JISE

A two-stage pricing and inventory optimization model for both ameliorating and deteriorating items in a competing environment

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Abstract

This paper develops a pricing and inventory model of a supply chain composing one supplier and two manufacturers in which the supplier sells its product to two competing manufacturers. Each manufacturer faces a deterministic demand that depends on his/her own and the other competitor sale price. Selling price for the supplier is time dependent and increment function. Both amelioration and deterioration effects are seen simultaneously for this model. In this paper, it is assumed that shortages are not allowed in which the objective is maximizing the total profit of supplier and manufacturers in two stages. In first stage, the total profit of supplier is maximized and then in the second stage, two manufacturers maximize their total profits with regard to the optimal decision variables obtained in the first stage. A case study of trout fish breeding is presented and numerical example is solved by Mathematica optimization tool. Sensitivity analysis is conducted to study the effect of various parameters on the optimal solution.

Keywords: Pricing, inventory model, Deteriorating and ameliorating items, competing environment, Bertrand game

1- Introduction

Nowadays, with the development of the technology and the growth in consumers' demand for various products, the intensity of market competition is increasing. Moreover, shortening life cycle of the products due to the complexity and diversification of customers' needs and some characteristics of the products such as fashions and perishability, forces companies to be more deliberate in their decisions in inventory management. To give an example for perishable product, more than two-thirds of industrial stores in U.S. are supermarkets where the majority of sales revenue of these supermarkets consists of perishable products such as food items (Chen et al. 2014). This matter shows the high-intensity competition in such markets. So, pricing is an important and effective lever to manage the profitability of these type of products. Of course, pricing is a double-edged sword. A lower selling price attracts more consumers to buy the products, but causes lower profit, whereas higher price has the reverse effect. Consequently, especially in the case of the seasonal/perishable products, decision makers should be aware of the pricing effects to manage inventory efficiently and effectively from raw materials to finished products to reduce their costs like holding cost, deteriorating cost, shortage cost for backlogged items and the unit cost of lost sales, and so on. Hence in last decades, interest has been increased in problems involving joint pricing and inventory control.

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In the overwhelming of the pricing and inventory model, the demand for the given items is assumed to vary as a function of the price. Of course the demand rate may depend on other factors such as time, stock level, advertising, service level, quality of product, lead time, and so on. Moreover, the demand rate in some study is depends on more than one factors simultaneously. For example Heydari and Norouzinasab (2016) developed a two echelons supply chain consisting of one manufacturer and one retailer which have a price and lead time dependent demand function. In another study, Zhu (2015) has developed a decentralized supply chain consisting of a supplier and a retailer facing price and lead time sensitive demand. Bashiri and Heydari (2017) in a similar work have investigated a supply chain which sells non-green traditional and sustainable green products. Their demand is function of retail price as well as green quality of product and retailers' sale efforts. For more detailed survey of related works, we refer the reader to the reviews by Chen and Simchi-Levi (2012) broad classes of pricing problems under different inventory factors such as deterioration and amelioration of products, warehouse capacity constraint, type of shortage, deterministic and stochastic demand, VMI or EOQ approach, single or multi-product, production factors, effects of inflation and time value of money, and so on.

In connection with aforementioned, food items such as meat, vegetable, and fruit and some oil derivatives such as gas, gasoline, and gas oil often deteriorate over the time. Deterioration, in general, is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of marginal value of a product that results in decreasing usefulness from the original one (Wee 1999). Moreover, it is well known that the life time of some other perishable products like medicines, cosmetics, blood, etc. is fixed and these cannot be used after the prescribed time period i.e. after the expired date (Mondal et al 2003). On the opposite side, growing animals such as fish, chickens, sheep, and ducks ameliorate over time. Amelioration occurs when the utility or value of livestock increases over time (Mahata and Kumur De 2016). When these items stay at farm or pond or in the distribution center, the stock level increases due to growth and also decreases due to death.

There are several papers about price dependent demand for either amelioration, or deterioration, but not both. In this regard, one of the first studies to consider the pricing problem for perishable items was undertaken by Cohen (1977). He assumed a retailer who sells a perishable product which deteriorates at an exponential rate under instantaneous replenishment. Kang and Kim (1983) extended Cohen's model to consider the case for finite replenishment. Thereafter, Abad (1996) presented a dynamic pricing and lot-sizing model which a reseller sells a perishable product. His model is a generalization of the model proposed by Rajan et al. (1992) regarding Abad considered shortage can be backlogged partially. Wee (1999) developed a joint pricing and inventory model with quantity discount and partial backordering for deteriorating items. Another inventory model was developed for a deteriorating item with a price-dependent demand rate by Mukhopadhyay et al. (2004). Assuming a price-dependent demand rate, Teng et al. (2005) proposed an EOQ model for perishable item to obtain the optimal price and lot size for a retailer when the supplier allows delayed payments. Freshly, Zhang et al. (2016) addresses a joint pricing, preservation technology investment problem under common resource constraints for deteriorating items.

There is several researches for ameliorating or deteriorating items which considered the demand is the function of other factors. For example Moon et al. (2005), Sana (2010), and Valliathal and Uthayakumar (2010) have considered the demand function of both ameliorating and deteriorating items time dependent. For both time and price dependent demand, Valliathal and Uthayakumar (2011), Maihami and Nakhai Kamalabadi (2012), and Feng et al. (2015) developed some models for perishable products. Dye and Ouyang (2005) have considered a stock level selling rate for perishable items in their model. Zhang et al. (2016) have assumed in their paper a price and service level dependent demand function for deteriorating item. Rabbani et al. (2015) have proposed a dynamic pricing and inventory control model for deteriorating stock. They have considered the demand is function of price and advertisement.

As noted above, a few researches have been conducted on developing joint pricing and inventory models considering amelioration and deterioration effects simultaneously. Mondal et al. (2003) developed an inventory model for both ameliorating and deteriorating item with the price dependent demand rate. Sana et al. (2009) incorporated the effects of advertising, sale price, and stock level to an economic order quantity model where the item is ameliorating and deteriorating over time. Mahata and Kumar De (2016)

