

Competitive pricing in reliable supply chain with uncertainty

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Abstract

Natural catastrophes or man-made mistakes cause a great economic loss in various industries. Because of that, managing supply chains in a reliable manner has become a significant concern for decision makers. More precisely, any devastation in supply chain's elements reduces the overall revenue. In this study, we strive to maximize the total profit of distribution centers (DCs) with a set of retailers by considering risk of disruptions for facilities. Moreover, a game-theoretic approach is employed to simultaneously investigate the impacts of facility location, inventory management, breakdown of facilities and pricing strategies in a three-echelon supply chain. Various numerical experiments are generated to assess the accuracy of achieved solutions. In the following, a sensitivity analysis is performed for some specific parameters to recognize the most important factors affecting the system's performance. The results show that considering disruptions helps to provide a more accurate estimation for the system. Finally, the value of the stochastic solution (VSS) is used to justify that putting extra efforts into formulating and solving stochastic programming is worthwhile.

Keywords: facility location, disruption, uncertainty, pricing decision, competitive supply chain

1- Introduction

In Supply chain management (SCM), many organizations are incorporated so as to perform the different processes with the purpose of flowing some products and commodities to meet customer's demands and gain more profit as much as possible. In fact, we try to provide integration among different levels to operate at the lowest possible cost. In general, supply chain planning involves various challenges such as:

- Facility location problems and infrastructure deployment.
- Inventory management
- Price decisions and quantity decisions. (two main strategies in revenue management)

The decisions about facility location problems and infrastructure deployment are long-term investments and will not change during the time. In fact, the network configuration should be remained unchanged; while tactical decisions for instance quantity of production or pricing are made for shorter

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periods. Managing supply chain in a reliable manner is a vital key that could affect the overall revenue.

In the present business environments, natural and man-made incidents can make supply chains vulnerable. In particular, any disruption can impose human and financial losses on the system. Some global examples are provided as follows:

- In 1998, American vehicle-assembly operations closed because of a lack of parts. It eventuates 193,517 workers to be laid off at 27 of General-Motor's plants. It can be considered as one of the most expensive strikes in United State, which imposes \$1.1 billion and \$1.2 billion after-tax loss for General-Motor in the second and the third quarter of 1998, respectively (Herod 2000).
- In 2001, an eight-minute fire at the Phillips semiconductor plant causes a virtual standstill for Ericsson (Liberatore and Scaparra 2011).
- A devastating earthquake in Japan triggered powerful tsunami, which affected the Toyota Company and imposed basic risks to its operations (Canis 2011).

In two consecutive subsections, we discuss literature on both tactical and strategic decisions in SCM, distinctly.

1-1-The facility location in supply chain management (strategic level)

In Facility location problems, we aim to minimize a fixed set-up and transportation costs by determining the optimal number and the locations of new facilities for serving customers. In the simplest case, special (P) number of facilities should be located in its place in order to minimize total costs that called "p-median problem". Surprisingly, (Since some decades ago) researchers such as (Bramel and Simchi-Levi (2000), Daskin et al (2005), Reville et al (2008)) studied the un-capacitated fixed-charge location problem (UFL). Afterwards, some researches considered capacity extension constraints for distribution centers or even manufacturing plants. To this end, we can mention to Schultmann et al (2003), Garrido (2005), Fleischmann et al (2006), Trancosa and Ommeren al (2006), Srivastava (2008). Covering problem is another general model in facility location problems. In covering problem, the distance between facilities and customers is important for customers which should have assigned to facilities with acceptable coverage distance. Many studies are published in this filed, we can refer to Drezner et al (2004), Fallah et al (2009), Drezner, and Krass (2010b), Murray, Tong, and Kim (2010) Farahani et al (2012).

In traditional works, a located facility will be available incessantly and is non-failure in supplying commodities. This situation is not practical in real worlds since we may face many disruptions. Disruptions can be considered as several unpredictable events (completely or partially) that decrease the overall performance of supply chain's networks. However, due to the many natural disasters e.g. (earthquake, flood, etc.) and human mistakes such as fire, terroristic acts, industrial accidents, etc. companies may face various unexpected events that disturb supply chain elements and cause facilities to be inaccessible, therefore, it seems crucial to provide some protection precautions to preserve the network from disruptions and sudden incidents. In addition, considering uncertain data in many decisions making problems plays an important rule, especially, when the supply procedure may face various breakdowns.

Several studies in the field of disruptions have been already presented in the literature. Drezner (1987) for the first time studied disruption in the facility location. He presented a classical p-median facility location model and studied the probability of failure in each node. He also proposed a model, which strives to minimize the maximum cost of failure of facilities. In another work, the reliability fixed-charge location problem (RFLP) and the reliability p-median problem (RPMP) were introduced by Snyder and Daskin (2005). The same failure probability is considered for all facilities same as Drezner's model, in which customers are served by the nearest non-disrupted facility. They suggested a Lagrangian relaxation approach for solving the problem. Snyder et al (2006), Shen et al. (2011) and Zanhirani Farahani et al (2014), Ghavamifar and Makui (2016) applied a different approach for considering disruption by numerating the disruption scenarios and formulated the problem as a stochastic programming model. Lim et al. (2010, 2013) and Vahdani et al (2012) considered a backup facility in which customers should be assigned to reliable facilities in emergencies. In this formulation,

the fixed costs of locating the reliable facility is higher than unreliable one. Shishebori et al (2014) combined system reliability and budget constraints to design a supply chain network. By investigating the tradeoff between the nominal cost and system reliability, they conclude that it is possible to improve the system reliability with only slight increase in total cost. Azad et al (2013), Tavakkoli-Moghaddam et al (2016) developed mathematic models by considering incomplete disruption on capacity and transportation modes.

1-2- pricing decision and inventory management (tactical and operational level)

The tactical factors in supply chain are relevant to decisions which can be easily changed in a short time, like inventory policy, pricing decisions and etc. operational level involves short - impact decisions which may change weekly or monthly, such as routing and scheduling. In the current study, we consider two tactical decisions related to the following issues:

- Inventory management: the inventory management, as an element of the supply chain management includes various aspects such as controlling and finding optimal ordering inventory, storage of inventory in DCs, and controlling the product range for sale with considering the holdings and shortage costs.
- Pricing decisions: Pricing decisions are the process of determining the optimal prices for different commodities over the planning horizon. Pricing is one of the most important decisions in revenue management. It has a direct effect on supply chain's profit and customer's satisfaction. Due to the competitive environment, it is important for companies to be flexible about meeting customers' demand quickly with high quality. Therefore, it is crucial to provide the best pricing strategies in competitive markets.

Boyaci and Gallego (2002) studied the problem of coordinating the pricing and inventory replenishment policies in a supply chain with a wholesaler and multi retailers under deterministic price-sensitive demand. Melo et al. (2009) have presented a comprehensive review on the supply chain network design (SCND) studies and have introduced revenue management issues. Nagurney (2010a, 2010b) developed a model for equilibrium capacity and price decisions. Chen and Chang (2012) presented an analytical approach for a single-period inventory problem with stochastic price-dependent demand in competitive environment

In recent year, incorporating strategic and tactical decisions helps to reduce costs and promote the performance of the supply chain. Meanwhile, ignoring the effect of integration often eventuates sub-optimal solutions (Escalona, 2015).

A classical LIP (Location-inventory problem) is basically integrated with strategic and tactical level (facility location and inventory decision). This problem decides about the location of facilities together with assignments of customers to facilities and determining the ordering quantity. The objective function minimizes fixed location cost, holding inventory, shortage and transportation costs. Daskin et al. (2002) and Miranda and Garrido (2004a) developed facility location problems with inventory control decisions. In these studies, authors used EOQ model for ordering inventory. Shen (2002), Daskin, and Coullard (2003), Ozsen et al (2008) presented a location-inventory problem with risk pooling. Stochastic model with considering demand uncertainty are studied by Tsiakis, Shah & Panetedes(2001), Gabor and van Ommeren (2006), Shen & Qi (2007), Miranda and Garrido (2006, 2008), Ozsen et al. (2008), Liao (2009), Diabat (2013), Sadjadi (2015). Ahmadi-Javid and Hoseinpour (2015), considered capacity constraints for DCs. Shen (2006), Ahmadi Javid (2014), Kaya et al (2016) combined pricing decisions with location-inventory models and studied profit maximization. In these works, the location of DCs, the best strategy for assigning customers, quantity sequence and optimal product pricing are determined simultaneously in order to maximize the total supply chain profit.

