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Coordination of Information Sharing and Cooperative Advertising in a Decentralized Supply Chain with Competing Retailers Considering Free Riding Behavior

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Abstract

This paper studies a decentralized supply chain in which a manufacturer sells a common generic product through two traditional and online retailers under free riding market. We assume that the traditional retailer provides the value added services but the online retailer does not. Factors such as retail prices, local advertising of the retailers, and global advertising of the manufacturer and service level of the traditional retailer simultaneously has effect on market demand. This paper studies the cost information sharing between the manufacturer and traditional retailer and uses the cooperative advertising program as an incentive mechanism for information sharing under free riding. This paper also examines how the free riding phenomenon affects the information sharing between the manufacturer and traditional retailer and also the supply chain coordination. Our analysis shows that, through the cooperative advertising program, information sharing between the manufacturer and traditional retailer is always beneficial for all the supply chain members and therefore, the entire supply chain is coordinated except when the traditional retailer is not efficient and the degree of free riding is relatively small.

Keywords: supply chain coordination, information sharing, vertical cooperative advertising, online shopping, free riding; game theory

1- Introduction

With the increasing development of internet technologies, more and more firms use the Internet to sell their products. On the other hand, with the rapid growth of Internet shopping, free riding becomes very popular. In practice, however, a traditional retailer can add some value added services (such as, display of products, the explanation of product features, answering customer's questions, etc.) to the product to encourage customers purchasing product. Many consumers firstly can go to the traditional retail store and enjoy the retail services, and then purchase the product through the online shopping. Such behavior is called free riding (Zhengping & Yezheng, 2013). Free riding behavior is common in the experiential products (e.g. , electronic products, household appliances, and cars) because these products have some characteristics like low purchase frequency and relatively high price (Liu et al., 2014).

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The main innovations of our research are as follows: first, this is the first research which uses the cooperative advertising program as an incentive mechanism for information sharing. Second, this is the first model which analyzes information sharing in a distribution channel where the manufacturer sells his product through two traditional and online retailers when free riding problem exists. Third, this is the first study which investigates the impact of free riding behavior on information sharing. In the following, we review the literature is related to these fields.

Many authors focused on free riding. For instance, Wu et al. (2004) examined the competition between the information service-providing sellers and showed that sellers might obtain benefits from providing information service, even when there is free riding. They also indicated that sellers' incentives for providing information service are decreased by increasing competition. Van Baal & Dach (2005) examined the effect of free riding in two directions: from traditional retail store to online shops and vice versa. They indicated that over 20% of consumers are free riders. Shin (2007) indicated that in addition to the free riding retailer, free riding is beneficial for the service-providing retailer if consumers are heterogeneous in terms of their opportunity costs for shopping. Umit Kucuk & Maddux (2010) showed that the service-providing retailers' beliefs about online customers' choice of purchase outlet are mainly influenced by online retail price rather than accessibility of a variety of products on the Internet. Liu et al. (2014) investigated the influence of free riding behavior on manufacturer and retailer pricing and performance in a dual channel supply chain.

To improve the performance of the supply chain, the members of supply chain may act as a part of a unified system and coordinate their decisions(Arshinder et al., 2008). Information sharing is the most basic form of coordination in supply chains (Choi, 2010). Different members of the supply chain often obtain different benefits of information sharing. Therefore, the members who earned less benefit are not willing to share their information. Therefore, how to motivate the supply chain members for information sharing is an interesting research issue.

When there are only traditional retailers, for eliminate double marginal effect, many coordinative contracts such as revenue-sharing (Cachon & Lariviere, 2005; Qin, 2008; Alaei & Setak, 2015), buyback (Pasternack, 2008; Chen & Bell, 2011), quantity-flexibility (Tsay, 1999) and combined revenue sharing and quantity discount (Partha Sarathi et al., 2014) have been employed. Also, many researchers have studied information sharing in a supply chain (Hsiao & Shieh, 2006; Chiang & Feng, 2007; Shamir, 2012; Hall & Saygin, 2012). Instead, there are few researches which studied how to create an incentive for the supply chain members to share their information. For example, Yao et al. (2008) studied the retailers' incentives for value-added cost information sharing in a supply chain with one supplier and two heterogeneous retailers. They indicated that, side payment is not a suitable strategy to extract more accurate information from a downstream player. Wu et al. (2008)Investigated the incentives for demand and cost information sharing in a Cournot model with capacity constraints. They showed that the incentives can be reversed when some equilibrium solutions are binding on capacity and information sharing does not necessarily increase social welfare. Liu & Özer (2010) considered a distribution channel with a manufacturer and a retailer and indicated that the buyback contract always motivates the manufacturer to share his demand forecast information. Mahajan & Venugopal (2011)investigated the supply chain consist of one manufacturer and one retailer and showed that the reduction in the lead time to the retailer can be motivate the retailer to share his demand information. Qian et al. (2012) examined a two-echelon supply chain comprising a manufacturer with a limited production capacity and many retailers and showed that a discriminated allocation strategy will motivate the retailers to share their demand information. Zhang & Chen (2013) considered a supply chain with a supplier and a retailer and indicated that the revenue sharing contract is coordinative and ensures that the supplier and the retailer disclose their demand information completely. Liu et al. (2016) studied the retailer's choice on cost information sharing in a dual- channel structure consisting of a retail channel and a direct sales channel. They indicated that in both single- and dual-channel structures, the retailer has little motivation to share its private cost information which is verified to be valuable for the manufacturer.Dukes et al. (2017) evaluated the impact of information sharing on wholesale and retail pricing incentives as well as on the distribution of economic rents. They demonstrated that, information sharing arrangements in equilibrium require side payments and/or sufficient cost savings because the manufacturer benefits from more information sharing at the loss of downstream retailers and consumers.

