Coordination and profit sharing in a two-level supply chain under periodic review inventory policy with delay in payments contract

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Abstract

In this paper, a coordination model has been investigated for a two-level supply chain (SC) consisting of one retailer and one supplier under periodic review inventory system. The review period and the retailer’s safety factor are assumed to be decision variables. The retailer faces stochastic demand following a normal distribution with known mean and variance. Moreover, it is assumed that unmet demand will be backordered. Firstly, the investigated SC is modeled under the decentralized and centralized decision-making structures, afterwards, a coordination mechanism based on delay in payments is proposed for transition from the decentralized to centralized model. To fairly share the surplus profit obtained by coordination, a profit sharing strategy is developed which is based on the bargaining power of the two SC members. Finally, a set of numerical experiments and sensitivity analysis are carried out. Numerical examples indicate that the proposed delay in payments contract can achieve channel coordination and the whole SC cost will decrease under the coordination model while the costs of neither retailer nor the supplier will increase.

**Keywords:** Supply chain coordination, periodic review inventory system, delay in payments, profit sharing

1- Introduction

Nowadays, competition among supply chains has been replaced with competition among individual enterprises. In traditional business environments, each SC member acts as an independent economical entity which is called decentralized structure. In such a situation, each SC member tries to maximize its own profit regardless of other SC actors. This case leads to an inefficient SC. Conversely, under the centralized decision-making all SC members try to maximize their profits according to the whole SC viewpoint. Although all members in the centralized system are managed by one economical entity, it may not be realistic as it might incur losses for some SC members. To resolve the conflicts of interest under the decentralized model, an appropriate coordination mechanism should be used.
Coordination mechanisms can guarantee the participation of all SC actors in the coordination plan (Chaharsooghi et al., 2011; Mokhlesian and Zegordi, 2015; Heydari and Norouzinasab, 2016).

Different contracts as coordination mechanisms have been broadly exerted in the literature. These contracts are classified on the basis of quantity, time, quality, and price that share the risk from different sources of uncertainty (time, demand, and price) between the SC members (Giannoccaro and Pontrandolfo, 2004). The most popular contracts are revenue sharing (Giannoccaro and Pontrandolfo, 2004; Cachon and Lariviere, 2005; Linh and Hong, 2009), quantity discount (Munson and Rosenblatt, 2001; Li and Liu, 2006; Heydari, 2014), return policies (Ding and Chen, 2008; Xiong et al., 2011; Heydari et al., 2016), quantity flexibility (Tsai, 1999; Chung et al., 2014), sales rebate (Talor, 2002; Heydariand Asl-Najafi, 2016) and delay in payments (Chaharsooghi and Heydari, 2009; Heydari, 2014). According to (Chaharsooghi and Heydari, 2009) contracts can be appropriately used for coordinating supply chains. Among various contracts, delay in payments of contracts play an important role in the modern business environment. Based on this contract, any increases in the retailer’s costs will be compensated by the supplier as long as the retailer makes globally optimal decisions from the entire SC point of view. Also, the retailer can benefit from returns on investment during the period of credit (Gao et al., 2014).

Due to the crucial decisions in the supply chains such as replenishment, reorder point, order quantity, protection interval and so on (Chaharsooghi et al., 2011; Johari et al., 2017), many researchers have been vastly paid attention to coordinate SC decisions. The current paper investigates supply chain coordination (SSC) in order to coordinate SC decisions under periodic review inventory system. The periodic review inventory models can often be used in managing inventory cases such as small retail stores, pharmacies, and grocery stores (Annadurai& Uthayakumar, 2010). For example, due to the high number of drugs in pharmacies, the inventory level is reviewed every $T$ units of time and an enough stock is ordered up to the order-up-to level $R$. Pharmacies’ ordering decisions impact on both upstream and SC costs. By applying delay in payments contract as a coordination mechanism, the upstream offered the credit option to the downstream and it can decrease cost in the SC. Although a handful of studies have been conducted on the coordination of periodic review inventory systems within SC, the delay in payments contract as a coordination mechanism has not yet been developed for coordinating these systems.

The current study contributes to the literature by applying the delay in payments contract as an incentive mechanism for coordinating supply chain under periodic review inventory system. To this end, a two-level supply chain consisting of one retailer and one supplier with one type of product is considered. The retailer faces stochastic demand following a normal distribution and uses a periodic review order-up-to level inventory system $(T, R)$. The review period and safety factor are the retailer’s decision variables. The stock out is considered to occur for both the retailer and supplier. The decisions made by the retailer (i.e., review period and safety factor) not only impacts on his/her own inventory costs, but also influences the whole SC inventory costs. Therefore, coordinating these primary decisions throughout the SC is of high importance. Firstly, the decentralized and the centralized models are developed and optimal values of decision variables are calculated. Then, a delay in payments contract is proposed to coordinate the investigated SC. Finally, a profit sharing strategy is developed based on the two SC members' bargaining power. In addition, a set of numerical experiments and sensitivity analysis are carried out to evaluate the performance of the proposed models. The results indicate that the investigated delay in payments contract is capable of reducing the whole SC costs while the developed coordination scheme is mutually beneficial.

The rest of this article is arranged as follows. In the next section, a literature review of inventory control policies and SC coordination is given. Problem definition, the notation and assumptions are introduced in section 3. Section 4 presents the mathematical model for decentralized, centralized and coordination models and solution procedures. Section 5 contains numerical experiments and sensitivity analysis. Conclusions and future researches are provided in section 6.

2- Literature review

In this section, the related literature on delay in payments contract and periodic review inventory models is discussed and then the contributions of this study are expressed.
Incentive schemes guarantee that all SC members participate in the coordination model. By using delay in payments contracts as a coordination mechanism, the supplier by offering the credit period persuades the retailer to participate in the coordination model. Jaber and Osman (2006) considered delay in payments mechanism in a two-level supply chain. In the investigated model, the length of credit option was assumed as a decision variable. Chaharsooghi and Heydari (2009) proposed a delay in payments contract for the joint determination of order quantity and reorder point. Moreover, they developed a profit sharing strategy based on the members’ bargaining power. Duan et al. (2012) and Wu and Zhao (2014) used delay in payments contracts in a two-level supply chain for fixed lifetime products. The credit option as a coordination mechanism for replenishment decisions supposing truckload limitations was applied by Heydari (2014). The author considered different rates of return for two SC members to invest during the period of credit.

