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A mathematical model for sustainable probabilistic network design problem with construction scheduling considering social and environmental issues

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Abstract

Recent facility location allocation problems are engaged with social, environmental and many other aspects, besides cost objectives. Obtaining a sustainable solution for such problems requires development of new mathematical modeling and optimization algorithms. In this paper, an uncapacitated dynamic facility location-network design problem with random budget constraints is considered. Social issues such as public satisfaction as a function of construction time, number of missed jobs incurred in the regions under study and environmental considerations are incorporated in the model.Sincethe proposed model is expected to be capable of dealing with probabilistic network design, a new chance constraint formulation is proposed and manipulated to increase the applicability of the model in uncertain decisions. Moreover, the proposed method enables decision makers to determine the completion rate of projects through a time horizon while this notion is not achievable by applying other methods in the literature. The optimization of the proposed model is performed using anovel bi-section procedure in which a heuristic and a Simulated Annealing (SA) method are applied interactively. The efficiency of the proposed method is verified through a real world application of establishing a set of health care centers and the connecting links in Meshgin Shahr, Iran. The results of case study showed that all considered sustainable objectives come to a steady status in the fifth year. Also according to geographical data, the results about creating links are regional and the health centers have dispersed geographically in order to serve the demands of the whole under study region.

Keywords: Facility location-allocation, network design, sustainability, healthcare, chance constraint, bi-section optimization.

1- Introduction

Facility location problem in its initial function tries to locate the facilities likemanufacturing plants, service centers like hospitals (Vahidnia et al., 2009) and distribution units (Weber & Friedrich, 1929), in a way that not only the demands of customers are satisfied but also the cost, travel time, customer satisfaction and other similar objectives are optimized (Klose & Drexl, 2005).

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The question of whether a facility should be located in a point or not, and also that which customers are served by each facility, requires the investigation about how the network of facilities and customers should be designed. Hence, the ever growing body of literature in facility location network design problem has been increasingly considered in a wide range of applications (Shen ,2007).

The facility network location problem was initially studied by Daskin et al. (1993), in which they considered their problem under the assumption of uncapacitated facilities and networks. In another research, Melkote (1996)extended the Uncapacitated Facility Location Network Design Problem (UFLNDP) to capacitated facility location network design problem (CFLNDP) as well as maximum covering location network design problem (MCLNDP). de Assis Corrêa et al. (2007) proposed a new framework for probabilistic maximal covering problem. Although the mentioned researches were so applicable in real world, as the parameters of the model such as demands and costs change through time, a designed network at a certain timewould not work well in a future time with different parameter values. Hence, considering dynamism in the model turned in to a priority which attracted the attention of many researchers. Several categories of dynamic facility location problems were studied which are thoroughly reviewed by Torres Soto (2010)andFarahani and Hekmatfar (2009). Some other instances are Jena et al. (2015), Jena et al. (2016) and Marufuzzaman and Ekşioğlu (2016) and Rabbani et al. (2017). In similar studies, Makui and Ghavamifar (2016)and Hamta et al. (2017)investigated stochastic supply chain network problems. Moreover, Jabbarzadeh et al. (2016)developed a supply chain network problem in a multi-period space.

In the recent years, despite the prosperity of the conventional mathematical modeling in supply chain research area, it is no longer sufficient and attractive to only optimize a facility location problem solely based on cost based objectives. Apparently, decision makers incline to find out about a solution that can simultaneously consider global environmental challenges, social issues such as unemployment and public satisfaction as well as cost in the modeling. In other words, the facility location problems are expected to meet sustainable development targets. In such new approaches, decision makers are to set sustainable targets to lead the whole industry level to a more sustained position. Reducing time to reach sustainability (TTS) is one of the more challenging and interested problemin a sustainable facility location network design problem. Eskandarpour et al. (2015), presented a comprehensive discussion on facility location in supply chain networks considering different types of criteria affecting sustainable development.Corominas et al. (2015) used TTS approach for sustainable development of the industry supply chain mostly by considering several economic factors such as after tax profits, investment scenarios and other similar indices.

Considering to multiple objective decision making methods, generally two approaches are common in this field including trade off optimization (TO) (see Eskelinen and Miettinen (2012)) and goal programming (GP) methods. In TO, the multiple objectives are aggregated considering their weights in a single objective and consequently this objective is considered as the overall index of the problem to be optimized. Sheu et al. (2005), Neto et al. (2008) used this method. The main drawback to this approach is that it is not capable of considering time periods in its optimization procedure, hence, for dynamic cases the TO approach cannot be applied. The goal programming approach initially developed by Charnes and Cooper (1957), tries to minimize the overall distances of objectives from their goal values through optimization of a linear programming. Although this approach has been more effective than the TO method, it fails to demonstrate ability of considering time horizons in its optimization procedure. In a more recent attempt, specifically developed for sustainability studies,Kannegiesser et al. (2014) present a time to sustainability (TTS) approach in which, not only an efficient framework for multi-objective optimization is facilitated but also a comprehensive insight is given to decision makers about the time that a system needs to reach stability among several objectives. This has been tested in several problems in the area of facility location problems proven to be effective in sustainable development planning.

Although Ghaderi and Jabalameli (2013) and Ghaderi (2015) worked on uncapacitated facility location–network design problem, a sustainability analysis that considers a probabilistic parameter space through chance constraints, has not been studied so far. In this paper, a problem is discussedin which locating facilities and allocating customers to them through establishing the links between servers and clients are needed. Since, such development projects are tightly engaged with social issues such as impact on unemployment and the dissatisfaction of citizens during project construction time and environmental concerns as well as cost objectives, we investigate the time to sustainability in which all the objectives reach a stable state. Moreover, since the budget assignment to such problems does not necessarily occur as the plans, the proposed method presents a novel chance constraint method that enables decision makers to tackle the probabilistic impreciseness of the budget realizations.In the proposed method, the construction of facilities and links complete through a time horizon and the model is capable to determine what percentage of any project should be completed in each time units exactly as it happens in real world applications too. Consequently, the proposed method provides a general framework for sustainable facility location network problem.

The problem under study is a prototype of many planning applications. For instance, health care development projects, different civil and urban projects and many others. In this paper, we illustrate its application in a real world case of health care in a city in Iran. Since, the conventional UFNLD problems are strongly NP hard problems, considering the sustainable version is clearly an NP hard problem for which we develop a hybrid heuristic and simulated annealing optimization procedure.

More details about the contributions of this article are given as follow:

1. The mathematical modeling of sustainable DUFLNDP is presented in this article. This approach enables modelers to not only consider a dynamic optimization of the problem, but also it can be investigated that how long it takes for all objectives to reach a stable state. This provides an insight of how different objectives are realized in long term periods.
2. Through the sustainable framework of the paper, the dissatisfaction of the public is modeled as a function of project construction time. The longer the project takes, the more dissatisfaction of public should incur in the model. Moreover, that a facility is opened or not in a region can influent the number of created or missed jobs in the region which is a significant example of social issues affecting a facility location allocation network design.
3. Due to uncertainties of budget preparation for many real world applications, random variables are considered for budget constraints, for which a mathematical method is proposed for the procedure. The space of randomness provides a flexible area for decision makers to satisfy probabilistic budget constraints with a defined confidence level investigated through chance constraint models.
4. Within the framework of sustainable optimization procedure, some basic limitations of DUFLNDP are tackled. For instance, in previous studies, it was assumed that starting and finishing of building a facility or a link is completed exactly at a point of time. In other similar researches, the establishment of projects is treated using a binary approach while this attitude significantly differs from real world problem as the project’s completion rates range a percentage value in each time period. In the proposed method, due to the constraints of budget, the projects progresses are determined in the model and the project completion occurs in a time horizon as it does in real world applications. Hence, the model is capable of handling project management considerations.
5. The presented method includes a general framework in which sustainability, probabilistic uncertainties and dynamism are considered and more elaborative modelling and optimizations are introduced such as project planning forconstructing the links and service centres. Hence, this comprehensive approach proposes a more applicable strategy for a wide variety of real world problems.