Literature review demonstrates that there are few studies which discussed coordination in supply chain when free riding problem exists. For example, Zhengping & Yezheng (2013) used the revenue sharing contract to coordinate the various dual channel structures in case of free riding. They indicate that the whole supply chain of decentralized dual channel is coordinated by revenue sharing contract.

Cooperative advertising is mostly used in consumer goods industries and financial support from manufacturers is a major part of a retailer's budget for local advertising (Jørgensen & Zaccour, 2014). Vertical cooperative advertising is an agreement between a manufacturer and his retailers, where a manufacturer offers to share a certain percentage of his retailer's local advertising costs(Aust & Buscher, 2014a).Among the existing studies on cooperative advertising, a few works study a channel in which a single manufacturer sells a product through two or more competing retailers.Ghadimi et al. (2011) studied the coordination of cooperative advertising in a supply chain with one manufacturer and two retailers when the relationships among the manufacturer and two retailers are symmetric and they cooperate to increase their profits.Zhang & Xie (2012) investigated the impacts of the retailer's multiplicity on channel members' decisions and total channel efficiencies in a distribution channel with a manufacturer and multiple competing retailers.Alaei et al. (2014) studied a supply chain with one manufacturer and two retailers who can either compete or cooperate, and either Nash or Stackelberg game is played between the echelons. They determined the fraction of local advertising costs shared by the manufacturer through a bargaining game. Aust & Buscher (2014b) investigated either Vertical Nash-Horizontal Nash or Manufacturer Stackelberg-Horizontal Nash game in a distribution channel consisting of one manufacturer and two competing retailers, who offer substitutable products to consumers. Karray & Hassanzadeh Amin (2015) studied the effect of cooperative advertising in a supply chain with one manufacturer and two competing retailers. They indicated that cooperative advertising may not be beneficial for the retailers and the channel, especially when there are low levels of price competition and high level of advertising competition between retailers.Li et al. (2015) first considered a dyadic supply chain with a manufacturer and a traditional retailer, in which the manufacturer sets a direct channel, and then expand their model to a competitive case with one manufacturer and two independent retailers. They indicate that an appropriate advertising effort can coordinate the supply chain and lead to a win-win outcome for each channel.

This paper considers a two-echelon supply chain in which the manufacturer sells a common generic product through two traditional and online retailers. Two retailers compete on retail price and advertise their product. The traditional retailer adds some services to product to attract customers to his store. Some customers may enjoy the traditional retailer's services, but purchase the final products through the online retailer. Thus the free riding problem exists. In addition, the manufacturer has no complete knowledge of the traditional retailer's service cost information. Therefore, the manufacturer shares a certain percentage of the traditional retailer's advertising expenditure to motivate the traditional retailer sharing his information. These are some of the questions that explored in this paper:

1. How the cooperative advertising program motivates the traditional retailer to disclose his private information to the manufacturer?
2. Whether the information sharing between the manufacturer and traditional retailer coordinates the entire supply chain?
3. How the free riding phenomenon affects the information sharing between the manufacturer and traditional retailer and also the supply chain coordination?
4. How the free riding phenomenon affects the traditional retailer's service level and the supply chain members' profits?

Therefore, the organization of the paper is as follows. Section 2 describes the research model. Section 3 solves the model and determines the equilibrium outcomes. Section 4 presents an illustrative example. The managerial implications are discussed in Section 5. Finally, the main findings of our research are summarized in section 6.

2- Model formulation

Consider a market setting where a manufacturer sells a common generic product through two traditional and online retailers who compete on retail prices. Suppose that the unit production cost is c