A group of studies related to periodic review inventory models, Ouyang and Chuang (2000) proposed a periodic review inventory model in which the lead time and the review period were assumed as decision variables. In the proposed model, a service level constraint was considered instead of stock-out term in the objective function. Ouyang et al. (2007) developed a periodic review inventory model in two different cases: (1) the protection interval demand followed a normal distribution (2) the protection interval demand followed a distribution free. They employed lost-sales rate reduction in their study. Also target inventory level, length of a review period, and fraction of the shortage that will be lost were assumed as decision variables. Annadurai and Uthayakumar (2010) developed a probabilistic inventory model that the lost sale rate was reduced by more investment. The review period, lead time safety factor, and lost-sales rate were supposed as decision variables.

All above mentioned papers were done in a single-echelon inventory system. There are few articles in multi-echelon periodic review inventory systems. Matta and Sinha (1991) considered a two-level periodic review inventory model with stochastic demand followed a normal distribution function. Kanchanasuntorn and Techanitisawad (2006) developed a two-echelon inventory–distribution system with periodic review policy for fixed-life perishable products. They considered stock-out for the retailer in their work. Hsu and Lee (2009) considered an integrated inventory model for a two-level supply chain with single manufacturer and multiple retailers. In the proposed model, the decisions of replenishment and lead-time reduction were investigated. Lin (2010) developed an integrated supplier-retailer inventory problem with stochastic demand. In this study, Length of the protection interval, the backorder price discount, the numbers of shipments from the supplier to the retailer per production run and the lead time were assumed as control variables. All mentioned multi-level periodic review inventory models are expanded in an integrated supply chain. Taking a different methodology, Nematollahi et al. (2016) developed a coordination model for a two-level pharmaceutical supply chain under periodic review inventory policy. In their proposed model, coordination model was considered in two different scenarios, economic coordinative decision-making and social coordinative decision-making. Johari et al. (2016) proposed a coordination model in a manufacturer-retailer chain under periodic review inventory system. They used quantity discount contract as a mechanism of coordination. In their proposed model, review period (T), order-up-to-level (R) and the number of shipments from manufacturer to retailer per production run (n) were considered as decision variables.

In addition, most of the previously coordination models have focused on the continuous review inventory system, such as Jaber and Osman, 2004; Chaharsooghi and Heydari, 2009; Chaharsooghi et al., 2011...just a handful of studies have been conducted on the coordination of the periodic review inventory systems within supply chain as mentioned above. As a result, in this paper, the delay in payments contract as a coordination mechanism is developed to coordinate the periodic review inventory decisions in addition to the length of the credit period. To fairly share the surplus profit obtained by applying coordination scheme, a profit sharing strategy based on the bargaining power of the two SC members is developed.

3- Problem definition
This paper investigates a two-level supply chain consisting of one retailer and one supplier with one type of product. The customer's demand follows a normal distribution with known mean and variance. The stock out at two levels is backlogged and in the next period must be answered. The proposed
supply chain is shown in figure 1. The retailer uses a periodic review inventory policy \((T,R)\). For minimizing the supplier’s inventory cost, the supplier’s review period is supposed as an integer multiple of the retailer’s review period \((mT)\) where \(m=1,2,...\) (Jaber and Osman, 2006). Following assumptions are considered in the current study:

1. The inventory level is reviewed every \(T\) units of time. A sufficient quantity is ordered up to the order-up-to level \(R\), and the ordering quantity will be arrived after \(L\) units of time.
2. The length of the lead time \(L\) does not exceed an inventory cycle time \(T\).
3. Demand during the protection period \((T+L)\) has a normal distribution with mean \(\mu\)
4. The supplier’s review period is an integer multiple of the retailer’s review period \((mT)\).
5. The order-up-to level \(R_r\) for the retailer is equal to the sum of retailer’s expected demand during the protection period \(D(T+L)\) and the safety stock \((SS_r)\), in which \(SS_r = k_r \times \sigma(T+L)\) (standard deviation of protection interval demand) and consequently \(R_r = D(T+L) + k_r \sqrt{T+L}\).
6. The order-up-to level \(R_s\) for the supplier is equal to the sum of supplier’s expected demand during period \(mTD\) and the safety stock \((SS_s)\), in which \(SS_s = k_s \times \sigma(mT)\) (standard deviation of protection interval demand).

7. The supplier lead time is supposed to be zero.

\[ R_r = D(T+L) + k_r \sigma(T+L) \]
\[ R_s = mTD + k_s \sigma(mT) \]

**Fig. 1.** The investigated two-level supply chain model

![The investigated two-level supply chain model](image)

The models are developed according to the following notations and assumptions.

### 3-1- Notations

**Parameters**

\( D \) \quad The retailer’s expected demand per year
\( L \) \quad Length of lead time
\( x \) \quad The retailer’s demand during the protection period \((T+L)\), which has a normal distribution with mean \(D(T+L)\) and variance \(\sigma^2(T+L)\)
\( y \) \quad The supplier’s demand during period \((mT)\), the demand has a normal distribution with mean \(mTD\) and variance \(mT\sigma^2\)
\( R_r \) \quad Order-up-to level for retailer
\( R_s \) \quad Order-up-to level for supplier
\( A_r \) \quad Unit ordering cost per replenishment for retailer
\( A_s \) \quad Unit ordering cost per replenishment for supplier
\( h_r \) \quad Unit inventory holding cost per year for retailer
\( h_s \) \quad Unit inventory holding cost per year for supplier
\( \pi_r \) \quad Shortage cost per unit for retailer
\( \pi_s \) \quad Shortage cost per unit for supplier
\( F_r \) \quad Fixed transportation cost for retailer
\( k_s \) \quad Safety factor for supplier, \(k_s \geq 0\)
\begin{itemize}
\item $\beta$ Percentage of holding costs are due to the investment cost
\item $\alpha$ The bargaining power of retailer
\item $m$ The number of supplier replenishment cycles as a multiple of retailer review period that is a positive integer $\geq 1$
\end{itemize}

**Decision variables**

- $T$ Length of a review period
- $k_r$ Safety factor for retailer, $k_r \geq 0$
- $CT$ Length of credit option

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#### 4- Model formulation and solution procedures

In this section, firstly the decentralized case is developed. Under the decentralized decision-making, each supply chain member makes decisions individually and tries to minimize its own cost function regardless of the other member. Then, the centralized case is considered. In the centralized decision-making, supply chain is considered as integrated SC. Optimal values of decision variables are calculated at each case. Finally, the delay in payments contract as an incentive scheme is developed. Coordination case is based on the joint decision-making in a decentralized structure. Finally, a profit sharing strategy is developed based on the two SC members’ bargaining power and the proposed models are evaluated.