This paper is organized as follows: In the next section, the problem definition and mathematical modeling is discussed. In the third step, the solution procedure and the corresponding algorithm is explained through a real case study. Finally, the conclusions are drawn in the fourth section.

2- Problem definition and mathematical modeling

In this section, the problem under study is introduced in details and the corresponding mathematical modeling is presented. The sustainable UFLNDP includes a set of customers with defined demands for which a set of potential locations exist for establishing a number of facilities to serve the clients. In this problem, new facilities are locatedand the clients are assigned to them to be served. In the problem under study, the development of new service centers, roads, the concentration of customers in the area and other parameters of the model occur in different time periods. Hence, the modeling should consider a dynamic approach. While, the establishment of service centers and the linking roads may long for a few years, the budget assignment, customer demands and etc., also need to be revised through time. Hence the dynamism is a crucial property of the problem under study. For this purpose, it is also of the interest to determine whether there would be a need to create a link between the established facilities.

The following are the assumptions folded in our model. First, we consider a time varying framework for the parameters of the model for which at different time horizons different values of parameters are considered. Secondly, the links and facilities are assumed to be uncapacitated. Next, the opened facilities remain open until the end of planning horizon. Moreover, the opening of a link or a facility happens in the beginning of a period. It should be noted that the DUFLNDP is considered as a mixed integer non-linear programming (MINLP).

2-1- Notations

The corresponding notations are given as follows:

Symbols

Sets

N	The set of nodes of the network $i,j \in \{1,2,\dots,\lvert N\rvert\}$, $k \in \{1,2,\dots,\lvert N\rvert\}$
N^0	The set of opened facilities at the available network , $N^0 \in \{1,2,\dots,\lvert N\rvert\}$
LE^t	Set of available network links at the t_{th} period $(i,j) \in LE^t$
LP^t	Set of potential links in the t_{th} period $(i,j) \in LP^t$
L^T	Set of network links in the t_{th} period, $(i,j) \in L^t, L^t = L_E^t \cup L_P^t$
L^0	Set of links available in the network
T	Set of time periods, $t \in \{1,2,\dots,\lvert T\rvert\}$
M	A large number

Parameters

d_k^t	Demand of k_{th} customer in the t_{th} period
m_{ij}	Length of link i to j
c_{ij}^t	Cost of establishing a link between node i and j in the t_{th} period
ρ_{ij}^t	Cost of travelling per unit flow on between node i and j in the t_{th} period
r_{ij}^{kt}	Cost of travelling between node i and j if all of the demand of the k_{th} customer passes through this link in the t_{th} period, $r_{ij}^{kt} = \rho_{ij}^t * d_k^t$
f_i^t	Operational cost of the opened facility in the i_{th} period
h_{ij}^t	The operational cost of the opened link between node i and j in the t_{th} period
$\bar{B}^t$	The available budget for investment of facilities in the t_{th} period which follows normal distribution by the mean $\bar{\bar{\beta}}^t$ and variance σ_H^t
$\hat{B}^t$	The available budget for investment of network links in the t_{th} period which follows normal distribution by the mean of $\bar{\hat{\beta}}^t$ and the variance of σ_R^t
ψ_t^i	Maintenance cost for the i_{th} facility in the t_{th} period
α_{qt}	Penalty factor due to the overflow of KPI_q in $period_t$
KPI_q	The key performance index relating to the q_{th} objective
$Base_q$	Base value for KPI_q
$Target_q$	Target value for KPI_q shown as a percentage of $Base_q$
UB_q	Maximum valid increase for KPI_q shown as a percentage of $Base_q$
E_{it}^1	Pollution resulted by opening a facility in the t_{th} period
E_{ij}^{2t}	Pollution resulted by creating a link between node i and j in the t_{th} period
E_{it}^3	Pollution resulted from waste disposal of facility i in the t_{th} period

N_{it}	Number of missed job opportunities in case of not opening the i_{th} facility in the t_{th} period
λ_i^t	The dissatisfaction of citizens living in the vicinity of the construction plant for i_{th} facility in the in the t_{th} period
τ_H^t	The least probability of budget provision for establishing the hospital from the beginning of the construction to t_{th} period
τ_R^t	The least probability of budget provision for establishing the network links from the beginning of the construction to t_{th} period

Variables

z_t^i	Binary variable, equals to 1 if facility i is opened in the t_{th} period, otherwise 0, ($z_i^t \in \{0,1\}$)
x_{ij}^t	Binary variable, equals to 1 if the link between node i and j is opened in the t_{th} period, otherwise 0, ($x_{ij}^t \in \{0,1\}$)
y_{ij}^{kt}	Proportion of demands of k_{th} customer that passes through link (i,j) in the t_{th} period, ($0 \le y_{ij}^{kt} \le 1$)
w_i^{kt}	Proportion of demands of k_{th} customer that is served by the i_{th} facility in the t$_{th}$ period, ($0 \le w_i^{kt} \le 1$)
$\tilde{\pi}_{qt} \ge 0$	The normalized total overflow due to KPI_q in the t_{th} period
η_i^t	The completion percentage of constructing facility i that is constructed in the t_{th} period, ($0 \le \eta_i^t \le 1$)

Note that N represents the set of nodes which denote cities in the problem under study. Since we attempt to locate facilities in some of these cities and relatively these facilities will be chosen to serve the customers of the cities, hence the set of all possible facilities would be interchangeably the same as set of customers.

Moreover, the notations given in the table above from α_{qt} to ψ_t^i are concerned with the sustainable optimization procedure. While α_{qt}, $Base_q$, $Target_q$, UB_q, τ_H^t, τ_R^t and determined based on the experts' choice, the E_{it}^1 , E_{ij}^{2t} , E_{it}^3, N_{it} , λ_i^t are mostly driven due to statistical data collection efforts.

2-2- Problem modeling

Considering the above notations, the mathematical modeling of the problem is given in equations (1) to (33) as follows:

$$\min\ \tilde{Z} = \sum_{q\in Q}\sum_{t\in T} \tilde{\pi}_{qt}.\alpha_{qt} \qquad q\in Q=\{1,2,3,4\},\ t\in T \tag{1}$$

$s.t$

$$\tilde{\pi}_{1t} \ge \left[\frac{\sum_{(i,j)\in L^t}\sum_{k\in N} r_{ij}^{kt} Y_{ij}^{kt} + \sum_{i\in N} f_i^t \eta_i^t + \sum_{(i,j)\in L^t : i<j} h_{ij}^t x_{ij}^t + \sum_{i\in N} \psi_t^i z_i^t - Target_C.Base_C}{(UB_C - Target_C).Base_C} \right], \qquad \forall t \in T, \tag{2}$$

$$\tilde{\pi}_{2t} \ge \left[\frac{E_t^{make} + E_R + E_t^{WD} - Target_E.Base_E}{(UB_E - Target_E).Base_E} \right], \qquad \forall t \in T, \tag{3}$$

$$\tilde{\pi}_{3t} \ge \left[\frac{\sum_i (1 - z_i^t) N_{it} - Target_N.Base_N}{(UB_N - Target_N).Base_N} \right], \qquad \forall t \in T, \tag{4}$$

$$\tilde{\pi}_{4t} \geq \left[\frac{\sum_i UN_{it} - Target_{UN}.Base_{UN}}{(UB_{UN} - Target_{UN}).Base_{UN}} \right], \qquad \forall t \in T, \qquad (5)$$

$$z_i^t + \sum_{j \in N} Y_{ij}^{it} = 1, \qquad \forall i \in N, \forall t \in T, \qquad (6)$$

$$\sum_{j \in N} Y_{ji}^{kt} = \sum_{j \in N} Y_{ij}^{kt} + w_i^{kt}, \qquad \forall i,k \in N : i \neq k, \forall t \in T, \qquad (7)$$