Another kind of integration in supply chain decisions is location-pricing models. As we know, the facility location is a strategic decision with a huge investment that is hard to change. Meanwhile, pricing is short-term decisions can change easily. For incorporating these decisions, we employ a two-stage stochastic programming to characterize the uncertainty of demands in a stochastic environment. In the first-stage those variables that not affected by randomness, should be specified, while variables that are influenced by uncertain parameters belong to the second stage. Therefore, facilities are located in first

stage and in the second stage; the price of the product is obtained. This type of problem was introduced by Wagner and Falkson (1975), who defined price sensitive demand relationship at various locations. Afterward, Fernandez et al (2006) proposed a model in which a new company decides on the location of its facilities and competed with other existing firms for the market share under discriminatory pricing. Schutz et al. (2009) formulated two stage stochastic model for designing supply chain. The model decides about locations in first stage and the second stage consists of operational decisions. The objective aims at minimizing the sum of investment costs and expected operating costs of the supply chain. Sarmah et al (2010) studied competition and coordination subject in a two-stage stochastic programming model with considering price competition with and without channel coordination. Huang et al (2009) studied the price decisions in the three-level reverse supply chain (RSC) included a manufacturer, a maintenance center and a retailer. They validated their conclusions by numerical simulation. Rezapour and Farahani (2015) also studied reverse network design. They presented a bi-level model for the strategic and tactical planning decisions in a closed-loop and single-period supply chain with price-dependent market demand. Competition was considered between two chains producing commodities in a same market. Giri and Sarker (2016) studied pricing decisions and service levels with two competitive retailers. The demand at each retailer is stochastic and influenced by the prices and service levels of both the retailers. They formulated the problem as a Stackelberg game. They also considered production disruption in the manufacture. Wei and Jing (2017) studied a retailer's Stackelberg supply chain, in which the retailer sells a product in two quality-differentiated brands to customers. They investigated the impact of channel integration on price and quality competition between two brands and showed that the quality difference and the relative efficiency of the two brands have a great effect on the performance of the supply chain structure. Chen et al (2017) also developed game-theoretic model to study a retailer's Stackelberg supply chain, and obtain equilibrium prices for a dual-channel. They showed that a dual-channel supply chain could enhance the profits of the manufacturer and the supply chain.

We prepared a review about related articles in table 1. To the best of our knowledge from a review of literature, there is no existing two stage stochastic model that incorporates the pricing, location, inventory decisions and facility breakdowns in competitive three-echelon supply chains. Our proposed model investigates the impacts of distribution center's location, inventory management and pricing decisions on profits simultaneously. We assume that disruptions occur on DCs, thus we provide backup facilities to deal with failures. Because of uncertainty of demands, a two-stage stochastic approach is applied to model the problem. The main contributions of this study are summarized as follows:

1. Integrating strategic and operational decisions in supply chain by employing two stage stochastic programming with finite number of possible realizations for demands.
2. Discussing more actual situations by considering disruptions in the distribution centers.
3. Determining pricing in competitive environment. Stackelberg game model is applied where DCs are leaders and retailers are followers.

The remainder of this paper is organized as follows. Section 2 contains the problem description, assumptions and notations. Mathematical formulation is presented in section 3. In Section 4 we provide some numerical examples for the problem, moreover a sensitivity analysis around several important parameters are prepared. Finally, Section 5 includes conclusion and future directions.

Table 1. Reviewed articles related to competitive supply chain and disruption

Authors	Demand		Type of competition		Competition basis				output				disruption	
	certainly	Un certainly	Nash	Stackelberg	Static	price	Service	other	strategic	operational			Reliable	scenario
									Facility location	price	Service level	Production flow		
<i>Boyaci and Gallego(2004)</i>	✓		✓				✓			✓		✓		
<i>Bernstein and Federgrun (2007)</i>		✓	✓				✓			✓	✓	✓		
<i>Fernandez et al(2007)</i>	✓				✓	✓		✓	✓	✓				
<i>Aboolian et al(2007)</i>		✓			✓			✓	✓					
<i>Plastria and Vanhaverbeke(2008)</i>	✓			✓				✓	✓	✓				
<i>Zhang and Rushto(2008)</i>					✓			✓	✓					
<i>Jiang and Wang(2009)</i>		✓		✓		✓								
<i>Rezapour et al(2011)</i>		✓		✓		✓			✓	✓				
<i>Rezapour et al (2014)</i>	✓		✓			✓	✓			✓		✓		✓
<i>Rezapour and Farahani (2015)</i>		✓	✓			✓			✓	✓		✓		
<i>Gavamifar and Makui(2016)</i>		✓		✓		✓			✓	✓				✓
<i>Giri and Sarker(2016)</i>		✓		✓		✓				✓	✓			
<i>Chen et al(2017)</i>				✓		✓				✓				
<i>Wei and Jing (2017)</i>	✓			✓		✓				✓		✓		
<i>This paper</i>		✓		✓		✓			✓	✓			✓	

2- Problem description

Consider three-echelon supply chain, which contains one manufacturer, multi distribution centers and multi retailers. Manufacturer and retailers are located in the specified places and the model decides about the optimal locations for DCs. Because of unexpected events, we assume that some of DCs are unreliable and may be failed (become unavailable), so a reliable DC is needed to support the retailers. Therefore, at least one reliable DC should be established. Facility location decisions belong to the strategic planning in the network design, which cannot be changed at a short time. In return, pricing and inventory decisions are operational activities. As a result, we propose a two-stage stochastic model to formulate this problem with two different decision levels. In this study, we assume that there are about 2–4 scenarios for demand variation in the stochastic model. In addition, we incorporate a Stackelberg game model with the aim of determining the best price of retailers and customers in a competitive environment. In the proposed model, DCs act as leaders and retailers are followers.

The structure of the proposed three-echelon supply chain is illustrated in Figure 1. The manufacturer provides products to the located DCs. Then the products are transported to the retailers in order to fulfill the customer's demands. Wholesale price and sale price should be determined considering inventory and transportation costs. Because of unpredictable events, two kinds of distribution centers are considered: reliable and unreliable DC. If an unreliable DC is established, then the proposed model provides a link that connects related customers to an additional reliable DC to cope with emergency situations.

