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Two echelon-two indenture extended warranty distribution network under imperfect preventive maintenance policies

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Abstract

Nowadays, offering extended warranty is considered as a lucrative source of income from the perspective of the after-sale service providers. Meanwhile, the main concern is presence or absence of base warranty and strategies adopted by the manufacturer during this period. Moreover, extended warranty structure must be responsive and customer oriented, which not only control the services cost, but also to handle the customers' requirement in a timely manner. In this paper, an extended warranty distribution network is designed from the perspective of a third party (3P) for supporting multi-indenture products in conjunction with base warranty. The proposed network is two-echelon; in this regard, a depot repair center is considered as the first echelon and a number of operational repair centers are selected at the second echelon. In order to decrease the cost of maintenance and spare parts logistics, a novel imperfect preventive maintenance approach is established based on the concept of virtual age. The third party aims to determine the optimum level of spare parts for each component of products at each repair centers in a way that: (1) total expected backorders is minimized (2) total maintenance and retrieval costs of product components are controlled. For optimizing the proposed model, an exact hybrid solution approach regarding Branch-and-Bound algorithm and Variable Neighborhood Search is presented. The obtained results showed the presence of a base warranty on a product has more advantages for third parties even without preventive maintenance.

Keywords: Extended warranty, warranty distribution network, imperfect preventive maintenance, branch and bound algorithm, variable neighborhood search algorithm, Monte - Carlo simulation

1- Introduction

Today, extended warranties (EWs) are provided for the majority of durable goods. Unlike the base warranty (BW), which is usually offered in a bundle with products, extended warranty is provided as a contract (Heese, 2012). It commits the provider to maintain and repair the damaged products for free or at a given cost during a limited period (2-5 years). Increased demand for extended warranty, along with its high profitability, has led to significant attention of warranty providers toward this topic. For example, while operating margin in the electronic industries is estimated at the range 5-10%, sale of EW has been reported 40-70% of net income (Shahanaghi, 2013).

In general, extended warranty literature is divided into two categories: (1) studies that consider extended warranty independent from the base warranty and (2) studies that investigate both of them through a supply chain of after sale services. In the first category, EW is evaluated only from the perspective of customers, manufacturer or third parties (Kumar and Chattopadhyay, 2004; Khiabani

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and Rangan, 2012; Gallego et al., 2014). Meanwhile, strategies applied during extended warranties could have significant impacts on manufacturer policies during base warranty or get affected by them. Therefore, secondary studies are presented. In this regard, Jiang and Zhang (2011), for the first time, evaluated the impact of extended warranty proposed by a third party on the base warranty offered from a manufacturer. According to their results, customers feel more quality of the product when the extended warranty is presented. Heese (2012) demonstrated that although extending the base warranty period of a product strengthens its position against competing products, customers are less willing to obtain extended warranties. This could be due to the overlap of base and extended warranties. In another study Esmaeili et al. (2014) modeled the interaction between manufacturers, third parties and customers in the form of a game theory approach in two non-cooperative and semi-cooperative modes. Nash equilibrium was obtained in the non-cooperative game, where manufacturer, third party and customer separately adopt their strategies. In the semi-cooperative game, manufacturer and third party act as united providers against customer, where manufacturer is regarded as the Stekelburg game leader.

According to the reviewed studies, optimizing extended warranty programs needs to investigate in conjunction with base warranty. Given the fact that warranty service costs have a significant impact on profit of after-sale service providers, finding an effective strategy is a big challenge. To overcome this shortcoming, two general solutions can be presented: (1) Applying high reliability components in products and (2) using maintenance policies. In the first method, reliability of the system could be elevated to a certain level by exploiting high reliability components. However, this is impossible for some cases or could be very costly (Yeh and Hsieh, 2011). In the second method, maintenance has significant roles in keeping product in an appropriate level of usability and reliability. In this regard, Wu and Longhurst (2011) studied product life costs in the presence of base and extended warranties and opportunistic maintenance. Bouguerra et al. (2012) presented a mathematical model by considering six maintenance policies so that the maximum additional cost of extended warranty which is handled by customers and minimum selling price of extended warranties provided by manufacturer, could be determined. Chang and Lin (2012), assumed *minimal* corrective maintenance (CM) is applied at time of failure and *imperfect* preventive maintenances (PM) are performed during the extended warranty. Under such circumstances, optimal values of the extended warranty duration and the numbers of PM actions were determined to maximize profits of the EW provider. Tong et al. (2014a) presented a two-dimensional pricing model of EW under different maintenance policies when minimal repairs are performed. Huang et al. (2017) investigated periodic and non-periodic PM during extended warranty when customers classified based on their usage rate.

While most of studies on extended warranty investigated maintenance policies, none of them has referred on how to manage these policies. In other words, corrective and preventive maintenance require an efficient structure to apply the repairs. In this regard, warranty distribution network (WDN) has been presented as an appropriate approach. WDN is a maintenance network, which is responsible for collecting faulty or damaged products of customers, distributing these products between repair centers, correcting defects and returning the repaired products to their owners (Ashaiery et al., 2015). Optimization of extended warranty distribution network (EWDN) from the perspective of a third party alongside base warranty policies of manufacturer is an interesting topic, which has been neglected in the warranty literature.

Basic factors, which can affect the optimization of warranty distribution network, are: inventory management, network structure and maintenance policies. In previous studies, inventory management has been accomplished through the optimizing spare parts inventory. In this regard, the literature of spare parts could be evaluated from two perspectives: (1) studies that consider products to be single-indenture and (2) studies that regard products as multi-indenture. In the first perspective, the behavior of WND could be similar to a closed logistic network. Pishvaee et al. (2010) provided a bi-objective integer programming model to minimize the total costs and maximize responsiveness to customers in a closed supply chain network. In another study by Ashayeri and Tuzkaya (2011), a multi-objective optimization model was suggested for designing after sale service network in high technology industries. While their model aims to maximize responsiveness of maintenance centers, no attention was paid to the inventory control of flows within the network. In a similar approach, Hassanzadeh Amin and Zhang (2014) designed a multi-item, factory, marketing technologies, demand markets and collection centers for closed supply chain network. Firtzsche and Lasch (2012) simulated a logistic network for optimizing the maintenance of spare parts in the aviation industry through the artificial neural network approach. In another research, Ozkir and Baslıgıl (2012) presented an integer non-linear programming (INLP) for closed supply chain network, which included various marketing processes. Ashaiery et al. (2015) provided a non-linear mixed-integer programming model for redesigning warranty distribution network in a closed supply chain network. However, logistic costs of spare parts and cost of maintenance were not considered in their model. Batarfi et al. (2017) presented a closed reverse logistic network in which production, refurbishing and waste distribution are addressed. To improve system performance, they evaluated a dual return channel compare to a retail channel. The obtained results showed show the superiority of the network under the dual policy.

Although the mentioned research modeled failures through a logistic network, products were considered as a single component by a simplified hypothesis. It should be mentioned that a product contains various components and factors, such as structural dependence, utilization rate, failures and management of spare parts, which affect the performance the warranty distribution network. According to the best of our knowledge, no study has investigated warranty distribution network from this perspective. The most relevant discussion can be the second category of studies which design repair networks based on hierarchy product structure. In this regard, Sherbrooke (1968) presented a technique to evaluate Multi-Echelon Technique for Recoverable Item Control (METRIC) for aviation industry. In the METRIC model, the repair network contains two echelons, including a main warehouse and a number of local warehouses. System components can be repaired in any of the main or local warehouses. Sherbooke (1968) determined optimum level of spare parts in the main and local warehouses so that sum of expected backorders is minimized. Following that, Muckstadt (1973) presented a MOD-METRIC model, in which items have a hierarchical structure consisting of two indentures. Slay (1984) established the VARI-METRIC model, in which the number of items under repair, follows negative binomial distribution by considering the equality of mean and variance. Sherbrooke (1986) developed the VARI-METRIC model when items were two indentures and repair network was two echelons. After that, the standard METRIC models have been developed by other researchers (Wang et al., 2000; Wong et al., 2005; Basten et al. 2013; Costantino et al., 2013). Recently Topan et al. (2017) modeled a multi-item two-echelon spare parts inventory model which aims at minimizing costs of inventory holding and fixed ordering. They presented an efferent heuristic to find optimal solutions for large-scale problems. Interested readers refer to Eruguz et al. (2017) for more detail explanation on maintenance logistics management and METRIC models.

The mentioned studies based on METRIC models are strategic approaches for supporting sensitive and capital-intensive systems, including aviation industry, in which the availability of items is a necessity. Applying the concept of METRIC models for optimizing warranty distribution network can result in timely response to customers demand and more realistic spare parts management through regarding hierarchy structure of products.

In this paper, an INLP model is presented to optimize EWDN from the perspective of a third party for supporting multi-indenture products. The Product is a multi-component system, which is sold by the manufacturer to a series of customers along with the base warranty. After the expiration of base warranty, third party optimizes the extended warranty policies according to the applied strategies by the manufacturer. To do so, a two-echelon extended warranty network based on the METRIC model is designed, which contains a depot repair center in the first echelon and a number of operational repair centers in the second echelon. In order to reduce repair costs, a novel imperfect preventive maintenance approach is established regarding virtual age concept in the proposed EWDN. In such conditions, the purpose of the model is to determine the optimum level of spare parts for every single component of the product in each of the operational and depot repair centers in a way that: (1) total expected backorders is minimized; (2) total maintenance and retrieval cost of product components are controlled.

Optimization of the proposed model is a challenging act. Since for medium and large scale models of INLP attaining the global optimum is practically impossible due to the presence of numerous local optima. To overcome the mentioned challenge an exact hybrid solution approach is developed based on Branch and Bound (B&B) algorithm and Variable Neighborhood Search (VNS) approach. This research could add the following contributions to the warranty literature:

1- Warranty distribution network is established for optimization of extended warranty polices.

2- Warranty distribution network is developed for multi-indenture systems. Previous studies have only used WND for single-indenture products.

3- Costs of warranty distribution network will be controlled by a novel imperfect maintenance policy. In this regard, a new virtual age model is presented based on the rejuvenation process during the base and extended warranty periods.

The problem definition is expressed in the second part of the paper. In the third section, model components including symbols, incomplete maintenance policy, Chi-square test by Monte Carlo simulation (for evaluation of Poisson hypothesis) and two-echelon two-indenture warranty distribution network are established. The completed model is mentioned in the fourth section. In the fifth part, an exact hybrid solution approach is proposed. Numeral example is presented in the sixth section, along with sensitivity analysis. Conclusion remarks and future research are discussed in the seventh section of the article.

2- Problem definition

Consider a repairable product which is sold to a set of customers by a manufacturer along with base warranty for time period$W$. After expiration of the base warranty period, customers can benefit from extended warranty by a third party (3P) for time EW. The product is a series system consisting of K linear replaceable units (LRU) which in the case of any failure, the whole system stops working. A LRU is a defective part that is detachable from the main system and is replaceable with an intact unit (spare part). The LRU failures follow a known statistical distribution. Figure 1 shows the structure of the product.

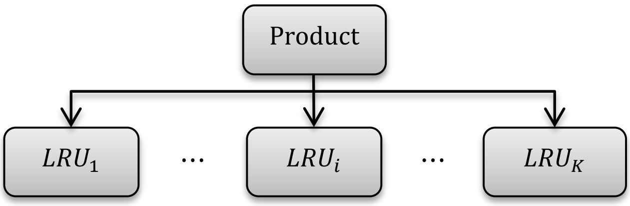

Figure 1. The structure of product consisted of K units of LRU

The selected policies by the manufacturer during the base warranty period as well as the need to respond promptly to customers' demand impact on extended warranty period policies of the third party. For this purpose, 3P benefits from a two level warranty network distribution (WND), in the first level there is a depot repair center ($j = 0$) and in the second level there are M ($j \in \{1,2,\ldots,M\}$) operational repair centers. The jth operational repair center covers L_j customer at jth customer zones.

There are two types of maintenance in operational repair centers: (1) preventive maintenance (PM) and (2) corrective maintenance (CM). At the time of failure, LRU is returned to the *as bad as old* status by *minimal* CM so that it would be rejuvenated by an *imperfect repair* at time of PM. This is because during preventive maintenance all LRUs used in product would be inspected and, if necessary, maintained. Hence, there is time and cost savings compare to when any type of LRU has a separate plan for PM and CM.

There is a certain level of cumulative damage for LRU type i at which doing maintenance is not possible and it should be replaced with a new one. In these circumstances LRU type i is transmitted to the jth operation repair center and if there is at least one spare part ($S_{ij} \geq 1$) replacement would immediately done, Otherwise, backorder occurs. The outdated LRU i with probability of u_{ij} is recoverable in operational repair center j and has retrieval time equal to t_{ij}. Also with probability of $(1 - u_{ij})$ it will be sent to the depot repair center. Therein, if there is spare part (S_{i0}) it is immediately sent to operational repair center j, otherwise backorder occurs. In this case, the retrieval time can be assumed as t_{i0} and it is based on the *First in First out* (FIFO) method and independent of the operational repair center j. The order, shipping and receiving time of LRU items from the depot center to the operational repair center j is considered as v_{0j}.If the demanded LRU is in backorder, the customer needs to wait; this waiting time has a negative effect on the customer' satisfaction of the extended warranty distribution network.

The time interval between PM actions ($\Delta \in [\Delta^{min}, \Delta^{max}]$) and level of PM ($r_i \in [r^{min}, r^{max}]$) has a direct impact on demand of spare parts and as a result LRU backorders in operational repair centers. So, the aim of 3P is to determine S_{ij}, S_{i0}, Δ and r_i values in such a way that during the extended warranty period, in addition to control maintenance costs of the product, retrieval and procurement costs of spare parts, occurred backordered at the operational repair centers would be minimized.

3- Components of the model

In this section, the mathematical model of two-echelon two-indenture extended warranty network under imperfect preventive maintenance policy is presented. At first, variables and parameters are introduced. Afterwards, imperfect preventive maintenance policy is developed based on the concept of virtual age. Following that, Chi-square test is performed by means of Monte-Carlo simulation to evaluate the hypothesis that demand for spare parts in repair centers follow a Poisson distribution. Eventually, two-echelon two-indenture EWDN will be developed based on the METRIC model.

3-1- Symbols

Mathematical symbols used in proposed models in the present article are as follows.