proposed an EOQ model ameliorating and deteriorating items where the demand is price dependent and payment can be delayed. A key feature of these kinds of items is amelioration and deterioration rates. There are several cases studied in this field. In the early stage of the study, most of the deterioration or amelioration rates in the models are constant. Padmanabhan and Vrat (1995), Bhunia and Maiti (1999), Dye and Ouyang (2005), Chung and Liao (2006), Zhan Pang (2011), Taleizadeh et al. (2012), Maihami and Nakhai Kamalabadi (2012), Xiao and Xu (2013), and Taleizadeh et al. (2014) considered that the deterioration rate is constant in their inventory model. In recent research, more and more studies have begun to consider the relationship between deterioration or amelioration rate and time. For instance, some research used two-parameter Weibull distribution, because given items are assumed to have a vary deterioration or amelioration rate over time. Wee (1999) developed a joint pricing and inventory model with the Weibull distributed deterioration rate. Mondal et al. (2003) assumed that the amelioration follows two- parameter Weibull distribution and deterioration rate is constant. Wee et al. (2008) considered a two parameter Weibull distribution for both ameliorating and deteriorating items. Yu et al. (2011) proposed an inventory model that the deterioration rate is modeled by a Weibull distribution. In this type of products, in spite of lead-time importance, the majority of inventory control models in the literature are based on the assumption that the lead-time is zero, while in real life; lead time is a factual concept that should be considered especially when the product is deteriorating item. Since transportation from vendor to buyer often causes deterioration in quality of this type of products. In this regard, Mondal et al. (2003) developed an inventory model for both ameliorating and deteriorating items as mentioned. They considered in their model lead-time is known and constant. Chen et al. (2014) proposed a joint pricing and inventory control system for a perishable product over a finite horizon. In their model, demand is price dependent and lead-time is positive.

In this paper, we develop a two-stage pricing and inventory optimization model in two-echelon supply chain for both ameliorating and deteriorating items where deterioration rate is constant and amelioration follows two-parameter Weibull distribution. It is worthwhile to note that the deterioration rate during lead time between the supplier and the manufacturers is considered time dependent. In the first stage, a supplier maximize his/her profit by finding own optimal decision variables. Then in the second stage, two manufacturers maximize their own profit regarding the optimal decision variables of pervious stage in a competing environment independently, while each manufacturer's decision affects and is influenced by the other manufacturer decision in which the demand for manufacturer j is assumed decreasing in his/her sale price and increasing in that of the competitor. This type of competition is called Bertrand competition which was developed by Joseph Louis Francois Bertrand (1883) for the first time. His model argued interaction among sellers that set prices while their demand function is price sensitive. The price dependent demand function in Bertrand game is $D_j = a - b.p_j + \gamma.p_i$ where $j = 1,2$ and $i = 3 - j$, where p_j denotes that the sale price charged by manufacturer j to customers, whereas p_i is the sale price charged by the competitor e.g. manufacturer i. This demand pattern has been widely used in marketing research literature. Choi (1991) has developed a pricing model for two manufacturers and one retailer where the manufacturers sell their product to the retailer in a competing environment. Ingene and Parry (1995) have investigated coordination of a channel consisting of a manufacturer and two competing retailers. Padmanabhan and Png (1997) have incorporated return policies into a pricing model like Ingene and Parry (1995) model. Lee and Staelin (1997) discussed an industry model composed of two manufacturers selling products to two competing retailers. But on the other hand, there is a poor literature on application of Bertrand competition in pricing in supply chain. Bernstein and Federgruen (2003) have developed a dynamic pricing and inventory system in which a supplier sells a product to N competing retailers. Yao et al. (2008) have proposed a revenue sharing contract for coordinating a supply chain comprising a manufacturer as a Stackelberg leader and two competing retailers as followers. Dong et al. (2009) have incorporated Bertrand competition into a pricing model in a different way. They have considered a manufacturer produces and sells two complementary to a retailer. When the sale price of one product increases, therefore the demand of this product decreases, whereas the demand of complement product increases. Lin Jiang et al. (2014) have developed a decision and coordination system consisting

of a manufacturer, a third party logistics (3PL) provider, and two competing retailers. They have investigated the product distribution that may be implemented by 3PL provider or retailers under three contract policies. However, based on the above literature review, we failed to find a paper that consider two-echelon competing supply chain for both ameliorating and deteriorating items in which consider time-dependent deterioration rate over the lead time. Hence, our study may be the first to consider the competition in a pricing and inventory model for both ameliorating and deteriorating items.

Table 1. Summary of related papers

authors	demand pattern					deterioration rate			amelioration rate			shortage			lead time	other considerations
	constant	price	time	stock level	other	constant	weibull	other	constant	weibull	other	backlog	lost sale	partial		
P.L.Abad (1996)	*					*								*		-
Wee (1999)		*					*							*		quantity discount
Mondal et al. (2003)		*				*					*		*		*	-
Moon et al. (2003)			*			*			*			*				-
Mukhopadhyay et al. (2004)		*						*								-
Dye and Ouyang (2005)			*			*								*		-
Chung and Liao (2006)	*					*										trade credit
Wee et al. (2008)	*						*			*						time value of money
Valliathal and Uthayakumar (2010)		*						*			*	*				inflation and time discounting
Valliathal and Uthayakumar (2011)		*	*			*								*		-
Pang (2011)		*				*								*		-
Maihami and Nakhai (2012)		*	*			*								*		-
Xiao and Xu (2013)		*		*		*										revenue sharing
Taleizadeh et al. (2013)	*					*						*				quantity discount
Yu et al. (2013)		*	*													revenue sharing
Chen et al. (2014)		*						*				*	*		*	-
Taleizadeh and Noori-daryan (2015)		*				*										VMI approach
Mahata and and De (2016)		*				*			*							delayed payment
Feng et al. (2016)		*	*			*										limited production capacity
Rabbani et al. (2015)		*			*	*										Promotion on sales
Zhang et al. (2016)		*	*				*		*							resource constraint

Table 1. Continued

authors	demand pattern					deterioration rate			amelioration rate			shortage			lead time	other considerations
	constant	price	time	stock level	other	constant	weibull	other	constant	weibull	other	backlog	lost sale	partial		
this paper		*				*			*						*	competing environment

The remainder of the paper is structured as follows. In section 2, the problem is defined. In section3, assumptions and notations are presented. Thereafter, in section 4, we adopted the mathematical model. Next, section 5 illustrates Numerical results and then, in Section 6, sensitivity analyses of the obtained optimal solutions are represented. Managerial insights are listed in section 7 and finally Section 8 finishes the paper with conclusion, recommended future research directions.

