Journal of Industrial and Systems Engineering
Vol. 10, No. 3, pp 140-160
Summer (July) 2017

A location-allocation model in the multi-level supply chain with multi-objective evolutionary approach

Mohammad Saidi-Mehrabad[1*], Adel Aazami[1], Alireza Goli[2]

[1]*School of Industrial Engineering, Iran University Of Science and Technology, Tehran, Iran*
[2] *Department of Industrial Engineering, Yazd University, Yazd*

mehrabad@iust.ac.ir, a_aazami@ind.iust.ac.ir, a.goli@stu.yazd.ac.ir

Abstract

In the current competitive conditions, all the manufacturers' efforts are focused on increasing the customer satisfaction as well as reducing the production and delivery costs; thus, there is an increasing concentration on the structure and principles of supply chain (SC). Accordingly, the present research investigated simultaneous optimization of the total costs of a chain and customer satisfaction. The basic innovation of the present research is in the development of the hierarchical location problem of factories and warehouses in a four-level SC with multi-objective approach as well as the use of the multi-objective evolutionary metaheuristic algorithms. The main features of the resulting developed model would include determination of the number and location of the required factories, flow of the raw material from suppliers to factories, determination of the number and location of the distribution centers, flow of the material from factories to distribution centers, and finally allocation of the customers to distribution centers. In order to obtain optimal solutions of the model, a multi-objective hybrid particle swarm algorithm (MOHPSO) was presented; then, to assess performance of the algorithm, its results were compared with those of the NSGA-II algorithm. The numerical results showed that this algorithm had acceptable performance in terms of time and solution quality. On this basis, a real case study was implemented and analyzed for supplying the mountain bikes with the proposed algorithm.

Keywords: Location and allocation, multi-level supply chain, non-dominated solution, Pareto optimal solution, hybrid particle swarm algorithm, NSGA-II metaheuristic algorithm.

1- Introduction

Supply chain (SC) is an integrated system of interrelated equipment and activities, which serves in relation with process, transfer, and distribution of the products among costumers. SC management is a set of tools used to improve efficiency of the suppliers, manufacturing plants, warehouses, and ultimately retailers of the product. The objective of SC management is the proper allocation to the proper place at the proper time in order to minimize the system's total cost and provide satisfactory services to the customers.

*Corresponding author.
ISSN: 1735-8272,

This definition implies that a SC is consisted of several interdependent components, each of which attempts to maximize its objective function; in fact, we are faced with a problem with various objective functions that need to be satisfied at the same time. Such a problem is called multi-objective optimization with numerous Pareto optimal solutions; thus, the final decision is to establish balance in the entire chain based on all the criteria. Such a balance, which is obtained based on the criteria, is called trade-off.

So far, the success criteria for companies generally included reduced costs, shorter production time, shorter delivery time, less inventory maintenance, higher market share, increased reliability of delivery time, better customer services, higher quality, and effective coordination between demand, supply, and production. The exchange between the investment cost and service level might change over time; thus, investigating the SC performance requires continuous evaluation of the chain because, under such conditions, the managers can make appropriate decisions at the proper time. The key problems in SC are generally divided into three categories: **(1)** Supply chain design, **(2)** Supply chain planning and **(3)** Supply chain control.

In the chain design phase, strategic decisions such as location of the facilities and selection of the appropriate technology are taken. In order to design an efficient SC, appropriate location of the facilities is of special importance. Strategic decisions require high costs and so much time; therefore, implementation of such decisions is expected to be more durable. The environmental changes during the facilities' lifetime are considered as a serious caution for location of the facilities; therefore, the best definitive location for new equipment is one of the most important strategic challenges. Once the SC framework is formed, the attention will be led toward technical-operational decisions. Decisions on management of the management of raw materials, semi-finished materials, or final product as well as decisions on the product distribution within the chain are among the decisions in this category. In a typical SC management, the SC network's decisions are usually focused on an objective, minimization of costs, or maximization of profits; however, decision-making, planning, and scheduling of the projects usually seek to establish a balance between various incompatible objectives including fair distribution of the profit among all the chain members, appropriate level of customer services, appropriate reliability inventory, flexibility in the orders volume, and so on. Thus, in a real SC, it is attempted to optimize multiple objective at the same time. The main problem with the SC design is to select a set of optimal solutions for a multi-objective problem; thus, it is necessary to have an efficient algorithm, which can provide the best possible solutions. In this regard, studies have shown that the evolutionary algorithms have good performance, since they can ultimately lead to an appropriate solution for a multi-objective problem. In the proposed model, a non-dominant PSO algorithm was used for simultaneous optimization of two objectives, namely minimization of the total chain cost and maximization of the finishing rate in a SC with four-level structure. Considering these two objective functions, an efficient SC can be designed along with optimal transportation between its components.

In general, the main innovations of the present research included the following cases: the first and most important innovation of this research was the simultaneous optimization of the costs and customer satisfaction level in a multi-level chain; on this basis, a four-level chain was considered in which the decisions should be made on the key components including warehouse location, manufacturing plants location, as well as product and raw material distribution. The second part of the innovation in this research was defined as the use of multi-objective evolutionary optimization methods to optimize the problem; accordingly, the multi-objective particle swarm algorithm and multi-objective genetic algorithm were used. Furthermore, in order to show applicability of the present research, the developed model as well as its results were investigated and analyzed on a real case study.

The remaining sections are organized as follows. In Section 2, some of the most important previous studies on this field are described. In Section 3, the developed mathematical model is described in details. Section 4 investigates the multi-objective problem-solving approach. In Section 5, performance of the evolutionary particle swarm algorithm is examined. In Section 6, the results obtained from optimization of the case study by the developed model and evolutionary algorithm is investigated. And in the final Section, the conclusion and some suggestions for further studies are presented.

2- Research background

This section deals with the previous works conducted on location of facilities as well as multi-objective optimization along with the particle swarm algorithm. The major decisions that are made on the SC management include:

1. Which product should be produced, and how much? **2.** How much of the product should be maintained in the inventory system of each section? 3. Where are the factories and distribution centers located?

Moreover, location of the facilities is one of the most important and difficult decisions, which affects efficiency of the SC. The total chain costs and the level of services are mainly affected by the number, size, and location of the facilities; therefore, a large part of the studies have been conducted on improvement of the SC efficiency in relation with location. The first studies on the theory of location were primarily initiated by Webster in 1909, which were focused on locating a warehouse in the city in order to minimize the total costs of the costumers' traveling. Since then, one of the considerable studies on location was conducted by Hakimi (1964), who investigated location of the distribution center in a network as well as location of the police stations in a highway.