$$z_k^t + \sum_{i \in N : i \neq k} w_i^{kt} = 1, \qquad \forall k \in N, \forall t \in T, \qquad (8)$$

$$Y_{ij}^{kt} + Y_{ji}^{kt} \leq x_{ij}^t, \qquad \forall (i,j) \in L^t : i < j, \forall k \in N, \forall t \in T, \qquad (9)$$

$$w_i^{kt} \leq z_i^t, \qquad \forall i,k \in N : i \neq k, \forall t \in T, \qquad (10)$$

$$p\left(\sum_{t'=1}^{t} \sum_{i \in N} f_i^{t'} \eta_i^{t'} \leq \sum_{t'=1}^{t} \bar{\beta}^{t'}\right) \geq \tau_H^t, \qquad \forall t \in T, \qquad (11)$$

$$p\left(\sum_{t'=1}^{t} \sum_{(i,j) \in L^t} c_{ij}^{t'} x_{ij}^{t'} (1 - x_{ij}^{t'-1}) \leq \sum_{t'=1}^{t} \hat{\beta}^{t'}\right) \geq \tau_R^t, \qquad \forall t \in T, \qquad (12)$$

$$z_i^t \geq z_i^{t-1}, \forall i \in N, \qquad \forall t \in T, \qquad (13)$$

$$x_{ij}^t \geq x_{ij}^{t-1}, \qquad \forall (i,j) \in L^t : i < j, t \in T, \qquad (14)$$

$$z_i^{t-1} = 1, \qquad \forall i \in N^0, \forall t = 1, \qquad (15)$$

$$z_i^{t-1} = 0, \qquad \forall i \notin N^0, \forall t = 1, \qquad (16)$$

$$X_{ij}^{t-1} = 1, \qquad \forall (i,j) \in L^0 : i < j, \forall t = 1, \qquad (17)$$

$$X_{ij}^{t-1} = 0, \qquad \forall (i,j) \notin L^0 : i < j, \forall t = 1, \qquad (18)$$

$$\sum_{t'=1}^{t} \eta_i^{t'} \leq 1 \qquad \forall i \in N, \forall t \in T, \qquad (19)$$

$$z_i^t \leq \sum_{t'=1}^{t} \eta_i^{t'} \qquad \forall i \in N, \forall t \in T, \qquad (20)$$

$$z_i^t \geq \left(\sum_{t'=1}^{t} \eta_i^{t'} - 1\right) M + 1 \qquad \forall i \in N, \forall t \in T, \qquad (21)$$

$$\eta_i^t \geq 0 \qquad \forall i \in N, \forall t \in T, \qquad (22)$$

$$\eta_i^t \leq 1 \qquad \forall i \in N, \forall t \in T, \qquad (23)$$

$$Y_{ij}^{kt} \geq 0, \qquad \forall (i,j) \in L^t, \forall k \in N, \forall t \in T, \qquad (24)$$

$$W_i^{kt} \geq 0, \qquad \forall i,k \in N, : k \neq i, \forall t \in T, \qquad (25)$$

$$z_i^t \in \{0,1\}, \qquad \forall i \in N, \forall t \in T, \qquad (26)$$

$$x_{ij}^t \in \{0,1\}, \qquad \forall i \in N, \forall t \in T. \qquad (27)$$

where

$$E_{it}^{make} = \sum_i E_{it}^1 \eta_i^t, \qquad (28)$$

$$E_R = \sum_i \sum_j E_{ij}^{2t} x_{ij}^t, \qquad (29)$$

$$E_t^{WD} = \sum_i E_{it}^3 z_i^t, \tag{30}$$

$$UN_{it} = (1 - z_i^t)\lambda_i^t \Delta t_i^t, \tag{31}$$

$$\Delta t_i^t = t - t_i^F \tag{32}$$

$$t_i^F = \min\left\{ t \middle| \eta_i^t > 0 \right\} \tag{33}$$

In equation (1), the total overflows of the multiple objectives of cost, environmental issues, number of missed jobs and unemployment are considered. The corresponding equations of $\tilde{\pi}_{qt}$ for q=1, 2, 3, 4 are presented in equations (2) to (5) whereas the normalized value of each objective is calculated considering the most ideal and the least persuasive values of each objective. Equation (6) guarantees that in each time period *t*, the overall demand of the clients should be served either by an opened facility z_i^t or by the fractions of services of other facilities, $\sum_{j \in N} Y_{ij}^{it}$.

The equation (7) deals with the balance between the input flows to the facility *i*that should be equal to the flow out of this facility for the k_{th} customer in the t_{th} time period. Equation (8) tries to emphasize that the demand of customer *k* in the time period *t* should be satisfied whether by the z_i^t or by other facilities. Equation (9) shows that as long as the link of x_{ij} is not created, no flow can pass through the link and accordingly as long as a facility is not opened it cannot serve any client. Equations (10), (11) and (12) guarantee that the probability of meeting the budget constraints for establishing the facilities and roads are larger than τ_H^t, τ_R^t in each time period. Note that these equations are necessary due to that in real world applications; the budget allocation is not straight forward. Regarding to the uncertainties that overwhelm facility location problems, rarely are projects assigned budgets as planned.However, based on experts' choices it is possible to draw estimation for the chance of budget assignment. Practically, such an approach has been proved more effective in comparison with the modelling based on deterministic values. In the proposed method, Equations (10), (11) and (12) define a level of satisfaction for provision of budget, hence the obtained solutions will be less affected by unpredicted events.Equations (13) and (14) show that a facility or a road remains open in the time period after it is opened initially.

Nowadays, the financial supply of projects may realize in accordance to many uncertain events. For instance, in many cases even though a project budget is estimated prior to the construction start, due to unpredicted issues other expenditure criteria may be prioritized. In developing countries this matter frequently occurs. Hence, in the proposed model the assumption of probabilistic heightens the model applicability.

Equations (15) to (18) demonstrate that which facilities and roads exist in the available network. While Equation (19) shows that the complete percentage of establishing a facility is equal or less than number "1".

The z_i^t variable can be assigned 1 when the total construction percentages are equal to 1as explained in equations (20) and (21). Equations (22) to (27) show the variable types. Equations (28), (29) and (30) define the environmental issues resulted in construction of the facilities, roads as well as the pollutions incurred to environment in exploitation phase. Equations (31) to (33), define the dissatisfaction of the citizens during the construction period of the project which is proportional to the construction time length.

Since the budget constraints in the proposed method are considered in terms of chance constraint format, the linearization of these constraints is explained as follows.While we assume that $\bar{\beta}^t$ follows normal distribution by the parameters of $(\bar{\bar{\beta}}^t, \sigma_H^{2t})$ which is denoted as $\bar{\beta}^t \sim N(\bar{\bar{\beta}}^t, \sigma_H^{2t})$, the corresponding linearized constraint is given in equation (34).

$$\sum_{t'=1}^{t}\sum_{i\in N} g_i^{t'}\eta_i^{t'} - \sum_{t'=1}^{t}\bar{\bar{\beta}}^{t'} \le -(\phi^{-1}(\tau_H^t)\sqrt{(\sum_{t'=1}^{t}\sigma_H^{2^{t'}})}). \tag{34}$$

The equation (34) is the result of a proof that is given in the appendix A.
Similarly, the equation (12) is linearized as shown in equation (35).

$$\sum_{t'=1}^{t}\sum_{(i,j)\in L^t} c_{ij}^{t'} x_{ij}^{t'}(1-x_{ij}^{t'-1}) - \sum_{t'=1}^{t}\bar{\hat{\beta}}^{t'} \le -(\phi^{-1}(\tau_R^t)\sqrt{(\sum_{t'=1}^{t}\sigma_R^{2^{t'}})}) \tag{35}$$

This equation is a result of the proof as given in the appendix B.