2- Problem description

In this paper we develop a two echelon supply chain consisting of one supplier and two competing manufacturers. The graphical representation of the inventory level for supplier and two manufacturers is depicted in Fig.1. The supplier initiates the purchase of livestock from an outside prior supplier. The amount of initial livestock are assumed decision variable as the raw materials for supplier. Since the raw materials are young and fast growing, amelioration occurs as the livestock are raised in the supplier farm during time interval$[0, t_s]$.$t_s$is the time that the livestock level of supplier reaches to a predetermined amount U by weight. Afterward, the supplier sells the same amount of livestock to each manufacturer with the price w which is incremental with respect tot_s e.g.$w = d + c.t_s$, where d is raw material purchase cost for supplier and $c$is a coefficient which denotes increase in the price per unit of time per unit of weight. It is noteworthy that during the lead time from the supplier to the manufacturers (L_j), some livestock die with the rate of θ_L which is decreasing with respect tot_s. Since the more the age of livestock e.g.t_s, the lower the rate of deterioration during lead time would be. The amelioration of livestock in each manufacturer continued to a given$T$. But the inventory cycle in each manufacturer is divided into two intervals. In the first interval, the stock level is increased only because of the amelioration activation. In this period, the items are not sold. In the second interval, selling the livestock are started with price ofp_j. In the both interval, the deteriorating units are discarded. As mentioned above, Manufacturer j's price dependent demand is:

$$D_j = a - b.p_j + \gamma.p_i \qquad j = 1,2 \ and \ i = 3 - j \tag{1}$$

where a denotes primary demand for products, b denotes the sensitivity of costumers to sale price, and γ denotes the competitive factor, which is a measure of the sensitivity of manufacturer j's sale to change in manufacturer j's price. In this study, we assume$b \geq \gamma$, which is sensible because the demand for manufacturer j is more sensitive to its own sale price than that of the competitor. We consider that the two manufacturers have the same amelioration, deterioration, holding cost and they face symmetric demand function.
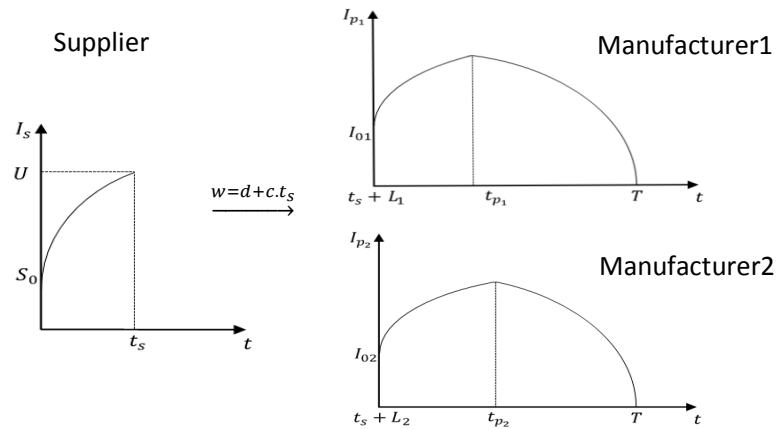
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Figure.1 inventory curve for supplier and manufacturers

This model has been solved in two stages. In the first stage, to achieve maximum profit for supplier, the optimum t_s would find by Taylor expansion approximation. In the second stage, the problem has been solved with the above characteristics by numerical method. Finally the optimum results are presented by a numerical example.

3- Model formulation

The following notation and assumptions are defined and will be used throughout the paper:

3-1- Parameters

α, β the parameters of Weibull distribution whose probability density is: $f(t) = \alpha\beta t^{\beta-1} exp(-\alpha t^\beta)$

In this study, β is considered a proper fraction (i.e. $0 < \beta < 1$). Since the livestock's growth is faster in the beginning and decline at later time periods.

$Y(t)$ the instantaneous rate of amelioration of the on hand inventory in the interval $0 \le t \le t_S$ which obeys the Weibull distribution. i.e. $Y(t) = \frac{f(t)}{1-F(t)} = \alpha\beta t^{\beta-1}$

I_{0j} the initial stock level for manufacturer j

L_j lead time between the supplier and manufacturer j

θ_S the constant deterioration rate for the items in the supplier site

θ_{P_j} the constant deterioration rate for the items in the manufacturer j's ($0 \le \theta_j \le 1$)

h_s the holding cost for supplier per unit per time

h_{p_j} the holding cost for manufacturer j per unit per time

C_b the raw material purchase cost for supplier

C_{as} the cost of amelioration per unit for supplier

C_{ap_j} the cost of amelioration per unit for manufacturer j

C_{ds} the cost of deterioration per unit for supplier

C_{dp_j} the cost of deterioration per unit for manufacturer j

U the stock level for supplier at t_S

T the prescribed cycle length

3-2- Decision variables

t_S the time that the meeting of the manufacturers' demand filled up

t_{Pj} the time that the meeting of the costumers' demand started by manufacturer j

3-3- Auxiliary variables

S_0 the initial stock level for supplier

p_j the optimal selling price for the manufacturer j product per unit

$w = c + d.t_S$ the optimal selling price for supplier product per unit

$\theta_L = qe^{-r.t_S}$ the deterioration rate for the items per unit during the lead time $(0 < q, r < 1)$

3-4- Assumption

a) The inventory system involves only one item.

b) The replenishment rate is instantaneous.

c) Shortage neither at supplier nor at manufacturers is permitted.

d) Time horizon is infinite.

e) Lead time is considered the same for both manufacturers.

f) No replacement or repairment is considered for deteriorated items during time period T.

g) The amelioration and deterioration occur when the item is effectively in stock.

h) The amelioration during lead time is assumed to be negligible.

4- Model formulation

The goal of this paper is to maximize the profit of supplier and manufacturers independently where manufacturers wish to maximize their profit in competing environment. To solve this model, a two stage optimization model is proposed. In the first stage, we aimed to find the optimal value of t_S and consequently maximize the supplier profit function. In the second stage, we want to maximize the manufacturers' profit function individually by finding t_{Pj}.

4-1- First stage

The following differential equations can describe the instantaneous state of $I_s(t)$:

$$\frac{dI_s(t)}{dt} = -\theta_S I_s(t) + \alpha\beta t^{\beta-1} I_s(t) \qquad 0 \le t \le t_S \tag{2}$$

with the boundary conditions $I_s(0) = S_0$, $I_s(t_S) = U$.

Solving the differential equation(1), we get the inventory level at time t as:

$$I_S = S_0.exp(\alpha t^\beta - \theta_S t) \qquad 0 \le t \le t_S \tag{3}$$

Further

$$I_s(t_S) = U = S_0.exp\left(\alpha t_s^\beta - \theta_S t_s\right) \qquad 0 \le t \le t_S \tag{4}$$

which implies

$$S_0 = \frac{U}{exp(\alpha t_s^\beta - \theta_S t_s)} \tag{5}$$

Now, we can obtain inventory costs and sales revenue per cycle for the supplier that consists of following five parts:

The purchasing cost:

$$TC_{SP} = C_B.S_0 = C_B.\left(\frac{U}{exp(\alpha t_s^\beta - \theta_S t_s)}\right) \tag{6}$$

The inventory holding cost:

$$TC_{SH} = h_s\left(\int_0^{t_s} S_0 exp(\alpha t^\beta - \theta_S t)dt\right) = c_h.H_S \tag{7}$$

The deterioration cost:

During the inventory cycle for supplier, the deteriorated items are withdrawn. So we have following equations:

$$TC_{SD} = c_d.\theta_S\left(\int_0^{t_s} S_0 exp(\alpha t^\beta - \theta_S t)dt\right) = c_d.\theta_S.H_S \tag{8}$$

The amelioration cost:

The inventory level of supplier varies during the interval $[0, t_s]$ due to two reasons: amelioration and deterioration. The inventory level increases because of amelioration, while deterioration reduces it. In this regard, the following equation can be obtained:

$$S_0 + A' - \theta_S H_S = U \tag{9}$$

which implies

$$A' = U + \theta_S H_S - S_0 \tag{10}$$

Therefore the amelioration cost is calculated as follows:

$$TC_{SA} = C_a.A' \tag{11}$$

The sales revenues:

$$TR_S = w.I_S(t_s) = (d + c.t_s)U \tag{12}$$

Consequently, the total profit of supplier is given by:

$$Z_S(t_s) = d.U + c.U.t_s - (C_b - C_{as})S_0 - C_{as}.U - (C_{as}.\theta_S + C_h + C_{ds}.\theta_S)H_S \tag{13}$$

The main purpose in this stage is to determine the optimal time t_s for selling the product to the manufacturers that correspond to maximize the total profit of supplier. To achieve our purpose, we should prove the concavity of the $Z_S(t_s)$.

Theorem1. The total profit function of supplier is a concave function with respect to t_s.

Proof of Theorem 1 is given in Appendix A.

Under concavity of total profit function $Z_S(t_s)$, the root of the first derivative of the objective function is non-real number with respect to t_s due to the exponents.

$$\frac{dZ_S(t_s)}{dt_s} = c.U - (C_b - C_{as}).U\left(\frac{-(\alpha\beta t_s^{\beta-1} - \theta_S)}{exp(\alpha t_s^\beta - \theta_S t_s)}\right) - (C_{as}\theta_S + C_h + C_d\theta_S)U = 0 \tag{14}$$

Hence we use Taylor expansion approximation around zero in order to find a real solution. That means we have:

$$\frac{dZ_S(t_s)}{dt_s} = c.U - (C_b - C_{as}).U\left(\frac{-(\alpha\beta t_s^{\beta-1} - \theta_S)}{1 + (\alpha t_s^\beta - \theta_S t_s) + \frac{(\alpha t_s^\beta - \theta_S t_s)^2}{2}}\right) - (C_{as}\theta_S + C_h + C_d\theta_S) \tag{15}$$
$$= 0$$

Solving the above equation, we got the best t_s that maximize the supplier profit function.

4-2- Second stage

The following differential equations represent the inventory status of manufacturer j:

$$\begin{cases} \dfrac{dI_{Pj}(t)}{dt} = -\theta_j I_{Pj}(t) + \alpha\beta t^{\beta-1}I_{Pj}(t)t_s + L_j \le t \le t_{Pj} \\ \dfrac{dI_{Pj}(t)}{dt} = -\theta_j I_{Pj}(t) + \alpha\beta t^{\beta-1}I_{Pj}(t) - D_j(p_j)t_{Pj} \le t \le T \end{cases} \tag{16}$$

With the boundary conditions $I_{Pj}\left(t_s + L_{P_j}\right) = I_{0j}$ and $I_{Pj}(T) = 0$, the (15) yield:

$$I_{Pj}(t) = \begin{cases} I_{0j}.exp\left(\alpha\left(t - (t_s + L_j)\right)^\beta - \theta\left(t - (t_s + L_j)\right)\right)t_s + L_j \le t \le t_{Pj} \\ D_j(p_j).exp\left(\alpha\left(t - (t_s + L_j)\right)^\beta - \theta\left(t - (t_s + L_j)\right)\right)\displaystyle\int_{t_{Pj}}^T exp(\theta t - \alpha t^\beta)dt \; t_{Pj} \le t \le T \end{cases} \tag{17}$$

From equation(16), we have

$$I_{Pj}(t_{Pj}) = I_{0j}.exp(\alpha t_{Pj}^\beta - \theta_j t_{Pj}) = D_j(p_j).exp(\alpha t_{Pj}^\beta - \theta t_{Pj})\int_{t_{Pj}}^T exp(\theta t - \alpha t^\beta)dt$$

which imply

$$D_j(p_j) = \frac{I_{0j}}{\int_{t_{Pj}}^T exp(\theta t - \alpha t^\beta)dt} \tag{18}$$

In addition, from equations $(1) - (18)$ the following equation can be obtained:

$$p_j = \frac{(b + \gamma.p_i)}{a} - \frac{I_{0j}}{a\int_{t_{Pj}}^{T} exp(\theta t - \alpha t^\beta)dt}, i = 3 - j \tag{19}$$

In this stage like previous stage, we divide the inventory cost and sale revenue per cycle for manufacturer j to the five following parts:

Purchasing cost is

$$TC_{P_jP} = w.\left(U/2\right) = (d + c.t_s)\left(U/2\right) \tag{20}$$

The total units held over the cycle in the manufacturer j's site is given by

$$H_{P_j} = \int_{t_s+L_j}^{t_{Pj}} I_{Pj}(t_{Pj})dt + \int_{t_{Pj}}^{T} I_{Pj}(t_{Pj})dt \tag{21}$$

Therefore, the inventory holding cost for manufacturer j can be express as

$$TC_{SH} = h_{p_j}.H_{P_j} \tag{22}$$

The deterioration cost consists of two parts. The first part is the deterioration units during lead-time, $\theta_L.L_j.\left(U/2\right)$, and the second is deterioration units during $[t_s + L_j, T]$, $\theta_{P_j}.H_{P_j}$.

Thus, the deterioration cost for manufacturer j is

$$TC_{P_jD} = C_{dp_j}\left(\theta_L.L_j.\left(U/2\right) + \theta_{P_j}.H_{P_j}\right) \tag{23}$$

The cost of amelioration is calculated like previous stage, but the demand is met during the interval $[t_{Pj}, T]$. So the units of amelioration can be obtained by

$$A' = D_j(p_j)(T - t_{Pj}) + \theta_{P_j}H - I_{0j} \tag{24}$$

Thus the amelioration cost for manufacturer j is

$$TC_{P_jA} = C_{ap_j}.A' \tag{25}$$

The sales revenue:

$$TR_{P_j} = D_j(p_j)(T - t_{Pj})p_j \tag{26}$$

Therefore the total profit of manufacturer j can be written as

$$Z_{p_j}(t_{Pj}) = TR_{P_j} - TC_{P_jA} - TC_{P_jD} - TC_{SH} - TC_{P_jP} = D_j(p_j)(T - t_{Pj})\left(p_j - C_{ap_j}\right)$$
$$- \left(C_{ap_j}.\theta_{P_j} + C_{dp_j}.\theta_{P_j} + h_{p_j}\right)H_{P_j} + C_{ap_j}.I_{0j} - \left(w + C_{dp_j}.\theta_L.L_j\right).\left(U/2\right) \tag{27}$$

Theorem2. The total profit function of manufacturer j is a concave function with respect to t_{Pj} when $a\left(p_j - C_{ap_j}\right) \geq D_j(p_j)$.