#### 4-1- Decentralized model

In this case, each SC member tries to minimize its own cost regardless of other SC members. Therefore, we model two different inventory problems for the retailer and supplier and obtain optimal solutions for the members.

The retailer uses a periodic review inventory policy $(T, R)$, in which review period $T$, and the safety factor $k_r$, are decision variables. Figure 2 shows the inventory level for the supplier and the retailer. As can be seen in figure 2, review period $T$ is considered as the time between the arrivals of two successive orders. The expected net inventory level at the beginning of the period is $R_r - DL$, and the expected net inventory level at the end of the period is $R_r - D(T + L)$. Therefore, the expected average inventory level is equal to:

$$\frac{(R_r - DL) + (R_r - D(T + L))}{2} = R_r - DL - \frac{DT}{2} \tag{1}$$

Now, putting value of $R_r$ into Eq. (1) will lead to the expected average inventory level as $\frac{DT}{2} + k_r \sigma \sqrt{T + L}$.

In this study, it is assumed that if the customer's demand cannot be met by the retailer immediately, the order is backlogged. The expected stock-out per replenishment cycle can be expressed as:

$$ E(x - R_r)^+ = \int_{R_r}^{\infty} (x - R_r) f_x(x) dx = \int_{k_r}^{\infty} \sigma \sqrt{T + L} (Z - K_r) f_Z(z) dz = \sigma \sqrt{T + L} G(K_r) > 0 \tag{2} $$

Where $G(K_r) \equiv \phi(K_r) - K_r[1 - \phi(K_r)]$, $\phi(K_r)$ and $\Phi(K_r)$ denote the Standard normal p.d.f and c.d.f, respectively.

The retailer’s inventory costs consist of ordering, inventory holding, stock-out, and fixed transportation costs. The expected total retailer’s cost function can be calculated as:

$$ TC_r^d(T, R_r) = \frac{A_r}{T} + h_r \left( \frac{DT}{2} + k_r \sigma \sqrt{T + L} \right) + \frac{p_r}{T} E(x - R_r)^+ + \frac{F_r}{T} \tag{3} $$

Putting $E(x - R_r)^+$ from equation (2) into equation (3), we get
In which the first term denotes the ordering cost, the second term denotes the inventory holding cost, the third term denotes the shortage cost and the last term denotes fixed transportation cost.

\[
TC_r^d(T^d, k_r^d) = \frac{A_r}{T} + h_r\left(\frac{DT}{2} + k_r\sigma\sqrt{T + L}\right) + \frac{\pi_r}{T}\sigma\sqrt{T + L}G(k_r) + \frac{F_r}{T}
\]

Fig. 2. Inventory level for supplier and retailer

**Proposition 1:** The retailer's cost function is convex with respect to \( T^d \) and \( k_r^d \).

**Proof:** see Appendix A for detailed proof.

To minimize the retailer’s cost function, taking the first order partial derivatives of \( TC_r^d(T^d, k_r^d) \) with respect to \( T^d \) and \( k_r^d \) gives:

\[
\frac{\partial TC_r^d}{\partial T^d} = -\frac{(A_r + F_r)}{T^2} + h_r\left(\frac{D}{2} + \frac{\sigma k_r}{2\sqrt{T + L}}\right) + \pi_r\sigma G(k_r)\left(\frac{1}{2T\sqrt{T + L}} - \frac{\sqrt{T + L}}{T^2}\right)
\]

\[
\frac{\partial TC_r^d}{\partial k_r^d} = h_r\sigma\sqrt{T + L} + \frac{\pi_r\sigma\sqrt{T + L}[\Phi(k_r) - 1]}{T}
\]

By setting equations (5) and (6) equal to zero, we obtain

\[
\frac{A_r + F_r}{T^2} + \frac{\pi_r\sigma G(k_r)\sqrt{T + L}}{T^2} = h_r\left(\frac{D}{2} + \frac{\sigma k_r}{2\sqrt{T + L}}\right) + \frac{\pi_r\sigma G(k_r)}{2T\sqrt{T + L}}
\]

and:

\[
1 - \Phi(k_r) = \frac{h_rT}{\pi_r}
\]

The following algorithm is used to find the optimal solution of \( T^d \) and \( k_r^d \).
Algorithm 1

**Step 1**: Get $T_2$, which is calculated based on Eq. (7), as follows:

$$T_2 = \sqrt{\frac{A_r + F_r + \pi_\sigma G(k_r)\sqrt{T + L}}{h_r \left(\frac{D}{2} + \frac{\sigma k_r}{2\sqrt{T + L}}\right) + \frac{\pi_\sigma G(k_r)}{2T\sqrt{T + L}}}}$$

**Step 2**: Set $k_r = 0$.

**Step 3**: Find $T$ through step 3-1 to step 3-4 as follows:

**Step 3-1**: Start with $y = 0$.

**Step 3-2**: Calculate $y$ using Eq. (9).

**Step 3-3**: Find value $-\infty < y < \infty$.

**Step 3-4**: If $T_2 - T \leq \varepsilon$ then substitute $T = T_2$ and go to step 4, otherwise substitute $T = T_2$ and go to step 3-2.

**Step 4**: Utilizing $T$ determines $k_r$ using Eq. (8).

**Step 5**: If two successive $T$ and $k_r$ are less than $\varepsilon$ simultaneously, then $T$ and $k_r$ get their optimum values. Otherwise, go to step 3.