Recently, many of the studies have been focused on location of facilities as the formulation of a static and deterministic system along with constant and identified inputs, which have finally led to an optimal solution for this formulation. Such problems that obtain the optimal solution through formulation are called average level problems. Besides, simultaneous location and allocation of multiple facilitates despite the flow of materials between the facilities and customers has been focused in researches. Such problems have been revised by Scott (1971). Excessive diversity of such problems in various industries and different SCs has been investigated by Warszawski (1973). In these models, a constant cost and a linear cost is considered for allocation and transportation, respectively; besides, it is assumed that each of the warehouses contains more than one type of product. Marianov and Serra (2001) presented the swarm index in the hierarchical location. According to this index, the hierarchical location models would attempt to locate the facilities of companies in regions with greater population concentration and supply all the customers' demands in the nearest center.

Yasenovskiy and Hodgson (2007) applied the p-median location in the hierarchical conditions; accordingly, a mathematical model was presented with the aim if reducing the total costs along with the accurate results of its solution. In another category of problems, location of facilities with one operator is considered, so that each facility is located in a potential place. In this category, there are two states: in the first state, location of the companies is carried out by considering their capacity, and in the second state, location is carried out regardless of their capacity. Such type of problems has been mainly investigated by Mirchandani and Francis (1990) and ReVelle et al. (2008). Dynamic location of the facilities is another type of location in the real world. Scott (1971) developed location of multiple facilities in dynamic form. Erlenkotter (1981) compared the performance of heuristic optimal solutions for the dynamic single-facility location problem.

Due to the complexity of the problems associated with location-allocation, and also their usage in supply chains, the necessity to use the approximate methods become clearer. The most important approximate methods that have been recently considered by researchers are meta-heuristic algorithms such as genetic algorithm and PSO (particle swarm optimization) algorithm. PSO was presented by Kennedy and Eberhart (1995) as the simulator of social behavior, and was introduced as a meta-heuristic algorithm in 1995. Parsopoulos and Vrahatis (2002) was one of the first researchers who attempted to work on the performance of PSO in multi-objective optimization and find the Pareto optimal solutions. Mostaghim and Teich (2004) presented a new method for extracting the new population in MOPSO (multi-objective particle swarm optimization algorithm) algorithm. In this algorithm, they attempted to fill the gap between the non-dominated solutions in the initial population and future populations.

In the following, this research is focused on relevant research in supply chain optimization with the purpose to use the approximate methods and meta-heuristic algorithms.

Tsou et al. (2011) presented a bi-level model, in which the orders were considered as constant, and there were missing sales. They optimized the periodic inventory investment and appropriate service level at the same time. In the optimal solution, they used an MOPSO-based algorithm to find the optimal inventory policies. Nguyen et al. (2012) studied the heuristic methods of location and distribution in bi-level chains. For this problem, they proposed three heuristic voracious algorithms for generating the initial solution; besides, in order to improve the solutions, they used the GRASP heuristic algorithm. Latha Shankar et al. (2013) tried to solve the problem of location, allocation, and distribution in a four-level SC. They carried out location of warehouses and factories, allocation of each warehouse to the factories, and determination of the commodities shipping rate at the SC level at the same time. Shahabi et al. (2013) developed a mathematical model for solving the problem of location and distribution by considering the warehouse inventory control in a four-level SC; accordingly, the hub was used for shipping the products at the SC level. The mathematical model proposed by these researchers determines three cases simultaneously: 1) location of warehouses and distribution hubs, 2) allocation of warehouses, retailers, and customers to the suppliers, warehouses, and retailers, respectively, and 3) decisions on inventory level.

Yu et al. (2015) considered the multi-product state in the location and distribution problem. They tried to determine the location of factories, production rate of each commodity, and rate of shipping to the central warehouses. Montoya et al. (2016) focused on the location of facilities with capacity constraints in the SC with regard to the environmental pollutions as well as the costs of production and implementation of the facilities. In this research, an integer linear mathematical model was presented to determine the optimal facility location. Haji abbas and Hosseininezhad (2016) developed a discrete covering location-allocation model for pharmaceutical centers. They considered two objectives; the first one minimizes the costs and the second one was the maximization of customer satisfaction by description of social justice. Jena et al. (2016) investigated the dynamic location in the SC. These researchers considered the possibility of opening and closing each facility at different periods for various facilities in the SC. Wang and Ouyang (2016) attempted to solve the problem of location in the SC in a dynamic manner. This research was aimed to determine the optimal number of facilities as well as the time of developing their capacity with regard to the SC costs. Li et al. (2017) investigated the multi-period hierarchical location for rural areas. In this regard, the purpose of their proposed mathematical model was to reduce total harmonic distances covered by the customers. This model was optimized by the proposed heuristic method. A summary of the reviewed studies is presented in table 1.

Table 1. Investigating some of the studies reviewed along with the present research

Researchers	Year	Subject	Objective	Solution approach
Weber	1909	Urban warehouses location	Reducing the costs	Precise solution
Scott	1971	Dynamic location of facilities	Reducing the costs	Precise solution
Marianov and Serra	2001	Hierarchical location	Increasing the service level with swarm index	Heuristic
Yasenovskiy and Hodgson	2007	P-median-based hierarchical location	Reducing the total cost	Precise solution
ReVelle et al.	2008	Bi-level location	Reducing the total cost	Precise solution
Tsou et al.	2011	Location and inventory in bi-level chain	Reducing the construction and inventory costs	Heuristic
Nguyen et al.	2012	Location and distribution in bi-level chain	Minimizing the chain cost	Presenting two new heuristic methods
Latha Shankar et al.	2013	Location, allocation, and distribution in SC	Minimizing the chain cost	Precise solution with GAMS
Shahabi et al.	2013	Location and distribution with regard to the warehouses' inventories	Reducing the construction and inventory costs	Precise solution with GAMS
Montoya et al.	2016	Location of warehouses with capacity constraints	Minimizing the environmental pollution and construction costs	Precise solution
Jena et al.	2016	Dynamic location in SC	Reducing the total chain cost	Lagrange release
Wang and Ouyang	2015	Dynamic location with regard to capacity development	Reducing the total chain cost	Continuous approximation
Li et al.	2017	Multi-period hierarchical location	Reducing the total cost	Heuristic
This research	-	Location-allocation in the four-level SC	Reducing the total cost-increasing the satisfaction level	Multi-objective hybrid particle swarm

By reviewing and comparing the previous studies, in accordance with table 1, one can consider that the basic innovation in the present study is introducing a multi-objective mathematical model for hierarchical location in a multi-level SC with respect to the simultaneous reduction of the total cost and increasing of the customer satisfaction. Furthermore, in order to solve this mathematical model, the evolutionary multi-objective meta-heuristic algorithms would be used; so that, according to the review of literature, such a research has not been conducted with the mentioned innovations so far.