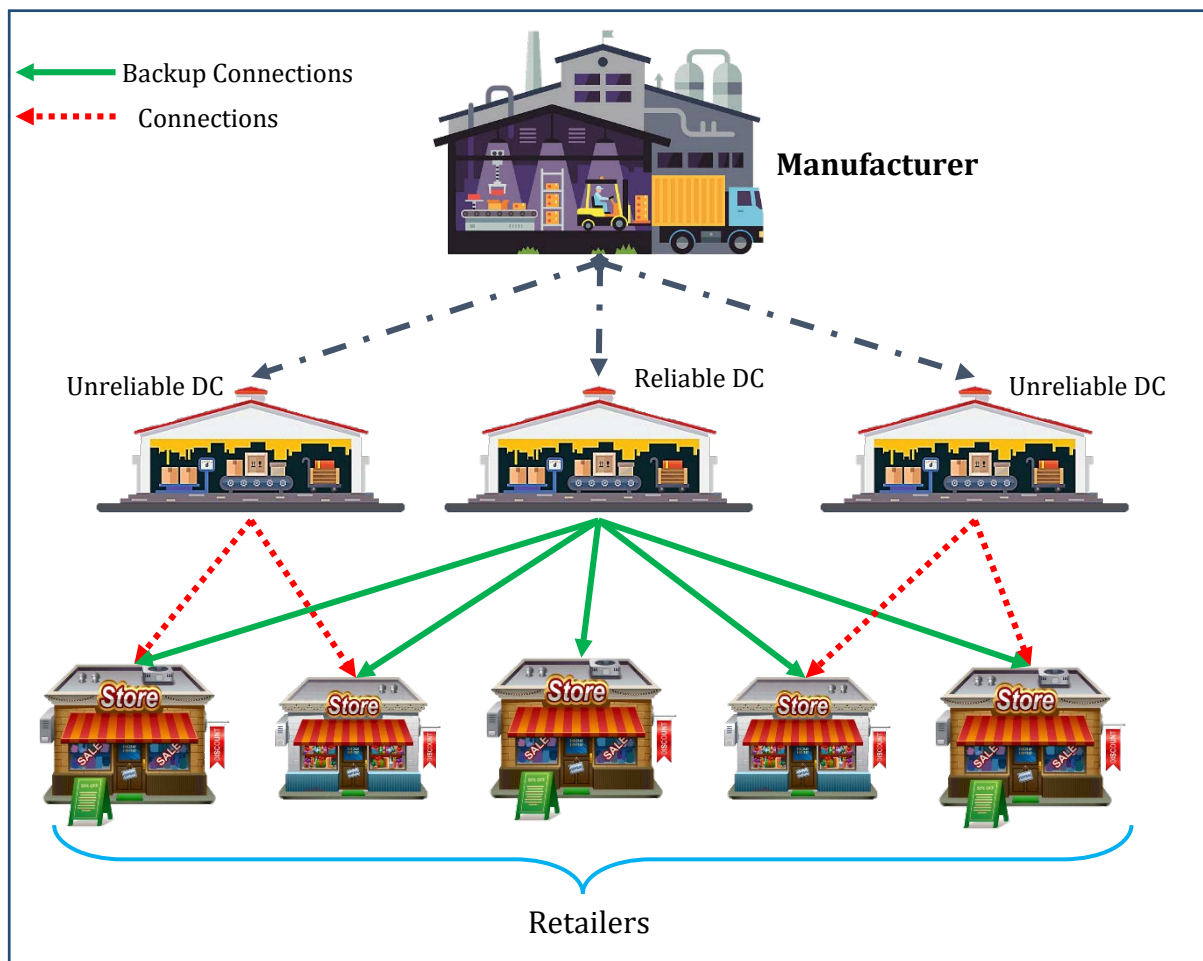


Fig1. Configuration of three echelon-supply chain network

Before presenting the model, let us explain the assumptions. Then we introduce notations of mathematical formulation.

2-1-Assumptions

Proposed profit maximization location inventory problem (PM-LIP) is formulated under some specific assumptions as follows:

- The number of DCs should be located is unknown and determined by model.
- Each retailer can be assigned to exactly one DC at each scenario.
- We consider fixed failure probability associated to each DC.
- Demand is considered to be uncertain; we assume that it has a discrete distribution with finite number of possible scenarios.
- DCs should serve all the retailers.
- Shortage is not allowed.
- Disruption could occurs in DC, however to tackle the failures, the model considers backup facilities for retailers.
- Capacity is considered at each DC and the manufacturer is not restricted by capacity limitation.

2-2-Sets

M Set of potential manufacturers, $m \in \{1. \dots M\}$

J Set of potential distribution centers, $j \in \{1. \dots J\}$

R Set of retailers, $r \in \{1. \dots R\}$

Ω Set of all possible scenarios, $\omega \in \{1. \dots \Omega\}$

2-3-Parameters

gr_j Fixed cost of locating a reliable distribution center j , which is not fail.

gu_j Fixed cost of locating an unreliable distribution center j .

f_m Fixed cost of selecting manufacturer m .

θp_{jr} Transportation cost for primary allocation between distribution center j and retailer r .

θb_{jr} Transportation cost for a backup assignment between distribution center j and retailer r .

θs_{jr} Transportation saving cost when retailer r assigned to a reliable distribution center j in primary assignment.

d_{jr} Distance (kilometer) from distribution j to retailer r

h_j Annual inventory holding cost of distribution center j (*per unit product*)

$De_r(\omega)$ Base market potential of retailer r in scenario ω .

$\pi(\omega)$ The probability for scenario ω .

h_r Annual inventory holding cost of retailer r

br Price elasticity of demand

cap_j Annual capacity of distribution j .

$w_j(\omega)$ Wholesale price of manufacturer m for scenario ω .

q_j Probability of failure state for an unreliable distribution j .

2.4 Decision variables

- O_{rj} $\left\{ \begin{array}{l} 1 \text{ if reliable distribution center } j \text{ is located} \\ 0 \text{ otherwise} \end{array} \right.$
- O_{uj} $\left\{ \begin{array}{l} 1 \text{ if unreliable distribution center } j \text{ is located} \\ 0 \text{ otherwise} \end{array} \right.$
- $y_{mj}(\omega)$ $\left\{ \begin{array}{l} 1 \text{ if distribution center } j \text{ assigned to manufacturer } m \text{ for scenario } \omega \\ 0 \text{ otherwise} \end{array} \right.$
- $Zp_{jr}(\omega)$ $\left\{ \begin{array}{l} 1 \text{ if retailer } r \text{ is assigned to distribution center } j \text{ as the primary site for} \\ \text{scenario } \omega \\ 0 \text{ otherwise} \end{array} \right.$
- $Zb_{jr}(\omega)$ $\left\{ \begin{array}{l} 1 \text{ if retailer } r \text{ is assigned to distribution center } j \text{ as a backup site for scenario } \omega \\ 0 \text{ otherwise} \end{array} \right.$
- $Zs_{jr}(\omega)$ $\left\{ \begin{array}{l} 1 \text{ if retailer } r \text{ is assigned to reliable distribution center } j \text{ as a primary site for} \\ \text{scenario } \omega \\ 0 \text{ otherwise} \end{array} \right.$
- $p_r(\omega)$ Price for retailer r offered by DC in scenario ω
- $pri_r(\omega)$ Sale price offered by retailer r in scenario ω
- $x_r(\omega)$ Annual demand of retailer r in scenario ω

3- Mathematical model

As discussed earlier, retailers try to determine the best selling price in order to maximize their profit. The profit function consisting purchase and inventory costs, can be formulated as equation (1) as follows:

$$\max f_r = \sum_{\omega} (pri_r(\omega) - p_r(\omega))x_r(\omega) - \sum_{\omega} h_r x_r(\omega) \quad (1)$$

$$x_r(\omega) = De_r(\omega) - br pri_r(\omega) \quad (2)$$

$$pri_r(\omega) \geq 0 \quad (3)$$

Constraint (2) indicates linear demand function and shows that price is sensitive to demand. This function is applied in many researches (Dong and Xu (2002), Chiang (2003), Sadjadie (2009), Anderson and Bao (2009)). Constraint (3) shows that prices are positive.

The profit of DSs contains total sale income, fixed location costs, purchase, and inventory and transportation costs. The formulation of the problem is as follow:

$$\max dc_j = E[Q(O, \xi(\omega))] - \sum_{j \in J} gu_j Ou_j - \sum_{j \in J} gr_j Or_j \quad (4)$$

$$\text{s.t} \quad Or_j, Ou_j \in \{0,1\} \quad (5)$$

$$Or_j + Ou_j \leq 1 \quad \forall j \quad (6)$$

$$\sum_{j \in J} Or_j \geq 1 \quad (7)$$

where $Q(O, \xi(\omega))$ is the optimal value of the following second stage problem:

Stage two :

$$\begin{aligned} \max \quad & \sum_{\omega \in \Omega} \sum_{j \in J} \sum_{r \in R} \left(p_r(\omega) x_r(\omega) pro(\omega) \right) + \sum_{j \in J} \sum_{r \in R} \sum_{\omega \in \Omega} q_j(\theta s_{jr} d_{jr}) pro(\omega) Zs_{jr}(\omega) \\ & - \sum_j \sum_r \sum_{\omega} (w_j(\omega) x_r(\omega) pro(\omega) Zp_{jr}(\omega)) \\ & - \sum_{j \in J} \sum_{r \in R} \sum_{\omega \in \Omega} x_r(\omega) pro(\omega) h_j Zp_{jr}(\omega) \\ & - \sum_{m \in M} \sum_{j \in J} \sum_{\omega \in \Omega} y_{mj}(\omega) pro(\omega) f_m \\ & - \sum_{j \in J} \sum_{r \in R} \sum_{\omega \in \Omega} q_j(\theta b_{jr} d_{jr}) pro(\omega) Zb_{jr}(\omega) \\ & - \sum_{j \in J} \sum_{r \in R} \sum_{\omega \in \Omega} (1 - q_j)(\theta p_{jr} d_{jr}) pro(\omega) Zp_{jr}(\omega) \end{aligned} \quad (8)$$