By applying Algorithm 1, the optimal solution of $T^d$ and $k^d_r$ for the retailer under the decentralized decision-making model will be obtained. The supplier also must solve its own problem separately. For minimizing the supplier’s inventory cost, the supplier’s review period is considered as an integer multiple of the retailer’s review period $mT$. The supplier’s costs consist of ordering, inventory holding and stock-out costs.

As illustrated in figure 2, the supplier’s average inventory level can be calculated as:

$$T((m - 1)DT + (m - 2)DT + \cdots + DT) + \int_0^{R_s} (R_s - y)f_\sigma(y)dy = \frac{DT^2(m - 1)m}{2mT} + k_s\sigma\sqrt{mT}$$

In which, the first term is the average inventory and the second term is the safety stock. At the supplier’s site, it is assumed that the unsatisfied demand will be backlogged. The expected supplier’s stock-out per replenishment cycle can be expressed as:

$$E(y - R_s)^+ = \int_{R_s}^\infty (y - R_s)f_\sigma(y)dy = \int_0^{\infty} \sigma\sqrt{mT} (Z - K)f_\sigma(Z)dz = \sigma\sqrt{mT}G(k_s) > 0$$

Therefore, the expected supplier’s cost function can be approximated as:

$$TC_s^d = \frac{A_s}{mT} + h_s \left(\frac{DT(m - 1)}{2} + k_s\sigma\sqrt{mT}\right) + \frac{\pi_s}{mT}E(y - R_s)^+$$

Putting $E(y - R_s)^+$ calculated in Eq. (10) into Eq. (11), we get:

$$TC_s^d = \frac{A_s}{mT} + h_s \left(\frac{DT(m - 1)}{2} + k_s\sigma\sqrt{mT}\right) + \frac{\pi_s}{mT}\sigma\sqrt{mT}G(k_s)$$

In which the first term denotes the ordering cost, the second and third terms denote inventory holding cost and shortage cost, respectively. The supplier’s cost function depends on the variable $T$, which was determined by the retailer. In the following, the centralized decision-making structure is investigated in which the value of review period is jointly determined.

4-2- Centralized model

In the centralized case, all SC members try to minimize their cost according to the whole SC viewpoint, accordingly the SC expected cost functions is the sum of the retailer's and supplier's costs functions.
To minimize the SC cost function, taking the first order partial derivatives of $TC_{\text{chain}}^c(T^c, k_r^c)$ with respect to $T^c$ and $k_r^c$ gives:

$$
\frac{\partial TC_{\text{chain}}^c}{\partial T^c} = \frac{A_r + F_r}{T^2} + h_r \left( \frac{D T}{2} + k_r \sigma \sqrt{T + L} \right) + \pi_r \sigma G(k_r) \left( \frac{1}{2T \sqrt{T + L}} - \frac{\sqrt{T + L}}{T^2} \right) - \frac{A_s}{mT^2} + h_s \left( \frac{D(m - 1)}{2} + k_s \sigma \right) + \pi_s \sigma G(k_s) \left( \frac{1}{2T \sqrt{mT}} - \frac{\sqrt{mT}}{mT^2} \right)
$$

And

$$
\frac{\partial TC_{\text{chain}}^c}{\partial k_r^c} = h_r \sigma \sqrt{T + L} + \pi_r \sigma \sqrt{T + L} \left[ \Phi(k_r) - 1 \right] - \frac{1}{T}
$$

By setting equations (14) and (15) equal to zero, we obtain

$$
\frac{A_r + F_r}{T^2} + \frac{\pi_r \sigma G(k_r) \sqrt{T + L}}{T} + \frac{A_s}{mT^2} + \frac{\pi_s \sigma G(k_s) \sqrt{mT}}{mT^2} = h_r \left( \frac{D}{2} + \frac{\sigma k_r}{2 \sqrt{T + L}} \right) + \frac{\pi_r \sigma G(k_r)}{2T \sqrt{T + L}} + h_s \left( \frac{D(m - 1)}{2} + \frac{k_s \sigma}{2 \sqrt{mT}} \right) + \frac{\pi_s \sigma G(k_s)}{2T \sqrt{mT}}
$$

And

$$
1 - \Phi(k_r) = \frac{h_r T}{\pi_r}
$$

The following Algorithm 2 is used to find the optimal solution of $T^c$ and $k_r^c$.

**Algorithm 2:**

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step1</td>
<td>Get $T_2$ based on Eq. (16) as follows:</td>
</tr>
<tr>
<td></td>
<td>$T_2 = \sqrt{\frac{m \left( \frac{A_r + F_r + \pi_r \sigma G(k_r) \sqrt{T + L}}{2} + A_s + \pi_s \sigma G(k_s) \sqrt{mT}}{\left( \frac{h_r \left( \frac{D}{2} + \frac{\sigma k_r}{2 \sqrt{T + L}} \right) + \pi_r \sigma G(k_r) \sqrt{T + L}}{2T \sqrt{T + L}} + h_s \left( \frac{D(m - 1)}{2} + \frac{k_s \sigma}{2 \sqrt{mT}} \right) + \pi_s \sigma G(k_s) \sqrt{mT}} \right)}}$ (II)</td>
</tr>
<tr>
<td>Step2</td>
<td>Set $k_r = 0$.</td>
</tr>
<tr>
<td>Step3</td>
<td>Find $T$ through step 3-1 to step 3-4 as follows:</td>
</tr>
<tr>
<td></td>
<td>Step3-1: Start with $T = 0.0001$.</td>
</tr>
<tr>
<td></td>
<td>Step3-2: Calculate $T_2$ using Eq. (II).</td>
</tr>
<tr>
<td></td>
<td>Step3-3: Find value $T_2 - T$.</td>
</tr>
<tr>
<td></td>
<td>Step3-4: If $T_2 - T \leq \epsilon$ then substitute $T = T_2$ and go to step 4. Otherwise, substitute $T = T_2$ and go to step 3-2.</td>
</tr>
<tr>
<td>Step4</td>
<td>Utilizing $T$ determines $\Phi(k_r)$ from the Eq. (17).</td>
</tr>
<tr>
<td>Step5</td>
<td>If two successive $T$ and $k_r$ are less than $\epsilon$ simultaneously, then $T$ and $k_r$ gets their optimum values. Otherwise, go to step 3.</td>
</tr>
</tbody>
</table>