Proof of Theorem 2 is given in Appendix B.

In this stage each manufacturer wishes to maximize their profit with respect to finding the optimal t_{Pj}. To obtain the optimal t_{Pj}, we should take the first derivative of the objective function of manufacturer j $\left(Z_{p_j}(t_{Pj})\right)$ and equate the result to zero.

The optimization tools used are Mathematica 10.2.

5- Case study of trout fish breeding

For numerical analysis, we present the case of and data Iran's trout fish supply chain. Generally, life cycle of market trout (250-300 gr) is divided into five stages: eggs, alevins, fry, fingerlings, and adult.
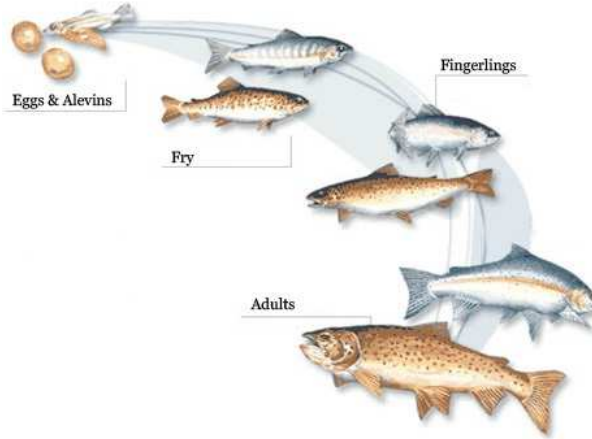

Figure 2. Trout fish life cycle

In our case, the supplier buys S_0 kg alevins from reproduction center as raw materials. Alevins grow rapidly in supplier cite to reach fry size. This stage, Depending on circumstances, takes between 7 to 15 weeks.In this interval for the alevins, amelioration and deterioration rate are defined. In the first stage of our problem, the supplier should find the optimal sale time of levies i.e. t_S and then S_0 and w with respect to purchase, amelioration, deterioration, and holding cost and predetermined stock level$U$ by weight.When the amount of fries reaches to$U$, they are transported to manufacturers' site. During this transportation, fries deteriorate with rate of θ_L. Because of transportation, this rate is higher in comparison to when the levies are in the supplier or manufacturer sites. Even though that this matter is clear and tangible in reality, to the best of our knowledge, there is no study in the literature in this field. However, we define $\theta_L = qe^{-r.t_S}$ and estimate the value of parameters q and r. For this purpose, if$t_S = 0$, then we have $\theta_L = q$ and it represents the deterioration rate of raw materials bought by supplier during lead time. Therefore, q should be greater thanθ_S. On the other hand, r is a parameter which is determined so that$\theta_L \geq \theta_{P_j}$. Therefore, in Table 2 we illustrate the acceptable values for q and r with respect to the values of θ_S andθ_{P_j}.

Table 2. Deterioration rate during lead time for different t_s, p and q

t_s	7				8				9			
q	0.075		0.08		0.075		0.08		0.075		0.08	
r	0.004	0.005	0.004	0.005	0.004	0.005	0.004	0.005	0.004	0.005	0.004	0.005
θ_L	0.073	0.072	0.078	0.077	0.073	0.072	0.077	0.077	0.072	0.072	0.077	0.076

t_s	10				11				12			
q	0.075		0.08		0.075		0.08		0.075		0.08	
r	0.004	0.005	0.004	0.005	0.004	0.005	0.004	0.005	0.004	0.005	0.004	0.005
θ_L	0.072	0.071	0.077	0.076	0.072	0.071	0.077	0.076	0.071	0.071	0.076	0.075

t_s	13				14				15			
q	0.075		0.08		0.075		0.08		0.075		0.08	
r	0.004	0.005	0.004	0.005	0.004	0.005	0.004	0.005	0.004	0.005	0.004	0.005
θ_L	0.071	0.070	0.076	0.075	0.071	-	0.076	0.075	0.070	-	0.075	0.074

According to the table 2, the deterioration rate during the lead time decreases by increasing t_s due to the higher age of live stocks. When $t_s = 14\ or\ 15$ in $q = 0.075$ and $r = 0.005$, the value of θ_L obtained less than θ_S which is not acceptable. It is noteworthy that in this study deterioration rate in supplier and manufacturers' sites was considered 0.07 and 0.04, respectively, based on Woynarovich et al (2011).

After, the fries grow in the manufacturers' ponds to reach to juvenile and adult size which is called market trout and is desirable for consumers to eat. Therefore, the manufacturers should determine the optimal time of starting sale interval i.e. t_{Pj} and then p_j with respect to their costs. These stages take approximately 10 to 12 months or 44 to 54 weeks (Woynarovich et al. 2011). In this study, T is considered 50 weeks. Finally, optimal t_S and t_{Pj} to maximize the total profit of supplier and manufacturer j, respectively, can be derived from above stages.

For illustration, following parameters are considered:

$$T = 50\ weeks, U = 300\ kg, \alpha = 0.5\ , \beta = 0.5\ , d = C_b = 8\ \$, c = 1\$, b = 600, a = 6,$$

$$\gamma = 3, L_1 = L_2 = 0.3\ week, \theta_S = 0.07, \theta_{P_1} = \theta_{P_2} = 0.04, h_s = 0.9\$/unit,$$

$$h_{p_1} = h_{p_2} = 0.7\ \$/unit, C_{as} = 0.9\ \$/unit, C_{ap_1} = C_{ap_2} = 1.2\$/unit,$$

$$C_{ds} = 1\$/unit, C_{dp_1} = C_{dp_2} = 1.5\$/unit, q = 0.08\ , r = 0.004$$

It is noticeable that aforementioned costs are equally magnified to show better the differences of results in sensitivity analysis. Therefore they aren't exactly real.

With these assumed parameters, the optimal solution of this system tabulate in table3:

Table3. The optimal solution of proposed system

	t_s	S_0	w	$t_{P1} = t_{P2}$	$p_1 = p_2$	Z_s	$Z_{P1} = Z_{P2}$
Optimal Solution	9.64	124.732	17.64	24.8	190.896	1603.81	104246

6- Sensitivity analysis

Now, we study the effect of changes in the above-mentioned inventory system on the following parameters and furthermore, based on sensitive analysis, we suggest some managerial implications can help the supplier and the two competing manufacturers to maximize their total profit.

6-1- Holding and amelioration cost for supplier and manufacturers

The main parts of costs of breeding live stocks include holding and amelioration costs. Holding cost consists of sentry, care and maintenance costs of live stocks, whereas amelioration cost consist of food and medicine costs. In this section according to Table 4, the changes of holding and ameliorating costs are investigated on decision variables and total profit of supplier.