3- Solution procedure

The problem under study is a mixed integer non-linear problem that is an NP-Hard problem. Hence, the available exact methods are not efficient enough to result permissible solutions. The general configuration of the proposed solution procedure includes a relaxation approach applying a metaheuristic method as well as a CPLEX optimization. In the proposed method, the variables of z_i^t are first determined using Simulated Annealing (SA) and then the relaxed problem is optimized in Gams software using CPLEX solver by which $x_{ij}^t, y_{ij}^{kt}, w_i^{kt}, \tilde{\pi}_q^{kt}, \eta_i^t$ are determined.

SA algorithm initially developed by Kirkpatrick et al. (1983), has been extensively applied in different fields of optimization. The algorithm has been inspired from metallurgy in which the optimization procedure simulates the annealing process of a metal. The main idea behind this algorithm is heating and controlled cooling of a metal to let its crystals shape optimally. This in turn, means that in the initial steps of optimization procedure more dispersed solutions are let to be generated and gradually in further steps more concentrated searches of solutions are applied. The steps of the proposed method are illustrated in figure 1.

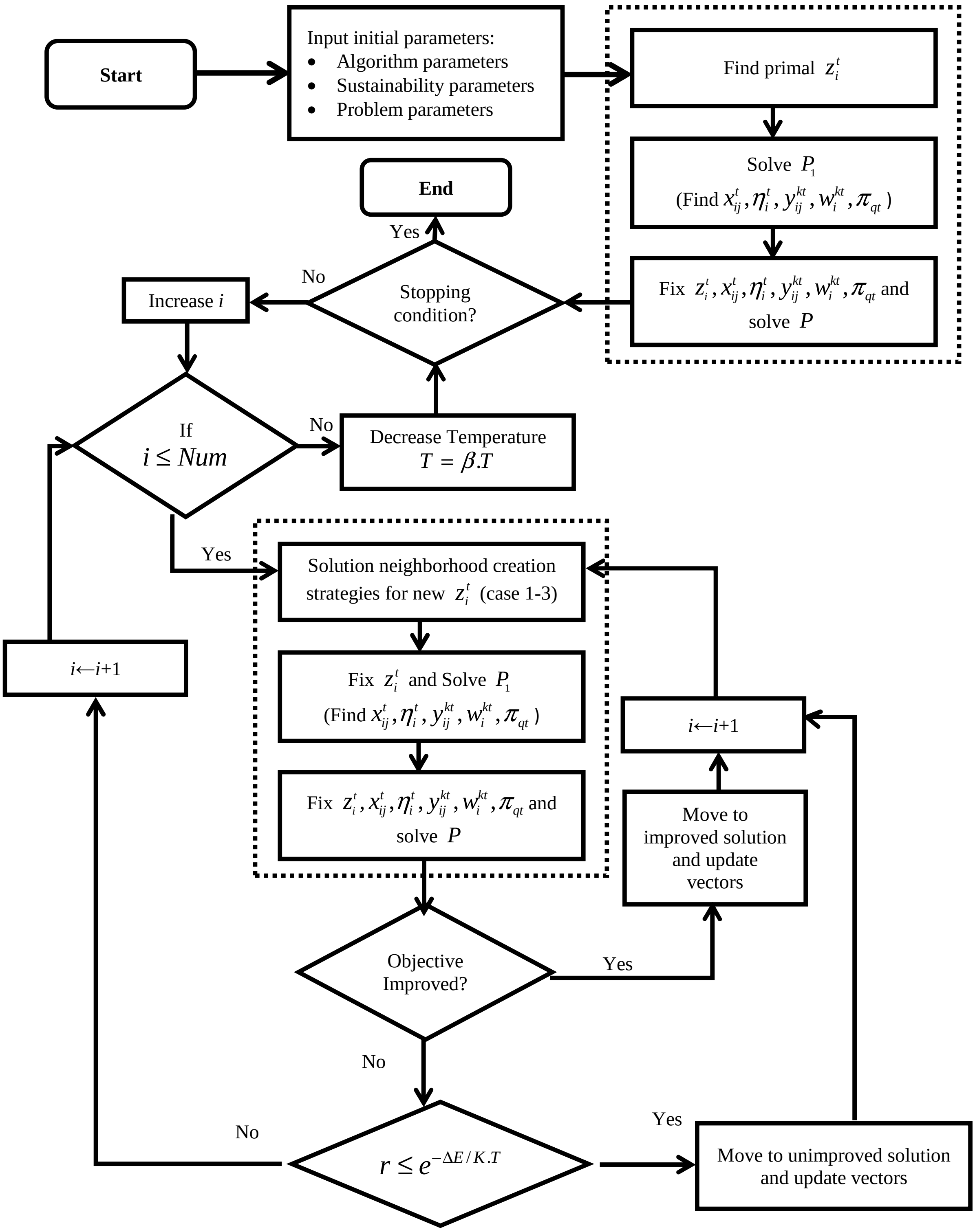

Fig 1. The steps of the proposed method

3-1- Solution representation

In this paper, a matrix based solution representation is introduced and applied within the SA framework randomly. In this solution representation, a matrix form is applied in which the rows of the matrix correspond to the i=1,2,…,m facilities and the columns are in association with t=1,2,3,…,n for the time periods. Each cell of this matrix is assigned a random binary value for which it represents the value of z_i^t in figure 2, this matrix is shown.

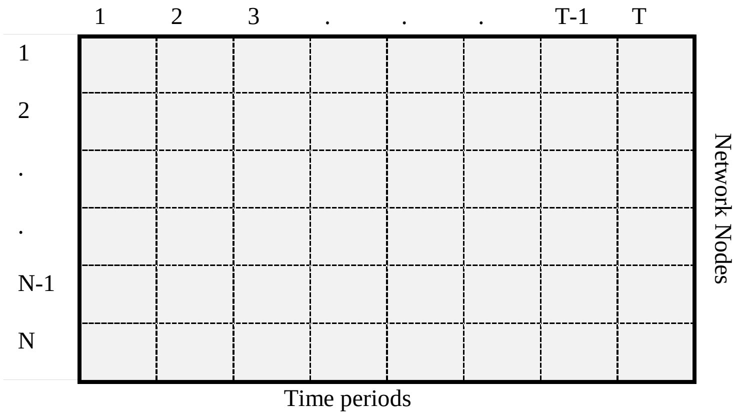

Fig2. A solution representation of z_i^t variables for the proposed algorithm

As mentioned before, when a facility is opened it remains open in the next time periods. Hence, when “1” appears in one cell it remains “1” in the next cells of the corresponding rows as well.
In this paper, three methods are proposed for generating neighborhood solutions. In the first method, a random city (row) is selected and its time of opening is changed randomly. In the second method, two random cities (rows) are considered and their corresponding rows are changed together. Finally, in the third method a city is chosen arbitrarily and the corresponding row is assigned with zeros in all cells. Note that the three neighborhood generation methods are applied together due to a random selection procedure.

3-2- Numerical results

In this section, some problems have been designed and solved to show validity of the proposed algorithm. Considered examples are designed in small and medium dimensions. In each instance, five problems have solved. Values of theparameters have been generated using uniform distribution as table 1.

Table 1. Discrete uniform distribution for parameters.

Parameter	Interval for discrete uniform distribution	Parameter	Interval for discrete uniform distribution
d_k^t	[100, 200]	$Base_1$	[13×10^6, 20×10^6]
m_{ij}	[20, 50]	$Base_2$	[0.7×10^6, 1.5×10^6]
c_{ij}^t	[10^8, 1.3×10^8]	$Base_3$	[700, 900]
ρ_{ij}^t	[1500, 5000]	$Base_4$	[70, 150]
r_{ij}^{kt}	$\rho_{ij}^t \times d_k^t$	$Target_q$	[15, 20]
f_i^t	[10^8, 2×10^8]	UB_q	[90, 110]
h_{ij}^t	[0.2×10^8, 0.4×10^8]	E_{ij}^1	[5000, 10000]
$\bar{\bar{\beta}}^t$	[3×10^8, 5×10^8]	E_{ij}^{2t}	[10000, 20000]
$\bar{\hat{\beta}}^t$	[3×10^8, 5×10^8]	E_{it}^3	[5000, 10000]
σ_H^t	[0.3×10^8, 0.5×10^8]	N_{it}	[10, 40]
σ_R^t	[0.3×10^8, 0.5×10^8]	λ_i^t	[10, 50]
ψ_t^i	[0.1×10^8, 0.4×10^8]	τ_H^t	0.95
α_{qt}	[50000, 400000]	τ_R^t	0.95

The proposed algorithm is coded using GAMS 24.1.2 and MATLAB R2013a software. The CPLEX solver is used for solving the model. The codes of proposed mathematical approach were executed on an ASUS laptop with Core i5 due CPU, 2.4 GHz, and Windows Seven using 4 GB of RAM. Each example solved by proposed metaheuristic algorithm 10 times and mean of results used for validation. Details are summarized in Table 2.

