shiraz</td>
<td>60</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>Karaj</td>
<td>75</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>Rasht</td>
<td>88</td>
<td>0.08</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>Kermanshah</td>
<td>45</td>
<td>0.05</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>Esfahan</td>
<td>215</td>
<td>0.07</td>
<td>0.2</td>
</tr>
<tr>
<td>6</td>
<td>Tabriz</td>
<td>120</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>7</td>
<td>Ahvaz</td>
<td>100</td>
<td>0.06</td>
<td>0.2</td>
</tr>
<tr>
<td>8</td>
<td>Zanjan</td>
<td>215</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>9</td>
<td>Yazd</td>
<td>185</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>10</td>
<td>Arak</td>
<td>211</td>
<td>0.07</td>
<td>0.15</td>
</tr>
<tr>
<td>11</td>
<td>Qom</td>
<td>180</td>
<td>0.08</td>
<td>0.1</td>
</tr>
<tr>
<td>12</td>
<td>Kerman</td>
<td>88</td>
<td>0.08</td>
<td>0.2</td>
</tr>
<tr>
<td>13</td>
<td>Ardebil</td>
<td>55</td>
<td>0.1</td>
<td>0.15</td>
</tr>
<tr>
<td>14</td>
<td>Golestan</td>
<td>300</td>
<td>0.05</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 4. Important values of DCs parameters considering \( p = 1 \) and \( s = 1 \)

<table>
<thead>
<tr>
<th>( j = k )</th>
<th>( F_j )</th>
<th>( F_k )</th>
<th>( Cap_{ks} )</th>
<th>( q_{kpx} )</th>
<th>( b_{kp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC1: Qom</td>
<td>10000</td>
<td>3000</td>
<td>150</td>
<td>100</td>
<td>45</td>
</tr>
<tr>
<td>DC2: Esfahan</td>
<td>7500</td>
<td>2400</td>
<td>140</td>
<td>52</td>
<td>42</td>
</tr>
<tr>
<td>DC3: Tabriz</td>
<td>9500</td>
<td>2800</td>
<td>120</td>
<td>60</td>
<td>38</td>
</tr>
<tr>
<td>DC4: Arak</td>
<td>8500</td>
<td>2200</td>
<td>110</td>
<td>80</td>
<td>40</td>
</tr>
<tr>
<td>DC5: Yazd</td>
<td>1100</td>
<td>500</td>
<td>130</td>
<td>63</td>
<td>45</td>
</tr>
<tr>
<td>DC6: Zanjan</td>
<td>1300</td>
<td>600</td>
<td>120</td>
<td>67</td>
<td>41</td>
</tr>
</tbody>
</table>

4-1-Sensitivity analyses

In this section, in order to gain more managerial insights, some sensitivity analyses based on the parameter values of table 2 have been conducted and discussed in-depth. In the direction of conducting sensitivity analysis, figure 2 illustrates the change of TCSC (total costs of the supply chain) vs. changes of \( \gamma \) as the reworking cost weights with \( \beta = 1, \theta = 0.8 \). As expected, the model shows linear and ascending behavior towards the parameter \( \gamma \), because as the reworking costs increase, the total cost increases subsequently.
Figure 2. Sensitivity analysis for $\gamma$

Figure 3 illustrates the change of TCSC vs. changes of transportation cost weight $\beta$ with $\gamma = 0.5, \theta = 0.8$. The model shows ascending linear behavior towards the changes of parameter $\beta$, which is normal and expected. Similarly, as can be seen in figure 3, $\beta$ varies in interval $[0,5]$ and because it has more variation in TCSC compared to the reworking cost weight, this indicates the fact that the proposed model is more sensitive to $\beta$ and should be carefully analyzed by managers and decision makers.

Figure 3. Sensitivity analysis for $\beta$

Figure 4 illustrates the change of TCSC vs. changes of $\theta$ as the inventory cost weight with considering $\gamma = 0.5, \beta = 1$. The model has nonlinear behavior towards the parameter $\theta$ and, therefore, a higher solution time, indicating the sensitivity of the model to this parameter, which is of high importance to managers.
Figure 4. Sensitivity analysis for $\theta$

Figure 5 illustrates the change of TCSC vs. changes of $q$ as demand quantity. The model is highly sensitive to the parameter $q$ and shows nonlinear behavior. In this regard, determining an appropriate value of this parameter may be so vital for managers and decision makers. Note that if the value of $q$ is selected from the range $[0,50]$, it incurs the highest cost to the proposed SC. Thus, increasing the amount of demand up to the suitable value would be an important goal for the decision makers of the SC.

Remind that in the proposed model, two kinds of events can be happened for the returned products: (1) disposing and (2) reworking. Regarding to the assumption that the disposal cost of per returned product is much lower than the reworking cost, the model expectedly prefers to dispose the most fraction of the returned products instead of sending back them to the forward logistics. Therefore, as $\eta$ (fraction of unrepairable products) increases, the TCSC decreases that have been illustrated in figure 6. Note that due to the high disposal costs of some products, it is more affordable to have reworking operations on them with the aim of returning to the forward logistics.
As expected, by investigating figure 7, it can be concluded that as fraction of returns from forward logistics $\delta$ increases, the respective costs of reverse logistics increases, which is the sufficient reason for increasing the TCSC.

Figure 8 illustrates the change of TCSC vs. changes of $\eta$ as a parameter that adjusts the robustness of the model. The model is highly sensitive to parameter $\eta$ in point 2. As can be concluded from figure 8, reaching the value of $\eta = 2$ may be an effective way to significantly reduce the costs of the existing SC. In this regard, appropriate plans from the managers’ side to reach this level have high priority in improving the performance of proposed model.
Figure 9 illustrates the change of TCSC vs. changes of $\Gamma_2$ as another parameter for adjusting the robustness. Similarly, the model is highly sensitive to parameter $\Gamma_2$ especially in point 1. Thus, managers should plan for achieving value 1 for $\Gamma_2$ because it produces the least total cost.

5 - Conclusion

In this paper, a resilient multi-echelon closed-loop location-allocation-inventory problem (RMCLIP) is presented in which some strategic and tactical decisions are analyzed in depth. In order to investigate the green part of the proposed SC, environmental impacts of different production technologies employed by manufacturers are taken into consideration in the form of costs. Furthermore, a pricing model based on quantity discount is proposed to represent the purchasing cost of raw materials taken from the supplier in the reverse channel. The capability of returning the reworked products to the forward logistics that can affect the ordering patterns of DCs is another significant issue under investigation. Furthermore, resilient capacity approach is used to provide a flexible SC toward the uncertainty of RCs and suppliers' capacity. Next, the problem was formulated as a mixed integer nonlinear location-allocation model. Finally, several sensitivity analyses were carried out to provide some managerial insights. Some extensions may be valuable for future researches, for example, considering the routing decisions besides the location-inventory problem. In addition, regarding the total time of SC (i.e. transportation time) as the second objective function along with the total cost can be a practical extension for the proposed model.
References