Table 4. Sensitive analysis with respect to holding and ameliorating costs for supplier

parameters	value	t_s	S_0	w	$t_{P1} = t_{P2}$	$p_1 = p_2$	Z_s	$Z_{P1} = Z_{P2}$
h_s	0.86	13.61	122.965	21.61	24.8	190.893	**1760.72**	103668
	0.88	11.35	123.198	19.35	24.8	190.895	**1690.33**	104006
	0.90	9.64	124.732	17.64	24.8	190.896	**1603.81**	104246
	0.92	8.32	126.971	16.32	24.8	190.897	**1517.52**	104432
	0.94	7.29	129.552	15.29	24.8	190.898	**1437.66**	104576
C_{as}	0.86	9.86	124.456	17.86	24.8	190.896	**1623.32**	104215
	0.88	9.75	124.591	17.75	24.8	190.896	**1613.59**	104231
	0.90	9.64	124.732	17.64	24.8	190.896	**1603.81**	104246
	0.92	9.52	124.893	17.52	24.8	190.896	**1593.71**	104263
	0.94	9.42	125.033	17.42	24.8	190.897	**1584.10**	104277

According to table 4, when the value of parameters h_s and C_{as} increases, the value of t_s decreases and as a result, the total profit of supplier decreases. Since t_s has direct dependence with the sale price of the supplier, the total profit of manufacturers increases by decreasing t_s value. Figure 3 shows the reverse relationship between amelioration and holding cost and supplier's total profit.
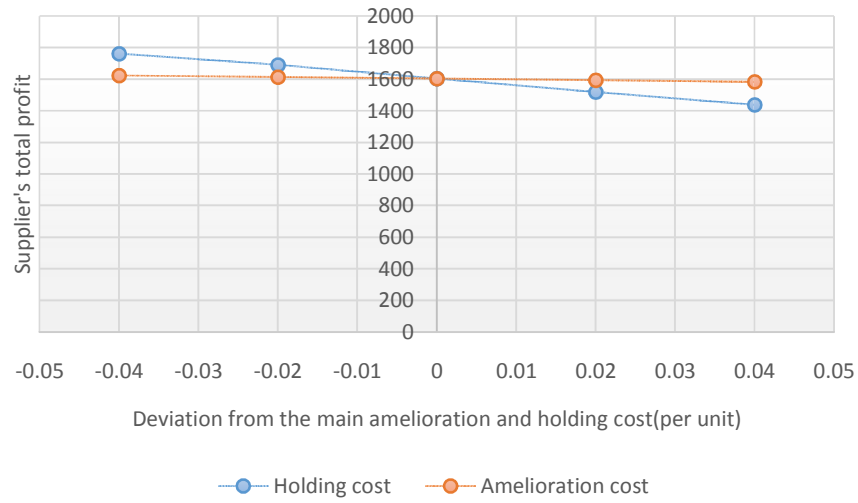
Figure 3. Effects of amelioration and holding cost on supplier's total profit changes

Table 5. Sensitive analysis with respect to holding and ameliorating costs

parameters	value	t_s	S_0	w	$t_{P1} = t_{P2}$	$p_1 = p_2$	Z_s	$Z_{P1} = Z_{P2}$
	0.60	9.64	124.732	17.64	24.8	190.896	1603.81	**107196**
	0.65	9.64	124.732	17.64	24.8	190.896	1603.81	**105721**
h_{Pj}	0.70	9.64	124.732	17.64	24.8	190.896	1603.81	**104246**
	0.75	9.64	124.732	17.64	24.8	190.896	1603.81	**102771**
	0.80	9.64	124.732	17.64	24.8	190.896	1603.81	**101296**
	1.10	9.64	124.732	17.64	24.8	190.896	1603.81	**104418**
	1.15	9.64	124.732	17.64	24.8	190.896	1603.81	**104332**
C_{ap_j}	1.20	9.64	124.732	17.64	24.8	190.896	1603.81	**104246**
	1.25	9.64	124.732	17.64	24.8	190.896	1603.81	**104160**
	1.30	9.64	124.732	17.64	24.8	190.896	1603.81	**104074**

Figure 4 illustrates the reverse relationship between amelioration and holding cost and manufacturers' total profit.
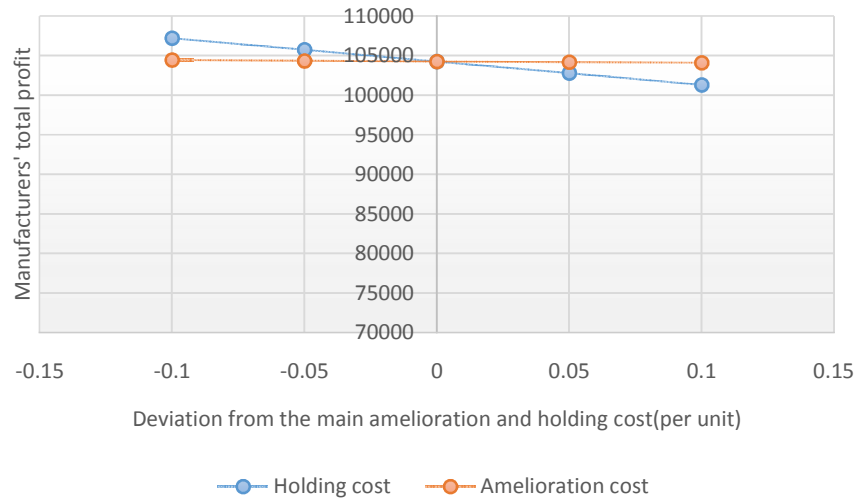
Figure 4. Effects of amelioration and holding cost on Manufacturers' total profit changes

On the other hand, regarding to the changes in the values of supplier and manufacturers total profit, the changes of holding cost has more effects on total profit than the amelioration cost per unit of change. E.g. according to the Fig 3 and Fig 4, the slope of blue line is more negative than the orange line. Hence from an economic view point, the breeding farms can increase their profit by decreasing the holding cost more than the melioration cost.

6-2- Asymmetrical manufacturer demand function

In previous section, we assume that the demand function is symmetric. In other words, if two competing manufacturers identify that equilibrium leads to the same sale price and other decision variables, then they act jointly on pricing. Therefore the demand function will become $D_j(p_j) = b - (a - \gamma)p_j$, where $j = 1,2$. But if two competing manufacturers encounter asymmetrical demand function, it has an asymmetrical impact on selling price and total profit of manufacturers.

As it can been seen in Table 6, we investigate the effect of different γ values on manufacturers' total profit with fixed a. Compared to the symmetrical condition at $a = 6$, the total profit of both manufacturers is declined. Furthermore, lowering the sensitivity of price i.e. a, leads to increase in the sale price and consequently the total profit increases. On the other hand, different competition factor in manufacturers demand function causes difference in value of sale price.