Table 2. Results on numerical examples

Dimension	Results of GAMS	Running time in GAMS (second)	Results of methaheuristic	Running time in methaheuristic (second)	Gap (%)
3	6.1327×10^{9}	0.1	6.1327×10^{10}	1.2	0
4	4.1411×10^{14}	1.3	4.1411×10^{11}	7.6	0
5	6.92483×10^{11}	3.6	6.9249×10^{11}	14.7	0. 1
6	2.3216×10^{12}	12.4	2.3218×10^{12}	53	0.95
7	3.5960×10^{12}	76.1	3.5962×10^{12}	112	0.8
8	1.4426×10^{13}	210	1.4428×10^{13}	145.2	1.6
9	8.0287×10^{13}	741	8.2053×10^{13}	183	2.2
10	1.17104×10^{14}	2892	1.1945×10^{14}	227	2.01
11	2.9801×10^{14}	3600*	2.9571×10^{14}	260.8	-
12	3.1301×10^{14}	3600*	3.1238×10^{14}	297.3	-

Note. * means that running have been terminated after 3600 second and best feasible solution reported

As it is clear from table 2, the proposed metaheuristic algorithm obtains an acceptable solution in a reasonable time with a negligible error. Figure 3 shows running time of exact solution versus metaheuristic solution with the increase in dimensions. It can be seen from figure 3 that with the increase in dimensions, running time will be increase exponentially. Therefore the proposed model is an NP-hard problem.

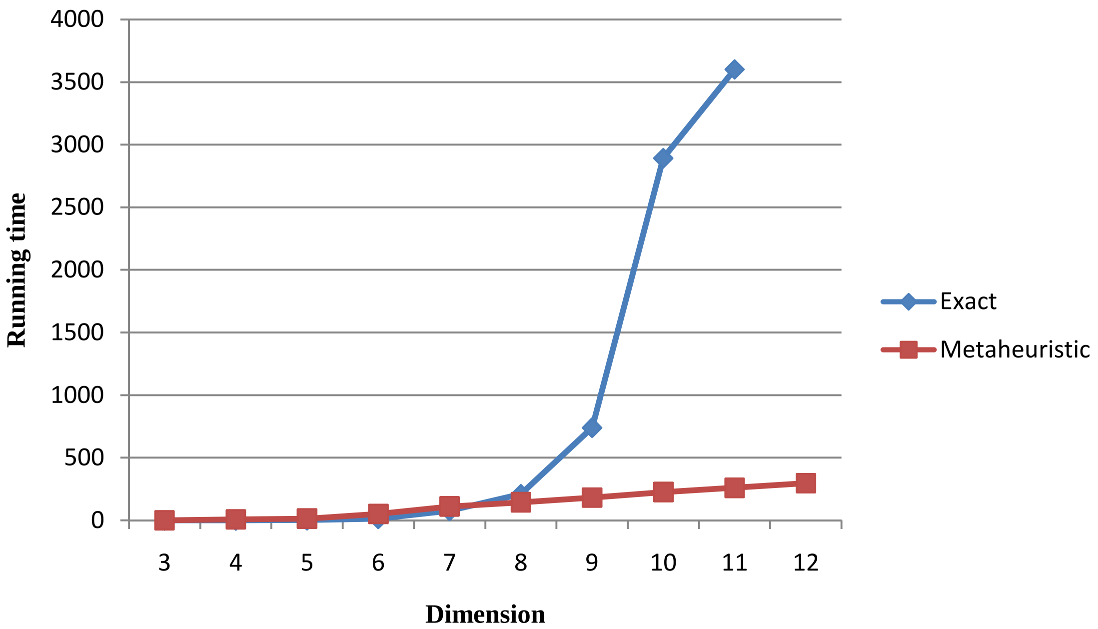

Figure 3. Running time to obtain exact solution.

3-3- Real world application

Facility location is a major field of regional planning. In related studies, different public services such as educational facilities, shopping centers, health centers and etc. are of the interest. In health care development studies, several researches have dealt with locating facilities. Mestre et al. (2015) worked on hospital network planning and similarly Davari et al. (2013) proposed a novel heuristic method for locating preventive health centers. These researches have not presented a comprehensive framework to consider several aspects together.

We consider locating hospitals in Meshgin Shahr in Iran which 12 rural regions are of the interest to be assigned health care centers. In the problem under study, 12 points have been nominated for opening the healthcare facilities. We used the database of local government to our problem.This case only is provided to demonstrate how the proposed method can be applied in practice. It could be very interested and logical to apply the proposed model in large scale healthcare problems. Unfortunately despite our enormous efforts, because of security reasons, we didn't access to more data and large scale data. The general sustainability framework that is applied in this article is illustrated in figure 4 as follows.

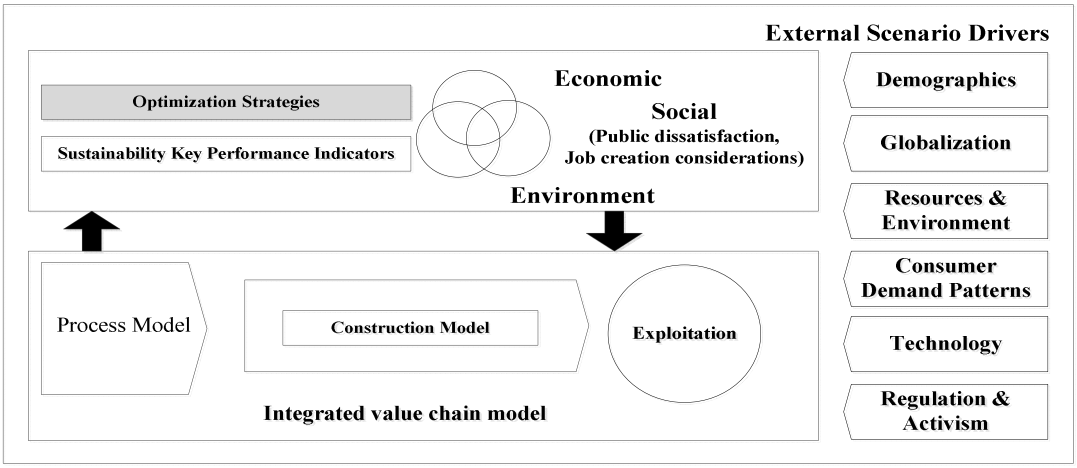

Figure 4. The sustainability framework

As shown in this framework, the process model, construction model and exploitation refer to the conventional procedure of planning a network design. This procedure interacts with sustainability optimization considerations by emphasizing on improvement of economic, social and environmental key performance Indices.

In this paper, the main social issues of the interest are the public dissatisfaction during the project construction time as well as job creation objectives. The external scenario drivers are those factors that can dynamically affect the whole model. For instance, the change of technology through time can violate the presumed parameters of the model. In our model, the main external drivers are those which affect the consumer demand and the cost parameters. Having gained a general insight on the sustainability property of our problem, more details of the real world application are discussed as below. Although in the literature are considerable researches which have dealt with locating health centers such as Skintzi et al. (2003), not much attention is focused on sustainable location allocation problems in this area.