3- Mathematical modeling and formulation

In this section, optimization of the network model objectives of an SC will be discussed, and then a mathematical model will be developed. While designing an SC network, the managers should make decisions on location and allocation of capacity to each of the facilities that is interrelated with others. In the developed model, a public SC network with four different levels is considered. The first level is the customer zone (CZ) that will be actually the place for selling the products to customers. The second level includes the distribution centers (DC), which will be indeed the place for transferring the products to customers. The third level is, in fact, the factory or the manufacturing sector. And in the fourth section, the suppliers are located. The income resulted from selling the products will be spent for the equipment, human force, transportation, purchasing the required material, and inventory. In order to reduce the complexity of the mathematical modeling and force the model to converge to optimal solution, a minimum demand is assumed for the given product, so that in all the scenarios, such minimum demand should be met. Figure 1 demonstrates the structure of the studied chain.

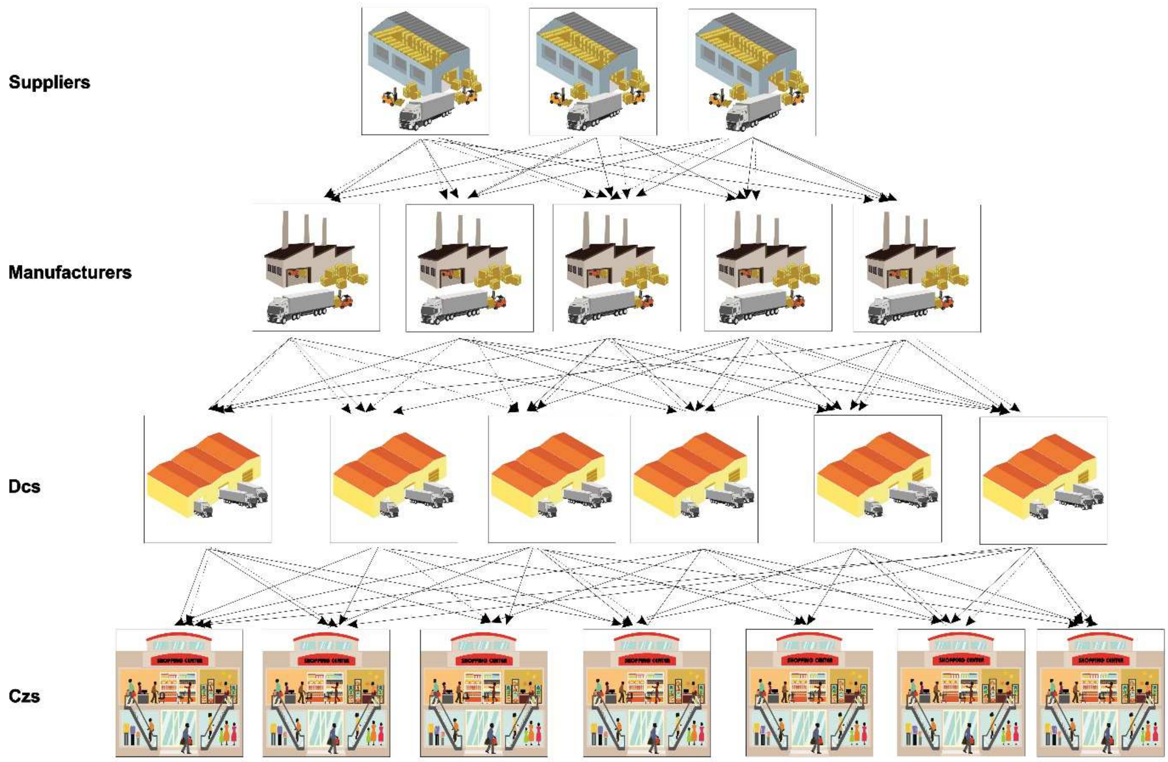

Figure 1. Structure of the studied chain

3-1- Hypotheses

- ✓ Manufacturing the product requires three raw materials.
- ✓ Capacity of the suppliers for raw materials varies, and the costs of production and transportation of each unit of raw material in each of the suppliers is specified.
- ✓ Potential location zones for factories and distribution centers are specified.

- ✓ Production and maintenance costs for each product and distribution costs for each factory and distribution center are constant.
- ✓ Inventory cost for each product and cost of transferring the product from each distribution center to each of the customer zones are constant.
- ✓ Minimum finishing rate should be maintained.

3-2- Model inputs

- ✓ Number of suppliers and their capacity for each raw material
- ✓ Number of potential zones of factories and distribution centers as well as their capacity
- ✓ Cost of creating each raw material in each of the suppliers as well as the cost of transferring from each supplier to each factory
- ✓ Costs of production and inventory in each factory
- ✓ Cost of transporting each unit of product from the factory to each distribution center
- ✓ Cost of efficiency of the product in each of these distribution centers and cost of transporting each product from that center to the customer zone
- ✓ Number of customer zones and their demands
- ✓ Minimum finishing rate (a percentage of the met demand) that should be maintained.

3-3- Model outputs

- ✓ Amount of each raw material that should be transported from a supplier to a factory.
- ✓ Number of factories and their location
- ✓ Flow of materials from the intended factory to distribution centers
- ✓ Number of distribution centers and their location
- ✓ Allocation of customer zones to distribution centers

3-4- Objective functions

- ✓ Minimizing the total SC cost, which includes raw material production, transportation, location of factories, production and maintenance, distribution of the products from factories to distribution centers, efficiency cost of the distribution centers, and cost of transporting from the distribution centers to the customer zone
- ✓ Maximizing the finishing rate

The production ability indicates the company’s ability to meet the customer’s order with regard to the available inventory. The inventory shortage occurs when the manufacturer receives the customer’s order but lacks sufficient inventory. The product’s finishing rate (FR) and cycle service level (CSL) are criteria for measuring the product’s availability. The product’s finishing rate is a fraction of the product demand, which is met through production and maintenance; however, the cycle service level is a fraction of the cycle replacement, which is finished by meeting the demands of all the customers. In the following sections, the decision parameters and variables in a problem will be presented.