Optimum values of $T^c$ and $k_r^c$ can be calculated by using the proposed algorithm 2. The centralized solution can be considered as a benchmark for the coordination model. Due to the optimization of the whole SC in the centralized model, entire SC costs are less than in decentralized model (Heydari, 2014; Chaharsooghi and Heydari, 2009).
4-3- Coordination model based on delay in payments scheme

A coordination model has two main objectives, (1) to increase the SC profit up to the centralized chain’s profit, and (2) to share the surplus profit among the SC members (Giannoccaro and Pontrandolfo, 2004). By applying incentive schemes in the coordination models, SC members will be motivated to follow globally optimum decisions for the entire SC (Nematollahi et al., 2016). In this section, a coordination model based on the delay in payments contract is developed. Under such a contract, the supplier persuades the retailer to participate in the coordination model by offering the credit period and any increases in the retailer’s costs will be compensated by the supplier.

4-3-1- Incentive scheme based on delay in payments

Incentive schemes guarantee that all SC members participate in the coordination model. Similar to Chaharsooghi and Heydari (2009), in this section, we define a set of operational coefficients for decision variables (i.e., $T$ and $k_r$) to decrease the cost of chain down to the centralized case. The review period coefficient $k_r$ to achieve channel coordination is:

$$k_r = \frac{T^c}{T^d}$$

The operational coefficient for the variable $k_r$ is defined as follows:

$$k_k = \frac{k_r^c}{k_r^d}$$

By applying $k_T$, $k_k$, the retailer’s review period shifts from $T^d$ to $T^c = k_r T^d$ and the retailer’s safety factor shifts from $k_r^d$ to $k_r^c = k_k k_r^d$. The use of these coefficients in the decentralized SC will lead to channel coordination, but the retailer’s cost will increase. In this section, an incentive scheme based on a credit option is developed. In this contract, the supplier persuades the retailer to participate in the coordination model by offering the credit period and any increases in the retailer’s costs will be compensated by the supplier.

The investment cost is one main cost of inventories that to base on the time value of money. At the retailer’s site, we assume that $\beta$% of inventory holding costs are due to the investment cost. As shown in figure 3, the retailer’s unit inventory holding cost in credit time is decreased by the coefficient $1 - \beta$. So by applying a credit option, the retailer’s inventory costs can be reduced and the extra costs can be compensated.

![Fig. 3. One average replenishment cycle of the retailer and the area of each part](image-url)
By applying the credit option, the retailer’s inventory holding costs decreases in the credit time. As shown in figure 3, the retailer’s inventory holding costs are decreased by $\beta\%$ in credit time. The expected cycle inventory holding cost changes from $h_r \left( \frac{DT_c}{2} \right)$ to $h_r \left( \frac{(D(T^c-CT))^2}{2T^c} + (1 - \beta) \left( D - \frac{DCT_c}{2T^c} \right) CT \right)$ and the expected safety stock cost changed from $h_r(K^c\sigma\sqrt{T^c} + L)$ to $h_r\left( K^c\sigma\sqrt{T^c} + L \left( 1 - \frac{\beta CT_c}{T^c} \right) \right)$. Therefore, the retailer’s cost function after participating in the coordination plan will be:

$$T_{C_i}^{\text{o}}(T^c, k^c, CT) = \frac{A_r}{T^c} + h_r \left( \frac{(D(T^c-CT))^2}{2T^c} + (1 - \beta) \left( D - \frac{DCT_c}{2T^c} \right) CT + K^c\sigma\sqrt{T^c} + L \left( 1 - \frac{\beta CT_c}{T^c} \right) \right) + \frac{\pi_r}{T^c} \sigma\sqrt{T^c} + LG(k^c) + \frac{F_r}{T^c} \tag{18}$$

In which the first term denotes the ordering cost, the second term shows the inventory holding cost, the third term represents the shortage cost and the last term denotes the fixed transportation cost. Therefore, the supplier’s cost function will be:

$$T_{C_s}^{\text{o}}(T^c, k^c, CT) = \frac{A_s}{mT^c} + h_s \left( \frac{DT^c(m-1)}{2} + k_s\sigma\sqrt{mT^c} \right) + \frac{\pi_s}{mT^c} \sigma\sqrt{mT^c}G(k_s) + \frac{\beta h_r}{T^c} \left[ CT \left( DT^c - \frac{1}{2} DCT + k^c\sigma\sqrt{T^c} + L \right) \right] \tag{19}$$

In equation (19), the first term shows the ordering cost, the second term represents the inventory holding cost, the third term denotes the shortage cost and the last term shows the additional cost due to the applying the delay in payments contract. According to figure 3, to calculate the last term, the surface under the retailer’s inventory level curve in the interval $[0, CT]$ is calculated and then the obtained value is multiplied by the investment cost factor, the retailer’s inventory holding cost and by the number of the retailer’s replenishment cycles in each year (i.e., $\frac{\beta h_r}{T^c}$).

Retailer participates in the coordination model, when its cost does not exceed that before participating; thus, we have:

$$T_{C_i}^{\text{o}}(T^c, k^c, CT) \leq T_{C_i}^{\text{d}}(T^d, k^d)$$

Therefore, the minimum amount of credit period that guarantees retailer’s participation in the coordination model can be calculated as:

$$CT_{\text{min}} = \left[ Dk^d T^d + k^d k^d\sigma\sqrt{k^dT^d} + L \right] \left( \frac{2A}{D} \right) - \sqrt{\frac{2A}{D} \left[ \frac{D}{2D} \right]} \tag{20}$$

Where
\[ A = \left[ D k_7 T^d + k_k k_7^d \sigma \sqrt{k_7 T^d + L} \right]^2 \]

\[ + \frac{2D}{\beta h_r} \left[ (A_r + F_r) \left( \frac{1}{T^d} - \frac{1}{k_7 T^d} \right) \right. \]

\[ + h_r \left( \frac{D T^d}{2} + k_k^d \sigma \sqrt{T^d + L} - \frac{D k_r T}{2} - k_k k_7^d \sigma \sqrt{k_7 T^d + L} \right) \]

\[ + \frac{\pi_r}{T^d} \sigma G(k_7^d) \sqrt{T^d + L} - \frac{\pi_r}{k_7 T^d} \sigma \sqrt{k_7 T^d + L} \] \( G(k_7^d) \]