Since Meshgin Shahr is 170 km far from the nearest big city, Tabriz, the sending of urgent patients to Tabriz is very threatening and the in-road mortality of critical patients is very high. Moreover, the problem even is more serious when patients have to travel from rural areas first to Meshgin Shahr and then they might be sent to Tabriz afterwards. Hence, establishing an efficient health center is critical for the area.

The selected points represent a comprehensive coverage of population living in the whole rural area. Note that, in this problem we assume that demand of each node is proportional to its' population size which changes over time by the determined growth rate. Moreover, all the costs of the model change according to the inflation rate. In table 3, the nodes and their population size is given and the corresponding map of the region is illustrated in figure 5.

Table 3. Nodes and populations.

City(Code)	Population	City(Code)	Population	City(Code)	Population
Lahroud (1)	2961	Ghareh Ghaya(5)	1713	Anar (9)	1786
Razi (2)	1749	Moradloo (6)	645	PariKhan (10)	3527
Ghasabe (3)	2110	Fakhr Abad (7)	1282	Alni (11)	3500
Jabdaraq (4)	2460	ArbabKandi (8)	903	MazraeKhalaf (12)	1144

Figure 5. The map of the region under study

Due to a new policy for keeping intercity roads safer, decision makers have decided to establish rural roads separated from roads that link larger cities. Hence, the problem includes locating health care centers as well as establishing roads.Considering the eco-tourism characteristics of the region and also green management issues of Meshgin Shahr natural resources, we aim to solve the facility network design by minimizing the damages incurred to environment as well as meeting the environmental requirements during the exploitations.

Moreover, since the state governor tends to incorporate job creation considerations within the project, another objective of the model is to understand how the opening of facilities can create more jobs in the region. Finally, the achievement of all above objectives is permissible while the project construction time is minimized and considerably the dissatisfaction of public in the region during the construction time is minimized.

In the region under study, ten important rural points are mapped and the provision of health services for these points is analyzed during five year plans. In the example under study, we assume that all cost parameters change with 10% inflation rate. The operational costs of facilities are defined as the corresponding estimated tables. Moreover, the matrix of the available links and potential links is considered as an input in the model and the travelling cost per each kilometer trip is 5000 Iranian Rials.

3-4- Solution discussion

The solution of the problem is obtained through a heuristic algorithm by applying SA and CPLEX for which the corresponding convergence graph is given in figure 6.

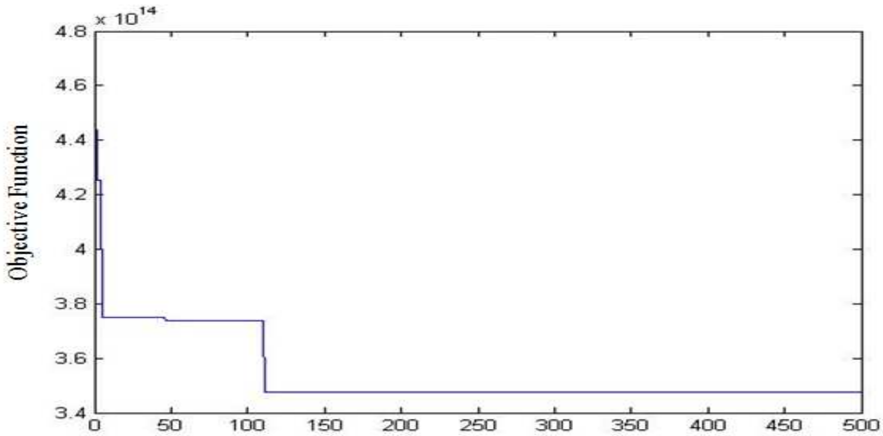

Fig6. The solution convergence of the model

According to the obtained solution, the variables of $z_2^1, z_3^2, z_7^1, z_{10}^1$ and z_{11}^3 are equal to 1. By having a general view on the obtained solution, it can be observed that the algorithm has located the health centers geographically dispersed enough so to serve the maximum demands of the whole region under study.

Then, the model has tried to improve the solutions in favor of other objectives for which locating a health center in Ghasabe in the second period and Alni in the third period are conducted. It should be noted that the regions in which the health centers are located, are the points which represent relatively congested areas. For instance, FakhrAbad town though being a populated area itself can serve other important towns such as Lahroud, Anar and ArbabKandi.

A similar strategy has taken place in the model for creating links as well. Considering the available links, having created the links of Ghasabeh to Parikhan, Moradloo to Parikhan and Razi to ArbabKandi in the first period, the generated links provide sufficient accessibility in whole network. While Parikhan is selected and its' link to Ghasabeh is created in the first period, similarly the Razey town and its link to ArbabKandi are created and a large proportion of the whole population of the region is served. In the second and the third period, some minor manipulations are made in the optimization model to reduce costs and improve the sustainability aims.

Moreover, about establishing the links, the model has tried to maximize the exploitation of the available roads and the two links of Moradloo to Alni and ArbabKandi to MazraehKhalaf are created. Besides, by creating the link of ArbabKandi to MazreaKhalaf the overall network can provide sufficient accessibility for all customers.Remember that the model was developed in a stochastic space where the budgets were to be supplied by 95% chance.

3-5- A sensitivity analysis

In this section, a single objective optimization of the model is considered to draw some conclusions of how the overall model might differ from a single objective approach. In the first step, while we only consider the cost function, the model locates the health centers in Lahrood, Razi, Ghasabeh, Jabdaraq, Parikhan and Alni. This pattern of assignment shows that the model has tried to consider more populated areas regardless of any consideration to minimize number of established facilities to decrease environmental losses.

A similar analysis performed for the second KPI of environmental issues, leads to opening only one health center in Fakhrabad to minimize damages incurred to environment while this single facility can serve the whole demand of the region. It should be reminded that Fakhrabad is located geographically in a position where it can be linked to all other parts of the network.

For the third objective, the model tries to maximize the number of created jobs. For this regard the budget is completely used to establish facilities corresponding to $z_1^1, z_2^1, z_3^5, z_4^5, z_{10}^3, z_{11}^1, z_{12}^1$ as well as creating the link of 6-11.

In the last analysis, the dissatisfaction of public is focused on. The obtained results show that the model assigns all the regions a health center except for the city of ArbabKandi. This strategy has been applied to finish all construction actions in one period so that the dissatisfaction of public was minimized. In figures7, 8, 9 and10the trend of reaching sustainable state of each objective is illustrated. As shown in figure 6, the sustainable state is achieved from the fourth period and for the pollution target this status has taken place from the fifth period.

A similar pattern happens for the public dissatisfaction as well as number of missed jobs in the fourth and fifth period respectively. Moreover, in figure 11, the comparison of the sustainability behavior of the four objectives is given. This graph illustrates that how the overall model comes to a steady status. The results show that, during 5 year and under obtained values by proposed model, all considered sustainability targets have been achieved.