3-5- Notations

Indices:

j	customer areas and demand zones
e	warehouses
i	potential zones of location of factories
h	suppliers
c	components (constituents of the final product)

Parameters:

D_j Demand of the customer j
k_i Potential capacity of the factory i
k_e Potential capacity of the warehouse e
s_{ch} Supply capacity in the supplier h for the component c
f_i Annual fixed cost to setup the factory i
f_e Annual fixed cost to setup the warehouse e
c_{chi} Cost of preparing and transporting the component c from the supplier h to the factory i
c_{ie} Cost of manufacturing and transporting from the factory i to the warehouse e
c_{ej} Cost of transporting from the warehouse e to the customer j zone
IC_i holding cost in the factory i
IE_e holding cost in warehouse e

Decision variables:

y_i 1 if the i is constructed, otherwise 0
y_e 1 if the warehouse e is constructed, otherwise 0
x_{hci} Amount of the component c transported from the supplier h to the factory i
x_{ie} Amount of the final product transported from the factory i to the warehouse e
x_{ej} Amount of the final product transported from the warehouse e to the customer j

3-6- Mathematical model

$$Min\, z_1 = \sum_{i=1}^{n} f_i . y_i + \sum_{e=1}^{t} f_e . y_e + \sum_{i=1}^{n}\sum_{h=1}^{l}\sum_{e=1}^{p}(c_{chi} + IC_i).x_{chi} + \sum_{i=1}^{n}\sum_{e=1}^{t}(c_{ie} + IE_i).x_{ie} + \sum_{e=1}^{t}\sum_{j=1}^{m} c_{ej}.x_{ej} \quad (1)$$

$$Max\; z_2 = \frac{\sum_{e=1}^{t}\sum_{j=1}^{m} x_{ej}}{\sum_{j=1}^{m} D_j} \quad (2)$$

$s.t.$

$$\sum_{i}^{n} x_{hci} \le S_{ch}.y_h \qquad \forall h,c \quad (3)$$

$$\sum_{e=1}^{t} x_{ej} \le D_j \qquad \forall j \quad (4)$$

$$\sum_{e=1}^{t} x_{ie} \le K_i . y_i \qquad \forall i \quad (5)$$

$$\sum_{j=1}^{m} x_{ej} \le K_e . y_e \qquad \forall e \quad (6)$$

$$\sum_{h=1}^{l} x_{hci} - \sum_{e=1}^{t} x_{ie} \ge 0 \qquad \forall i,c \quad (7)$$

$$\sum_{i=1}^{n} x_{ie} - \sum_{j}^{m} x_{ej} \geq 0 \qquad \forall e \tag{8}$$

$$0.8 \leq \frac{\sum_{e=1}^{t}\sum_{j=1}^{n} x_{ej}}{\sum_{j} D_j} \leq 1 \tag{9}$$

$$y_i, y_e, y_h \in \{0,1\} \qquad \forall i, e, h \tag{10}$$

$$x_{ie}, x_{ej}, x_{hci} \geq 0 \qquad \forall i, e, j, h, c \tag{11}$$

The objective function (1) demonstrates minimization of the total cost of implementing and operationalizing of network (including fixed and variable costs), and the objective function (2) represents maximization of the finishing rate.

Constraint (3) indicates that the total amount of the products sent from the supplier cannot be greater than the supplier's capacity. Constraint (4) implies that the demand should be met at any point in the market. Constraint (5) states that no factory can supply commodities more than its capacity; similarly, for the constraint (6), the supplier serves in the same way. Constraint (7) expresses that the amount sent from the factory should not be greater than the flow of the raw material entering the factory. Constraint (8) suggests that the amount sent from the warehouse should not exceed the amount entering the warehouse. Constraint (9) states that the level of meeting a demand, regarding the finishing rate, should be in the range of 80-100%. Finally, the constraints (10) and (11) determine the type of decision variables.

4- Solution approach

4.1. Multi-objective optimization concepts

General multi-objective optimization can be considered as a continuous process of optimizing two or more conflicting objectives with regard to certain constraints. Since the multi-objective optimization has multiple objective functions, the solution method seeks to find exchanges between the obtained solutions. The concept of Pareto optimality in the multi-objective optimization was introduced by Pareto in 1986 as follows:

Where the point $x^* \in \Omega$ will be the Pareto optimal solution if $(f_i(x^*) \leq f_i(x))\forall i \in I, x \in \Omega$, where $I = \{1, 2, ..., K\}$ and there is at least one $i \in I$ so that $(f_i(x^*) \prec f_i(x))\forall i \in I$.

This definition states that X^* is the Pareto optimal solution if there is no reasonable solution that can reduce some of the measures without simultaneous increase in at least one of the objectives. The multi-objective optimization algorithm uses the concept of dominance to obtain the optimal solution. In this algorithm, two solutions are compared with each other, so that it is evaluated whether one dominates the other one or not. If there are M objective functions, then the solution X will dominate the solution Y in case that both of the following conditions are true:

1. The solution X is worse than Y in none of the objectives.
2. The solution X is better than Y in at least one of the M objectives.

If only one of the above conditions is true, then the solution X will not dominate the solution Y. In the multi-objective optimization, since we face more than one objective for optimization, there is not only a single optimal solution that optimizes all the objectives. The output results include a set of optimal solutions that have different values in different functions. This set of solutions is called non-dominant set. The following two conditions should be true for each member of the non-dominant set:

1. **1.** Both non-dominance sets should be non-dominant relative to all of each other's members.
2. **2.** Any solution that doesn't belong to the non-dominant set is dominated by one of the members of the non-dominant set. Such non-dominant set is known as the set of Pareto optimal solutions.

Due to minimization of the total SC costs and maximization of the finishing rate and, as a result, the conflict of these two objectives in terms of their nature, it would be impossible to obtain an optimal solution for this problem; therefore, in such a case, it is attempted to obtain optimal solutions that determine appropriate policies for the chain. Thus, the set of Pareto solutions and its decision variables are designed such that the decision maker can select the Pareto solutions to meet his need.

4-2- Introduction of particle swarm algorithm

Particle swarm algorithm (PSO) is one of the newest population-centered techniques for optimization of the models (Kennedy and Eberhart, 1995). In this algorithm, some of the solutions are considered as the particles moving in the solution's space. PSO acts based on the behavior of the societies that are interrelated both socially and individually (such as the birds looking for food) (Coello, 1999). A bird might find its food whether through group cooperation with other birds or lonely.

In the PSO algorithm, each individual (particle) indicates a solution in the N-dimensional space; besides, each particle is informed of the best experience of his own and others. Each particle changes its path with regard to the equations (12) and (13) (Coello et al. 2002).

$$v_{ij} = w * v_{ij} + c_1 * r_1 * (p_{ij} - x_{ij}) + c_2 * r_2 * (p_{gi} - x_{ij}) \tag{12}$$

$$x_{ij} = x_{ij} + v_{ij} \tag{13}$$

In equations (12) and (13), w is a constant factor that is affected by the local and general ability of the algorithm, v_{ij} is the speed of the i^{th} particle in the j^{th} dimension, c_1 and c_2 are the weights that are affected by the personal and social movements, r_1 and r_2 are the random numbers from uniform distribution between 1 and 0, p_{ij} represents the best value found by the i^{th} particle, and p_{gi} indicates the best solution found by all the particles. Once the particle's speed is updated, the new position of the i^{th} particle is calculated in the j^{th} direction. Eventually, all the particles resemble a huge flock of birds that, in order to find their food, are moving toward the regions with more foods; in fact, they are approaching an optimal solution with more fitness function. The PSO algorithm is highly regarded due to its simplicity of implementation and capability of rapid convergence to the reasonable solution. In this algorithm, better searching and finding the optimal solution requires adjusting a few parameters.