$i \in \{1,2,\dots,K\}$	LRU items	T_i	Failure process of LRU type *i*
W	Duration of base warranty	$f_i(.)$	Failure probability function of LRU type *i*
EW	Duration of extended warranty	$F_i(.)$	Cumulative failure distribution function of LRU type *i*
$j = 0$	Depot repair center	$h_i(.)$	Failure function of LRU type *i* (Hazard rate)
$j \in \{1,2,\dots,M\}$	Operational repair centers	$h_i^{-1}(.)$	Inverse failure function of LRU type *i*
$\acute{\Delta}$	Duration between two consecutive PMs during base warranty	$v^i(t)$	Virtual age of LRU type *i* at time *t*
Δ	Duration between two consecutive PMs during extended warranty	$\acute{\psi}_i$	Reduction function of deterioration process for LRU type *i* during base warranty
$\acute{r}_i$	Level of applied PM during base warranty period from manufacturer for LRU type *i*	$\delta(\acute{r}_i)$	Decreasing function of the chosen PM level by manufacturer
r_i	Level of applied PM during extended warranty period from third party for LRU type *i*	ψ_i	Reduction function of deterioration process for LRU type *i* during extended warranty
$\acute{\tau}_q$	Time of applying qth PM during base warranty	$\delta(r_i)$	Decreasing function of the chosen PM level by third party
τ_p	Time of applying pth PM during extended warranty	EN_i	Expected number of failure for URL i during the extended warranty
n_1	Number of performed PM during base warranty	C_{ij}^{cm}	Cost of CM on the LRU i in jth operational repair center during the extended warranty
n_2	Number of performed PM during extended warranty	EC_{ij}^{cm}	Expected corrective maintenance cost on the LRU i in the jth repair center during the extended warranty
γ_i^{lower}	Lower permissible level of failure function for performing PM	C_{ij}^{pm}	Cost of PM on the LRU i in jth operational repair center during the extended warranty
γ_i^{upper}	Upper permissible level of failure function for performing PM	EC_{ij}^{pm}	Expected preventive maintenance cost on the LRU i in the jth repair center during the extended warranty
S_{ij}	The number of LRU type i in repair center *j*	y_{ip}	Binary variable that is one, if deterioration process of LRU i through pth preventive maintenance is between the lower and upper permissible limit, otherwise it is zero.
t_{ij}	Retrieval time of LRU type *i* at repair center *j*	Ω_{ij}	Scale parameter of PM costs associated with the LRU type i in operational repair center j
ϑ_{ij}	Transferring time of LRU *i* from operational repair center *j* to the depot repair center and vice versa	∂_{ij}	Shape parameter of PM costs associated with the LRU type i in operational repair center j
μ_{ij}	Demand spare parts of LRU i from operational repair center j	ES_i	Expected number of repairs for LRU type *i* during the extended warranty period
u_{ij}	The probability of repairing LRU *i* in repair center *j*	ES_{ij}^l	Expected number of required spare parts for customers *l* at region *j* for LRU type *i*
R_{ij}	Average time which are required to resupply an LRU type i to repair center j	z_{ip}	Binary variable that if the deterioration process in pth preventive maintenance is greater than or equal to the upper permissible limit for LRU i, it is one. Otherwise it is zero.
X_{i0}	The number of LRU type i backorders in the depot repair center	β_i	Shape parameter of Weibull distribution related to LRU type *i*
X_{ij}	The number of LRU type i backorder, due to operational repair center	θ_i	Scale parameter of Weibull distribution related to LRU type *i*
Y_{ij}	The random variable, showing the number of LRU i in resupply cycle to repair center *j*	c_i	Cost of providing one spare part of LRU type *i*
$EBO_{i0}(S_{i0})$	The expected backorder of LRU type *i* when S_{i0} spare parts present in the depot repair center	IC_j	The inspection cost in operational repair center *j*
$EBO_{ij}(S_{ij})$	The expected backorder of LRU type *i* when S_{ij} spare parts present in the operational repair center *j*	TIC_j	Total inspection cost in operational repair center *j*
$\partial\,(S_{i0})$	The average delay time of LRU type *i* when S_{i0} spare parts exist in depot repair center	C_{ij}^R	Retrieval cost of LRU type *i* at repair center *j*

$TCR_{ij}(\Delta,\acute{\Delta},r_i,\acute{r}_i)$	Total expected retrieval cost of LRU type *i* at repair center *j*	β	Value of budget
$TCM_j(\Delta,\acute{\Delta},r_i,\acute{r}_i)$	Total expected cost of corrective maintenance in repair center *j*	L_j	Number of customers of *j*th customer zone
$TPM_j(\Delta,\acute{\Delta},r_i,\acute{r}_i)$	Total expected cost of preventive maintenance in repair center *j*	ξ	A parameter in the [0,1] range, which set the level of preventive maintenance

3-2- Maintenance policy

The warranted product has a serial structure composed of K units of LRU ($i \in \{1,2,\ldots,K\}$). These items will fail as time goes by and as a result of deterioration process. Suppose that random variable of T_i indicates the LRU failure type i. Also $f_i(t)$ and $F_i(t)$ respectively represent the probability distribution function and the cumulative distribution function of failure for LRU type i. Accordingly, the failure rate function of LRU type i ($h_i(t)$) will be as follows:

$$h_i(t) = \frac{f_i(t)}{1-F_i(t)} \qquad \forall\, i = 0,1,2,\ldots,k \tag{1}$$

$h_i(.)$ is an increasing function whereby the deterioration process increases over time, so that leads to failure of item i. In order to reduce the failure costs of LRU i, the imperfect preventive maintenance policy can be applied during the warranty period (W) via the manufacturer as well as during the extended warranty period (EW) by 3P. Under this policy: (1) at the time of $\acute{\tau}_1,\acute{\tau}_2,\ldots,\acute{\tau}_{n_1}$ ($n_1 = \left[\frac{W}{\acute{\Delta}}\right]$) with fixed time intervals of $\acute{\Delta}$, the product is investigated by the manufacturer for preventive maintenance actions. (2) As a result of failure in one LRU, the product stops working. In this situation the defective LRU is sent to the manufacturer for doing minimal CM. (3) At times of $\tau_1,\tau_2,\ldots,\tau_{n_2}$ ($n_2 = \left[\frac{EW-W}{\Delta}\right]$) with fixed intervals of Δ, the customers' products of jth customer zone are forwarded to the operational repair center j to carry out inspection and imperfect PM. (4) Due to the failure in one LRU unit, the product stops working. In this situation the faulty unit is separated from the customer's product of customer zone j and is sent to the operational repair center j for doing minimal CM. Under this policy, there are four states for LRU type i:

I. The failure of LRU i has stopped the performance of the product. In this case, by applying a minimal CM, LRU type i is returned to the position before failure (as bad as old).
II. At the time of PM inspection, the deterioration process of LRU i is less than or equal to γ_i^{lower} ($i.e, h_i(t) \le \gamma_i^{lower}$). In this condition, there is no requirement to impose PM.
III. At the time of PM inspection, the deterioration process of LRU i is at a level between γ_i^{lower} and γ_i^{upper} ($i.e, \gamma_i^{lower} < h_i(t) < \gamma_i^{upper}$). In this case, by applying an imperfect preventive maintenance at $r_i \in \left[r_i^{min}, r_i^{max}\right]$, LRU i is rejuvenated.
IV. At the time of PM inspection, the deterioration process of LRU i is at a level higher than γ_i^{upper} ($i.e, h_i(t) \ge \gamma_i^{upper}$). In this case, LRU i is replaced with a spare part.

Figure 2 shows the maintenance policies mentioned for the LRU type i in circumstances that the manufacturer at base warranty period has applied no PM. At the time τ_1, the deterioration process falls into range of $\left(\gamma_i^{lower}, \gamma_i^{upper}\right)$. As a result, by applying a PM in level r_i, LRU type i will be rejuvenated. During the second planned pause (τ_2), the deterioration process is higher than γ_i^{upper}.So LRU i is replaced with a spare part. At the time of τ_3, the deterioration process is in a level lower than γ_i^{lower}, thus it is not needed to carry out PM.

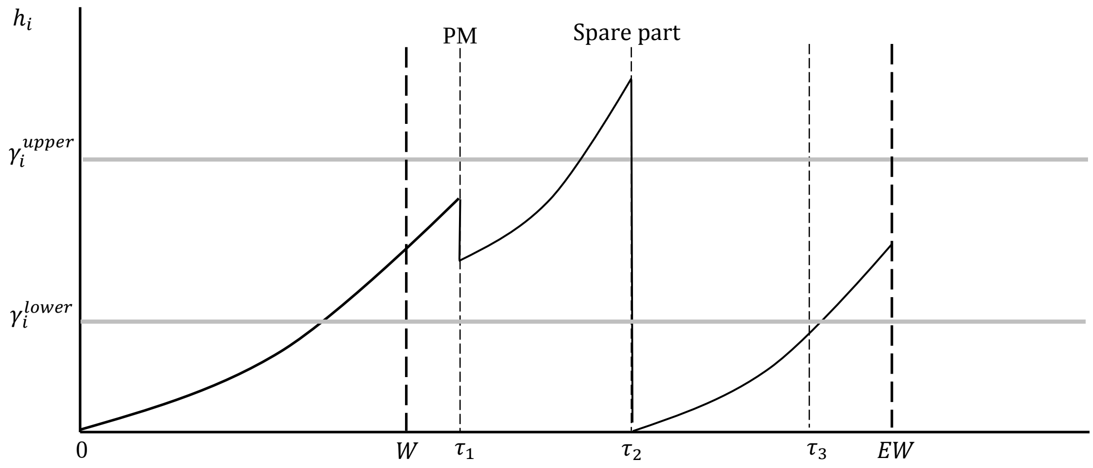

Figure 2. Maintenance policy applied by the third-party during the extended warranty period for LRU type i

The suggested preventive maintenance policy, using the concepts of age reduction and virtual age will be modeled during the base warranty period under the strategies of the manufacturer and also during the extended warranty period from the third party perspective.

3-2-1- Maintenance policy during the warranty period

Suppose $v^i(t)$ indicates the virtual age of LRU i at time t. Until the first PM, the virtual and real age of ith LRU are equal. As a result, we have:

$$v^i(t) = t \qquad 0 \le t < \acute{\tau}_1, i = 1,2,\dots,K \tag{2}$$

$$h_i\left(v^i(t)\right) = h_i(t) \qquad 0 \le t < \acute{\tau}_1, i = 1,2,\dots,K \tag{3}$$

After applying the first PM, the virtual age of ith LRU (v_1^i) is obtained as following equation:

$$v_1^i = h_i^{-1}\left(\acute{\psi}_\iota h_i(\acute{\tau}_1)\right) \qquad t = \acute{\tau}_1 \tag{4}$$

In equation (4), $h_i^{-1}(.)$ is the inverse failure function and $\acute{\psi}_\iota$ is a function that expresses the reduction in the deterioration process and is calculated based on equation (5):

$$\acute{\psi}_\iota = \begin{cases} 1 & h(a) \le \gamma_i^{lower} \\ \delta(\acute{r}_i) & \gamma_i^{lower} < h(a) < \gamma_i^{upper} \\ 0 & \gamma_i^{upper} \le h(a) \end{cases}, i = 1,2,\dots,K \tag{5}$$

According to equation (5) if the deterioration process at the moment a ($i.e, h\,(a)$), is less than γ_i^{lower}, no PM action would be taken from the manufacturer. If $\gamma_i^{lower} < h(a) < \gamma_i^{upper}$, the value of the deterioration process is reduced by a coefficient of $\delta(\acute{r}_i) \in [0,1]$ and if the deterioration process is more than γ_i^{upper}, LRU i is replaced with a spare part. In this regard, $\delta(\acute{r}_i)$ is a decreasing function of chosen PM level by the manufacturer ($i.e, \acute{r}_i \in \left[\acute{r}^{min}, \acute{r}^{max}\right]$), so that $\delta(0) = 1$, transfers the under maintenance LRU to as bad as old and $\delta(\acute{r}^{max}) = 0$ transfers it to as good as new.

According to equation (4) after failure rate of LRU i was reduced to $\acute{\psi}_\iota h_i(\acute{\tau}_1)$, at moment $\acute{\tau}_1$, the function of $h_i^{-1}(.)$ determines the virtual age corresponding to this level of failure rate. After qth PM, the values of failure rate function ($h_i(v_q^i)$) and virtual age (v_q^i) are as equations (6) and (7):

$$h_i(v_q^i) = \acute{\psi}_\iota h_i(v_{q-1}^i + \acute{\Delta}) \qquad q = 2,\dots,n_1 - 1, \quad i = 1,2,\dots,K \tag{6}$$

$$v_q^i = h_i^{-1}\left(\acute{\psi}_\iota h_i\left(v_{q-1}^i + (\acute{\tau}_q - \acute{\tau}_{q-1})\right)\right) = h_i^{-1}\left(\acute{\psi}_\iota h_i(v_{q-1}^i + \acute{\Delta})\right) \qquad q = 2,\dots,n_1 - 1, \quad i = 1,2,\dots,K \tag{7}$$

As can be seen from equations (6) and (7), the failure rate function after applying qth preventive maintenance is a fraction of the failure rate before that. In this situation, the function $h_i^{-1}(.)$ indicates the virtual age corresponding to that failure level. The values of virtual age and failure rate in the range of $\acute{\tau}_q \le t < \acute{\tau}_{q+1}$ are:

$$v^i(t) = v_q^i + (t - \acute{\tau}_q) \qquad \acute{\tau}_q \le t < \acute{\tau}_{q+1}, \quad q = 2,\dots,n_1 - 1, \quad i = 1,2,\dots,K \tag{8}$$

$$h_i\left(v^i(t)\right) = h_i\left(v_q^i + (t - \acute{\tau}_q)\right) \qquad \acute{\tau}_q \le t < \acute{\tau}_{q+1}, \quad q = 2,\dots,n_1 - 1, \quad i = 1,2,\dots,K \tag{9}$$

Finally, the equations (10) and (11) indicate the virtual age and failure rate values in the range of $\acute{\tau}_{n_1} \leq t \leq W$.

$$v^i(t) = v^i_{n_1} + \left(t - \acute{\tau}_{n_1}\right) \qquad \acute{\tau}_{n_1} \leq t \leq W, \qquad i = 1,2,\dots,K \tag{10}$$

$$h_i\left(v^i(t)\right) = h_i\left(v^i_{n_1} + \left(t - \acute{\tau}_{n_1}\right)\right) \quad \acute{\tau}_{n_1} \leq t \leq W, \qquad i = 1,2,\dots,K \tag{11}$$