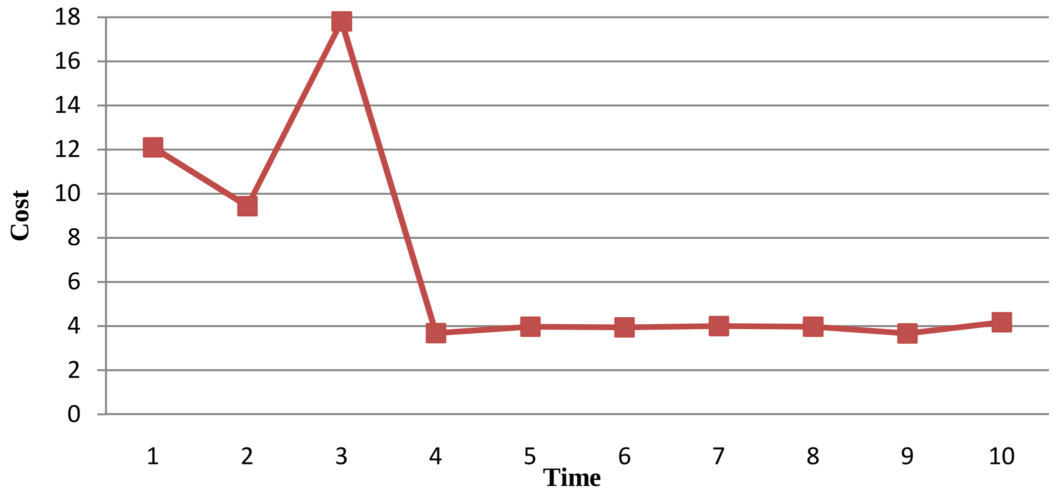

Figure 7. The time to reach sustainability for cost ($\times 10^8$)

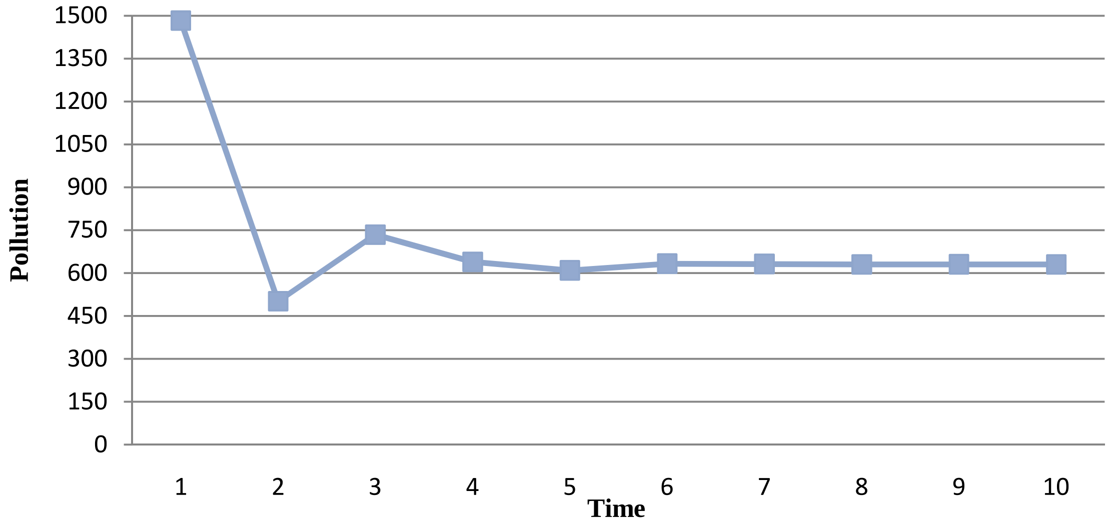

Figure 8. The time to reach sustainability for pollution

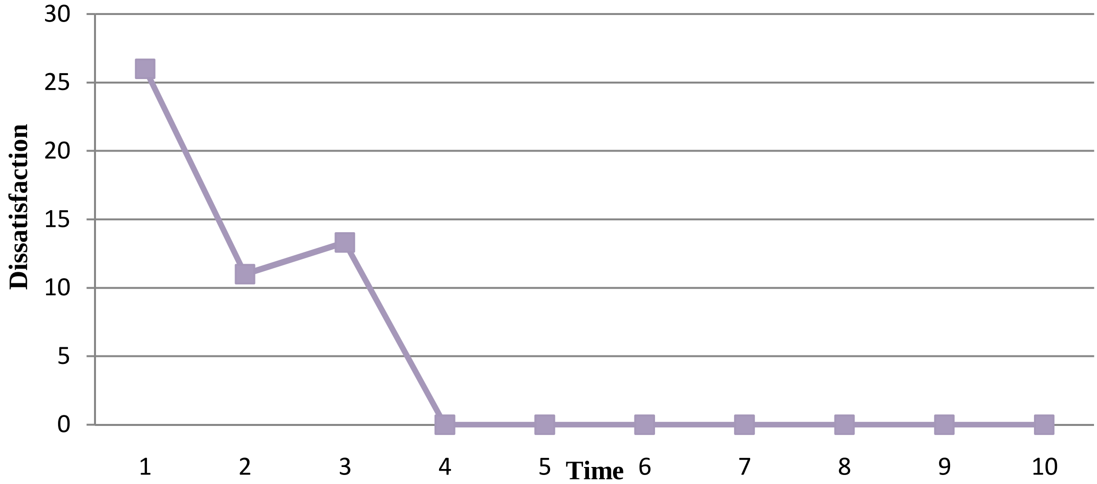

Figure 9. The time to reach sustainability for Dissatisfaction

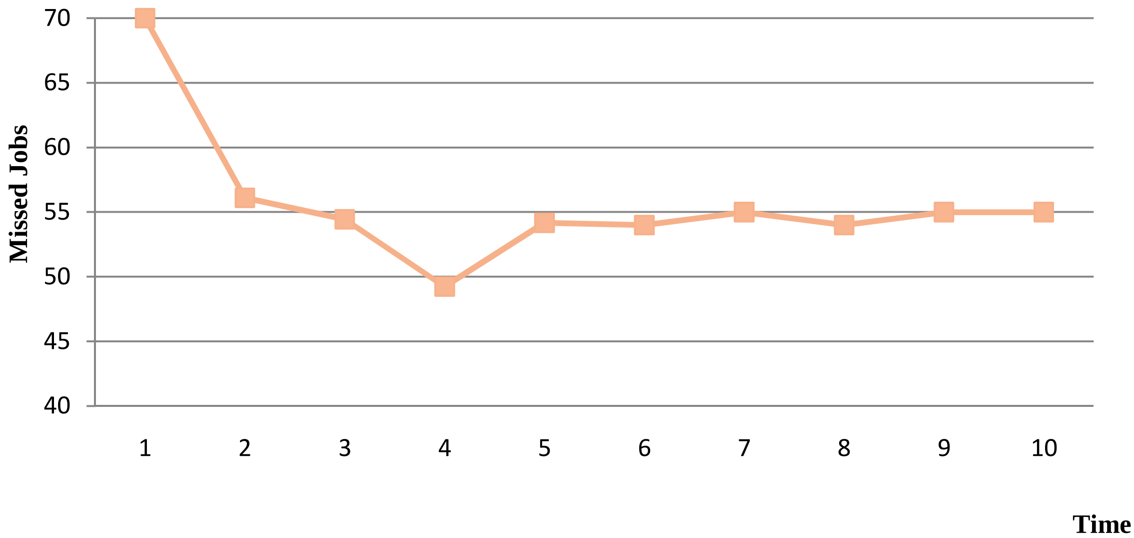

Figure 10. The time to reach sustainability for number of missed jobs

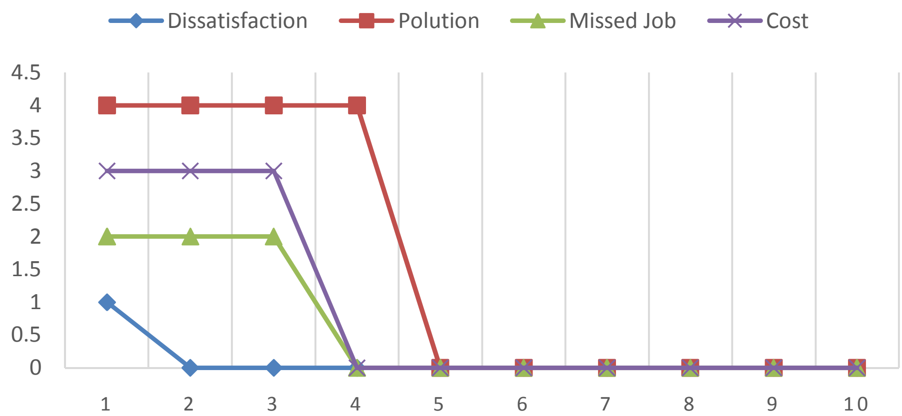

Figure 11. A comparison of the trends of the multiple objectives

4- Summary and conclusion

In this paper, a sustainable optimization framework was used for dynamic facility location-network design problem, considering random budget constraints. In the developed framework, economic, social and environmental issues were considered in dynamic space under randomness of some input variables. The effectiveness of the proposed framework was verified in a real world application of locating healthcare facilities in Meshgin Shahr a city in Ardabil province in Iran, focusing on establishing healthcare centers in the rural regions. The conducted analyses and discussions on the sustainability of the model demonstrated persuasive results proving the model reliability. Moreover, the determination of the time to sustainability of objectives provided reliable insight on the issue for the decision makers. The results showed that all considered sustainable objectives come to a steady status in the fifth year. Also according to geographical data, the results about creating links are regional.It can be observed that the algorithm has located the health centers geographically dispersed enough so to serve the demands of the whole region under study.