Operators of the MOHPSO algorithm should be selected properly. In this algorithm, the speed should be converted into the probabilistic mode, which is the chance of getting a value of 1 for the particle. Here, the particle's speed was calculated using equations (14), (15), and (16), where c_1 and c_2 are constant and equal numbers, $pBest_l$ is the best solution of any particle 1, $nBest_t$ is the best total solution (leader), w is the constant inertia value and equal to 0.5, r_1 and r_2 are random numbers, V_{it} is the particle's speed, x_{it} is particle position, $V_{\max}$ is equal to 4, and sp_l is the probability between 0 and 1. Then, the random number ρ is produced in [0, 1], and the particle's new position is determined using equation (17) (Che, 2012).

$$V_{it} = w.V_{l,t-1} + c_1.r_1.\left(pBest_l - x_{l,t}\right) + c_2.r_2\left(nBest_t - x_{l,t}\right) \tag{14}$$

$$-V_{\max} \le V_{l,t} \le V_{\max} \tag{15}$$

$$sp_1 = \frac{1}{1+e^{-V_{l,t}}} \tag{16}$$

$$x_{l,t} = \begin{cases} 1 & \rho \le sp_1 \\ 0 & Otherwise \end{cases} \tag{17}$$

In this algorithm, the random mutation with rate of 0.2 was used. In general, the MOHPSO algorithm's steps can be expressed as follows:

Step-one: Generating the initial solutions randomly

Step-two: Calculating the fitness value for each of the initial solutions based on the defined objective functions

Step-three: Determining the non-dominant solutions generated in the set of initial solutions

Step-four: Determining the leader solution from among the available solutions to create neighborhood in the solutions

Step-five: Creating new solutions based on equations (14) and (15)

Step-six: Updating the best personal experience of each particle. If the particle's new position dominates the best experience, then the new position will replace best experience, and if none of them dominate the other one, one of the above positions will be randomly considered as the best experience.

Step-seven: Adding the non-dominated members of the current population to the external memory

Step-eight: Eliminating the non-dominated members of the external memory

Step-nine: Eliminating the members exceeding the external memory's capacity

The probability of elimination of the members exceeding the external memory's capacity is obtained through Equation (18). In this equation, ii is the cell number. After determining the probabilities of elimination of the additional solutions by Roulette Wheel method, the additional solutions are removed.

$$del\_prob_i = \frac{e^{n_{ii}}}{\sum_j je^{n_{ii}}} \ , 0 \le del\_prob_{ii} \le 1, \sum_i iq_{ii} = 1 \tag{18}$$

Step-ten: Stop in case of fulfillment of the finishing condition, otherwise going to the third step.

In figure 2, the MOHPSO algorithm's steps are represented as flowchart.

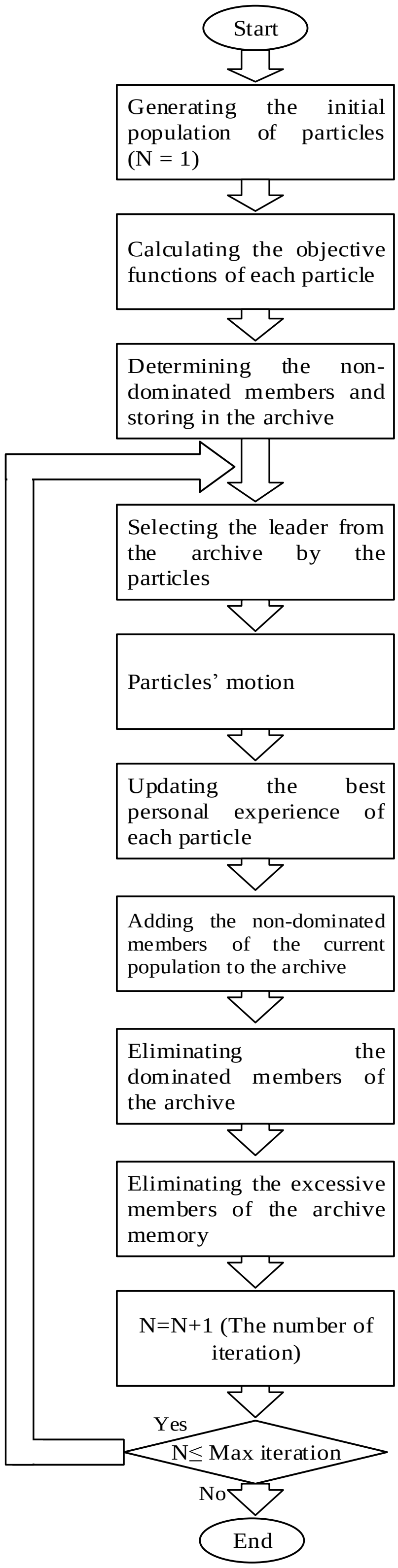

Figure 2. MOHPSO algorithm's flowchart

4-3-Two-objective genetic algorithm (NSGA II)

Genetic algorithm is one of the heuristic algorithms for solving the problems, which has been derived from biological modeling of the animals’ population. In this algorithm, characteristics of the animals’ generation is resembled to the value of the objective functions and improvement of the generations’ characteristics over time, and emersion of the new generations from intercourse of the previous generations is analogized to the improvement of the value of the objective function; in other words, this algorithm uses the Darwin's natural selection principle to find a formula or the optimal solution in order to predict or compare the pattern. The NSGA II general algorithm, as one of the multi-objective genetic modes, is as follows:

1. Creating the initial population
2. Calculating the fitting criteria
3. Sorting the population based on the dominance conditions
4. Calculating the swarm distance
5. Selection; as soon as the initial population is sorted based on the dominance conditions, the swarm distance will be calculated, and selection from among the initial population will begin. This selection is carried out based on two elements:
    - Population ranking: populations are selected at lower ranks
    - Distance calculation: Assuming that p and q are two members of a same rank, the member with higher swarm distance will be selected. It should be noted that the selection priority is primarily based on the ranking and then on the swarm distance.
6. Generating new children through crossover and mutation, and integration of the initial population with the population obtained from crossover and mutation
7. Replacing the parent population with the best members of the population integrated in the previous steps

In the first step, members with lower ranks are replaced for the previous parents, and then are sorted based on the swarm distance. The initial population and the population resulted from crossover and mutations are sorted primarily based on ranking, and then those with lower ranks are eliminated. In the next step, the remaining population is sorted based on the swarm distance (Coello et al. 2007).

5- Numerical results and analysis

5-1-Results of generated numerical examples and comparing two algorithms

In order to evaluate performance of the proposed algorithm, the MOHPSO algorithm’s performance should be assessed on the examples. According to the multi-objective structure of the mathematical model as well as the proposed algorithm, the comparison is carried out by one of the most powerful multi-objective optimization algorithms, namely the NSGA II algorithm. So the structure of the two algorithms was encoded in MATLAB R2014 software.