3-2-2- Maintenance policy during the extended warranty period

The maintenance policy which is applied by the manufacturer during the base warranty period (contains the values of $\acute{\Delta}$ and $\acute{r}_i$) has a direct impact on virtual age of product units and consequently on maintenance policy of the third party. Therefore, virtual age of LRU type i during the extended warranty period $(i.e, v^i(t), W < t \leq EW)$ is determined according to base warranty duration (W), the time interval between the preventive maintenance actions during the base warranty period $(\acute{\Delta})$ and the level of PM actions $(\acute{r}_i)$. During EW, the customers product of jth customer zones is sent to the operational repair center j for executing minimal CM or doing PM at time $\tau_1, \tau_2, \dots, \tau_{n_2}$($n_2 = \left[\frac{EW-W}{\Delta}\right]$) with fixed intervals of Δ. In this case, the virtual age and failure rate of LRU i are as follows:

$$v^i(t) = v^i(W) + (t - W) \qquad W < t < \tau_1, \qquad i = 1,2,\dots,K \tag{12}$$

$$h_i\left(v^i(t)\right) = h_i\left(v^i(W) + (t - W)\right) \qquad W < t < \tau_1, \qquad i = 1,2,\dots,K \tag{13}$$

In the (12) and (13) equations, the value of $v^i(W)$ reveals the virtual age of LRU i at the end of base warranty period, obtained based on (10) equation. After applying the first PM in extended warranty duration, virtual age and value of failure process are as following:

$$h_i\left(v^i_1\right) = \psi_i h_i\left(v^i(W + \tau_1)\right) \quad , i = 1,2,\dots,K \tag{14}$$

$$v^i_1 = h_i^{-1}\left(\psi_i h_i\left(v^i(W + \tau_1)\right)\right) \qquad t = \tau_1, i = 1,2,\dots,K \tag{15}$$

In equations (14) and (15), ψ_i is a function that expresses the reduction in the deterioration process and is obtained based on equation (16):

$$\psi_i = \begin{cases} 1 & h(a) \leq \gamma_i^{lower} \\ \delta(r_i) & \gamma_i^{lower} < h(a) < \gamma_i^{upper} \\ 0 & \gamma_i^{upper} \leq h(a) \end{cases} \tag{16}$$

According to equation (16) if the deterioration process at the moment a, $h(a)$, is less than γ_i^{lower}, no PM action would be taken by the third party. If $\gamma_i^{lower} < h(a) < \gamma_i^{upper}$, the value of the deterioration process by a coefficient of $\delta(r_i) \in [0,1]$ is reduced. If the deterioration process is more thanγ_i^{upper}, LRU i is immediately replaced in case of existence of spare parts in repair center, otherwise a backorder happens. After pth PM, virtual age and the failure function will be as follows:

$$h_i\left(v^i_p\right) = \psi_i h_i\left(v^i_{p-1} + \left(\tau_p - \tau_{p-1}\right)\right) = \psi_i h_i\left(v^i_{p-1} + \Delta\right) \qquad p = 2,\dots,n_2 - 1, i = 1,2,\dots,K \tag{17}$$

$$v^i_p = h_i^{-1}\left(\psi_i h_i\left(v^i_{p-1} + \left(\tau_p - \tau_{p-1}\right)\right)\right) = h_i^{-1}\left(\psi_i h_i\left(v^i_{p-1} + \Delta\right)\right) \quad p = 2,\dots,n_2 - 1, i = 1,2,\dots,K \tag{18}$$

As equations (17) indicate, the failure rate of the under repair LRU after applying the pth preventive maintenance, is a fraction of pre-failure rate. The values of virtual age and failure rate in the range of $\tau_p \leq t < \tau_{p+1}$ are:

$$v^i(t) = v^i_p + \left(t - \tau_p\right) \qquad \tau_p \leq t < \tau_{p+1} \qquad p = 2,\dots,n_2 - 1, i = 1,2,\dots,K \tag{19}$$

$$h_i\left(v^i(t)\right) = h_i\left(v^i_p + \left(t - \tau_p\right)\right) \qquad \tau_p \leq t < \tau_{p+1} \qquad p = 2,\dots,n_2 - 1, i = 1,2,\dots,K \tag{20}$$

Finally, equations (21) and (22) indicate the virtual age and failure rate values in the range of $\tau_{n_2} \leq t \leq EW$:

$$v^i(t) = v^i_{n_2} + \left(t - \tau_{n_2}\right) \qquad \tau_{n_2} \leq t \leq EW \quad , i = 1,2,\dots,K \tag{21}$$

$$h_i\left(v^i(t)\right) = h_i\left(v^i_{n_2} + \left(t - \tau_{n_2}\right)\right) \quad \tau_{n_2} \leq t \leq EW \quad , i = 1,2,\dots,K \tag{22}$$

Since the occurred failures in the LRU items are minimal repaired with negligible time, the expected number of failure in each time interval is obtained by the integral of failure function during that interval. Hence, the expected number of failure for URL i during the extended warranty is attained as $EN_i = \int_W^{EW} h_i\left(v^i(t)\right) dt$. Therefore, we have:

$$EN_i = \int_W^{EW} h_i\left(v^i(t)\right) dt$$
$$= \int_W^{\tau_1} h_i\left(v^i(W) + (t - W)\right) dt + \sum_{p=1}^{n_2-1} \int_{\tau_p}^{\tau_{p+1}} h_i\left(v_p^i + \left(t - \tau_p\right)\right) dt \qquad (23)$$
$$+ \int_{\tau_{n_2}}^{EW} h_i\left(v_{n_2}^i + \left(t - \tau_{n_2}\right)\right) dt$$

Suppose C_{ij}^{cm} is defined as the cost of CM on the LRU i in jth operational repair center during the extended warranty period. Also EN_{ij} shows the expected number of failed LRU type i which is sent to jth operational repair center corresponding to equation (23). Then the expected corrective maintenance cost (EC_{ij}^{cm}) in the jth repair center during the extended warranty period is determined as below:

$$EC_{ij}^{cm} = C_{ij}^{cm} EN_{ij} = C_{ij}^{cm} \int_W^{EW} h_i\left(v^i(t)\right) dt$$
$$= C_{ij}^{cm} \int_W^{\tau_1} h_i\left(v^i(W) + (t - W)\right) dt + C_{ij}^{cm} \sum_{p=1}^{n_2-1} \int_{\tau_p}^{\tau_{p+1}} h_i\left(v_p^i + \left(t - \tau_p\right)\right) dt \qquad (24)$$
$$+ C_{ij}^{cm} \int_{\tau_{n_2}}^{EW} h_i\left(v_{n_2}^i + \left(t - \tau_{n_2}\right)\right) dt$$

If $C_{ij}^{pm}$is defined as cost of PM on the LRU type i in jth repair center during the extended warranty period. Then the expected preventive maintenance cost of LRU i for the jth repair center (EC_{ij}^{pm}) is determined as follows:

$$EC_{ij}^{pm} = \sum_{p=1}^{n_2} C_{ij}^{pm} y_{ip} \quad , i = 1,2, \dots , K \qquad (25)$$

In equation (25), y_{ip} is a binary variable that is one, if deterioration process of LRU i through pth preventive maintenance is between the lower and upper permissible limit, otherwise it is zero.

$$y_{ip} = \begin{cases} 1 & if \quad \gamma_i^{lower} < h_i\left(v_{p-1}^i + \Delta\right) < \gamma_i^{upper} \\ 0 & otherwise \end{cases} \quad p = 1, 2, \dots , n_2, \qquad i = 1,2, \dots , K \qquad (26)$$

In equation (26), preventive maintenance cost (C_{ij}^{pm}) depends on the PM level (r_i). Therefore, C_{ij}^{pm} is defined based on equation (27).

$$C_{ij}^{pm} = \Omega_{ij} e^{\partial_{ij} r_i}, \qquad \Omega_i \geq 0, \partial_i \geq 0, \qquad i = 1,2, \dots , K \qquad (27)$$

The values ∂_{ij} and Ω_{ij} are respectively scale and shape parameter of preventive maintenance costs associated with the LRU type i in operational repair center j. According to equation (27), the increase in value of r_i causes more PM costs for the third party. Moreover, the increase of parameter ∂ has more effect on PM cost growth than increase in parameter Ω. The average number of spare parts during the extended warranty period for the LRU i (ES_i) is obtained as follows.

$$ES_i = \sum_{p=1}^{n_2} z_{ip} \qquad , i = 1,2, \dots , K \qquad (28)$$

In equation (28) z_{ip} is a binary variable that if the deterioration process in pth PM is greater than or equal to the upper permissible limit for LRU i, it is one. Otherwise it is zero.

$$z_{ip} = \begin{cases} 1 & if \quad h_i\left(v_{p-1}^i + \Delta\right) \geq \gamma_i^{upper} \\ 0 & otherwise \end{cases} \quad p = 1, 2, \dots , n_2, \qquad i = 1,2, \dots , K \qquad (29)$$

Since the warranted product has K types of LRU, the total expected cost of corrective maintenance ($TCM_j(\Delta, \acute{\Delta}, r_i, \acute{r}_i)$) and total expected cost of preventive maintenance ($TPM_j(\Delta, \acute{\Delta}, r_i, \acute{r}_i)$) for jth operational repair center during the extended warranty period are obtained as below:

$$TCM_j(\Delta, \acute{\Delta}, r_i, \acute{r}_i) = \sum_{i=1}^{K} EC_{ij}^{cm} \qquad (30)$$

$$TPM_j(\Delta, \acute{\Delta}, r_i, \acute{r}_i) = \sum_{i=1}^{K} EC_{ij}^{pm} \qquad (31)$$

As equations (30) and (31) indicate, the total expected cost of CM and PM are a function of maintenance policies adopted by the manufacturer during the warranty period (including values of $\acute{\Delta}$ and $\acute{r}_i$) and the third party maintenance policies during the extended warranty (including values of Δ

and r_i). Due to this fact that the maintenance network is two-echelon, calculation of the spare parts' cost is complex. In this regard, Sherbrooke (1968) for the first time, suggested METRIC in aviation industry. In the METRIC model, maintenance network consists of two levels including a main warehouse in the first echelon and some local warehouse in the second echelon. Maintenance operations of system can be done in any of the main or local centers. In this paper, spare parts logistic management is modeled based on METRIC approach. The main challenge is Poisson assumption of spare parts demand from the operational repair centers, while in the proposed model the failure function of LRUs ($h_i(.)$) is in accordance with an increasing process. Therefore, in the next section, Monte-Carlo simulation is applied to show that under what conditions the demand for spare parts from the operational repair centers is estimable by a Homogeneous Poisson Process (HPP).

3-3- Chi-square test based on Monte Carlo simulation

Consider a service provider which supports l series systems (LRU). According to Cox and Smith (1954), if every time that a system is not working, it gets repairs and would be as good as new, the process consisted of total failures is called renewed accumulation process (SRP). Cox and Smith (1954) showed that although SRP is not a renewal process, SRP will be a HPP, if l approaches infinity. Wang (2012), through simulation expressed, if $l \geq 10$, then consecutive times between two events follow the exponential distribution which results in a HPP. According to the proposed maintenance policy, the values of $\acute{\Delta}$, $\acute{r}_i$, Δ and r_i impact on the spare parts demand of LRU items. In this section the requirements, under which the demand for spare part of LRU items would be from homogeneous Poisson process, is estimated based on Monte-Carlo simulation approach. For this purpose, it is assumed that the probability function of failure process for LRU i is a two parameter Weibull distribution, (in accordance with equation 32) with shape parameter $\beta_i$and scale parameter θ_i.

$$f_i(t) = \frac{\beta_i}{\theta_i^{\beta_i}} t^{\beta_i - 1} e^{-\left(\frac{t}{\theta_i}\right)^{\beta_i}} \quad , \quad t \geq 0 \, , \beta_i > 0 \tag{32}$$

The function $\delta(r_i)$ is defined as below:

$$\delta(r_i) = 1 - \xi r_i \tag{33}$$

In equation (33) parameter ξ is in range of $[0,1]$ and set the level of preventive maintenance. In this regard, for any value of ξ, increasing in the level of PM causes reduction in the function $\delta(r_i)$. Function $\delta(\acute{r}_i)$ is defined similar to equation (33), the only difference is that instead of r_i, variable $\acute{r}_i$ is used. The higher level of PM is, the more reduction in virtual age, which is equivalent to LRU rejuvenation. The proposed Monte-Carlo simulation approach is as follows:

Step1- Consider null for data ($Data \leftarrow \emptyset$) and repeat the following steps $simulation^{max}$ times.

Step2- Consider l units of LRU type i and repeat steps 3 to 15 for l times.

Step3- Take into account the values $z_{ip} = 0$, $last^i = 0, v^i = 0, t_{Simulation} = 0, t_i = 0, t_{pm} = 0$

Step4- Repeat steps 5 to 9 until the stimulation time is less than warranty time ($t_{Simulation} < W$).

Step5- Repeat the following processes for LRU items:

5-1- Create the failure time of LRU type i (t_i), according to Weibull distribution with shape parameter $\beta_i$and scale parameter θ_i.

Step 6- Increase the time of preventive maintenance (t_{pm}) as much as $\acute{\Delta}$ ($t_{pm} \leftarrow t_{pm} + \acute{\Delta}$)

Step 7- $t = \min\{t_i, t_{pm}\}$

Step 8- Update $v^i$for LRU type i ($v^i \leftarrow v^i + t - last^i$)

Step 9- Compare t_{pm} with t:

If t is equal to t_{pm} then repeat below steps:

9-1- Put the time forward and set in t_{pm} ($t_{Simulation} \leftarrow t_{pm}$)

9-2- Calculate v_q^i and $h_i(v_q^i)$ according to equations (6) and (7).

Repeat below steps if t is equal to t_i.

9-3- Put the time forward and set in t_i ($t_{Simulation} \leftarrow t_i$)

9-4- $last^i \leftarrow t_i$

9-5- Create the next failure time of LRU type i according to Weibull distribution with shape parameter $\beta_i$and scale parameter θ_i and add it to $t_{Simulation}$ ($t_i \leftarrow t_i + t_{Simulation}$).

Step 10- Repeat steps 11 to 14 until the stimulation time is less than extended warranty duration ($t_{Simulation} < EW$)
Step 11- $t = \min\{t_i, t_{pm}\}$
Step 12- Consider v^i for LRU type i ($v^i \leftarrow v^i + t - last^i$)
Step 13- Compare t_{pm} with t.