Table 6. Optimal solution with asymmetric demand function in competition intensity factor

Demand functions	a	t_{P_1}	t_{P_2}	p_1	p_2	Z_{p_1}	Z_{p_2}
$D_1 = b - ap_1 + 3p_2$ $D_2 = b - ap_2 + 2.5p_1$	6.5	24.8	24.8	156.56	148.32	80616	74945
	6	24.8	24.8	180.85	170.80	97331	90416
	5.5	24.8	24.8	213.97	201.39	120127	111465

In this study, if the competition intensity factor in the demand function of a manufacturer is less than the other, a price obtained is less than the other competitor price. It means the changes in other competitor price have lower impact intensity on own demand function in comparison to the other demand function.

An analogous analysis has done for the effect of different a on the total profit of each manufacturer with fixed γ. Regarding to the Table 7, how much γ has greater value, a value of price obtained would be more. In other words, the low competition intensity factor means that the manufacturers can sell more

products with a lower sale price. In addition, it is noteworthy that being deference in price sensitivity factor causes more difference between the sale prices of manufacturers in comparison to being deference in competition intensity factor in the same magnitude of change.

Table 7. Optimal solution with asymmetric demand function in sensitivity of price factor

Demand functions	γ	t_{P_1}	t_{P_2}	p_1	p_2	Z_{p_1}	Z_{p_2}
$D_1 = b - 6p_1 + \gamma p_2$	3.5	24.8	24.8	274.24	306.05	161610	183816
$D_2 = b - 5p_2 + \gamma p_1$	3	24.8	24.8	218.16	245.43	123015	141784
	2.5	24.8	24.8	180.84	204.96	97331	113927

7- Managerial insights

This paper studied a pricing and inventory model for both ameliorating and deteriorating items like fish and broiler chicken in which that its results can be helpful for farm owner/manager.

In the sensitivity analysis part, we investigated the effect of amelioration and holding cost in the same magnitude of change on the total profit function of supplier and manufacturers. We observed that changes in holding cost per unit affect more on the total profit function than ameliorating cost. In the other words, farm owner/manager to have more consideration about holding cost rather than amelioration cost.

In the other sensitivity analysis, the changes of price sensitivity were investigated. It showed that the higher value of price sensitivity causes less profit for vendors. Because how much price sensitivity has greater value, the vendors are more aware of their sale price because of sensitivity of costumers to the change of the price. Because price sensitivity varies based on industry, competitor marketing, needs of customers, and so on, it is difficult to handle the selling products by tuning the price sensitivity. Nevertheless, overriding the desire and developing the loyalty of customers can help the vendors to overcome high price sensitivity.

In the end in the similar analysis, we assumed the manufacturers' market asymmetric. It also showed that the greater the value of competitive intensity, the higher the value of sale price would be. In other words, the low competitive intensity factor means that the manufacturers can sell more product with a lower sale price. Of course the value of competitive intensity depends on some factors such as the number of competitors in the market which is out of the control, but the vendors can pursue some strategies like advertisement, improvement in quality and so on that gain more customers attention in a market with many competitors.(Caldart and Oliveira 2010)

The present pricing and inventory model also is of very importance in the developing country, like Iran, India, Pakistan, etc. Since the cultural of life stoke such as high breed broiler and fish has been grown nowadays in a very large scale.

8- Conclusion

In this paper, a two stages pricing and inventory optimization model is proposed for two echelon supply chain consisting a supplier and two manufacturers. Both manufacturers face price dependent demand in a competing environment in which increase in price of each manufacturers causes reduction in own demand and increase in the other competitor demand function. In this supply chain, the supplier sells predetermined amount of live stocks to the manufacturers so that the sale price is time dependent function. Since the product is both ameliorating and deteriorating item, the deterioration and amelioration rate and its costs are assumed for the supplier and the manufacturers. Transportation of the live stokes from supplier to manufacturers takes some time which called lead time. During this time, we define a deterioration rate which depends on age of live stocks. The greater the live stocks age, the deterioration

rate reduces. Growth of live stocks in manufacturers' sites continues. Then the sale of live stocks starts after a time that manufacturers should determine as a decision variables. After proving the concavity of the supplier and manufacturers' total profit function, the times to sale for the supplier and the manufacturers are obtained and the initial stock for the supplier and sale price for the manufacturers are determined.

Solving this optimization model, we consider the market is symmetric. Therefore, the price sensitivity and competition intensity factor are the same for each manufacturers' demand function. Hence, in sensitivity analysis we assumed the market asymmetric and re-optimize the model. The analysis indicated that the sale price of manufacturers increase by reduction of price sensitivity and decrease by reduction of the competition intensity. The analysis also showed that the impact of sensitivity price change in demand is greater than the impact of competition intensity factor.

This model can be further improved by incorporating quantity discount, inflation, effect of advertisement in the demand, coordination policies, etc.

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Appendix A:

Proving the concavity of supplier profit function $Z_S(t_s)$:

$$Z_S(t_s) = d.U + c.U.t_s - (C_b - C_{as})S_0 - C_{as}.U - (C_{as}.\theta_S + C_h + C_d.\theta_S)H_S \tag{A.1}$$

where $H_S = \int_0^{t_s} I_S \, dt = \int_0^{t_s} S_0 exp(\alpha t^\beta - \theta_S t)dt$

Now,

$$\frac{d^2 Z_S(t_s)}{dt_s^2} = (C_b - C_{as}).U \frac{\alpha\beta(\beta - 1)t^{\beta-2} - (\alpha\beta t_s^{\beta-1} - \theta_S)^2}{exp(\alpha t_s^\beta - \theta_S t_s)} \tag{A.2}$$

As $\beta < 1$, therefore $(\beta - 1) < 0$,

and as $C_b > C_{as}$

thus

$$\frac{d^2 Z_S(t_s)}{dt_s^2} = (C_b - C_{as}).U \frac{\alpha\beta(\beta - 1)t^{\beta-2} - (\alpha\beta t_s^{\beta-1} - \theta_S)^2}{exp(\alpha t_s^\beta - \theta_S t_s)} < 0 \tag{A.3}$$

So, the supplier's profit function is concave with respect to t_s.