$$Zp_{jr}(\omega) \leq \sum_{m \in M} y_{mj}(\omega) \quad \forall j, r, \omega \quad (9)$$

$$Zb_{jr}(\omega) \leq \sum_{m \in M} y_{mj}(\omega) \quad \forall j, r, \omega \quad (10)$$

$$\sum_{m \in M} y_{mj}(\omega) \leq Ou_j + Or_j \quad \forall j, \omega \quad (11)$$

$$\sum_{j \in J} Zb_{jr}(\omega) = 1 \quad \forall r, \omega \quad (12)$$

$$\sum_{j \in J} Zp_{jr}(\omega) = 1 \quad \forall r, \omega \quad (13)$$

$$Zb_{jr}(\omega) \leq Or_j \quad \forall j, r \quad (14)$$

$$Zp_{jr}(\omega) \leq Or_j + Ou_j \quad \forall j, r, \omega \quad (15)$$

$$Zs_{jr}(\omega) \leq Zp_{jr}(\omega) \quad \forall j, r, \omega \quad (16)$$

$$Zs_{jr}(\omega) \leq Zb_{jr}(\omega) \quad \forall j, r, \omega \quad (17)$$

$$\sum_{r \in R} De_r(\omega) Zp_{jr}(\omega) \leq cap_j \quad \forall j, r, \omega \quad (18)$$

$$Zb_{jr}(\omega), Zp_{jr}(\omega), Zs_{jr}(\omega), y_{mj}(\omega) \in \{0,1\} \quad (19)$$

$$x_r(\omega), p_r(\omega) > 0 \quad (20)$$

Objective function for leader (DCs) is two-stage stochastic programming. In this model location of DCs are strategic and should be decided in the first stage then the assignments and pricing decisions that are dependent to the scenarios and should be determined in the second stage. Constraint (6) insures that only one distribution center (reliable or unreliable) can be located at node j. Constraint (7) enforces that at least a reliable distribution center j should be located.

In the second stage, the objective function contains sale products to DCs and saving costs of assigning retailers to reliable DCs as primary allocation. The next terms are purchase, inventory and selecting manufacturer costs. Two last terms of function, indicate the expected transportation costs when retailers are served by their primary or backup distribution center facility. Constraint (9) and (10) express both reliable and unreliable DCs should be assigned to the selected manufacture. Constraint (11) ensures that each opened DC should be assigned to the selected manufacturer. Constraint (12) and (13) show that each retailer should be assigned to both primary and backup distribution center j. Constraints (14) and (15) ensure that the primary assignment should be considered for an open distribution center (reliable or un reliable) and the backup assignment is related to a reliable facility. Constraint (16) and (17) state that the savings associated with any assignment can only be realized if the retailer r is assigned to the same distribution center j as its primary and its backup facility. Capacity constraint of DCs is considered in constraint (18). Constraint (5), (19) and (20) define non-negative and binary variables.

3-1- Stackelberg game model

For calculating the equilibrium in game model, we use backward induction (calculation moves 'backwards'). First, we calculate the best response functions of the retailer. Then the best reaction function for DCs are obtained by considering the retailer's reaction.

As the second derivative of F_r is $\frac{\partial^2 F_r}{\partial pri_r(\omega)^2} = -2br$, we obtain optimal $pri_r(\omega)$ by setting the first derivative of retailer's function and put it equal to 0.

So we get:

$$pri_r(\omega) = \frac{De_r(\omega) + p_r(\omega)br + brh_r}{2Br} \quad (21)$$

$$x_r(\omega) = \frac{De_r(\omega) - p_r(\omega)br - brh_r}{2} \quad (22)$$

We replace equation (21) into equation (2) to achieve and equation (22). Then we set equation (22) in DC's objective function and rewrite the second stage of the model.

Stage two :

$$\begin{aligned}
& \max \sum_{\omega \in \Omega} \sum_{j \in J} \sum_{r \in R} \left(p_r(\omega) \left(\frac{De_r(\omega) - p_r(\omega)br - brh_r}{2} \right) pro(\omega) \right. \\
& + \sum_{j \in J} \sum_{r \in R} \sum_{\omega \in \Omega} q_j(\theta s_{jr} d_{jr}) pro(\omega) Zs_{jr}(\omega) \left. \right) \\
& - \left(\frac{De_r(\omega) - p_r(\omega)br - brh_r}{2} \right) \left(\sum_j \sum_r \sum_{\omega} (w_j(\omega) pro(\omega) Zp_{jr}(\omega)) \right. \\
& \left. - \sum_{j \in J} \sum_{r \in R} \sum_{\omega \in \Omega} pro(\omega) h_j Zp_{jr}(\omega) \right) - \sum_{m \in M} \sum_{j \in J} \sum_{\omega \in \Omega} y_{mj}(\omega) pro(\omega) f_m \\
& - \sum_{j \in J} \sum_{r \in R} \sum_{\omega \in \Omega} q_j(\theta b_{jr} d_{jr}) pro(\omega) Zb_{jr}(\omega) \\
& - \sum_{j \in J} \sum_{r \in R} \sum_{\omega \in \Omega} (1 - q_j)(\theta p_{jr} d_{jr}) pro(\omega) Zp_{jr}(\omega)
\end{aligned} \tag{23}$$

St.

Constraint (9)-(20)

As it is clear, objective function (8) contains nonlinear terms, which are difficult to deal with them. Nevertheless, in the reformulation scheme, quadratic term is appearing and we can use conic quadratic approach for solving the problem. The deterministic equivalent program (DEP) of proposed model is presented in appendix 1.

4-Numerical experiment

In this section, we prepare some computational experiments to test the accuracy of the model. All examples were coded in 24.1.3GAMS using BONMIN solver and implemented on the computer with the following configuration: processor is Intel Core i7 Duo CPU @ 1.80 GHz 2.40 GHz with 6 G memory. Data for provided instances are generated according table2. Fixed location costs, inventory costs, fixed selecting manufacturer costs and transportation costs are generated randomly according to uniform distribution. Distance between manufacturer to DCs and DCs to retailers are calculated by Euclidean distances. The results of solving the problem with different sizes and features are presented in table 3.

Table2. Data generation

annual base market	$de_r(\omega) = U[250000.700000]$
Distribution center capacities	$cap_j = U[3000000.10000000]$
Price elasticity of demand	$br = 85$
Fixed cost for locating reliable DC	$gr_j = U[90000.120000]$
Fixed cost for locating un reliable DC	$gu_j = U[65000.90000]$
Fixed cost of selecting manufacturer	$f_j = U[900.1500]$
Annual inventory cost in DC	$h_j = U[70.90]$
probability that an unreliable distribution may be failed	$q_j = U[0.1]$
Transportation cost for primary allocation	$\theta_{jr} = U[0.01.0.03]$

Table 3. Numerical examples

NO.	Size of problem			Location of reliable DC	Location of unreliable DC	Objective value	Time(second)	Gap
	M	J	R					
1	2	2	5	or_2	-	1.529153E+9	0.85	0
2	2	3	7	or_1	-	2.078840E+9	3.24	0
3	2	3	10	or_2	-	3.729261E+9	2	0
4	3	3	10	or_2		3.813310E+9	15	0
5	3	4	15	or_3	-	5.594888E+9	43	0
6	3	4	20	or_5	ou_1	6.415639E+9	100	0
7	4	4	20	or_1	ou_4	7.709541E+9	120	0
8	3	5	20	or_5	ou_2	6.644004E+9	132	0
9	4	6	25	or_1, or_5	-	9.367021E+9	180	0
10	4	5	25	or_4	ou_2	9.103945E+9	188	0
11	3	5	30	or_1	ou_5	1.07796E+10	190	0
12	3	6	35	or_5	ou_2	1.35872E+10	600	0
13	3	6	40	or_1	ou_2, ou_5	1.43067E+10	3600*	0.000015
14	4	7	45	or_6, or_1	ou_3	1.51325E+10	3600*	0.37
15	4	8	50	or_4, or_1	ou_5	1.93216E+10	3600*	0.56
16	5	7	53	or_5, or_6	ou_2	2.384860E+10	7200*	0.022
17	5	8	55	or_2, or_3, or_4	-	2.502368E+10	7200*	0.0026
18	4	8	57	or_4, or_7	ou_5	2.708291E+10	7200*	0.0017
19	4	9	60	or_5, or_1	ou_3, ou_4	3.292707E+10	7200*	0.0033
20	5	12	65	or_5, or_4	ou_{12}, ou_6	3.971502E+10	7200*	0.032
21	6	12	67	or_3	ou_{10}, ou_5, ou_9	4.284641E+10	7200*	0.0016
22	6	13	70	or_4, or_5, or_7	ou_9	4.776628E+10	7200*	0.0068
23	5	12	73	or_7, or_4	ou_{11}, ou_9	5.490514E+10	7200*	0.034
24	5	14	75	or_5, or_1	ou_{13}, ou_6	5.951820E+10	7200*	0.015
25	6	14	80	or_{14}, or_6, or_7	ou_{10}, ou_2	6.282946E+10	7200*	0.32
26	7	15	85	or_7, or_4	ou_{12}, ou_9, ou_5	6.57385E+10	10800*	0.0013

* GAMS cannot find the optimal solution during the reported time

Table 3 listed the results of proposed model. As can be seen in the table GAMS was able to find the optimal solutions for small and average size of instances.