Repeat below steps if t is equal to t_{pm}.
13-1- Put the time forward and set in t_{pm} ($t_{Simulation} \leftarrow t_{pm}$)
13-2- calculate v_p^i and $h_i(v_p^i)$ according to equations (17) and (18).
13-3- If $\psi_i = 1$ then consider 1 for z_{ip} ($z_{ip} \leftarrow 1$)
Repeat below steps if t is equal to t_i.
13-4- Put the time forward and set in t_i ($t_{Simulation} \leftarrow t_i$)
13-5- $last^i \leftarrow t_i$
13-6- Create the next failure time of LRU type i in according to Weibull distribution with shape parameter β_i and scale parameter θ_i and add it to $t_{Simulation}$ ($t_i \leftarrow t_i + t_{Simulation}$).

Step 14- Calculate ES_i according to equation (27).
Step 15- $Data \leftarrow Data \cup ES_i$
Step 16- Do Chi-square test on data and show the results.

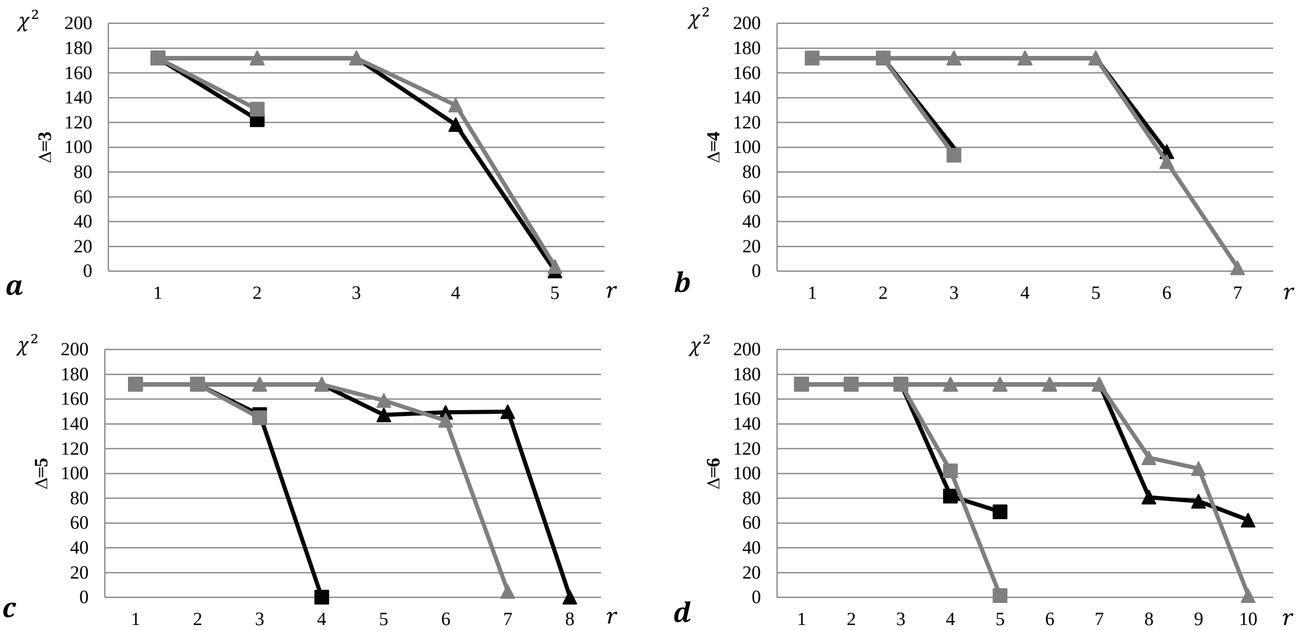

Figure 3. Chi-square error value changes against variation of ξ and r under two scenarios. Scenario1 (black color): Performing of PM during the warranty period by manufacturer. Scenario2 (gray): without doing PM by the manufacturer. In both scenarios the triangle mark is considered for $\xi = 0.025$ and square mark is defined for $\xi = 0.050$.

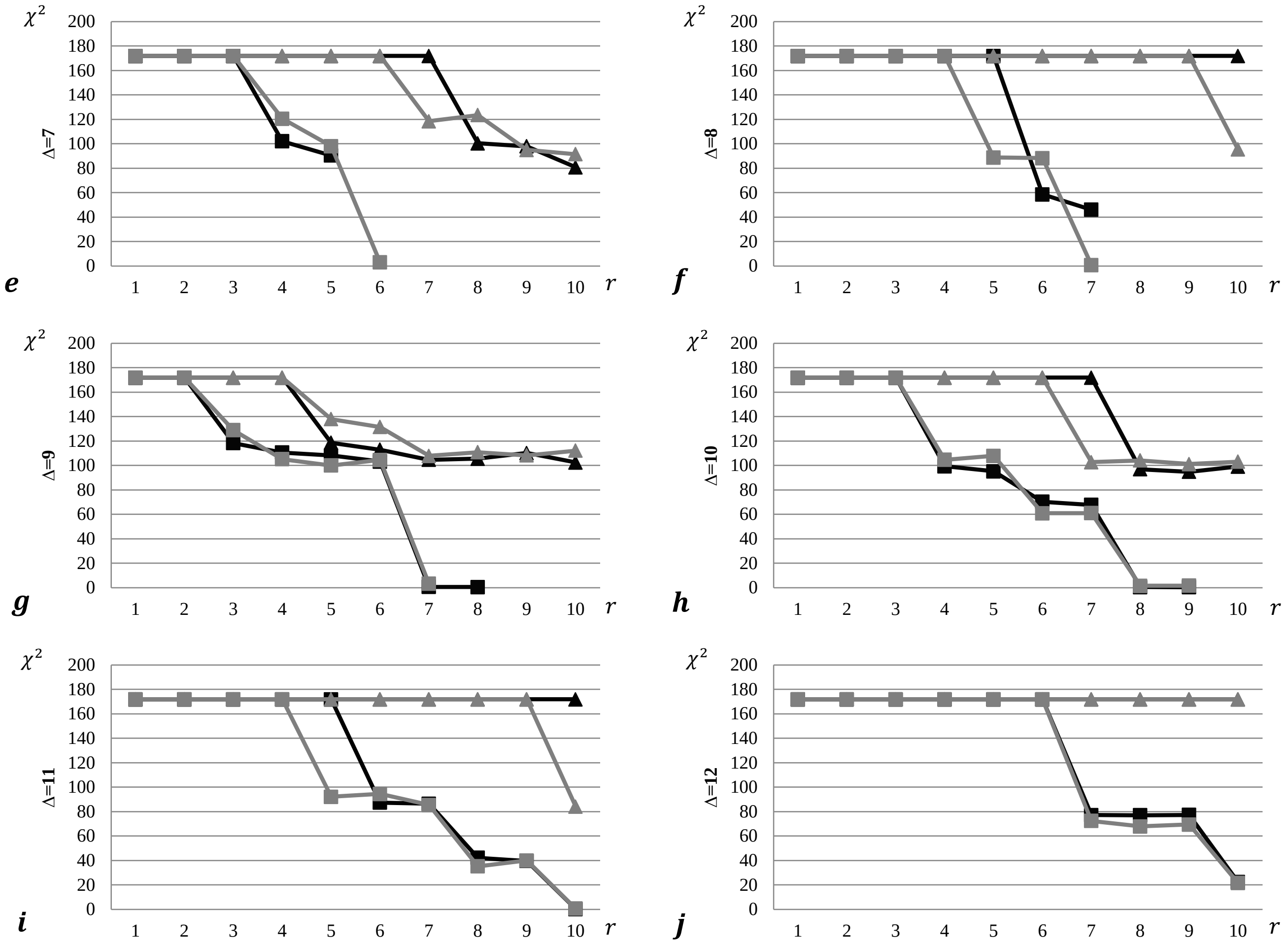

Figure 3. Chi-square error value changes against variation of ξ and r under two scenarios (continue)

The proposed Monte-Carlo simulation algorithm was modeled for l units for one type of LRU that has a failure rate function according to Weibull distribution with shape parameter $\beta = 3$ and the scale parameter $\theta = 1000$. In this regard, duration of the base warranty period is considered 1 year ($W = 365\, days$) and duration of extended warranty is considered 2 years ($EW = 730\, days$) after expiration of the base warranty. The values of $\gamma^{upper} = 0.002$, $\gamma^{lower} = 0.0001$ are assumed. The distance between two consecutive PM are defined according to months and in the range of $\Delta \in \{3, 4, \dots, 12\}$ and preventive maintenance level in the range of $r \in \{1,2, \dots,10\}$. The simulated operation was performed under two scenarios: (1) during the warranty period, maintenance policy of manufacturer includes PM actions and is the same as the third-party maintenance policy during the extended warranty period (*i.e*, $\Delta = \acute{\Delta}$ and $r = \acute{r}$). (2) Manufacturer applies only CM during the base warranty and the PM actions are performed only by the third party during the extended warranty period. Figure 1 shows the results of Monte-Carlo simulation to investigate the effect of preventive maintenance level and the time interval between two consecutive PM on Chi square errors. Therefore, only one unit of LRU is considered ($l = 1$).

Figure 3 indicates that in both scenarios with enhancing effectiveness of preventive maintenance, Chi-square error (χ^2) will be reduced. Because through enhancing the mentioned values, the rejuvenation process will intensify and hence the demand for spare parts will diminish. Therefore, more HPP conditions will be made. As another consequence, it can be seen that by increasing the value of Δ, more PM level (r) is needed. For example, when $\Delta = 3$ and $r = 5$ the value of χ^2 tends to zero (figure 3–a) while for the $\Delta = 5$ and $r = 8$ the value of χ^2 tends to zero (figure 3–f). Moreover by increasing the value of Δ, to reduce χ^2, the higher amount of ξ is required. For example, for $\Delta = 12$ if $\xi = 0.025$ the amount of χ^2 even at high levels of PM is significant. While for $\xi = 0.05$ the reduction of χ^2 on $r \geq 7$ is possible (figure 3–j).

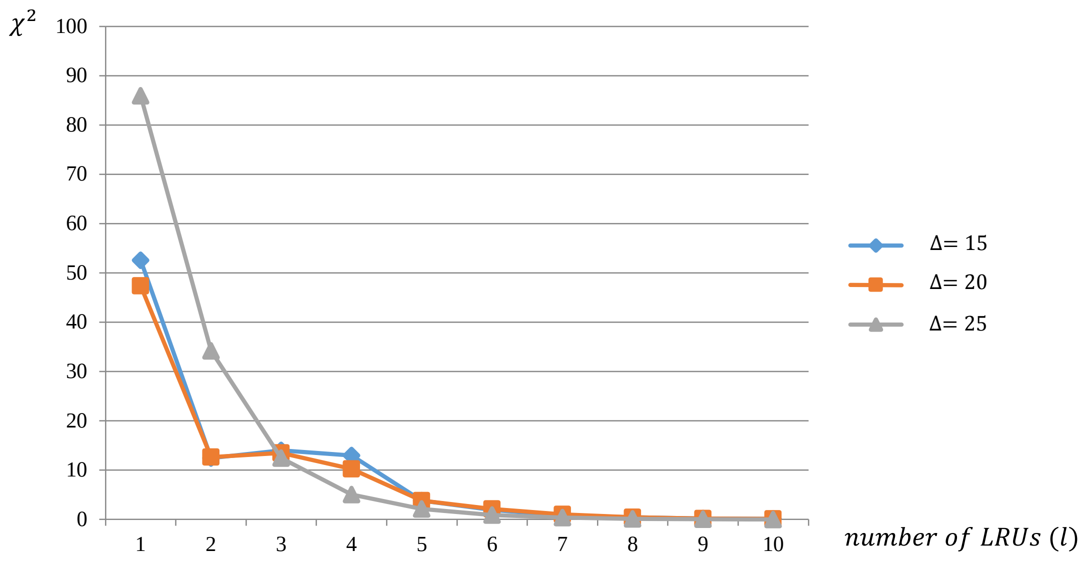

Figure 4. Variation of the Chi-square error against increase in the number of LRU units and Δ

In order to investigate the effect of increasing number of LRU on value of χ^2, Monte-Carlo simulation was performed. For this purpose, the base warranty period is assumed one year (W = 365 $days$) and the extended warranty period is considered 2 years (EW = 730 $days$) after expiration of the base warranty period. Values of $\gamma^{upper} = 0.002$, $\gamma^{lower} = 0.0001$ and the time interval between two consecutive PM actions were defined based on weeks and in range of $\Delta \in \{15,20,25\}$. The values of $r = 5$, $\xi = 0.05$ and $l \in \{1,2,\dots,10\}$ are considered. Moreover, during the warranty period, maintenance policy of manufacturer includes PM and is the same as the third-party maintenance policy during the extended warranty period (ie $\Delta = \acute{\Delta}$ and $r = \acute{r}$). Figure 4 shows the results of Monte-Carlo simulation.

As it is evident in figure 4, by increasing the number of LRU items, their cumulative demand has less χ^2error and it is closer to homogeneous Poisson process. Results of figure 4 is in agreement with the study of Wang (2012). Based on these results, if $l \geq 10$ the consecutive time interval between two events follows the exponential distributions that the result will be a HPP.

According to the obtained results in this section, it can be noted, if the jth customers' zone has at least 10 customer ($L_j \geq 10$) then the demand spare parts of LRU i from operational repair center j is a HPP with average rate of $\mu_{ij}(\Delta, r_i)$ as below:

$$\mu_{ij}(\Delta, r_i) = \frac{\sum_{l=1}^{L_j} ES_{ij}^l}{EW} \quad , i = 1,2,\dots,K, j = 1,2,\dots,M, L_j \geq 10 \tag{34}$$

In equation (34) ES_{ij}^l is the average number of spare part of LRU type i required for customer l that belongs to customer zone j. ES_{ij}^l is calculated in accordance with equation (27). As shown in the previous section, the value of ES_{ij}^l depends on time interval between PM actions and PM levels. Hence, $\mu_{ij}$is a function of Δ and r_i. In the next section, the logistic management of spare parts is modeled based on the METRIC approach.

3-4- Two-echelon two-indenture extended warranty distribution network

Warranty distribution network owned by 3P, is a two-echelon maintenance network in which a depot repair center ($j = 0$) is located at level one and M ($j = 1,2,\dots,M$) operational repair centers are located in the second echelon. The network supports a product that is sold to the set of customers' zones. Operational repair center j, covers L_j customer in jth customer zone. In the repair centers, maintenance policy is according to Section 3-3. Accordingly, if the failure level of LRU type i is greater than or equal to upper permissible limit, doing PM is not possible and it should be replaced by a spare part. In this case, if a spare part inventory of LRU type i in the repair center j (S_{ij}) is sufficient, replacement would takes place. Otherwise, there is a backorder. In continue first, we calculate the expected number of backorders for spare parts in the operational repair centers. Then the costs of maintenance and inspection of defective LRU will be described. It should be noted, the time interval between preventive maintenance (Δ) and the PM level (r_i) have a direct impact upon spare parts demands and consequently the deficiencies of LRU items in repair centers.