Hence, 10 examples were designed with different dimensions, and solved by MOHPSO and NSGA II algorithms. Since there is no accurate benchmark for the investigated problem, the sample examples were randomly generated from a uniform distribution. In table 2, dimensions of the examples are specified, and also the results of implementation the two given algorithms are compared.

Table 2. Dimensions of examples and comparing the efficiency of MOHPSO and NSGA II algorithms

Problem dimension						MOHPSO			NSGA II		
	J	E	I	H	C	NOS	MD	T	NOS	MD	T
P1	5	3	2	1	1	2	32.1	0.24	2	32.1	1.27
P2	10	5	4	3	2	4	187.4	3.79	4	187.4	8.67
P3	15	7	5	4	4	11	891.7	7.16	10	9016.2	12.47
P4	20	12	10	9	5	21	1428.3	9.15	25	1539.3	19.33
P5	25	15	13	11	9	27	2781.6	13.87	18	26146.1	29.14
P6	30	17	15	12	13	48	5472.1	21.1	31	5503.6	37.16
P7	35	20	20	14	20	73	8105.2	26.48	48	8133.2	51.11
P8	40	25	22	16	30	94	10741.1	55.71	63	10942.8	66.43
P9	50	30	25	20	40	128	11247.1	89.41	99	11019.5	83.56
P10	60	35	30	25	50	214	119007.6	103.19	157	11143.8	96.79
Average						62.2	15989.42	33.01	45.7	8366.4	40.593

In table 2, dimensions of the problem show the number of customers, number of warehouses, number of factories, number of suppliers, and number of components, respectively. The studied algorithms were compared using MD and NOS indices. The NOS states the number of Pareto solutions in the final output of the meta-heuristic algorithm, and MD expresses the maximum expansion. This index is calculated through Equation (19):

$$MD = \sqrt{(Z_1^{\max} - Z_1^{\min})^2 + (Z_2^{\max} - Z_2^{\min})^2} \qquad (19)$$

Also, T is the time of algorithm implementation in seconds. Clearly, the higher is the value of NOS and MD, the higher the efficiency of the algorithm in finding the Pareto frontier.

According to table 2, comparing the MOHPSO and NSGA II algorithms based on the NOS index revealed that the MPHOSP algorithm and NSGA II algorithm found nearly 62 and 45 solutions at the Pareto frontier, respectively. Thus, the MOHPSO algorithm had the superiority in this index. A more detailed investigation showed that the NSGA II algorithm has had better NOS index merely in the P4 problem, and in other cases, the MOHPSO had been better. In Figure 3 presents the values of the NOS index for both algorithms in different examples.

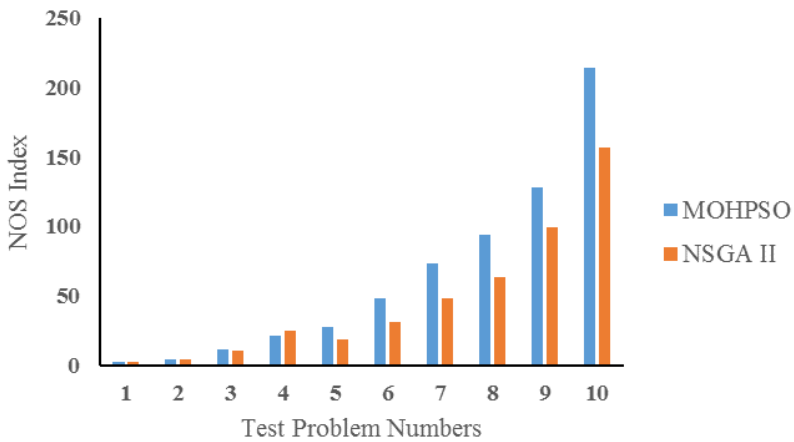

Figure 3. Comparing NOS index in two MOHPSO and NSGA II algorithms

Also, according to table 2, the MD index is the same as NOS. In 8 cases out of the 10 examples, the MOHPSO algorithm had better MD value; however, in general, there is an average difference of 7600 units between the two algorithms in terms of the average of this index. Thus, in this index, the MOHPSO algorithm has had the superiority as well.

However, besides the solution quality, the solution time is another dimension of the meta-heuristic algorithm, which must be analyzed and assessed. Figure 4 represents the solution time in the two algorithms.

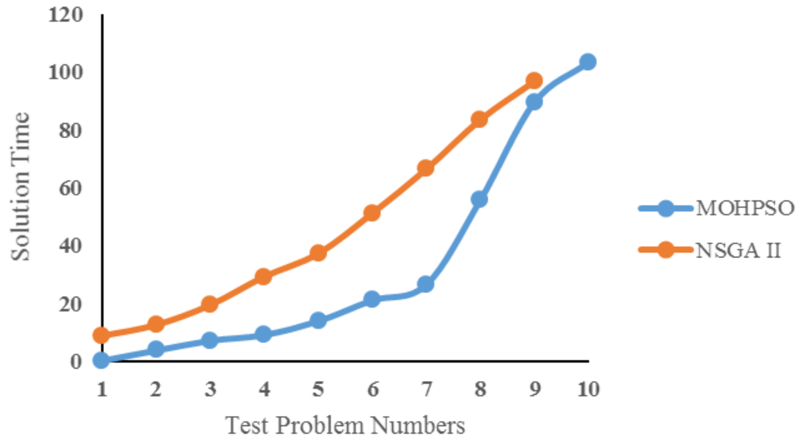

Figure 4. Solution time in MOHPSO and NSGA II algorithms

As shown in figure 4, also in terms of the solution time, the MOHPSO algorithm could have the relative superiority to the NSGA II algorithm and provided the solutions in the shortest possible time.

Therefore, according to the comparisons, efficiency of the MOHPSO algorithm is proved in terms of the solution quality and speed; therefore, in the following section, a real case study is implemented by this algorithm.

5-2- Case study results

To show capability of the proposed algorithm, a real case study is considered on supplying the mountain bikes from a four-level supply chain with a network structure with 3 suppliers, 5 factories, 6 distribution centers, and 7 customer zones. The primary components are sent by the domestic and foreign suppliers to the relevant factories. In factories, these components are assembled and the final bicycle is ready to be transferred to the warehouse and then delivered to the customers. In this problem, the company is seeks to locate the 5 manufacturing plants with a total capacity of 142 units per month, so that it can meet the 30-unit demand of 7 areas using 6 warehouses with a total capacity of 82 units. At the first level of the SC network, the suppliers provide all the three types of raw material for manufacturing a product. The capacity of suppliers, factories, warehouses, as well as the fixed costs for each factory are shown in table 3. The production costs of a component and transportation costs of each unit to the manufacturing plants are shown in the table 4. The production variable inventory, manufacturing costs, and transportation costs of each demanded shipment to each customer are shown in table 5. The maintenance and transportation costs as well as the manufacturing costs for each unit at the third level of the chain are shown in table 6.