4-1- Sensitivity analysis

In this section, sensitivity analysis is performed for some specified parameters to recognize the most significant factors affecting the system's efficiency and to suggest useful guidelines.

4-2-The price elasticity of demand

The price elasticity of demand (br) is one of the significant parameters that has an impressive effect on supply chain's revenue. The primary value is considered to be 85 and percentages of improvements and reduction are equating to (20%, 30%, 40%). As it shows in figure 2 and figure 3(a, b), increase in br , causes reductions in the price and profit of retailers and DCs. Contrariwise, decrease in br eventuates the increase of prices and profits in both DCs and retailers.

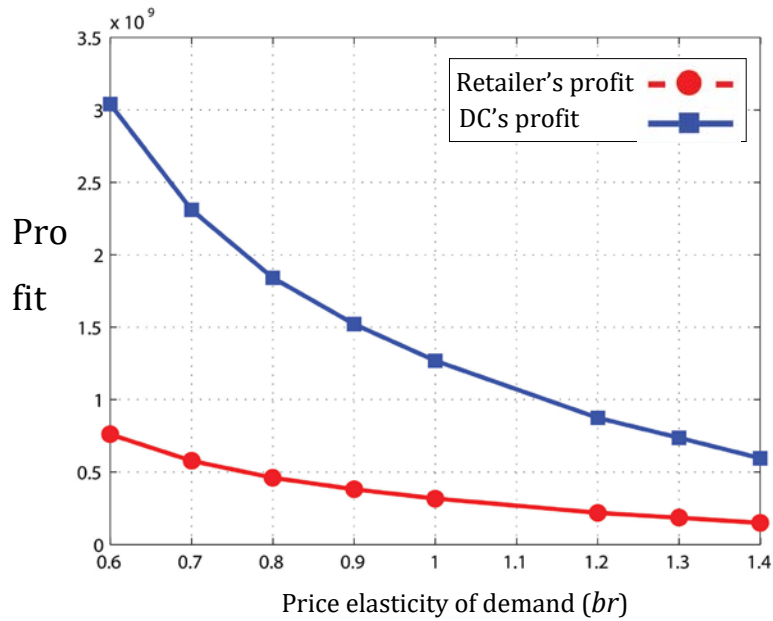


Fig2. Impact of changing br on DCs and retailers profit

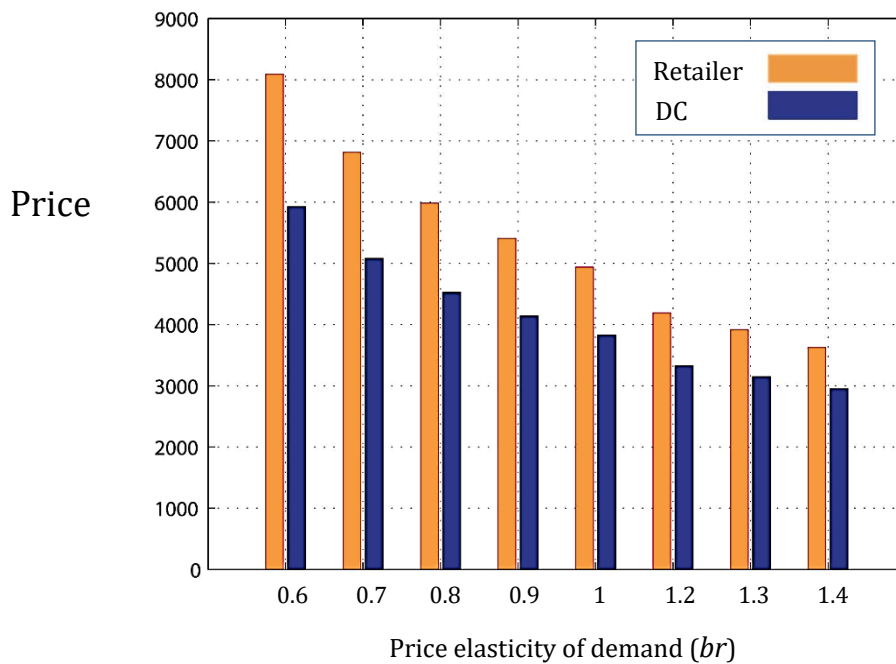


Fig 3 (a). The result of changing br on the price of DCs and retailers in the first scenario

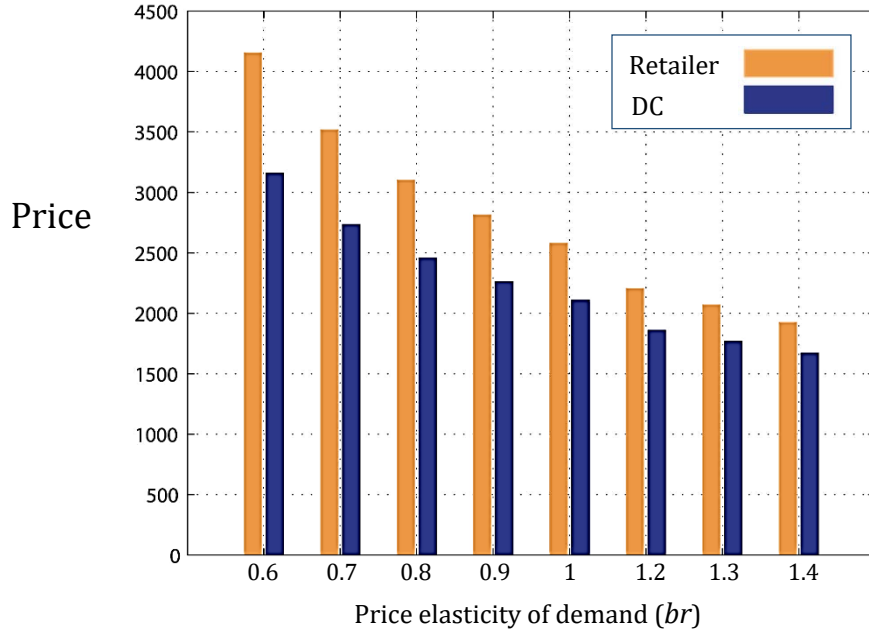
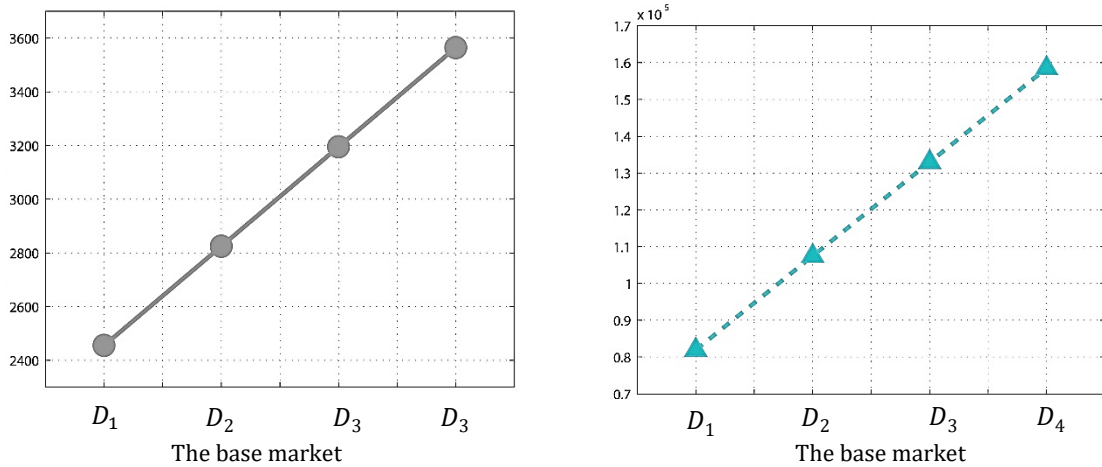


Fig3 (b). The result of changing br on the price of DCs and retailers in second scenario

4-3- The Base market potential of retailers

We provide a sensitivity analysis for the base market (de) in price and retailers demand in figure 4. The result describes that by increase in de , the price and demand increase. For example, when the base market increases from 250000 to 350000, the price increases from 2500 to 3200 (by 28%).



$$D_1 = 250000, D_2 = 300000, D_3 = 350000, D_4 = 400000$$

Fig4. Impact of base market on retailer's demand and price.