\( (21) \)

Supplier participates in the coordination model if its costs after applying the delay in payments do not exceed its costs under the decentralized model; thus, we have:

\[ TC_s^c(T, k, CT) \leq TC_s^d \]

Therefore, the maximum credit period that guarantees supplier’s participation in the coordination scheme can be calculated as:

\[ CT_{\text{max}} = \left[ \frac{D k_7 T^d + k_k k_7^d \sigma \sqrt{k_7 T^d + L}}{D} \right] - \frac{2M}{\sqrt{D}} \]

(22)

Where

\[ M = \left[ \frac{D k_7 T^d + k_k k_7^d \sigma \sqrt{k_7 T^d + L}}{2D} \right]^2 \]

\[ - \frac{k_7 T^d}{\beta h_r} \left[ A_s \left( \frac{1}{m T^d} - \frac{1}{m k_7 T^d} \right) \right. \]

\[ + h_s \left( \frac{D T^d (m - 1)}{2} (1 - k_7) + k_s \sigma \left( \sqrt{m T^d} - \sqrt{m k_7 T^d} \right) \right) \]

\[ + \pi_s \left( \frac{\sigma \sqrt{m T^d} G(k_s)}{m T^d} - \frac{\sigma \sqrt{m k_7 T^d} G(k_s)}{m k_7 T^d} \right) \]

(23)

Each amount of credit time in this interval \([CT_{\text{min}}, CT_{\text{max}}]\) can lead to channel coordination. At \( CT_{\text{min}} \), all coordination benefits will be achieved by the supplier, and at \( CT_{\text{max}} \) all coordination benefits will be obtained by the retailer. In the following, it is presented a profit sharing strategy to calculate the exact credit time.

**4-3-2- Profit sharing strategy**

The surplus benefit obtained by a coordination mechanism should fairly share among the SC members. Accordingly, in the following, a profit sharing strategy based on the bargaining power of the two SC members is developed. We define the retailer’s bargaining power against the supplier as \( \alpha \), and therefore the supplier’s bargaining power will be \( 1 - \alpha \).

The amount of reduced SC cost resulting by using the coordinated model can be calculated as:
\[ \Delta TC = TC_{\text{chain}}^d(T^d, k_r^d) - TC_{\text{chain}}^c(T^c, k_r^c) \]
\[ = \left( \frac{A_r}{T_d} + h_r \left( \frac{DT_d}{2} + k_r \sigma \sqrt{T_d + L} \right) + \frac{\pi_r}{T_d} \sigma \sqrt{T_d + LG(k_r)} + \frac{F_r}{T_d} + \frac{A_s}{mT_d} \right) \]
\[ + h_s \left( \frac{DT_d(m - 1)}{2} + k_s \sigma \sqrt{mT_d} \right) + \frac{\pi_s}{mT_d} \sigma \sqrt{mT_d} G(k_s) \]
\[ - \left( \frac{A_r}{T_c} + h_r \left( \frac{DT_c}{2} + k_r \sigma \sqrt{T_c + L} \right) + \frac{\pi_r}{T_c} \sigma \sqrt{T_c + LG(k_r)} + \frac{F_r}{T_c} + \frac{A_s}{mT_c} \right) \]
\[ + h_s \left( \frac{DT_c(m - 1)}{2} + k_s \sigma \sqrt{mT_c} \right) + \frac{\pi_s}{mT_c} \sigma \sqrt{mT_c} G(k_s) \]  
\[ (24) \]

By considering the retailer’s bargaining power (i.e., \( \alpha \)), it is expected that \( \alpha\% \) of the amount of reduced SC cost will be transferred to the retailer, we have:
\[ TC_{\text{co}}(T^c, k_r^c, CT) = TC_{\text{co}}^d(T^d, k_r^d) - \alpha \Delta TC \]  
\[ (25) \]

Putting equations (4) and (18) into Eq. (25), we get:
\[ \alpha \Delta TC = TC_{\text{co}}^d(T^d, k_r^d) - TC_{\text{co}}^c(T^c, k_r^c, CT) \]
\[ = (A_r + F_r) \left( \frac{1}{T_d} - \frac{1}{T_c} \right) + h_r \left( \frac{DT_d}{2} + k_r \sigma \sqrt{T_d + L} \right) + \frac{\pi_r}{T_d} \sigma \sqrt{T_d + LG(k_r)} \]
\[ - h_r \left( \frac{D(T_c - CT)^2}{2T_c} + (1 - \beta) \left( D - \frac{D \cdot CT}{2T_c} \right) \cdot CT + K^c \sigma \sqrt{T_c + L (1 - \beta \cdot CT/T_c)} \right) \]
\[ - \frac{\pi_r}{T_c} \sigma \sqrt{T_c + LG(k_c)} \]  
\[ (26) \]

By some simplifying the above equation, we have:
\[ CT = \frac{Dk_r T_d + k_r k^d \sigma \sqrt{k_r T_d + L}}{D} \pm \frac{2A_r}{D} \frac{2k_r T_d \alpha \Delta TC}{D \beta h_r} \]  
\[ (27) \]

The obtained credit time is acceptable if and only if it be in the interval \([CT_{\text{min}}, CT_{\text{max}}]\). Using the calculated \( CT \) as the credit time, the coordination plan will benefit both members based on their bargaining power.