Information such as the components of the product, consumption, and supply chain capacity has been determined through interviews with relevant experts of the bicycle production company. The transport costs are determined with respect to position of each of the supply chain components and also the distance between them.

This problem and the final results were analyzed by considering two conflicting objectives.

Table 3. Capacity of suppliers, capacity of factories, fixed costs for each factory, and capacity of warehouses

Supplier capacity	Element		
	c1	c2	c3
1	36	62	50
2	40	65	55
3	42	70	60

Factory	Capacity	Fixed cost
p1	18	7650
p2	24	3500
p3	37	500
p4	22	4100
p5	41	2200

Warehouse	Capacity
wh1	15
wh2	12
wh3	14
wh4	13
wh5	12
wh6	16

Table 4. Production costs of a component and transportation cost of each unit to the manufacturing plants

Supplier	Element	Preparation cost for supplier	Transportation cost for factory				
			1	2	3	4	5
1	c1	300	10	13	8	11	15
	c2	115	6	7	5	8	4
	c3	90	3	4	5	4	5
2	c1	320	17	14	12	12	15
	c2	120	6	5	7	5	7
	c3	85	6	6	5	6	4
3	c1	290	13	12	14	11	9
	c2	125	6	5	3	4	5
	c3	75	3	6	3	2	3

Table 5. Production variable inventory as well as the manufacturing and transportation costs of each shipment unit to each of the customer zones

Factory	Warehouse						Production cost	Inventory cost
	wh1	wh2	wh3	wh4	wh5	wh6		
1	7	12	18	17	18	20	3900	50
2	12	10	11	13	15	17	2010	45
3	8	10	14	15	18	21	1945	55
4	10	12	13	14	13	18	1855	48
5	8	10	11	15	11	12	1975	52

Table 6. Holding and transportation costs as well as the manufacturing costs for each unit at the third level of the chain

Warehouse	Customer							Holding cost
	1	2	3	4	5	6	7	
wh1	8	3	9	6	7	3	4	55
wh2	5	8	7	6	3	2	8	50
wh3	9	3	8	6	7	5	4	60
wh4	3	9	2	2	5	4	8	54
wh5	7	6	3	9	4	9	4	55
wh6	5	6	7	8	3	2	9	45
Monthly demand	3	5	4	6	4	5	3	

For this problem, minimizing the total costs and maximizing the finishing rate are carried out by considering that the minimum finishing rate should be 80% of the real demand. On the whole, there are 128 decision variables and 48 constraints. Solution of this problem solution is to obtain 7 Pareto optimal solutions. The final decision is that ranking is performed out of these solutions based on some certain criteria. This ranking is one of the most important decision problems for the decision maker. This decision might also be changed every year with regard to the demand and costs variations.

In table 7, the Pareto solutions of the particle swarm algorithm are specified. Each of the non-dominant solutions is in the form of a scenario for implementation and execution. Table 8 shows the optimal flow of the material from the suppliers to the factory with regard to Pareto.

Table 7: Resulted Pareto

Non-Dominated solution (scenario)	Supply chain cost	The satisfied demand level	Real demand
1	159306	30	30
2	149385	29	30
3	140629	28	30
4	129904	27	30
5	117818	26	30
6	107290	25	30
7	100480	24	30

Table 8: Optimized flow of the materials in 7 scenarios

Factory	Element 1			Element 1			Element 1		
	1	2	3	1	2	3	1	2	3
Scenario 1									
1	0	0	8	10	0	0	0	12	0
2	5	0	11	10	12	0	0	22	0
3	15	0	0	18	0	0	0	12	0
4	11	0	0	0	12	0	15	0	0
5	0	0	0	0	0	0	0	0	0
Scenario 2									
1	0	1	4	0	0	8	0	0	0
2	0	18	0	0	0	9	12	0	0
3	10	9	0	17	0	0	0	20	20
4	0	0	0	0	0	0	0	0	0
5	0	11	0	10	0	0	9	0	0
Scenario 3									
1	7	0	10	5	0	5	0	14	14
2	0	0	6	8	0	0	0	9	9
3	0	0	0	0	0	0	0	0	0
4	8	0	0	0	0	7	0	0	0
5	0	0	0	0	0	0	0	0	0
Scenario 4									
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	23	0	0	0	0	21	0	0	0
4	0	0	15	16	0	0	0	18	18
5	0	0	17	0	0	0	0	22	22
Scenario 5									
1	0	11	0	0	12	8	3	0	0
2	0	6	0	6	0	0	0	7	7
3	0	0	0	0	0	0	0	0	0
4	10	0	0	0	2	12	0	0	0
5	0	0	0	0	0	0	0	0	0
Scenario 6									
1	0	0	0	0	0	0	0	0	0
2	0	7	11	0	14	19	0	0	0
3	6	5	0	0	0	12	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	9	12	0	0	7	0	0
Scenario 7									
1	0	0	9	0	10	0	0	11	11
2	0	0	0	0	0	0	0	0	0
3	17	0	0	0	0	0	20	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0

Subsequently, for a more detailed investigation of the results of the case study, first, the Pareto chart of solutions derived from the case study should be presented. In Figure 5, the Pareto chart of the case study is presented.

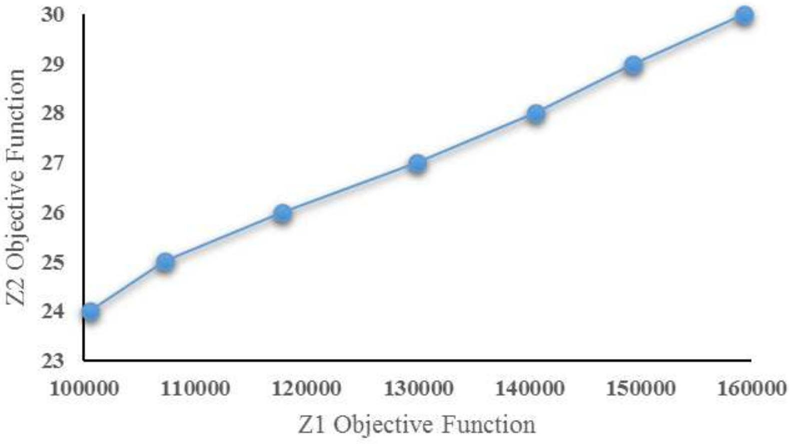

Figure 5. Pareto chart of the solutions obtained from the case study

As seen in figure 5, in case of looking for the scenarios with lower cost, the customer satisfaction level is reduced as well; thus, the conflicts of the objectives presented in this research can be shown properly. On the other hand, with respect to the 80% meeting of the demand, only 7 scenarios are presented by the given meta-heuristic algorithm. If this constraint is reduced to lower than 80%, the number of the found Pareto solutions will be evidently larger.