4-4- Impact of disruption

In this subsection, we investigate the impact of considering disruptions in competitive supply chain. First, we assumed that the probability of disruption is equal to zero and solved the model without failure in distribution centers. Then, we considered the failure probability and solved the model again. The achieved result of DC's profit with and without considering disruption is represented in figure 5

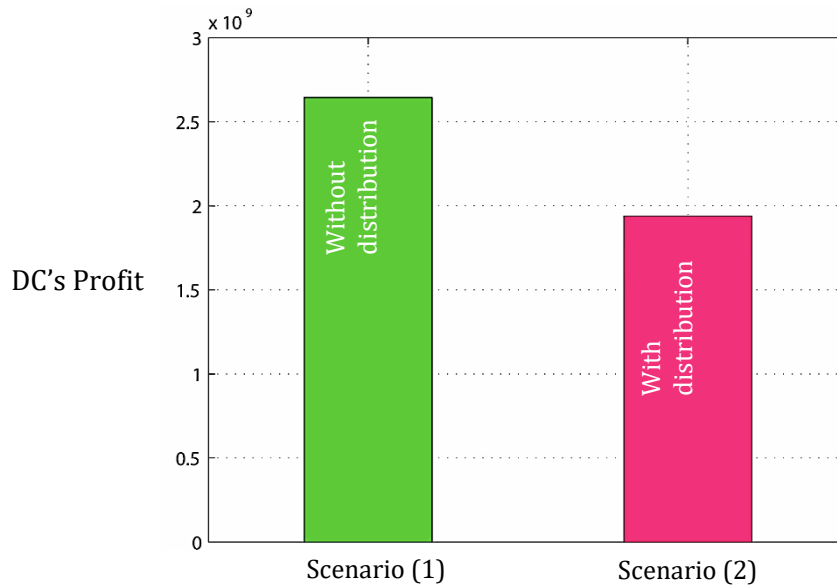


Fig5. Impact of considering disruption on DC's profit

As can be seen from the figure, disruption has a great effect on the performance of supply chain and decrease the profit of DCs. In conclusion, failures of facilities in supply's elements affect the performance of supply chain, substantially. Thus, consideration of disruptions helps to estimate and design the supply chains in an efficient way.

3-4-1-Impact of disruption probability on reliable and unreliable DCs

In this section, we study that how the disruption probability affects the number of reliable and unreliable DCs. To this end, we focused on two instances with 60 and 80 numbers of retailers. The results are depicted in figure 6. It can be concluded from the figure that by increase in q value the optimum number of opened reliable DCs increase, in return, the optimum number of opened unreliable DCs decrease. For example, figure 6 shows that at size 80 and $q = 0.1$, there are 2 and 3 opened reliable and unreliable DCs, respectively, whereas at $q = 0.4$, there are 4 and 1 opened reliable and unreliable DCs. Similar results are also found at size 60.

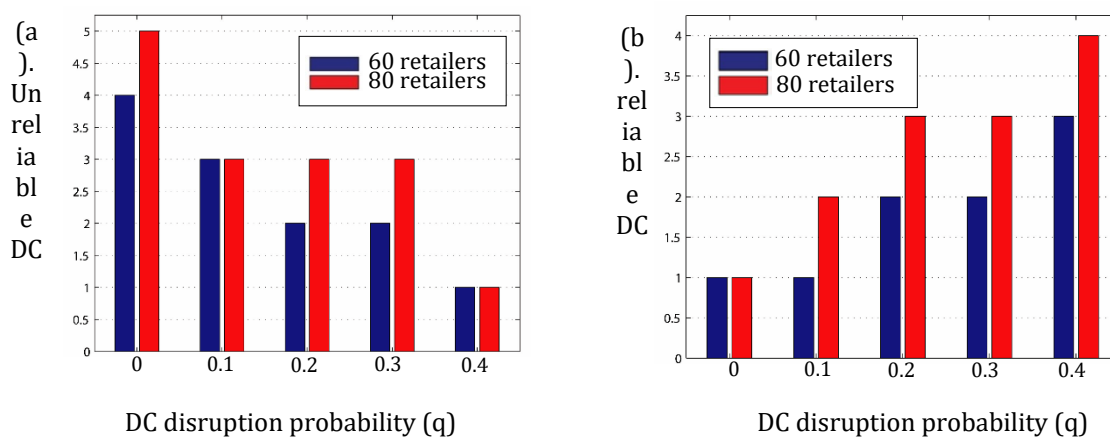


Fig 6. Impact of disruption probability on unreliable DCs (a) and reliable DCs(b)

4- 5- Value of stochastic programming

In order to evaluate the advantage of considering uncertainty in the model, we implemented the well-known value of the stochastic programming (VSS) factor. In this approach, instead of random variables, the corresponded average values are applied. In this case, we employed the average of demands in different scenarios to model the problem. Let $\bar{z}(\bar{\xi})$ be the optimal decisions in the first stage of the problem. The VSS is then calculated as $SS = Q^* - EEV$, where Q^* is the objective value of the stochastic programming and EEV is expected result of using EV (expected value program) solution and computed by $EEV = E_{\xi} (Q(\bar{z}(\bar{\xi}), \xi))$ (Tikani et al. 2016).

The results of solving stochastic model and the calculated VSS are compared in Table4. Higher values of VSS implies the inefficiency of deterministic model and explained that, applying stochastic programming model is more valuable and justify the use of more sophisticated modeling and solving techniques.

Table4. Computational results for VSS and the stochastic programming solutions.

NO	Size of problem			Q^*	EEV	VSS
	$ M $	$ J $	$ R $			
1	2	2	5	1.696634E+9	1.122728E+9	5.739060E+8
2	2	3	10	3.688538E+9	2.370339E+9	1.318199E+9
3	3	4	15	5.178219E+9	3.413224E+9	1.764995E+9
4	3	4	17	6.149202E+9	4.146100E+9	2.003102E+9

5-Conclusion

In this paper, a two stage stochastic programming model is presented to incorporate game-theoretic approach and facility failures in multi-level supply chain. To deal with the uncertainty of demands, we assume that demand has a discrete distribution with number of possible scenarios. In particular, in the first stage of the model, we decide about the location of DCs (strategic planning) which are not affected by randomness of stochastic environment while second stage deals with tactical decisions that vary regard to randomness reality. In details, proposed model strives to maximize DCs and retailers profit by following issues:

- Facility location problem, to determine the number and location of distribution centers.
- Allocation strategies, to determine the best assignments of retailers to DCs and DCs to selected manufacture in both normal and emergency situations.
- Pricing decision, to find the optimal price strategy in competitive market for retailers and customers, considering costs of inventories and transportations.

In emergent conditions, due to the natural disasters or human mistakes DCs may face various unexpected events that disturb supply chain. Thus, backup facilities are considered for responding retailer's demands in failure situations. In the following, various numerical experiments is provided to show the efficiency of the model. In addition, Sensitivity analysis is performed to provide insights regarding the effect of parameters. Finally, we calculated Value of the Stochastic Solution factor (VSS) for some examples to justify use of more sophisticated modeling techniques and extra computational efforts.