There are some suggestions for the future researches. We considered uncapacitated network assumption to build the model. In most of real world problems, this assumption may not realize. Therefore extending the proposed framework for capacitated networks could be interesting. SA algorithm was used for model solving. As another suggestion, using other algorithms to solve the proposed model and comparing their results are challenging. Because of state limitations, it was difficult to access large scale datasets. Researches can apply the proposed method in other case studies or in large scale cases to examine its' performance. The proposed framework is capable to be extended to other types of networks for instance the capacitated ones, or being considered with fuzzy parameters.

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Appendix A

Proof of the equation (34):

$$p\left(\sum_{t'=1}^{t}\sum_{i\in N} g_i^{t'}\eta_i^{t'} \le \sum_{t'=1}^{t}\bar{\beta}^{t'}\right) \ge \tau_H^t, \qquad \forall t\in T,$$

$$where$$

$$\bar{\beta}^t \sim N(\bar{\bar{\beta}}^t, \sigma_H^{2^t})$$

$$\sum_{t'=1}^{t}\bar{\beta}^{t'} \sim N\left(\sum_{t'=1}^{t}\bar{\bar{\beta}}^{t'}, \sum_{t'=1}^{t}\sigma_H^{2^{t'}}\right)$$

$$Y = \sum_{t'=1}^{t}\sum_{i\in N} g_i^{t'}\eta_i^{t'} - \sum_{t'=1}^{t}\bar{\beta}^{t'} \le 0$$

$$E(Y) = E\left(\sum_{t'=1}^{t}\sum_{i\in N} g_i^{t'}\eta_i^{t'} - \sum_{t'=1}^{t}\bar{\beta}^{t'}\right) = \sum_{t'=1}^{t}\sum_{i\in N} g_i^{t'}\eta_i^{t'} - E\left(\sum_{t'=1}^{t}\bar{\beta}^{t'}\right)$$

$$= \sum_{t'=1}^{t}\sum_{i\in N} g_i^{t'}\eta_i^{t'} - \sum_{t'=1}^{t}\bar{\bar{\beta}}^{t'}$$

$$Var(Y) = Var\left(\sum_{t'=1}^{t}\sum_{i\in N} g_i^{t'}\eta_i^{t'} - \sum_{t'=1}^{t}\bar{\beta}^{t'}\right) = \sum_{t'=1}^{t}\sigma_H^{2^{t'}}$$

$$\frac{y-E(y)}{\sqrt{Var(Y)}} \sim N(0,1)$$

$$Y \le 0 \Rightarrow \frac{y-E(y)}{\sqrt{Var(Y)}} \le \frac{-E(y)}{\sqrt{Var(Y)}}$$

$$p\left\{\frac{y-E(y)}{\sqrt{Var(Y)}} \le \frac{-E(y)}{\sqrt{Var(Y)}}\right\} \ge \tau_H^t$$

$$\Rightarrow \phi^{-1}(\tau_H^t) \le \frac{-E(y)}{\sqrt{Var(Y)}}$$

where ϕ denotes the standardized normal distribution. Therefore:

$$\phi^{-1}(\tau_H^t)\sqrt{Var(Y)} \le -E(y)$$

$$\Rightarrow E(y) \le -(\phi^{-1}(\tau_H^t)\sqrt{Var(Y)})$$

$$\Rightarrow \sum_{t'=1}^{t}\sum_{i\in N} g_i^{t'}\eta_i^{t'} - \sum_{t'=1}^{t}\bar{\bar{\beta}}^{t'} \le -\left(\phi^{-1}(\tau_H^t)\sqrt{\left(\sum_{t'=1}^{t}\sigma_H^{2^{t'}}\right)}\right)$$

Appendix B

Proof of the equation (35):

$$p\left(\sum_{t'=1}^{t}\sum_{(i,j)\in L^t} c_{ij}^{t'}x_{ij}^{t'}(1-x_{ij}^{t'-1}) \le \sum_{t'=1}^{t}\hat{\beta}^{t'}\right) \ge \tau_R^t, \qquad \forall t\in T,$$

$$where$$

$$\hat{\beta}^t \sim N(\bar{\hat{\beta}}^t, \sigma_R^{2^t})$$

$$\sum_{t'=1}^{t} \hat{\beta}^{t'} \sim N(\sum_{t'=1}^{t} \bar{\hat{\beta}}^{t'}, \sum_{t'=1}^{t} \sigma_R^{2^{t'}})$$

$$Y = \sum_{t'=1}^{t} \sum_{(i,j)\in L^t} c_{ij}^{t'} x_{ij}^{t'} (1 - x_{ij}^{t'-1}) - \sum_{t'=1}^{t} \hat{\beta}^{t'} \leq 0$$

$$E(Y) = E(\sum_{t'=1}^{t} \sum_{(i,j)\in L^t} c_{ij}^{t'} x_{ij}^{t'} (1 - x_{ij}^{t'-1}) - \sum_{t'=1}^{t} \hat{\beta}^{t'}) = \sum_{t'=1}^{t} \sum_{(i,j)\in L^t} c_{ij}^{t'} x_{ij}^{t'} (1 - x_{ij}^{t'-1}) - E(\sum_{t'=1}^{t} \hat{\beta}^{t'})$$

$$= \sum_{t'=1}^{t} \sum_{(i,j)\in L^t} c_{ij}^{t'} x_{ij}^{t'} (1 - x_{ij}^{t'-1}) - \sum_{t'=1}^{t} \bar{\hat{\beta}}^{t'}$$

$$Var(Y) = Var(\sum_{t'=1}^{t} \sum_{(i,j)\in L^t} c_{ij}^{t'} x_{ij}^{t'} (1 - x_{ij}^{t'-1}) - \sum_{t'=1}^{t} \hat{\beta}^{t'}) = \sum_{t'=1}^{t} \sigma_R^{2^{t'}}$$

$$\frac{y - E(y)}{\sqrt{Var(Y)}} \sim N(0,1)$$

$$Y \leq 0 \Rightarrow \frac{y - E(y)}{\sqrt{Var(Y)}} \leq \frac{-E(y)}{\sqrt{Var(Y)}}$$

$$p\left\{\frac{y - E(y)}{\sqrt{Var(Y)}} \leq \frac{-E(y)}{\sqrt{Var(Y)}}\right\} \geq \tau_R^t$$

$$\Rightarrow \phi^{-1}(\tau_R^t) \leq \frac{-E(y)}{\sqrt{Var(Y)}}$$

where ϕ is the standardized normal distribution. Therefore:

$$\phi^{-1}(\tau_R^t)\sqrt{Var(Y)} \leq -E(y)$$

$$\Rightarrow E(y) \leq -(\phi^{-1}(\tau_R^t)\sqrt{Var(Y)})$$

$$\Rightarrow \sum_{t'=1}^{t} \sum_{(i,j)\in L^t} c_{ij}^{t'} x_{ij}^{t'} (1 - x_{ij}^{t'-1}) - \sum_{t'=1}^{t} \bar{\hat{\beta}}^{t'} \leq -(\phi^{-1}(\tau_R^t)\sqrt{(\sum_{t'=1}^{t} \sigma_R^{2^{t'}})})$$