Also, since several optimal solutions have been found for this problem, the SC management would have the opportunity to evaluate different scenarios in terms of executability as well as the hidden aspects of the problem, which cannot be presented in the structure of the mathematical models, and finally choose the most appropriate one.

6- Conclusion

In this research, a mathematical model was developed for location and allocation of the facilities in a four-level supply chain. In this model, capacity of the factories and distribution centers, as well as the production and maintenance costs had certain and constant values; furthermore, the inventory and transportation costs were considered besides this location. The basic innovation of this research was in the simultaneous optimization of the costs and customer satisfaction in the hierarchical location of the SC as well as the use of multi-objective evolutionary meta-heuristic algorithms for optimization. The algorithm presented in this research, which was based on the particle swarm algorithm, was developed for the model in order to find the Pareto optimal solutions, ultimately leading to a number of non-dominant solutions. The obtained solutions showed that by trade-off between the objectives, different optimal solutions can be provided. The comparisons between this algorithm and NSGA II algorithm in table 2 indicated high efficiency of the proposed algorithm in solving the problems related to the hierarchical location in the supply chain (also see figure 3 and figure 4). At the end, the case study of mountain bike using meta-heuristic MOHPSO algorithm showed that the set of Pareto solutions were executable and could achieve the customer satisfaction (see figure 5 and table 7). As further research on the proposed model, it is suggested to consider the reliability inventory and risk factor for the chain as well as the reliability costs.

References

Che, Z. H. (2012). A particle swarm optimization algorithm for solving unbalanced supply chain planning problems. *Applied Soft Computing Journal*, 1279–1287.

Coello, C. a. C. (1999). A Comprehensive Survey of Evolutionary-Based Multiobjective Optimization Techniques. *Knowledge and Information Systems*, 1(3), 269–308.

Coello Coello, C. A., Lamont, G. B. and Veldhuizen, D. a Van (2007). Evolutionary Algorithms for Solving Multi-Objective Problems (Vol. 5). *New York: Springer*.

Coello Coello, C. A. and Lechuga, M. S. (2002). MOPSO: A proposal for multiple objective particle swarm optimization. *Proceedings of the 2002 Congress on Evolutionary Computation, CEC 2002*, 1051–1056.

Erlenkotter, D. (1981). A comparative study of approaches to dynamic location problems. *European Journal of Operational Research*, 6(2), 133–143.

Haji abbas, M. and Hosseininezhad, S. J. (2016). A robust approach to multi period covering location-allocation problem in pharmaceutical supply chain. *Journal of Industrial and Systems Engineering*, 9(special issue on location allocation and hub modeling), 71–84.

Hakimi, S. L. (1964). Optimum Locations of Switching Centers and the Absolute Centers and Medians of a Graph. *Operations Research*. INFORMS, 12(3), 450–459.

Jena, S. D., Cordeau, J. F. and Gendron, B. (2016). Solving a dynamic facility location problem with partial closing and reopening. *Computers and Operations Research*, 67, 143–154.

Kennedy, J. and Eberhart, R. (1995). Particle swarm optimization. *Neural Networks, 1995. Proceedings., IEEE International Conference on*, 4, 1942–1948.

Latha Shankar, B., Basavarajappa, S., Chen, J. C. H. and Kadadevaramath, R. S. (2013). Location and allocation decisions for multi-echelon supply chain network - A multi-objective evolutionary approach. *Expert Systems with Applications*, 40(2), 551–562.

Li, T., Song, R., He, S., Bi, M., Yin, W. and Zhang, Y. (2017). Multiperiod Hierarchical Location Problem of Transit Hub in Urban Agglomeration Area. *Mathematical Problems in Engineering*. Hindawi Publishing Corporation, 2017.

Marianov, V. and Serra, D. (2001). Hierarchical location-allocation models for congested systems. *European Journal of Operational Research*, 135(1), 195–208.

Mirchandani, P. B. and Francis, R. L. (1990). *Discrete Location Theory*.

Montoya, A., Vélez–Gallego, M. C. and Villegas, J. G. (2016). Multi-product capacitated facility location problem with general production and building costs. *NETNOMICS: Economic Research and Electronic Networking*, 17(1), 47–70.

Mostaghim, S. and Teich, J. (2004). Covering Pareto-optimal fronts by subswarms in multi-objective particle swarm optimization. *Proceedings of the 2004 Congress on Evolutionary Computation (IEEE Cat. No.04TH8753)*, 2, 1404–1411.

Nguyen, V.-P., Prins, C. and Prodhon, C. (2012). A multi-start iterated local search with tabu list and path relinking for the two-echelon location-routing problem. *Engineering Applications of Artificial Intelligence*, 25(1), 56–71.

Parsopoulos, K. E. and Vrahatis, M. N. (2002). Particle swarm optimization method in multiobjective problems. *2002 ACM Symposium on Applied Computing (SAC 2002)*, 603–607.

ReVelle, C. S., Eiselt, H. A. and Daskin, M. S. (2008). A bibliography for some fundamental problem categories in discrete location science. *European Journal of Operational Research*, 184(3), 817–848.

Scott, A. J. (1971). Dynamic Location-Allocation Systems: Some Basic Planning Strategies. *Environment and Planning A*. SAGE Publications, 3(1), 73–82.

Shahabi, M., Akbarinasaji, S., Unnikrishnan, A. and James, R. (2013). Integrated Inventory Control and Facility Location Decisions in a Multi-Echelon Supply Chain Network with Hubs. *Networks and Spatial Economics*, 13(4), 497–514.

Tsou, C. S., Yang, D. Y., Chen, J. H. and Lee, Y. H. (2011). Estimating exchange curve for inventory management through evolutionary multi-objective optimization. *African Journal of Business Management*, 5(12), 4847–4852.

Wang, X. and Ouyang, Y. (2015). A continuum approximation approach to competitive facility location design under facility disruption risks. *Transportation Research Part B: Methodological*, 50, 90–103.

Warszawski, A. (1973). Multi-Dimensional Location Problems. *Journal of the Operational Research Society*, 24(2), 165–179.

Weber, A. (1909). *Theory of industrial location.*

Yasenovskiy, V. and Hodgson, J. (2007). Hierarchical location-allocation with spatial choice interaction modeling. *Annals of the Association of American Geographers*, 97(3), 496–511.

Yu, V. F., Normasari, N. M. E. and Luong, H. T. (2015). Integrated location-production-distribution planning in a multiproducts supply chain network design model. *Mathematical Problems in Engineering*, 2015.