3-4-1- Calculation the expected backorders of spare parts in depot and operational repair centers

Occurring failure in LRU items is a stochastic process; consequently, in order to obtain the deficiencies value of spare parts, the mathematical expectation of the backorders in repair centers should be calculated. For this purpose, first the probability distribution function of those LRU units which are exist in the resupply cycle to operational repair center j, should be determined. Suppose that R_{ij} is the average time which are required to resupply one LRU type i to repair center j. In this situation we have:

$$R_{ij} = u_{ij}t_{ij} + (1-u_{ij})\vartheta_{ij} + (1-u_{ij})\vartheta_{ij}\partial\ (S_{i0}) \quad (35)$$

The first part in equation (35) states with the possibility of u_{ij}, LRU i is repairable in repair center j with the retrieval time equals t_{ij}. Moreover, with probability of $(1-u_{ij})$, LRU i will be sent to the depot repair center. As a result, the average retrieval cycle is ϑ_{ij} plus the expected delay time caused by spare parts deficiency in the depot repair center ($\partial\ (S_{ij})$). The main question now is about how to calculate$\partial\ (S_{ij})$.

As it was mentioned, if jth customer zone has a minimum of 10 customers ($L_j \geq 10$) then demand rate of spare parts from operational repair center j is HPP with average rate of $\mu_{ij}(\Delta, r_i)$. In this section for ease of exposition, $\mu_{ij}(\Delta, r_i)$ is applied as μ_{ij}. Then the entrance rate of LRU items that have the possibility of recovery in the jth operational repair center is a HPP with $u_{ij}\mu_{ij}$ rate. Also the average demand rate of LRU items through the depot repair center (μ_{i0}) is calculated as follows:

$$\mu_{i0} = \sum_{j=1}^{M}(1-u_{ij})\mu_{ij}, \quad \forall i \in \{1,2,\ldots,K\} \quad (36)$$

In addition $\partial\ (S_{i0})$ represents the average delay or waiting time for LRU item, when the number of spare parts at the depot repair center is defined as S_{i0}. According to Little's Law:

$$\partial\ (S_{i0}) = \frac{EBO_{i0}(S_{i0})}{\mu_{i0}}, \quad \forall i \in \{1,2,\ldots,K\} \quad (37)$$

In equation (37), $EBO_{i0}(S_{i0})$ expresses the expected number of backorders regarding S_{i0} unit of LRU inventory items in the depot repair center and μ_{i0} is calculated according to (36). If the amount of demand during the LRU recovery in the depot repair center is less than the amount of spare parts in stock ($\mu_{i0}t_{i0} \leq S_{i0}$), then the $EBO_{i0}(S_{i0})$ tends to zero. In order to calculate $EBO_{i0}(S_{i0})$, the probability distribution function of LRU items in resupply cycle to the operational repair center j should be determined. Suppose Y_{ij} is defined as a random variable which represents the number of LRU units in the resupply cycle to operational repair center j. In this situation, mathematical expectation of Y_{ij} ($E(Y_{ij})$) are calculated according to equation (38).

$$E(Y_{ij}) = \mu_{ij}R_{ij} = u_{ij}\mu_{ij}t_{ij} + (1-u_{ij})\vartheta_{ij}\mu_{ij} + (1-u_{ij})\frac{\mu_{ij}}{\mu_{i0}}EBO_{i0}(S_{i0}) \quad (38)$$

To calculate $Var\ (Y_{0j})$, consider the number of backorders of LRU type i at the depot repair center and the number of backorders caused by the operational repair center j in the depot repair center as X_{i0} and X_{ij}, respectively. In these circumstances, the probability of $X_{ij} = x_{ij}$ in condition of $X_{i0} = x_{i0}$ and $x_{ij} \leq x_{i0}$ has a binomial distribution with an average $x_{i0}\left(\frac{\hat{\mu}_{ij}}{\mu_{i0}}\right)$ and variance $x_{i0}\left(\frac{\hat{\mu}_{ij}}{\mu_{i0}}\right)\left(1-\frac{\hat{\mu}_{ij}}{\mu_{i0}}\right)$. So, we have:

$$P\{X_{ij} = x_{ij} | X_{i0} = x_{i0}\} = \binom{x_{i0}}{x_{ij}}\left(\frac{\hat{\mu}_{ij}}{\mu_{i0}}\right)^{x_{ij}}\left(1-\frac{\hat{\mu}_{ij}}{\mu_{i0}}\right)^{x_{i0}-x_{ij}}, \quad (39)$$

Where $\hat{\mu}_{ij} = (1-u_{ij})\mu_{ij}$ is defined. According to equation (39), the expected number of LRU backorders in the operational repair center j will be as follows:

$$E[X_{ij}|S_{i0}] = E[X_{ij}] = E_{X_{i0}}\left[E_{X_{ij}}[X_{ij}|X_{i0}]\right] = E_{X_{i0}}\left[\frac{\hat{\mu}_{ij}}{\mu_{i0}}X_{i0}\right] = \frac{\hat{\mu}_{ij}}{\mu_{i0}}EBO_{i0}(S_{i0}) \quad (40)$$

Equation (40) shows that the expected number of LRU backorders in the jth operational repair center is a fraction of the expectation backorders in the depot repair center ($EBO_{i0}(S_{i0})$). Now, to calculate the variance of X_{ij} we have:

$$Var[X_{ij}|S_{i0}] = E[X_{ij}{}^2|S_{i0}] - E[X_{ij}|S_{i0}]^2 \tag{41}$$

According to Muckstadt (2005), equation (41) can be obtained as below:

$$Var[X_{ij}|S_{i0}] = \frac{\hat{\mu}_{ij}}{\mu_{i0}}\left(1 - \frac{\hat{\mu}_{ij}}{\mu_{i0}}\right) EBO_{i0}(S_{i0}) + \left(\frac{\hat{\mu}_{ij}}{\mu_{i0}}\right)^2 Var(X_{i0}|S_{i0}) \tag{42}$$

In equation (42) the value of $Var(X_{i0}|S_{i0})$ is calculated according to equation (43).

$$Var(X_{i0}|S_{i0}) = E_{X_{i0}}(X_{i0}{}^2|S_{i0}) - EBO_{i0}(S_{i0})^2 \tag{43}$$

In equation (43) the amount of $E_{X_{i0}}(X_{i0}{}^2|S_{i0})$ is unknown. It should be noted that X_{i0} indicates the number of backorders of LRU type i in the depot repair center. If $X_{i0} = x_{i0}$, the number of requested LRU i from the depot repair center is as many as x_{i0} units more than warehouse inventory. If y represents the number of LRU i in the resupply cycle which is requested from the depot repair center, then $y - S_{i0} = x_{i0}$.

$$E_{X_{i0}}(X_{i0}{}^2|S_{i0}) = \sum_{y>S_{i0}} (y - S_{i0})^2 Pr(y|\mu_{i0}t_{i0}) \sum_{y>S_{i0}} (y - S_{i0})^2 Pr(y|\mu_{i0}t_{i0}) \tag{44}$$

In equation (44), $Pr(y|\mu_{i0}t_{i0})$ indicates the probability that y units of LRU i have been requested in the resupply process of the depot repair center. According to the Palm theory, $Pr(Y|\mu_{i0}t_{i0}) = \frac{e^{-\mu_{i0}t_{i0}}(\mu_{i0}t_{i0})^y}{y!}$. Sherbrooke (1986) showed that equation (44) can be calculated as a recurrence relation. Equation (42), indicates a part of the variance related to the number of items LRU type i in the resupply cycle to the repair center j which is located in the depot repair center. To calculate the exact value of variance, $Var(Y_{ij})$, it is necessary to add the variance of the number of LRU i at the order, shipping and receiving time from the depot repair center to jth operational repair center (ϑ_{ij}) and also variance for the number of LRU i under repair time (t_{ij}) in operational repair center j. In this case we have:

$$\begin{aligned} Var(Y_{ij}) = {} & u_{ij}\mu_{ij}t_{ij} + (1 - u_{ij})\mu_{ij}\vartheta_{ij} + (1 - u_{ij})\frac{\mu_{ij}}{\mu_{i0}}\left(1 - (1 - u_{ij})\frac{\mu_{ij}}{\mu_{i0}}\right) EBO_{i0}(S_{i0}) \\ & + \frac{\left((1 - u_{ij})\mu_{ij}\right)^2}{\mu_{i0}{}^2} Var(X_{i0}|S_{i0}) \end{aligned} \tag{45}$$

Sherbrooke (1986) showed that Y_{ij} has a negative binomial distribution with $E(Y_{ij}) = \frac{r(1-p)}{p}$ and $Var(Y_{ij}) = \frac{r(1-p)}{p^2}$. So we have:

$$p = \frac{E(Y_{ij})}{Var(Y_{ij})}, \quad r = \frac{pE(Y_{ij})}{1 - p} \tag{46}$$

In equation (46), $E(Y_{ij})$ and $Var(Y_{ij})$ respectively are obtained from equations (38) and (45). So, the different probabilities of Y_{ij} will be calculated as follows:

$$Pr(Y_{ij} = 0) = p^r \tag{47}$$

$$Pr(Y_{ij} = y) = Pr(Y_{ij} = y - 1)\frac{(r + y - 1)(1 - p)}{y} \tag{48}$$

As mentioned earlier, the more deficiency in demanded LRU items, the more dissatisfaction customers have. Because they have to spend more time to repair of their defective product. $EBO_{ij}(S_{ij}|S_{i0})$ is the expected number of backorders of LRU type i in the operational repair center j, which indicates the customers' dissatisfaction of customer zones j. Therefore, S_{ij} should be determined in such a way that $EBO_{ij}(S_{ij}|S_{i0})$ would be minimized. In order to calculate the expected number of backorders of LRU i in the operational repair center j, the probability function of Y_{ij} should be defined. As previously shown, Y_{ij} has been approximated by negative binomial distribution with parameters $r$and p according to equation (42). Therefore, $EBO_{ij}(S_{ij}|S_{i0})$ can be obtained as follows:

$$EBO_{ij}(S_{ij}|S_{i0}) = \sum_{y>s_{ij}} (y - S_{ij}) Pr(Y_{ij} = y|S_{i0}) = \sum_{y>s_{ij}} (y - (S_{ij} - 1) - 1) Pr(Y_{ij} = y|S_{i0}) = \tag{49}$$

$$\sum_{y>s_{ij}} \left(y-(S_{ij}-1)\right) Pr\left(Y_{ij}=y|S_{i0}\right) - \sum_{y>s_{ij}} Pr\left(Y_{ij}=y|S_{i0}\right)$$

$$= \left(\sum_{y>s_{ij}} \left(y-(S_{ij}-1)\right) Pr\left(Y_{ij}=y|S_{i0}\right) + \left(S_{ij}-(S_{ij}-1)\right) Pr\left(Y_{ij}=S_{ij}|S_{i0}\right)\right) \tag{50}$$
$$- \sum_{y>s_{ij}} Pr\left(Y_{ij}=y\right) - \left(S_{ij}-(S_{ij}-1)\right) Pr\left(Y_{ij}=S_{ij}|S_{i0}\right)$$

$$= \sum_{y>s_{ij}-1} \left(y-(S_{ij}-1)\right) Pr\left(Y_{ij}=y|S_{i0}\right) - \sum_{y>s_{0j}-1} Pr\left(Y_{ij}=y|S_{i0}\right) \tag{51}$$

$$EBO_{ij}\left(S_{ij}|S_{i0}\right) = EBO_{ij}\left(S_{ij}-1|S_{i0}\right) - \left(1-\sum_{y\le s_{ij}-1} Pr\left(Y_{ij}=y|S_{i0}\right)\right) \tag{52}$$

According to equation (52) with the addition of a spare unit to repair center j, the expected number of backorders for LRU items, reduces as much as $\left(1-\sum_{y\le s_{ij}-1} Pr\left(Y_{ij}=y|S_{i0}\right)\right)$. Also $Pr\left(Y_{ij}=y|S_{i0}\right)$ is obtained according to equations (47) and (48). Equation (52) is a recurrence relation which at point $S_{ij}=0$ is calculated as below:

$$EBO_{ij}(0|S_{i0}) = \sum_{y>0} (y-0)\, Pr\left(Y_{ij}=y|S_{i0}\right) = E(Y_{ij}) \tag{53}$$

The value of $EBO_{ij}(0|S_{i0})$ can be calculated recursively according to equations (52) and (53). It is worth mentioning that values of $E(Y_{ij})$ and $Var(Y_{ij})$ are related to $EBO_{i0}(S_{i0})$. Moreover, $EBO_{i0}(S_{i0})$ is attained according to similar proof.