5-Numerical experiments

In this section, by using a set of numerical experiments, the performance of the proposed coordination model is evaluated. To this end, the results of decentralized, centralized and coordination models are obtained. In addition, a set of sensitivity analysis with respect to the primary parameters are conducted. Table 1 displays the data for the five test problems. We try to cover a wide range of parameters in the numerical examples.
By running the model in mathematical software, the results of decision variables and the cost functions in the decentralized, centralized, and coordinated models are calculated. As illustrated in Table 2, by comparing the results in the decentralized and centralized models, it is observed that the centralized model decreases the cost of whole supply chain and the length of period review, also increases the value of safety factor in all test problems. However, the retailer’s cost increases by shifting from the decentralized to the centralized mode. As shown in Table 2, parameters of the coordination model are calculated. The results indicate that the interval $[CT_{min}, CT_{max}]$ is a non-empty interval and therefore channel coordination is accessible in all test problems. Moreover, using the Profit sharing strategy, the exact value of $CT$ is obtained on the basis of the bargaining power of SC members. By using the obtained credit option in the coordination model, the retailer's and supplier's costs are less than those ones in the decentralized mode. Therefore, applying the credit period guarantees that both members participate in coordination model. Also Table 2 shows improvement of coordination model versus the decentralized model in all numerical experiments.
Table 2. Results of running decentralized, centralized, and coordinated models

<table>
<thead>
<tr>
<th>Test problem</th>
<th>Test problem</th>
<th>Test problem</th>
<th>Test problem</th>
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<td></td>
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<td><strong>Coordination parameter (day) (year = 365 days)</strong></td>
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<td>9.42</td>
<td>5.28</td>
<td>9.15</td>
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In the following, a set of sensitivity analysis with respect to the demand uncertainties and the members’ costs are conducted. Using the data of numerical example 2, the results of running the model for various values of $\sigma$ are shown in Table 3. We change the value of $\sigma$ between $\sigma-\%40$ and $\sigma+\%40$. For the all values of $\sigma$, the SC cost in the coordination model is less than the decentralized model.
Table 3. Results of sensitivity analysis with respect to the demand uncertainties

<table>
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<tr>
<th>σ</th>
<th>σ -%40</th>
<th>σ -%30</th>
<th>σ -%20</th>
<th>σ -%10</th>
<th>σ</th>
<th>σ +%10</th>
<th>σ +%20</th>
<th>σ +%30</th>
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<td>1600</td>
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<td>2000</td>
<td>2200</td>
<td>2400</td>
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<td>2800</td>
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<td>TC£\text{co_chain}</td>
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<td>19604</td>
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<td>21819</td>
<td>24095</td>
<td>25237</td>
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<tr>
<td>ΔTC</td>
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<td>24632</td>
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<tr>
<td>Improvement%</td>
<td>8.47</td>
<td>8.70</td>
<td>8.92</td>
<td>9.16</td>
<td>9.42</td>
<td>9.70</td>
<td>9.98</td>
<td>10.27</td>
<td>10.62</td>
</tr>
</tbody>
</table>

As shown in figure 4, by increasing the value of σ, the proposed coordination model has better performance compared to the decentralized model. Moreover, in the high values of uncertainty, the difference between the supply chain’s cost in both decentralized and coordinated models increases. Therefore, applying the coordination model is of high significance under demand uncertainty.

![Fig. 4. The supply chain’s cost in the decentralized and coordination modes by increasing σ](image)

According to the numerical example 2, the changes of SC members’ cost in the coordination and decentralized models under various values of σ are shown in figure 5. As σ increases, the cost function of two members improve. In the other words, by increasing σ, difference between the profitability under the coordination and decentralized models increases. Accordingly, it can be concluded that the developed coordination model is of great importance under demand uncertainty. It is noteworthy that the improvement index is calculated as \( \frac{TC_{\text{chain}} - TC_{\text{co\_chain}}}{TC_{\text{chain}}} \times 100 \).
Figure 6, indicates the changes of SC members’ cost after applying the coordination model under various values of the retailer’s shortage cost in the numerical example 1. As shown in figure 6, by increasing \( \pi_r \), the profitability of the model for both members will decrease. In the other words, the members’ profitability is high in the low values of \( \pi_r \). Moreover, for the various values of \( \pi_r \), improvement of the supplier’s cost will be more than the retailer.

According to the numerical example 1, the improvement of both SC members after applying the coordination model under various values of \( h_r \) are shown in figure 7. As \( h_r \) increases the profitability of the model for both members decreases. Therefore, in the high values of \( h_r \), the SC members prefer to hold less inventory.
Table 4 shows the results of sensitivity analysis with respect to 10% increase and decrease the retailer's and supplier's holding cost in numerical example 2. As shown in Table 4, increasing the retailer’s holding cost reduces the value of $CT$ in most cases and this is undesirable for the retailer, also increasing the supplier’s holding cost leads to increase resulting $CT$ in most cases which is undesirable for the supplier. So the coordination model encourages SC members to keep down their holding cost.

Table 6, indicates the values of credit option with respect to 10% increase and decrease $\beta$ in the five test problems. As shown in Table 6, by increasing $\beta$, credit option decreases in the five test problems, which is expected and this is undesirable for the retailer.
6-Conclusion

In this paper, a coordination model based on the delay in payments contract for a two-level supply chain has been developed. The customer's demand followed a normally distributed function and backorder shortage was allowable in the system. The retailer used a periodic review inventory model and decided simultaneous on the review period and the safety factor. Firstly, the investigated SC was modeled under the decentralized and centralized decision-making structure, afterwards, a delay in payments contract as an incentive mechanism was proposed to guarantee the retailer's participates in the coordination plan. The main contribution of this paper is to coordinate the review period and safety factor simultaneously in a supplier-retailer SC under periodic review inventory system. Although a handful of studies have been conducted on coordination of periodic review inventory systems within SC, the delay in payments contract as a coordination mechanism has not yet been developed for coordinating these systems. The length of credit option is assumed to be decision variable in current study. A profit sharing strategy to fairly share the surplus profit obtained by coordination was considered according to the bargaining power of the two SC members and the exact value of $CT$ is calculated. By running the models, the results of decision variables and the cost functions in the decentralized, centralized, and coordinated models were calculated. The results indicate that changes of the members' holding cost impact on the credit time. However, the proposed coordination contract is capable of coordinating the SC under various values of holding cost. Also, in the high demand uncertainty (i.e., by increasing $\sigma$), the results indicate that the proposed coordination model performed quite well and resolved the conflict of interests.