Appendix B:

Proving the concavity of manufacturer j profit function $Z_j(t_{p_1})$:

$$Z_{p_j}(t_{Pj}) = D_j(p_j)(T - t_{Pj})(p_j - C_{ap_j}) - (C_a.\theta_j + c_d.\theta_S + c_h)H_{p_j} + C_a.I_{0j} - c_p.I_{0j} \\ - c_{LT}.\theta_L\left(\frac{U}{2}\right) \tag{B.1}$$

where

$$H_{p_j} = \int_{t_s}^{t_{Pj}} I_{Pj}(t_{Pj})dt + \int_{t_{Pj}}^{T} I_{Pj}(t_{Pj})dt =$$

$$\int_{t_s}^{t_{Pj}} I_{0j} \cdot exp(\alpha t^\beta - \theta t)dt + \int_{t_{Pj}}^{T} D_j(p_j) \cdot exp(\alpha t^\beta - \theta t)\left(\int_{t_{Pj}}^{T} exp(\theta t - \alpha t^\beta)dt\right)dt \qquad (B.2)$$

Now,

$$\frac{\partial^2 Z_{p_j}(t_{Pj})}{\partial t_{Pj}{}^2} = \frac{d^2 D_j(p_j)}{dt_{p_j}^2}(T - t_{Pj})\left(p_j - C_{ap_j}\right) - 2\frac{dD_j(p_j)}{dt_{Pj}}\left(p_j - C_{ap_j}\right) + 2\frac{dD_j(p_j)}{dt_{Pj}}(T - t_{Pj});$$

$$+ D_j(p_j)(T - t_{Pj})\frac{d^2 p_j}{dt_{p_j}^2} - 2D_j(p_j)\frac{dp_j}{dt_{Pj}} \qquad (B.3)$$

$$- \left(C_{ap_j} \cdot \theta_j + c_d \cdot \theta_S + c_h\right)\left(I_{0j} - 1\right)\left(\left(\alpha\beta t_{p_j}^{\beta-1} - \theta\right) exp\left(\alpha t_{p_j}^\beta - \theta t_{p_j}\right)\right)$$

where

$$\frac{dD_j(p_j)}{dt_{Pj}} = I_{0j}\frac{exp\left(\theta t_{Pj} - \alpha t_{p_j}^\beta\right)}{\left(\int_{t_{Pj}}^{T} exp(\theta t - \alpha t^\beta)dt\right)^2} > 0 \qquad (B.4)$$

$$\frac{d^2 D_j(p_j)}{dt_{p_j}^2} = I_{0j}\frac{exp(\theta t - \alpha t^\beta)\left(\left(\theta - \alpha\beta t^{\beta-1}\right)\left(\int_{t_{p_j}}^{T} exp(\theta t - \alpha t^\beta)dt\right) - 2exp(\theta t - \alpha t^\beta)\right)}{\left(\int_{t_{Pj}}^{T} exp(\theta t - \alpha t^\beta)dt\right)^3} \qquad (B.5)$$

$$< 0$$

as $\theta < \alpha\beta t_{p_j}^{\beta-1}$

$$\frac{dp_j}{dt_{Pj}} = -\frac{I_{0j}}{a}\frac{exp\left(\theta t_{Pj} - \alpha t_{p_j}^\beta\right)}{\left(\int_{t_{Pj}}^{T} exp(\theta t - \alpha t^\beta)dt\right)^2} < 0 \qquad (B.6)$$

$$\frac{d^2 p_j}{dt_{p_j}^2} = -\frac{I_{0j}}{a}\frac{exp(\theta t - \alpha t^\beta)\left(\left(\theta - \alpha\beta t^{\beta-1}\right)\left(\int_{t_{Pj}}^{T} exp(\theta t - \alpha t^\beta)dt\right) - 2exp(\theta t - \alpha t^\beta)\right)}{\left(\int_{t_{Pj}}^{T} exp(\theta t - \alpha t^\beta)dt\right)^3} \qquad (B.7)$$

$$> 0$$

as $\theta < \alpha\beta t_{p_j}^{\beta-1}$

The second-order derivative of $Z_{p_j}(t_{Pj})$ i.e. $\frac{\partial^2 Z_{p_j}(t_{Pj})}{\partial t_{Pj}{}^2}$, involves six terms. Now, we investigate it term by term:

1^{st}term:$\dfrac{d^2D_j(p_j)}{dt_{p_j}^2}(T - t_{Pj})\left(p_j - C_{ap_j}\right) < 0$according to (B.5)

2nd term:$-2\dfrac{dD_j(p_j)}{dt_{Pj}}\left(p_j - C_{ap_j}\right) < 0$according to(B.7)

3rd term:$+2\dfrac{dD_j(p_j)}{dt_{Pj}}(T - t_{Pj})\dfrac{dp_j}{dt_{Pj}} < 0$according to (B.4) and (B.6)

$4^{\text{th}}\&5^{\text{th}}$ terms:$+D_j(p_j)(T - t_{Pj})\dfrac{d^2p_j}{dt_{p_j}^2} - 2D_j(p_j)\dfrac{dp_j}{dt_{Pj}}$

The above two terms are positive. Because of high complication in these terms, we compare 4th and 5th terms with previous similar terms 1st and 2nd, respectively. Next, by assuming $a\left(p_j - C_{ap_j}\right) \geq D_j(p_j)$, we proof that the absolute value of 1st and 2^{nd}terms are greater than absolute value4^{th} and 5th, respectively. Consequently, sum of 1st, 2nd, 4th, and 5th terms is negative.

$$\frac{\text{1st term}}{\text{4th term}} = \frac{\left|\dfrac{d^2D_j(p_j)}{dt_{p_j}^2}\right|(T - t_{Pj})\left(p_j - C_{ap_j}\right)}{D_j(p_j)(T - t_{Pj})\left|\dfrac{d^2p_j}{dt_{p_j}^2}\right|} \geq 1 \rightarrow \left|\frac{d^2D_j(p_j)}{dt_{p_j}^2}\right|\left(p_j - C_{ap_j}\right) \geq D_j(p_j)\left|\frac{d^2p_j}{dt_{p_j}^2}\right| \quad \text{(B.8)}$$
$$\rightarrow a\left(p_j - C_{ap_j}\right) \geq D_j(p_j)$$

$$\frac{\text{2nd term}}{\text{5th term}} = \frac{2\left|\dfrac{dD_j(p_j)}{dt_{Pj}}\right|\left(p_j - C_{ap_j}\right)}{2D_j(p_j)\left|\dfrac{dp_j}{dt_{Pj}}\right|} \geq 1 \rightarrow \left|\frac{dD_j(p_j)}{dt_{Pj}}\right|\left(p_j - C_{ap_j}\right) \geq D_j(p_j)\left|\frac{dp_j}{dt_{Pj}}\right| \quad \text{(B.9)}$$
$$\rightarrow a\left(p_j - C_{ap_j}\right) \geq D_j(p_j)$$

6th term:$-\left(C_{ap_j}.\theta_j + c_d.\theta_S + c_h\right)(I_{0j} - 1)\left(\left(\alpha\beta t_{p_j}^{\beta-1} - \theta\right)exp\left(\alpha t_{p_j}^{\beta} - \theta t_{p_j}\right)\right) < 0$ as

$$\theta < \alpha\beta t_{p_j}^{\beta-1}$$

So, the manufacturer j's total profit function is concave with respect to t_{p_j}.

Hence, the supplier and manufacturers objective functions are concave, we can conclude that the two optimal points t_S^* and $t_{p_j}^*$ are global maximum for our optimization model.