3-4-2- Retrieval costs of spare parts in two-echelon two indenture warranty distribution network

Due to the proposed maintenance policy, during the extended warranty period, products of customer zone j at time intervals of Δ are sent to operational repair center j for doing PM. In this center at first the product is inspected that its cost is equivalent to IC_j. The total inspection cost of products for customer zone j is obtained as follows:

$$TIC_j = IC_j n_2 L_j \quad \forall j = 1,2,\dots,M \tag{54}$$

Based on equation (16), if recovery of LRU type i is required then with probability of u_{ij} it is done in operational repair center j that has costs of C_{ij}^R and with the probability $(1-u_{ij})$, the retrieval action takes place at the depot repair center with C_{i0}^R cost. As a result, for LRU i the total retrieval costs in operational and depot repair centers are attained according to the relations (55) and (56), respectively.

$$TCR_{ij}(\Delta,\acute{\Delta},r_i,\acute{r}_i) = C_{ij}^R \mu_{ij}\left(\Delta,\acute{\Delta},r_i,\acute{r}_i\right) EWu_{ij} \quad i=1,2,\dots,K, \forall j=1,2,\dots,M \tag{55}$$

$$TCR_{i0}(\Delta,\acute{\Delta},r_i,\acute{r}_i) = C_{i0}^R \mu_{i0}\left(\Delta,\acute{\Delta},r_i,\acute{r}_i\right) EWu_{i0} \quad i=1,2,\dots,K \tag{56}$$

4- Two echelon-two indenture extended warranty distribution network model under imperfect preventive maintenance policies

After presenting the model components including maintenance policy and spare parts logistic, two-echelon two-indenture extended warranty distribution network model under imperfect preventive maintenance policies can be expressed from the third party perspective. In this regard, 3P purpose is to determination of S_{ij}, S_{i0}, Δ and r_i so that during the extended warranty period in addition to controlling of maintaining and logistic cost of product component, the expected number of backorders in the operational repair centers would be minimized.

$$\min \sum_{i=1}^{K}\sum_{j=1}^{M} EBO_{ij}\left(S_{ij}|S_{i0},\Delta,\acute{\Delta},r_i,\acute{r}_i\right) \tag{57}$$

Subject to:

$$\sum_{i=1}^{K} c_i\left(S_{i0}+\sum_{j=1}^{M} S_{ij}\right)+\sum_{j=0}^{M} TCM_j(\Delta,\acute{\Delta},r_i,\acute{r}_i)+\sum_{j=0}^{M} TPM_j\,(\Delta,\acute{\Delta},r_i,\acute{r}_i)+\sum_{j=1}^{M} TIC_j + \sum_{i=1}^{K}\sum_{j=0}^{M} TCR_{ij}(\Delta,\acute{\Delta},r_i,\acute{r}_i)\le \beta \tag{58}$$

(30)،(31)، (49)-(56)

$$0\le S_{ij}\le S^{max}, \Delta^{min}\le \Delta\le \Delta^{max}, r^{min}\le r_i\le r^{max}\ \ intger \quad \forall i\in\{1,2,\dots,K\}, \forall j\in\{0,1,\dots,M\} \tag{59}$$

The objective function seeks to minimize the total expected backorders of LRU items in the operational repair centers that leads to minimize the total customer dissatisfaction supported by 3P. c_i indicates the cost of providing a single LRU type i. Constraint (58) states that total costs of retrieval and providing of spare parts at operational and depot repair centers, corrective and preventive maintenance costs and inspection costs should not exceed predetermined budget (β). As it is evident in equation (59) the amounts of spare parts in operational repair centers (S_{ij}) and depot repair centers (S_{i0}), the PM level imposed by third party (r_i) and the time interval between two consecutive PM (Δ) are variables that their optimal values are determined by the third party. It should be noted, the level of PM imposed by the manufacturer ($\acute{r}_i$) and the distance between two consecutive PM during the warranty period ($\acute{\Delta}$) impact on extended warranty period policies. Thus, the values of expected backorders of LRU items ($EBO_{ij}\left(S_{ij}|S_{i0},\Delta,\acute{\Delta},r_i,\acute{r}_i\right)$), expected costs of retrieval, CM and PM (equations (57) and (58)) are determined according to the mentioned values.

5-The exact hybrid Branch and Bound-Variable Neighborhood Search algorithm

The proposed model is an integer non-linear programming. In this kind of problem as there is a high possibility of local optimum points, determination of the optimum values in medium and large scale problems is practically impossible. In order to meet the challenges in this section an exact hybrid solution approach based on Branch and Bound algorithm is developed that sought to achieve the optimal solution by reducing the search space. For this purpose, the suggested method applies some instruments such as the lower bound estimation, different cutting rules and Variable Neighborhood Search algorithm to estimate the upper bound. In the proposed B&B branching strategy is depth first. The search tree can be in the form of a four-part series and considered as figure 5.

1st segment	2st segment	3st segment			4st segment										
		S_{i0}			S_{ij}										
Δ	r	S_{10}	$\cdots$	S_{K0}	S_{11}	$\cdots$	S_{1M}	$\cdots$	S_{i1}	$\cdots$	S_{iM}	$\cdots$	S_{K1}	$\cdots$	S_{KM}
Depth															
$D=0$	$D=1$														

Figure 5. Display of search tree

The process of searching begins from the depth of one (D = 1), where the 1st segment is located and includes Δ values. The higher depths (2st segment to 4st segment) which respectively indicate r, S_{i0}, S_{ij} have their lowest value ($r=r^{min}, S_{i0}=0, S_{ij}=0$). The algorithm first searches different values of Δ in depth 1, then it goes one depth higher ($D = 2$) and promote variable r to one unit upper and follows the search process from the first depth with amount of Δ^{min}.This procedure will continue until r reaches the maximum value (r^{max}). Then a unit is added to the variable at depth of three ($S_{10}=1$) and the algorithm goes back to depth one while $\Delta=\Delta^{min}$ and $r=r^{min}$. In general, if the variable in depth $D = d-1$ reaches to its maximum value, the algorithm first increases the variable in depth $D = d$ one unit upper, then set all variables in the lower depths on the lowest value ($\Delta=\Delta^{min}, r=r^{min}, 0,\dots,0, c_d+1,\dots,c_{K(M+1)+2}$) and continues the search process from the first depth. Figure 6 shows the described process. According to K different value for the index i, (M + 1) values for the index j, one value for Δ and r, the search tree includes the maximum depth of $K(M+1)+2$.

Depth								
$D=1$	$D=2$	$D=3$	$\cdots$	$D=d-1$	$D=d$	$D=d+1$	$\cdots$	$D=K(M+1)+2$
Δ^{max}	r^{max}	S_1^{max}	$\cdots$	S_{d-2}^{max}	c_d	c_{d+1}	$\cdots$	$c_{K(M+1)+2}$

Δ^{min}	r^{min}	0	$\cdots$	0	c_d+1	c_{d+1}	$\cdots$	$c_{K(M+1)+2}$

Figure 6. Search process of proposed Branch and Bound algorithm

Suppose $(\Delta, r, S)_d$ indicates branch of $(c_1, c_2, \dots, c_d, c_{d+1}, ..)$ in depth of $D = d$. For this branch we have the following three conditions:

I. If the objective function value is greater than the upper bound ($UB < f_{(\Delta,r,S)_d}$), then branch $(\Delta, r, S)_d$ will cut.

II. If the lower bound of that branch is higher than the upper bound ($UB < LB((\Delta, r, S)_d)$) then branch $(\Delta, r, S)_d$ will cut.

III. If the branch $(\Delta, r, S)_d$ become infeasible, then it will cut.

In the mentioned cutting policies, the upper and lower bound values (UB and $LB((\Delta, r, S)_d)$) are respectively obtained, in accordance with sections 5-1 and 5-2.

5-1- The upper bound estimation of Branch and Bound algorithm using Variable Neighborhood Searching algorithm

One of the most important factors affecting the performance of the B&B algorithm is determination of proper bounds for it. To do this, using metaheuristic algorithms in optimization literature is customary (Meyer et al., 2009). One of the efficient metaheuristics in this regard is variable neighborhood searching approach (VNS) that has a suitable search speed through the search space; VNS is inspired by the fact that using different neighborhoods through search of the solution space can lead to different optimal solutions and since the global optimum solution is a local optimum solution, then it is more possible to achieve it (Talbi, 2009). If $NH_e (e = 1, \dots, e_{max})$ is defined as the set of neighborhood structures, the algorithm at first starts to search with NH_1, If there is no improvement using it, the next neighborhood structure is applied. As soon as improvement is seen, the algorithm goes back to the first neighborhood structure. In other words, VNS searches for solutions with higher quality by sequential use of predefined neighborhood structures. In this paper, VNS approach is used as upper bound estimator of B&B algorithm and has the following steps:

Step 0 (solution representation): in the proposed VNS algorithm, values of (S_{i0}, S_{ij}) in the form of a matrix $K \times (M + 1)$ are displayed.

$$\left[\begin{array}{c|cccccc} S_{10} & S_{11} & \cdots & S_{1j} & \cdots & S_{1M} \\ \vdots & \vdots & & \vdots & & \vdots \\ \boldsymbol{S_{i0}} & S_{i1} & \cdots & \boldsymbol{S_{ij}} & \cdots & S_{iM} \\ \vdots & \vdots & & \vdots & & \vdots \\ S_{K0} & S_{K1} & \cdots & S_{Kj} & \cdots & S_{KM} \end{array}\right]$$

Figure 7. Display of (S_{i0}, S_{ij}) values in coded space

As it is seen in figure 7, the coded matrix has two parts, each set includes S_{i0} and S_{ij}.

Step 1: Put $NE_e \leftarrow 1$ and for solution (Δ, r, S) calculate the value of the objective function ($f_{(\Delta,r,S)}$).

Step 2: Repeat steps 3 to 11 IT_{max} iterations.

Step 3: If $NE_e = 1$ then do below steps, otherwise go to Step 4.

Step 3-1- Create the random number i in range of $[1, K]$

Step 3-2- Create the random number j in range of $[0, M]$

Step 3-3- Create the random number s in range of $[0, S^{max}]$
Step 3-4- If $S(i,j) - s \geq 0$ then $S(i,j) \leftarrow S(i,j) - s$
Step 3-5- $(\Delta, r, S)^{new} \leftarrow S(i,j) \cup \{\Delta, r\}$
Step 4- If $NH_e = 2$ then do below steps, otherwise go to Step 5
Step 4-1- Create the random number i in range of $[1, K]$
Step 4-2- Create the random number j in range of $[0, M]$
Step 4-3- Create the random number s in range of $[0, S^{max}]$
Step 4-4- If $S(i,j) + s \leq S^{max}$ then $S(i,j) \leftarrow S(i,j) + s$
Step 4-5- $(\Delta, r, S)^{new} \leftarrow S(i,j) \cup \{\Delta, r\}$
Step 5- if $NH_e = 3$ then do below steps, otherwise go to Step 6
Step 5-1- Create the random number i in range $[\Delta^{min}, \Delta^{max}]$
Step 5-2- If $\Delta - i \geq \Delta^{min}$ then $\Delta \leftarrow \Delta - i$
Step 5-3- $(\Delta, r, S)^{new} \leftarrow \Delta \cup \{S, r\}$
Step 6- If $NH_e = 4$ then do below steps, otherwise go to Step 7
Step 6-1- Create the random number i in range $[\Delta^{min}, \Delta^{max}]$
Step 6-2- If $\Delta + i \leq \Delta^{max}$ then $\Delta \leftarrow \Delta + i$
Step 6-3- $(\Delta, r, S)^{new} \leftarrow \Delta \cup \{S, r\}$
Step 7- If $NH_e = 5$ then do below steps, otherwise go to Step 8
Step 7-1- Create the random number i in range $[r^{min}, r^{max}]$
Step 7-2- If $r - i \geq r^{min}$ then $r \leftarrow r - i$
Step 7-3- $(\Delta, r, S)^{new} \leftarrow r \cup \{S, \Delta\}$
Step8- If $NH_e = 6$ then do below steps, otherwise go to Step 9
Step 8-1- Create the random number i in range $[r^{min}, r^{max}]$
Step 8-2- If $r + i \leq r^{max}$ then $r \leftarrow r + i$
Step 8-3- $(\Delta, r, S)^{new} \leftarrow r \cup \{S, \Delta\}$
Step9- Calculate the objective function of the new solution ($f^{new}_{(\Delta,r,S)}$)
Step 10- If $f^{new}_{(\Delta,r,S)} < f_{(\Delta,r,S)}$ then
Step 10-1- Put $(\Delta, r, S) \leftarrow (\Delta, r, S)^{new}$ and $f_{(\Delta,r,S)} \leftarrow f^{new}_{(\Delta,r,S)}$
Step 10-2- Put $NH_e \leftarrow 1$
Step 11- If $f^{new}_{(S,\Delta)} \geq f_{(\Delta,r,S)}$ then put $NE_e \leftarrow NE_e + 1$ and go to Step 3
Step 12- Display (Δ, r, S) and $f_{(\Delta,r,S)}$

5-2- Estimation of lower bound

The lower bound of the proposed Branch and Bound algorithm (LB) is determined based on two factors: (1) Depth-first strategy (2) The values of S_d^{max}, Δ^{max} and r^{max}. Consider state of c in depth d, in this situation by optimizing the following mathematical programming, LB is obtained for solution of $\mathcal{S} = (S_1^{max}, S_2^{max}, \dots, S_{d-3}^{max}, c, 0, \dots, 0)$.

$$LB(\mathcal{S}, \Delta) = min \sum_{j=1}^{M} EBO_{ij}(\mathcal{S}, \Delta, r), \tag{60}$$

Subject to:

$$, \Delta^{min} \leq \Delta \leq \Delta^{max}, r^{min} \leq r \leq r^{max} \; integer \tag{61}$$

According to equation (60), the lower bound of the B&B is obtained by minimizing the total backorders of LRU items in operational repair centers in the different values of Δ and r when value of spare parts are in the $\mathcal{S}$ level.

6- Numerical results

In this section in order to evaluate the two echelon-two indenture extended warranty distribution network model under imperfect preventive maintenance using exact hybrid algorithm of B&B and VNS, a set of numerical examples are introduced. For this purpose, at first extended warranty distribution network including set of customers, supported product, structure of depot and operational repair centers is described. Then the results of the model optimization and sensitive analysis will be expressed. The proposed solution algorithm is coded in MTALAB R2013a and all the calculations were conducted on a Core i5/ CPU 2.4 GHz/ RAM 4GB system.

6-1-Description of numerical example

Extended warranty distribution network consists of two operational repair centers and one depot repair center, so that the first repair operational center covers 1000 customers ($L_1 = 1000$) and the second one covers 500 customers ($L_2 = 500$). The sold product is a series system consisting of two LRUs, each of which has the failure process in accordance with the Weibull distribution. Table 1 indicates the details of each LRU, parameters of EWDN, proposed maintenance approach and the related costs.

Table 1. Parameters of the numerical example

Parameter	Value	Parameter	value	Parameter	Value	Parameter	Value	Parameter	Value
γ_1^{lower}	0.00001	C_{12}^R	1500	θ_2	1000	u_{21}	0.75	t_{11}	3 days
γ_1^{upper}	0.00020	C_{22}^R	1300	β_1	5	u_{22}	0.60	t_{12}	2 days
γ_2^{lower}	0.000001	C_{10}^R	1500	θ_2	1000	ϑ_{11}	1 day	t_{20}	2 days
γ_2^{lower}	0.00001	C_{20}^R	2000	c_1	1500	ϑ_{12}	2 days	t_{21}	4 days
C_{11}^R	1000	ξ	0.1	c_2	2000	ϑ_{21}	1 day	t_{22}	3 days
C_{21}^R	900	β_1	3	u_{11}	0.70	ϑ_{21}	2 days	IT_{max}	100
				u_{12}	0.60	t_{10}	1 day	∂_{ij}	0.2

In order to evaluate the proposed model properly, the numerical examples are considered under two general scenarios, each of which is divided into 4 sub-scenarios. In the first scenario, the duration of the base warranty period is 1 year ($W = 1$) and the manufacturer has applied no PM actions. Also the duration of the extended warranty period of the third party is 2 years ($EW = 2$) and it is exerted after expiration of the base warranty period. In second scenario the manufacturer does not provide the basic warranty along with the product ($W = 0$) and extended warranty period is considered 3 years ($EW = 3$). Table 2 shows the details of scenarios.