Interesting avenues of the paper for future researches consists in addressing this problem with larger sizes and use of proper algorithms for solving it, and considering partial facility disruptions as well as partial hardening options (with less investment) or employing other pricing policies such as zone pricing.

References

- Aboolian, Robert, Oded Berman, and Dmitry Krass. "Competitive facility location and design problem." *European Journal of Operational Research* 182.1 (2007): 40-62.
- Ahmadi-Javid, Amir, and Pooya Hoseinpour. "Incorporating location, inventory and price decisions into a supply chain distribution network design problem." *Computers & Operations Research* 56 (2015): 110-119.
- Anderson, Edward J., and Yong Bao. "Price competition with integrated and decentralized supply chains." *European journal of Operational research* 200.1 (2010): 227-234.
- Azad, Nader, et al. "Strategies for protecting supply chain networks against facility and transportation disruptions: an improved Benders decomposition approach." *Annals of Operations Research* 210.1 (2013): 125-163.
- Baghalian, Atefeh, Shabnam Rezapour, and Reza Zanjirani Farahani. "Robust supply chain network design with service level against disruptions and demand uncertainties: A real-life case." *European Journal of Operational Research* 227.1 (2013): 199-215.
- Berman, Oded, Zvi Drezner, and Dmitry Krass. "Generalized coverage: New developments in covering location models." *Computers & Operations Research* 37.10 (2010): 1675-1687.
- Bernstein, Fernando, and Awi Federgruen. "Coordination mechanisms for supply chains under price and service competition." *Manufacturing & Service Operations Management* 9.3 (2007): 242-262.
- Boyacı, Tamer, and Guillermo Gallego. "Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers." *International Journal of Production Economics* 77.2 (2002): 95-111.
- Bramel, Julien, and David Simchi-Levi. "The logic of logistic." *Theory, Algorithms, and Applications for Logistics* (2000).
- Canis, Bill. *Motor Vehicle Supply Chain: Effects of the Japanese Earthquake and Tsunami*. Diane Publishing, 2011.
- Chen, Jen-Ming, and C. I. Chang. "The economics of a closed-loop supply chain with remanufacturing." *Journal of the Operational Research Society* 63.10 (2012): 1323-1335.
- Chen, Xi, et al. "Optimal decisions in a retailer Stackelberg supply chain." *International Journal of Production Economics* 187 (2017): 260-270.
- Chiang, Wei-yu Kevin, Dilip Chhajed, and James D. Hess. "Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design." *Management science* 49.1 (2003): 1-20.
- Daskin, Mark S., Collette R. Coullard, and Zuo-Jun Max Shen. "An inventory-location model: Formulation, solution algorithm and computational results." *Annals of operations research* 110.1 (2002): 83-106.
- Daskin, Mark S., Lawrence V. Snyder, and Rosemary T. Berger. "Facility location in supply chain design." *Logistics systems: Design and optimization* (2005): 39-65.
- Diabat, Ali, Jean-Philippe Richard, and Craig W. Codrington. "A Lagrangian relaxation approach to simultaneous strategic and tactical planning in supply chain design." *Annals of Operations Research* (2013): 1-26.

Dong, Yan, and Kefeng Xu. "A supply chain model of vendor managed inventory." *Transportation research part E: logistics and transportation review* 38.2 (2002): 75-95.

Drezner, Zvi. "Heuristic solution methods for two location problems with unreliable facilities." *Journal of the Operational Research Society* (1987): 509-514.

Drezner, Zvi, George O. Wesolowsky, and Tammy Drezner. "The gradual covering problem." *Naval Research Logistics (NRL)* 51.6 (2004): 841-855.

Escalona, P., F. Ordóñez, and V. Marianov. "Joint location-inventory problem with differentiated service levels using critical level policy." *Transportation Research Part E: Logistics and Transportation Review* 83 (2015): 141-157.

Farahani, Reza Zanjirani, et al. "Covering problems in facility location: A review." *Computers & Industrial Engineering* 62.1 (2012): 368-407.

Fallah, Hamed, Ali Naimi Sadigh, and Marjan Aslanzadeh. "Covering problem." *Facility Location*. Physica-Verlag HD, 2009. 145-176.

Fernández, José, et al. "Solving a Huff-like competitive location and design model for profit maximization in the plane." *European Journal of Operational Research* 179.3 (2007): 1274-1287.

Fleischmann, Bernhard, Sonja Ferber, and Peter Henrich. "Strategic planning of BMW's global production network." *Interfaces* 36.3 (2006): 194-208.

Graves, Stephen C., AHG Rinnooy Kan, and Paul Herbert Zipkin, eds. *Logistics of production and inventory*. Vol. 4. Elsevier, 1993.

Giri, B. C., and Bhaba R. Sarker. "Coordinating a two-echelon supply chain under production disruption when retailers compete with price and service level." *Operational Research* 16.1 (2016): 71-88.

Herod, Andrew. "Implications of just-in-time production for union strategy: Lessons from the 1998 General Motors-United Auto Workers dispute." *Annals of the Association of American Geographers* 90.3 (2000): 521-547.

HUANG, Ying-ying, Gen-gui ZHOU, and Jian CAO. "Study on Price and Incentive Mechanism of Three-level Reverse Supply Chain for Electronic Products [J]." *Industrial Engineering and Management* 3 (2009): 88-97.

Jiang, Li, and Yunzeng Wang. "Supplier competition in decentralized assembly systems with price-sensitive and uncertain demand." *Manufacturing & Service Operations Management* 12.1 (2010): 93-101.

Kaya, Onur, and Busra Urek. "A mixed integer nonlinear programming model and heuristic solutions for location, inventory and pricing decisions in a closed loop supply chain." *Computers & Operations Research* 65 (2016): 93-103.

Li, Wei, and Jing Chen. "Backward integration strategy in a retailer Stackelberg supply chain." *Omega* (2017).

- Liao, Shu-Hsien, and Chia-Lin Hsieh. "A capacitated inventory-location model: Formulation, solution approach and preliminary computational results." *Next-Generation Applied Intelligence* (2009): 323-332.
- Lim, Michael K., et al. "Facility location decisions with random disruptions and imperfect estimation." *Manufacturing & Service Operations Management* 15.2 (2013): 239-249.
- Lim, Michael, et al. "A facility reliability problem: Formulation, properties, and algorithm." *Naval Research Logistics (NRL)* 57.1 (2010): 58-70.
- Liberatore, Federico, and Maria Paola Scaparra. "Optimizing protection strategies for supply chains: comparing classic decision-making criteria in an uncertain environment." *Annals of the Association of American Geographers* 101.6 (2011): 1241-1258.
- Makui, Ahmad, and Ali Ghavamifar. "Benders Decomposition Algorithm for Competitive Supply Chain Network Design under Risk of Disruption and Uncertainty." *Journal of Industrial and Systems Engineering* 9 (2016): 30-50.
- Melo, M. Teresa, Stefan Nickel, and Francisco Saldanha-Da-Gama. "Facility location and supply chain management—A review." *European journal of operational research* 196.2 (2009): 401-412.
- Miranda, Pablo A., and Rodrigo A. Garrido. "A simultaneous inventory control and facility location model with stochastic capacity constraints." *Networks and Spatial Economics* 6.1 (2006): 39-53.
- Miranda, Pablo A., and Rodrigo A. Garrido. "Incorporating inventory control decisions into a strategic distribution network design model with stochastic demand." *Transportation Research Part E: Logistics and Transportation Review* 40.3 (2004): 183-207.
- Miranda, Pablo A., and Rodrigo A. Garrido. "Valid inequalities for Lagrangian relaxation in an inventory location problem with stochastic capacity." *Transportation Research Part E: Logistics and Transportation Review* 44.1 (2008): 47-65.
- Nagurney, Anna. "Optimal supply chain network design and redesign at minimal total cost and with demand satisfaction." *International Journal of Production Economics* 128.1 (2010): 200-208.
- Nagurney, Anna. "Supply chain network design under profit maximization and oligopolistic competition." *Transportation Research Part E: Logistics and Transportation Review* 46.3 (2010): 281-294.
- Namdar, J., et al. "DESIGNING A RELIABLE DISTRIBUTION NETWORK WITH FACILITY FORTIFICATION AND TRANSSHIPMENT UNDER PARTIAL AND COMPLETE DISRUPTIONS." *International Journal of Engineering-Transactions C: Aspects* 29.9 (2016): 1273.
- Ozsen, Leyla, Collette R. Coullard, and Mark S. Daskin. "Capacitated warehouse location model with risk pooling." *Naval Research Logistics (NRL)* 55.4 (2008): 295-312.
- Plastria, Frank, and Lieselot Vanhaverbeke. "Discrete models for competitive location with foresight." *Computers & Operations Research* 35.3 (2008): 683-700.
- Revelle, Charles S., Horst A. Eiselt, and Mark S. Daskin. "A bibliography for some fundamental problem categories in discrete location science." *European Journal of Operational Research* 184.3 (2008): 817-848.
- Rezapour, Shabnam, and Reza Zanjirani Farahani. "Strategic design of competing centralized supply chain networks for markets with deterministic demands." *Advances in Engineering Software* 41.5 (2010): 810-822.