Managerial implications from the proposed model can be summarized as: (1) In a two-level supply chain under periodic review inventory policy, the centralized decision making decreased the whole SC cost but increased the retailer’s cost. By using the delay in payments contract as a coordination mechanism, the whole SC cost and the retailer and supplier’s cost were less than those under the decentralized decision making. Therefore, the proposed delay in payments contract guaranteed that both SC members would participate in the coordination model. (2) Based on the sensitivity analyses, the SC members’ holding cost impact on the value of $CT$. As by increasing the retailer’s holding cost reduces the value of $CT$ in most cases and this is undesirable for the retailer, also increasing the supplier’s holding cost leads to increase in resulting $CT$ in most cases which is undesirable for the supplier. So the coordination model encourages SC members to keep down their holding cost. (3) The results indicate that by increasing the value of $\sigma$, in the high demand uncertainty, the difference between the profitability of the coordinated and decentralized models increases. Therefore, applying the coordination model is of high significance under high demand uncertainty. (4) Results of sensitivity analysis indicate that the members’ profitability is high in the low values of $\pi_r$. Moreover, for the various values of $\pi_r$, improvement of the supplier’s cost will be more than the retailer. As future research, it is suggested to assume the number of supplier replenishment cycles and the lead time as decision variables that the length of lead time can shorten at an extra crashing cost. In addition, this model can apply lost-sale inventory models.
References


Appendix A

Convexity of the retailer expected annual cost function in the decentralized model

Proof: To prove convexity, it is essential to compute the Hessian matrix of the retailer expected annual cost function with respect to $k_r$ and $T$. We have:

$$H = \begin{pmatrix}
\frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial k_r^2} & \frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial T \partial k} \\
\frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial T \partial k} & \frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial T^2}
\end{pmatrix}$$

Then

$$\frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial k_r^2} = \frac{\pi_r\sigma(T + L)^{1/2}f(k)}{T} = |H_{11}| > 0$$

$$\frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial T^2} = \frac{2(A_r + F_r)}{T^3} + \pi_r\sigma G(k_r)\left(\frac{2\sqrt{T + L}}{T^3} - \frac{1}{T^2\sqrt{T + L}} - \frac{1}{4T(T + L)^{3/2}}\right) - \frac{h_r k_r \sigma}{4(T + L)^{3/2}}$$

$$\frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial T \partial k} = \frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial k \partial T} = \frac{h_r \sigma}{2\sqrt{T + L}} + \pi_r\sigma \left(\frac{1}{2T\sqrt{T + L}} - \frac{\sqrt{T + L}}{T^2}\right) [\phi(k_r) - 1]$$
By calculating the above Hessian matrix can be observed that the first principal minor Hessian has a positive value. Under the condition of problem, the second principal minor is positive. The problem is tested with the various numerical examples which cover a wide range of reasonable parameters and it is observed that it has a positive value for all numerical examples.

Appendix B
Convexity of supply chain cost function in the centralized model
To prove convexity of supply chain cost function in the centralized model, the Hessian matrix is calculated with respect to \( k_r \) and \( T \) as follows:

\[
H = \begin{pmatrix}
\frac{\partial^2 T C^c_{\text{chain}}(T^c, k_r^c)}{\partial k_r^2} & \frac{\partial^2 T C^c_{\text{chain}}(T^c, k_r^c)}{\partial k_r \partial T} \\
\frac{\partial^2 T C^c_{\text{chain}}(T^c, k_r^c)}{\partial T \partial k_r} & \frac{\partial^2 T C^c_{\text{chain}}(T^c, k_r^c)}{\partial T^2}
\end{pmatrix}
\]

\[
\frac{\partial^2 T C^c_{\text{chain}}(T^c, k_r^c)}{\partial k_r^2} = \frac{\pi_r \sigma (T + L)^{1/2} f(k_r)}{T} = |H_{11}| > 0
\]

\[
\frac{\partial^2 T C^c_{\text{chain}}(T^c, k_r^c)}{\partial T^2} = \frac{2(A_r + F_r)}{T^3} + \pi_r \sigma G(k_r) \left( \frac{2\sqrt{T + L}}{T^3} - \frac{1}{T^2\sqrt{T + L}} - \frac{1}{4T(T + L)^{3/2}} \right) - \frac{h_r k_r \sigma}{4(T + L)^{3/2}}
\]

\[+ \frac{2A_s}{mT^3} - \frac{h_s k_s \sigma}{4T\sqrt{ml}} + \pi_s \sigma G(k_s) \left( -1 \frac{1}{4T\sqrt{ml}} + \frac{1}{T^2\sqrt{ml}} \right)
\]
\[ \frac{\partial^2 T C_{\text{chain}}^C(T^c, k_x^c)}{\partial T \partial k} = \frac{\partial^2 T C_{\text{chain}}^C(T^c, k_x^c)}{\partial k^2} \frac{\partial T C_{\text{chain}}^C(T^c, k_x^c)}{\partial T} \frac{\partial^2 T C_{\text{chain}}^C(T^c, k_x^c)}{\partial k \partial T} = \frac{h_r \sigma}{2 \sqrt{T + L}} + \frac{\pi_r \sigma}{2(2T \sqrt{T + L})} \left( \frac{1}{T^2} - \frac{\sqrt{T + L}}{T^2} \right) \left( \frac{1}{T} - \frac{\sqrt{T + L}}{T^2} \right) \left[ \varphi(k_r) - 1 \right] \]

\[ |H_{22}| = \frac{\partial^2 T C_{\text{chain}}^C(T^c, k_x^c)}{\partial T^2} \times \frac{\partial^2 T C_{\text{chain}}^C(T^c, k_x^c)}{\partial k^2} \times \frac{\partial^2 T C_{\text{chain}}^C(T^c, k_x^c)}{\partial k \partial T} \]

\[ = \frac{2(A_r + F_r)}{T^3} + \frac{h_r k_s \sigma}{4T \sqrt{mT}} + \frac{\pi_x \sigma G(k_s)}{\sqrt{mT}} \left( \frac{1}{4T \sqrt{mT}} - \frac{\sqrt{T + L}}{T^2} \right) \left[ \varphi(k_r) - 1 \right]^2 \]

The first principal minor Hessian has a positive value. Under the condition of problem, the second principal minor is positive. The problem is tested with the various numerical examples which cover a wide range of reasonable parameters and it is observed that it has a positive value for all numerical examples.