Table 3 provided the results of optimizing two-echelon two-indenture warranty distribution network under imperfect preventive maintenance using the proposed exact hybrid solution approach of B&B and VNS on the described numerical examples. In table 3, Z^* column shows optimum value of the objective function for each sub-scenario. The β column demonstrates the budget value allocated to each product unit. Moreover, S^* column shows the optimum values of spare parts at two operational and main repair centers. In this regard, a six element row vector was used, revealing the first element as S_{10}, the second and third elements as S_{11} and S_{12}, respectively, and fourth, fifth and sixth elements as S_{20}, $S_{21}$and S_{22}. Columns of Δ^* and U^* express the optimum values of time intervals between two consecutive PMs and PM level. Eventually, running time in second of the proposed algorithm is indicated in CPU column. It should be noted, the infeasible solutions are also presented in table 4 with a dark line.

Table 2. Scenarios information

Scenario	W	EW	$\Omega_{11}, \Omega_{12}, \Omega_{21}, \Omega_{22}$	$C_{11}^{cm}, C_{12}^{cm}, C_{21}^{cm}, C_{22}^{cm}$	IC
P11	1	2	100,90,110,100	350,300,250,200	100
P12	1	2	50,40,60,50	350,300,250,200	100
P13	1	2	100,90,110,100	550,500,450,400	100
P14	1	2	100,90,110,100	350,300,250,200	50
P21	0	3	100,90,110,100	350,300,250,200	100
P22	0	3	50,40,60,50	350,300,250,200	100
P23	0	3	100,90,110,100	550,500,450,400	100
P24	0	3	100,90,110,100	350,300,250,200	50

Table 3 indicates that:

1. In all sub-scenarios, the expected number of backorders in operational repair centers decreases, as budget increases.
2. Although the value of base warranty in the first scenario was one year and the manufacturer would not perform PM actions, this condition caused less costs for third party, compared to the second scenario. In the second scenario, third party must pay the cost of PM actions in addition to CM costs during base warranty period. It should be noted, failure function of LRU items is additive and using PM actions during the first days of life for these items has little economic justification. Therefore, the required currency for each product in the second scenario is 1400-200 more than what is required in the first scenario.
3. At any level of backorder, increased durations between PM actions (Δ) led to performing higher levels of PM (r) by third party, as observed in figure 8-(b). This might be due to the fact that high-quality preventive maintenance leads to more rejuvenation of the product items. Hence, the time interval between two consecutive PMs can be increased.
4. Inspection cost has significant effects on the budget value and the expected number of backorder for LRU items in operational repair centers. This is more important when the associated budget is low. This is due to the fact that the involved budget to inspection process could be decreased by technological change and the saved budget is allocated to purchase more spare parts, which leads to reduction of total expected backorders of LRU items.
5. When a high level of PM actions is performed, decreasing value of Δ intensely enhances the required budget, compared to when a lower level of PM is applied. As a result, in a high level of required PM, third party increases the time intervals between two consecutive PMs, which prevents the growth of needed budget (see figure 8-(a)).
6. The expected number of backorders of LRU items significantly increases in the presence of a higher CM cost. This issue is more important when manufacturer does not provide base warranty for the customers (second scenario) as third party has to take the responsibility of CM repairs during base warranty.

The highest running time of problem solving was 1227 seconds, which was related to the P24 scenario. It should be mentioned that exhaustive search of this situation requires $10^6 * 52 * 10 = 520000000$ iterations of problem solving, in which the expected number of backorders and costs of system during two periods of base and extended warranties must be determined. Therefore, it could be observed that the proposed exact hybrid approach has high performance in the path of EWDN optimization. Variation of the objective function and needed budget against variations of Δ and r is presented in figure 8.

Table 3. Results of optimizing two-echelon two-indenture warranty distribution network under imperfect preventive maintenance using the proposed exact hybrid solution approach of B&B and VNS

P11	Z^*	β	S^*	Δ^*	U^*	**CPU(s)**	**P21**	Z^*	β	S^*	Δ^*	U^*	**CPU(s)**
	0	5000	[8 9 4 1 9 8]	27	10	0.013		0	7000	[1 0 0 0 0 0]	32	10	0.056
	0.7358	4000	[2 9 6 7 10 10]	29	5	391		0.1201	6000	[2 10 7 1 9 7]	20	5	993
	0.7358	3000	[5 9 5 7 10 10]	35	10	383		0.1201	5000	[2 10 8 1 9 9]	27	5	984
	0.7358	2000	[1 8 5 7 10 10]	35	1	607		0.1201	4000	[2 10 7 1 9 5]	27	1	976
	0.7358	1900	[1 8 5 7 10 10]	35	1	588		1.1831	3400	[9 10 10 1 9 5]	40	1	970
	0.7361	1890	[1 8 5 6 10 10]	35	1	559		1.1831	3370	[9 10 10 1 9 5]	40	1	1109
	0.8067	1880	[1 7 4 3 10 7]	35	1	547		1.3160	3360	[4 10 8 1 8 4]	40	1	752
	1.9742	1870	[1 5 2 1 10 5]	35	1	238		2.7260	3350	[1 10 5 1 6 3]	40	1	357
	6.5042	1860	[1 4 1 1 7 2]	35	1	133		7.2281	3340	[1 9 4 1 2 1]	40	1	245
	13.2796	1850	[1 2 1 1 3 0]	35	1	104		14.7580	3330	[1 6 2 0 0 0]	40	1	91
	-	1840	-	-	-	-		-	3320	-	-	-	-
P12	Z^*	β	S^*	Δ^*	U^*	**CPU(s)**	**P22**	Z^*	β	S^*	Δ^*	U^*	**CPU(s)**
	0	3000	[5 6 9 1 9 8]	29	10	0.013		0	5000	[1 0 0 0 0 0]	23	9	0.084
	0.7358	2000	[1 9 9 7 10 10]	36	5	422		0.014	4000	[1 0 0 0 0 0]	32	10	0.054
	0.7358	1900	[1 8 5 7 10 10]	35	5	447		0.1201	3500	[2 10 7 1 9 5]	27	1	978
	0.7358	1830	[1 8 5 7 10 10]	35	1	562		0.1201	3200	[2 10 7 1 9 5]	27	1	970
	0.7895	1820	[1 8 4 3 10 7]	35	1	411		0.1201	3140	[2 10 7 1 9 5]	27	1	918
	1.7940	1810	[1 5 3 1 10 5]	35	1	226		0.1258	3130	[1 10 7 1 9 5]	27	1	681
	5.8858	1800	[0 3 0 1 9 3]	35	1	138		0.9527	3120	[1 9 4 1 7 3]	27	1	354
	12.4435	1790	[0 1 0 1 5 1]	35	1	90		4.3197	3110	[1 8 3 1 3 1]	27	1	126
	-	1780	-	-	-	-		10.8420	3100	[1 5 1 0 0 1]	27	1	96
								-	3090	-	-	-	-
P13	Z^*	β	S^*	Δ^*	U^*	**CPU(s)**	**P23**	Z^*	β	S^*	Δ^*	U^*	**CPU(s)**
	0	5000	[9 0 0 2 10 7]	27	10	0.061		0	7000	[1 0 0 0 0 0]	32	10	0.059
	0.7358	4000	[1 8 9 7 10 10]	29	5	376		0.1201	6000	[2 10 7 1 9 8]	20	5	979
	0.7358	3000	[2 9 8 7 10 10]	27	5	395		0.1201	5000	[2 10 8 1 9 8]	28	5	1043
	0.7538	2000	[1 8 5 7 10 10]	35	1	349		0.1201	4000	[2 10 7 1 9 5]	27	1	1081
	0.7358	1930	[1 8 5 7 10 10]	35	1	317		1.1831	3500	[10 10 10 1 9 5]	40	1	1193
	0.7765	1920	[1 8 5 3 10 7]	35	1	233		1.1954	3400	[4 10 10 1 9 5]	40	1	881
	1.6457	1910	[1 5 2 2 10 5]	35	1	221		1.7553	3390	[2 10 7 1 7 3]	40	1	529
	5.6974	1900	[1 4 1 1 7 3]	35	1	140		1.9661	3380	[2 10 7 1 6 3]	40	1	510
	12.2909	1890	[1 2 1 1 4 0]	35	1	90		10.7487	3370	[1 9 4 0 0 0]	40	1	101
	-	1880	-	-	-	-		19.6232	3360	[1 3 0 0 0 0]	40	1	100
								21.6213	3350	[1 1 0 0 0 0]	40	1	100
								-	3340	-	-	-	-
P14	Z^*	β	S^*	Δ^*	U^*	**CPU(s)**	**P24**	Z^*	β	S^*	Δ^*	U^*	**CPU(s)**
	0	5000	[0 5 5 1 9 6]	30	10	0.046		0	7000	[1 0 0 0 0 0]	32	10	0.052
	0.7358	4000	[2 7 0 7 10 10]	35	10	398		0.1201	6000	[2 10 10 1 9 5]	20	5	989
	0.7358	3000	[1 9 8 7 10 10]	35	5	397		0.1201	5000	[2 10 10 1 9 8]	29	5	996
	0.7358	2000	[2 9 9 7 10 10]	36	5	414		0.1201	4000	[2 10 7 1 9 5]	27	1	991
	0.7358	1800	[1 8 5 7 10 10]	35	1	588		0.1201	3500	[2 10 7 1 9 5]	27	1	968
	0.7361	1790	[1 8 5 6 10 10]	35	1	556		0.5879	3300	[1 10 5 1 7 3]	27	1	1227
	0.8067	1780	[1 7 4 3 10 7]	35	1	396		2.7260	3200	[1 10 5 1 6 3]	40	1	348
	1.9742	1770	[1 5 2 1 10 5]	35	1	231		7.2281	3190	[1 9 4 1 2 1]	40	1	131
	6.5042	1760	[1 4 1 1 7 2]	35	1	133		8.6511	3180	[1 8 3 1 2 1]	40	1	118
	13.2796	1750	[1 2 1 1 3 0]	35	1	100		16.6511	3170	[1 5 1 0 0 0]	40	1	92
	-	1740	-	-	-	-		-	3160	-	-	-	-

6-2- Analyzing effect of manufacturer maintenance policies on third party strategies

In the two main scenarios proposed in the previous section, it was assumed that manufacturer did not use PM actions during the period of base warranty. In this section, the effect of manufacturer maintenance policies during the base warranty period, including the interval between two consecutive PMs ($\acute{\Delta}$) and level of PM actions ($\acute{r}$), on third party strategies during the extended warranty period are evaluated. For this purpose, regarding the information of P11 scenario and by considering $\acute{\Delta} \in \{12,26,40,52\}$ (in weeks) and $\acute{r} \in \{2,5,8\}$, the model is optimized under 12 new scenarios. The optimization results are provided in table 4.

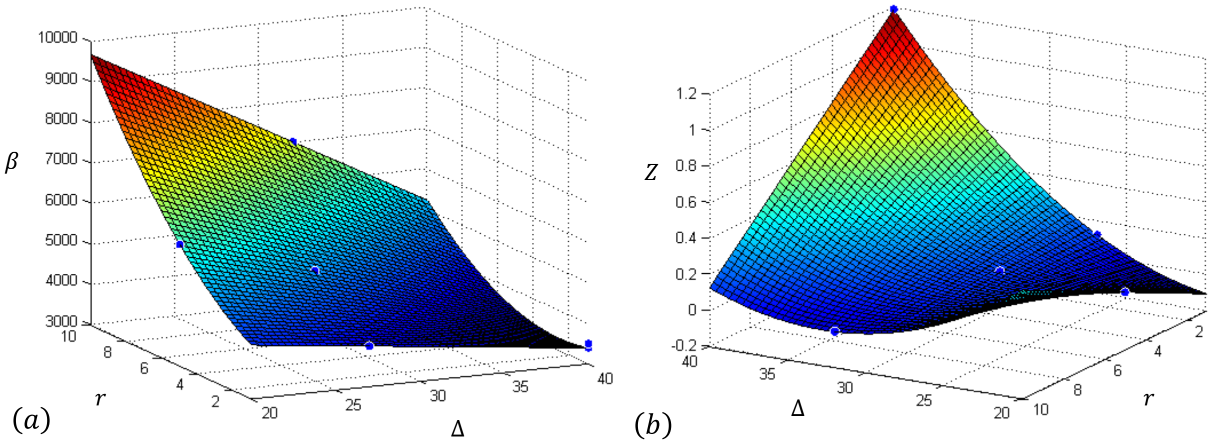

Figure 8. Variation of the objective function and needed budget against variations of Δ and r

Table 4. Results of analyzing effect of manufacturer maintenance policies during warranty period on third party strategies during extended warranty period