- Rezapour, Shabnam, et al. "Competitive closed-loop supply chain network design with price-dependent demands." *Journal of Cleaner Production* 93 (2015): 251-272.
- Sadjadi, Seyed Jafar, et al. "Applying queuing approach for a stochastic location-inventory problem with two different mean inventory considerations." *Applied Mathematical Modelling* 40.1 (2016): 578-596.
- Sajadieh, Mohsen S., and Mohammad R. Akbari Jokar. "Optimizing shipment, ordering and pricing policies in a two-stage supply chain with price-sensitive demand." *Transportation Research Part E: Logistics and Transportation Review* 45.4 (2009): 564-571.
- Schultmann, Frank, Bernd Engels, and Otto Rentz. "Closed-loop supply chains for spent batteries." *Interfaces* 33.6 (2003): 57-71.
- Schütz, Peter, Asgeir Tomasgard, and Shabbir Ahmed. "Supply chain design under uncertainty using sample average approximation and dual decomposition." *European Journal of Operational Research* 199.2 (2009): 409-419.
- Shen, Zuo-Jun Max, and Lian Qi. "Incorporating inventory and routing costs in strategic location models." *European journal of operational research* 179.2 (2007): 372-389.
- Shen, Zuo-Jun Max, Collette Coullard, and Mark S. Daskin. "A joint location-inventory model." *Transportation science* 37.1 (2003): 40-55.
- Shen, Zuo-Jun Max, Roger Lezhou Zhan, and Jiawei Zhang. "The reliable facility location problem: Formulations, heuristics, and approximation algorithms." *INFORMS Journal on Computing* 23.3 (2011): 470-482.
- Shen, Zuo-Jun Max. "A profit-maximizing supply chain network design model with demand choice flexibility." *Operations Research Letters* 34.6 (2006): 673-682.
- Shishebori, Davood, Lawrence V. Snyder, and Mohammad Saeed Jabalameli. "A reliable budget-constrained fl/nd problem with unreliable facilities." *Networks and Spatial Economics* 14.3-4 (2014): 549-580.
- Sinha, Santanu, and S. P. Sarmah. "Coordination and price competition in a duopoly common retailer supply chain." *Computers & Industrial Engineering* 59.2 (2010): 280-295.
- Snyder, Lawrence V., and Mark S. Daskin. "Reliability models for facility location: the expected failure cost case." *Transportation Science* 39.3 (2005): 400-416.
- Snyder, Lawrence V., and Zuo-Jun Max Shen. "Supply and demand uncertainty in multi-echelon supply chains." *Submitted for publication, Lehigh University* 15 (2006).
- Srivastava, Samir K. "Network design for reverse logistics." *Omega* 36.4 (2008): 535-548.
- Tikani, Hamid, Mahboobeh Honarvar, and Yahia Zare Mehrjerdi. "Joint Optimization of Star P-hub Median Problem and Seat Inventory Control Decisions Considering a Hybrid Routing Transportation System." *International Journal of Supply and Operations Management* 3.3 (2016): 1.
- Troncoso, Juan J., and Rodrigo A. Garrido. "Forestry production and logistics planning: an analysis using mixed-integer programming." *Forest Policy and Economics* 7.4 (2005): 625-633.

Tsiakis, Panagiotis, Nilay Shah, and Constantinos C. Pantelides. "Design of multi-echelon supply chain networks under demand uncertainty." *Industrial & Engineering Chemistry Research* 40.16 (2001): 3585-3604.

Vahdani, Behnam, et al. "Reliable design of a forward/reverse logistics network under uncertainty: a robust-M/M/c queuing model." *Transportation Research Part E: Logistics and Transportation Review* 48.6 (2012): 1152-1168.

Van Ommeren, J. C. W., Adriana Felicia Bumb, and A. V. Sleptchenko. "Locating repair shops in a stochastic environment." *Computers & Operations Research* 33.6 (2006): 1575-1594.

Wagner, J. L., and L. M. Falkson. "The Optimal Nodal Location of Public Facilities With Price-Sensitive Demand." *Geographical Analysis* 7.1 (1975): 69-83.

Zhang, Lixun, and Gerard Rushton. "Optimizing the size and locations of facilities in competitive multi-site service systems." *Computers & Operations Research* 35.2 (2008): 327-338.

Zipkin, Paul Herbert, and Paul H. Zipkin. *Foundations of inventory management*. 2000.

Appendix 1

Two stage stochastic model by assuming finite number of scenarios probabilities $\pi(\omega)$ is provided as follows:

$$\begin{aligned} \max \sum_{\omega \in \Omega} \pi(\omega) \sum_{j \in J} \sum_{r \in R} & \left(p_r(\omega) x_r(\omega) + q_j (\theta s_{jr} d_{jr} Z s_{jr}(\omega)) - (w_j(\omega) x_r(\omega) Z p_{jr}(\omega)) \right. \\ & - x_r(\omega) h_j Z p_{jr}(\omega) - \sum_{m \in M} f_m y_{mj}(\omega) - q_j (\theta b_{jr} d_{jr} Z b_{jr}(\omega)) \\ & \left. - (1 - q_j) (\theta p_{jr} d_{jr} Z p_{jr}(\omega)) \right) - \sum_{j \in J} g u_j O u_j - \sum_{j \in J} g r_j O r_j \end{aligned} \quad (\text{a-1})$$

$$Z p_{jr}(\omega) \leq \sum_{m \in M} y_{mj}(\omega) \quad \forall j, r, \omega \quad (\text{a-2})$$

$$Z b_{jr}(\omega) \leq \sum_{m \in M} y_{mj}(\omega) \quad \forall j, r, \omega \quad (\text{a-3})$$

$$\sum_{m \in M} y_{mj}(\omega) \leq O u_j + O r_j \quad \forall j, \omega \quad (\text{a-4})$$

$$O r_j + O u_j \leq 1 \quad \forall j \quad (\text{a-5})$$

$$\sum_{j \in J} O r_j \geq 1 \quad (\text{a-6})$$

$$\sum_{j \in J} Z b_{jr}(\omega) = 1 \quad \forall r, \omega \quad (\text{a-7})$$

$$\sum_{j \in J} Z p_{jr}(\omega) = 1 \quad \forall r, \omega \quad (\text{a-8})$$

$$Z b_{jr}(\omega) \leq O r_j \quad \forall j, r \quad (\text{a-9})$$

$$Z p_{jr}(\omega) \leq O r_j + O u_j \quad \forall j, r, \omega \quad (\text{a-10})$$

$$Z s_{jr}(\omega) \leq Z p_{jr}(\omega) \quad \forall j, r, \omega \quad (\text{a-11})$$

$$Z s_{jr}(\omega) \leq Z b_{jr}(\omega) \quad \forall j, r, \omega \quad (\text{a-12})$$

$$\sum_{r \in R} D e_r(\omega) Z p_{jr}(\omega) \leq c a p_j \quad \forall j, r, \omega \quad (\text{a-13})$$

$$x_r(\omega) = \frac{D e_r(\omega) - p_r(\omega) b r - b r h_r}{2} \quad \forall r, \omega \quad (\text{a-14})$$

$$O r_j, O u_j, Z b_{jr}(\omega), Z p_{jr}(\omega), Z s_{jr}(\omega), y_{mj}(\omega) \in \{0,1\} \quad (\text{a-15})$$

$$x_r(\omega), p_r(\omega) > 0 \quad (\text{a-16})$$