			Scenario 1								Scenario 2				
$\acute{\Delta}$	$\acute{r}$	Z^*	β	S^*	Δ^*	U^*	CPU(s)	$\acute{\Delta}$	$\acute{r}$	Z^*	β	S^*	Δ^*	U^*	CPU(s)
12	2	0	6000	[1 9 8 1 9 9]	30	10	0.062	26	2	0	6000	[1 9 8 1 9 9]	30	10	0.060
12	2	0	5000	[1 2 1 1 3 0]	35	10	0.048	26	2	0	5000	[7 6 0 1 9 9]	30	10	0.046
12	2	0.7358	4000	[3 9 6 7 10 10]	35	10	393	26	2	0.7358	4000	[2 10 7 1 9 5]	35	10	377
12	2	0.7358	3000	[2 9 6 7 10 10]	29	5	495	26	2	0.7358	3000	[2 9 6 7 10 10]	29	5	398
12	2	0.7358	2000	[1 8 5 7 10 10]	35	1	624	26	2	0.7358	2000	[2 9 6 7 10 10]	29	5	398
12	2	0.7358	1900	[1 8 5 7 10 10]	35	1	609	26	2	0.7358	1900	[1 8 5 7 10 10]	35	1	590
12	2	0.7361	1890	[1 8 5 6 10 10]	35	1	559	26	2	0.7361	1890	[1 8 5 6 10 10]	35	1	554
12	2	0.8067	1880	[1 7 4 3 10 7]	35	1	526	26	2	0.8067	1880	[1 7 4 3 10 7]	35	1	392
12	2	1.9742	1870	[1 5 2 1 10 5]	35	1	214	26	2	1.9742	1870	[1 5 2 1 10 5]	35	1	205
12	2	6.5042	1860	[1 4 1 1 7 2]	35	1	134	26	2	6.5042	1860	[1 4 1 1 7 2]	35	1	130
12	2	13.2796	1850	[1 2 1 1 3 0]	35	1	92	26	2	13.2796	1850	[1 2 1 1 3 0]	35	1	89
12	2	-	1840	-	-	-	-	26	2	-	1840	-	-	-	-
			Scenario 3								Scenario 4				
$\acute{\Delta}$	$\acute{r}$	Z^*	β^*	S^*	Δ^*	U^*	CPU(s)	$\acute{\Delta}$	$\acute{r}$	Z^*	β^*	S^*	Δ^*	U^*	CPU(s)
12	5	0	6000	[2 9 9 1 10 6]	29	10	0.142	26	5	0	6000	[1 9 8 1 9 9]	35	10	0.055
12	5	0	5000	[2 9 6 1 9 8]	30	10	0.048	26	5	0	5000	[7 6 0 1 9 9]	30	10	0.046
12	5	0.7358	4000	[2 9 8 7 10 10]	39	4	408	26	5	0.7358	4000	[3 9 6 7 10 10]	35	10	379
12	5	0.7358	3000	[1 8 8 7 10 10]	30	5	377	26	5	0.7358	3000	[2 9 6 7 10 10]	29	5	394
12	5	0.7358	2000	[18 5 7 10 10]	35	1	625	26	5	0.7358	2000	[1 8 5 7 10 10]	35	1	603
12	5	0.7358	1940	[1 8 5 7 10 10]	35	1	624	26	5	0.7358	1900	[1 8 5 7 10 10]	35	1	594
12	5	0.7389	1930	[1 8 5 6 10 9]	35	1	561	26	5	0.7361	1890	[1 8 5 6 10 10]	35	1	478
12	5	0.8851	1920	[1 4 1 1 10 5]	35	1	366	26	5	0.8067	1880	[1 7 4 3 10 7]	35	1	393
12	5	2.6606	1910	[1 5 3 1 10 5]	35	1	195	26	5	1.9742	1870	[1 5 2 1 10 5]	35	1	205
12	5	7.6136	1900	[1 2 1 1 7 2]	35	1	120	26	5	6.5042	1860	[1 4 1 1 7 2]	35	1	130
12	5	14.5037	1890	[0 2 0 1 3 0]	35	1	95	26	5	13.2796	1850	[1 2 1 1 3 0]	35	1	89
12	5	-	1880	-	-	-	-	26	5	-	1840	-	-	-	-
			Scenario 5								Scenario 6				
$\acute{\Delta}$	$\acute{r}$	Z^*	β^*	S^*	Δ^*	U^*	CPU(s)	$\acute{\Delta}$	$\acute{r}$	Z^*	β^*	S^*	Δ^*	U^*	CPU(s)
12	8	0	6000	[1 9 8 1 9 9]	30	10	0.055	26	8	0.8559	6000	[2 10 9 7 10 10]	29	5	973
12	8	0	5000	[7 6 0 1 9 9]	30	10	0.061	26	8	0.8559	5000	[2 10 9 7 10 10]	27	5	979
12	8	0.7358	4000	[3 9 6 7 10 10]	35	10	387	26	8	0.8559	4000	[2 10 8 7 10 10]	30	5	988
12	8	0.7358	3000	[2 9 6 7 10 10]	29	5	413	26	8	0.8589	3100	[2 10 7 7 10 10]	27	1	959
12	8	0.7538	2000	[1 8 5 7 10 10]	35	1	669	26	8	0.8589	3060	[2 10 7 7 10 10]	27	1	952
12	8	0.7358	1900	[1 8 5 7 10 10]	35	1	646	26	8	0.8653	3050	[1 10 7 5 10 9]	27	1	922
12	8	0.7361	1890	[1 8 5 6 10 10]	35	1	578	26	8	1.1878	3040	[1 10 5 2 10 6]	27	1	742
12	8	0.8067	1880	[1 7 4 3 10 7]	35	1	412	26	8	3.7162	3030	[1 8 3 1 9 4]	27	1	389
12	8	1.9742	1870	[1 5 2 1 10 5]	35	1	215	26	8	9.3391	3020	[1 6 3 1 5 2]	27	1	145
12	8	6.5042	1860	[1 4 1 1 7 2]	35	1	135	26	8	16.4705	3010	[1 5 2 0 0 2]	27	1	89
12	8	13.2796	1850	[1 2 1 1 3 0]	35	1	97	26	8	-	3000	-	-	-	-
12	8	-	1840	-	-	-	-								

Table 4. Results of analyzing effect of manufacturer maintenance policies during warranty period on third party strategies during extended warranty period (continue)

Scenario 7								Scenario 8							
$\acute{\Delta}$	$\acute{r}$	Z^*	β	S^*	Δ^*	U^*	CPU(s)	$\acute{\Delta}$	$\acute{r}$	Z^*	β	S^*	Δ^*	U^*	CPU(s)
40	2	0	6000	[1 2 1 1 5 2]	35	1	0.072	52	2	0	6000	[1 4 1 1 9 4]	35	1	0.065
40	2	0	5000	[1 2 1 1 5 2]	35	1	0.014	52	2	0	5000	[1 4 1 1 9 4]	35	1	0.013
40	2	0.7358	4000	[2 9 7 7 10 10]	28	5	421	52	2	0.7358	4000	[9 4 5 7 10 10]	36	10	377
40	2	0.7358	3000	[0 0 4 7 10 10]	36	10	435	52	2	0.7358	3000	[2 9 8 7 10 10]	27	5	402
40	2	0.7358	2000	[1 8 5 7 10 10]	35	1	634	52	2	0.7358	2000	[1 8 5 7 10 10]	35	1	421
40	2	0.7358	1900	[1 8 5 7 10 10]	35	1	691	52	2	0.7358	1900	[1 8 5 7 10 10]	35	1	588
40	2	0.7361	1890	[1 8 5 6 10 10]	35	1	643	52	2	0.7361	1890	[1 8 5 6 10 10]	35	1	554
40	2	0.8067	1880	[1 7 4 3 10 7]	35	1	454	52	2	0.8067	1880	[1 7 4 3 10 7]	35	1	394
40	2	1.9742	1870	[1 5 2 1 10 5]	35	1	238	52	2	1.9742	1870	[1 5 2 1 10 5]	35	1	206
40	2	6.5042	1860	[1 4 1 1 7 2]	35	1	151	52	2	6.5042	1860	[1 4 1 1 7 2]	35	1	131
40	2	13.2796	1850	[1 2 1 1 3 0]	35	1	96	52	2	13.2796	1850	[1 2 1 1 3 0]	35	1	88
40	2	-	1840	-	-	-	-	52	2	-	1840	-	-	-	-
Scenario 9								Scenario 10							
$\acute{\Delta}$	$\acute{r}$	Z^*	β^*	S^*	Δ^*	U^*	CPU(s)	$\acute{\Delta}$	$\acute{r}$	Z^*	β^*	S^*	Δ^*	U^*	CPU(s)
40	5	0	6000	[8 8 9 8 7 0]	29	10	0.047	52	5	0	6000	[1 8 8 1 9 6]	27	10	0.063
40	5	0	5000	[4 7 9 0 0 0]	28	10	0.045	52	5	0	5000	[6 3 0 1 9 8]	27	10	0.063
40	5	0	4000	[1 8 6 1 9 7]	27	5	0.105	52	5	0.7358	4000	[9 0 6 7 10 10]	36	10	408
40	5	0	3000	[2 9 8 1 9 7]	29	5	0.459	52	5	0.7358	3000	[4 6 6 7 10 10]	36	10	427
40	5	0	2000	[18 5 1 9 5]	27	1	513	52	5	0.7358	2000	[1 8 5 7 10 10]	35	10	602
40	5	0	1820	[1 8 5 1 9 5]	27	1	384	52	5	0.7358	1900	[1 8 5 7 10 10]	35	1	597
40	5	0.4474	1810	[1 6 4 1 7 3]	27	1	241	52	5	0.7361	1890	[1 8 5 6 10 10]	35	1	562
40	5	2.8514	1800	[1 3 1 1 5 2]	27	1	104	52	5	0.8067	1880	[1 7 4 3 10 7]	35	1	396
40	5	8.5326	1790	[1 2 0 0 2 0]	27	1	89	52	5	1.9742	1870	[1 5 2 1 10 5]	35	1	209
40	5	-	-	-	-	-	-	52	5	6.5042	1860	[1 4 1 1 7 2]	35	1	131
								52	5	13.2796	1850	[1 2 1 1 3 0]	35	1	90
								52	5	-	1840	-	-	-	-
Scenario 11								Scenario 12							
$\acute{\Delta}$	$\acute{r}$	Z^*	β^*	S^*	Δ^*	U^*	CPU(s)	$\acute{\Delta}$	$\acute{r}$	Z^*	β^*	S^*	Δ^*	U^*	CPU(s)
40	8	0	6000	[1 4 1 1 9 4]	35	1	0.013	52	8	0	6000	[5 9 3 1 10 8]	27	10	0.045
40	8	0	5000	[1 4 1 1 9 4]	35	1	0.060	52	8	0	5000	[0 8 0 1 9 7]	27	10	0.046
40	8	0.7358	4000	[7 7 7 7 10 10]	35	10	400	52	8	0.7358	4000	[1 9 5 7 10 10]	30	5	408
40	8	0.7358	3000	[0 3 0 7 10 10]	35	10	383	52	8	0.7358	3000	[2 9 8 7 10 10]	28	5	396
40	8	0.7538	2000	[1 8 5 7 10 10]	35	1	605	52	8	0.7358	2000	[1 8 5 7 10 10]	35	1	589
40	8	0.7358	1900	[1 8 5 7 10 10]	35	1	642	52	8	0.7358	1900	[1 8 5 7 10 10]	35	1	588
40	8	0.7361	1890	[1 8 5 6 10 10]	35	1	557	52	8	0.7361	1890	[1 8 5 6 10 10]	35	1	551
40	8	0.8067	1880	[1 7 4 3 10 7]	35	1	399	52	8	0.8067	1880	[1 7 4 3 10 7]	35	1	389
40	8	1.9742	1870	[1 5 2 1 10 5]	35	1	209	52	8	1.9742	1870	[1 5 2 1 10 5]	35	1	208
40	8	6.5042	1860	[1 4 1 1 7 2]	35	1	131	52	8	6.5042	1860	[1 4 1 1 7 2]	35	1	128
40	8	13.2796	1850	[1 2 1 1 3 0]	35	1	90	52	8	13.2796	1850	[1 2 1 1 3 0]	35	1	89
40	8	-	1840	-	-	-	-	52	8	-	1840	-	-	-	-

According to table 4, it can be concluded that high level of PM action ($\acute{r} = 8$) taken by manufacturer did not necessarily lead to reduction of third party cost. Because not only the time intervals between PM actions ($\acute{\Delta}$) have significant role in this regard, but also the level of applied PM is important as well. According to scenario 6, performing PM actions every 26 weeks at level of $\acute{r} = 8$ by manufacturer increases the expenses of third party. Since doing PM at high levels in early life of LRU items led to delivering these items at high failure rate to third party from the starting point of extended warranty, which increases CM costs.

Moreover, 3P must decrease the value of Δ to reduce failure rate, which leads to increased costs of PM, inspection and requirement of spare parts. According to scenario 9, the best condition for third party is performing PM actions by manufacturer at a medium level $\acute{r} = 5$ during the final stage of base warranty $\acute{\Delta} = 40$. This condition not only decreases the expenses of third party during extended warranty period, but also reduces the total expected of backorders for LRU items in operational repair centers.

Performing PM at a low level $\acute{r} = 2$ by manufacturer had no significant effect on the performance of third party since no significant improvement was observed in failure rate of LRU items. This conclusion is equally true for doing PM actions in high value of $\acute{\Delta}= 52$. In such circumstances, change in PM level $(\acute{r})$ had no impact on improved condition of third party due to the passing of appropriate time of PMs.

7- Conclusion remarks and further research

In this paper, an INLP model was presented for optimizing extended warranty distribution network from the perspective of third party to support multi-indenture products when manufacturer offer base warranty. A novel imperfect preventive maintenance approach was introduced based on the concept of virtual age to decrease repair cost of the network. In the proposed maintenance approach if the failure rate of a component is higher than the permissible level, it will be replaced with a spare part. Logistics management of spare parts is modeled based on METRIC approach. In this regard, Poisson assumption of spare parts demand from the operational repair was a challenge. Required conditions for establishment the mentioned assumption were estimated using Monte-Carlo simulation approach. Results indicated, as increasing the level of PM causes enhancement of rejuvenation level of faulty item, demand rate for spare part decreases and tends to Homogeneous Poisson Process. Moreover, it was observed by increasing time interval between PM actions, higher levels of these actions are required for the purpose that Poisson assumption of spare parts demand can be established. It was also determined that increasing the number of components to more than 10 items led to HPP in their cumulative demand, which is in congruence with the results obtained by Wang (2012).

For optimizing the presented model, Branch and Bound algorithm was introduced, in which the upper bound was calculated by metaheuristic approach of variable neighborhood search algorithm. Results showed the fact that taking high levels of PM actions by manufacturer did not necessarily lead to reduction third party expenses. Since, the level of applied PM is as important as the time intervals between PM actions. Performing high levels of early preventive maintenance by manufacturer may increase failure rate of product components upon delivery to third party in the beginning of extended warranty. Therefore, this strategy will increase CM costs. According to the current research, inspection cost has significant effects on needed budget and the number of expected backorders in operational repair center. This is more important when third party has a low budget. In this regard, the involved budget to inspection process could be decreased by technological change and the saved budget is allocated to purchase more spare parts, which decreased the expected number of backorders.

The highest running time was estimated at 1227 seconds, related to the condition where exhaustive search required 520000000 iterations problem solving. In each iteration, system costs during the periods of base and extended warranties and total expected backorders during extended warranty period must be determined. Therefore, it can be concluded that exact hybrid solution approach has a high performance through optimization of the model. This study can be developed, as follows:

1- In the proposed model, it was assumed that all customers would buy the extended warranty. Risk of buying the extended warranty and its impact on manufacturer and third party policies could be evaluated.
2- In this paper, the location of depot and operational repair centers were predetermined. By locating the depot and operational repairing centers and allocating customers to them, issues such as time of transferring failed items between depot and operational repair centers and demand rate from each repairing center can be adjusted in such a way that total expected of LRU backorders get minimized.
3- In current research, it was assumed that third party only supports one product and one manufacturer. Presence of several products or more than one manufacturer could be investigated to determine their effects on strategies of manufacturers or third parties.

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