and the manufacturer sets a common wholesale price for retailers ($w > c$). Factors such as retail prices, local advertising of the retailers and global advertising of the manufacturer simultaneously has effect on market demand. We consider the global advertising expenditure $A$for the manufacturer and the local advertising expenditure $a_i$for the retailer i ($i = t$denotes the traditional retailer and $i = e$denotes the online retailer). The traditional retailer offers services to customers to attract customers to his store. Some customers may enjoy the retail services offered by the traditional retailer, but purchase the final products through the online shopping. Therefore, besides the retail prices and the advertising expenditures of all the members, the customer demand is also influenced by the traditional retailer's service level. Here, traditional retailer has private information about the cost of the service level which is unknown to the manufacturer and the online retailer. The manufacturer decides to offer a vertical cooperative advertising program to encourage the traditional retailer sharing his information, whereby he can bear a fraction ($0 \leq \theta < 1$) of traditional retailer's local advertising expenditure. However, if the traditional retailer is unwilling to disclose his information, the manufacturer does not offer the cooperative advertising program. Furthermore, our main goal is to determine the condition under which cooperative advertising program and information sharing is beneficial not only for the manufacturer, but also for the traditional retailer. When a specific variable is optimized by one of the supply chain partners, the other partners' objectives may be ignored and correspondingly, the whole supply chain objectives may be ignored (Tsay, 1999). Therefore we modeled the advertising participation rate as a parameter and determined that for what values of manufacturer's participation rate information sharing can benefit both the manufacturer and the traditional retailer. In this field, we can refer to some studies in which the participation rates are exogenously given (Kunter, 2012; Zhang et al., 2013).

Throughout the paper we will use the following notations:
c: the unit production cost of the manufacturer
w: the unit wholesale price of the manufacturer
p_i: the retail price of retailer i , $i = t, e$
α_i: the market base of retailer i , $i = t, e$
s: the service level of the traditional retailer
a_i: the local advertising expenditure of retailer i , $i = t, e$
A: the global advertising expenditure of the manufacturer
π_i: the profit of player i , $i = t, e, M$
β_1: the demand sensitivity to the own price
β_2: the demand sensitivity to the service level
γ_1 : the substitutability coefficient of two retailers
μ: the free riding ratio
k_1 : the effectiveness of own local advertising
k_2: the effectiveness of rival's local advertising
k_3: the effectiveness of global advertising
θ: the advertising participation rate of the manufacturer
η: the service level cost efficiency of the traditional retailer
$\bar{\eta}$: the average service level cost efficiency
ε: the deviation service level cost efficiency

Similar to Aust and Buscher (2014b; 2014c), it can be assumed that the consumer demand $d_i(p_i, p_j, s, a_i, a_j, A)$ has the following form:

$$d_i(p_i, p_j, s, a_i, a_j, A) = f_i(p_i, p_j, s).\, g_i(a_i, a_j, A), \quad i, j = t, e, \quad i \neq j \tag{1}$$

where$f_i(p_i, p_j, s)$ shows the effect of retail prices and service level on demand, and $g_i(a_i, a_j, A)$ shows the effect of advertising costs on demand. We use a similar demand function as Raju et al. (1995), which is a well-known liner demand function in the literature. Then we add the service level to the traditional retailer's demand function as a decision variable (Tsay & Agrawal, 2000; Yao et al., 2008). The customers' perception of value and, therefore, their buying decisions, are influenced not only by the product's selling price, but also the amount of "service" that is provided with it. Here, the services that provided by the traditional retailer includes display of product, explaining the product features in detail, answering customer's questions, after-sales service, etc. These elements, which represent all forms of demand-enhancing strategies of a firm, are aggregated into a decision variable for the traditional retailer. Therefore, price and service are two attributes of the product and the traditional retailer chooses its own price and service. The nature of market demand is such that, if the traditional retailer reduces price or increases service, he will enjoy from sales growth (Tsay & Agrawal, 2000). Moreover, the traditional retailer's service level simultaneously has effect on the online retailer's demand, due to free riding. Therefore, $f_i(p_i, p_j, s)$for the traditional retailer and online retailer is as follows, respectively:

$$f_t(p_t, p_e, s) = (\alpha_t - \beta_1 p_t + (\beta_2 - \mu)s + \gamma_1(p_e - p_t)) \quad (2)$$

$$f_e(p_e, p_t, s) = (\alpha_e - \beta_1 p_e + \mu s + \gamma_1(p_t - p_e)) \quad (3)$$

Also, Similar to Xie & Wei (2009), SeyedEsfahani et al. (2011) and Aust and Buscher(2014b; 2014c), we model advertising effects on consumer demand as:

$$g_i(a_i, a_j, A) = (k_1\sqrt{a_i} + k_2\sqrt{a_j} + k_3\sqrt{A}), \;\; i, j = t, e, \;\; i \neq j \quad (4)$$

Therefore, the resulting total demand function for the traditional retailer and online retailer are respectively identified by:

$$d_t = (\alpha_t - \beta_1 p_t + (\beta_2 - \mu)s + \gamma_1(p_e - p_t))(k_1\sqrt{a_t} + k_2\sqrt{a_e} + k_3\sqrt{A}) \quad (5)$$

$$d_e = (\alpha_e - \beta_1 p_e + \mu s + \gamma_1(p_t - p_e))(k_1\sqrt{a_e} + k_2\sqrt{a_t} + k_3\sqrt{A}) \quad (6)$$

Furthermore, like many researchers (Iyer, 1998; Tsay & Agrawal, 2000; Yao et al., 2008; Xiao & Yang, 2008), we assume that, when the traditional retailer provides a service level$s$, the service cost of the traditional retailer is:

$$C(s) = \eta \frac{s^2}{2} \quad (7)$$

where the smaller theη, the more efficient traditional retailer. According to Yao et al. (2008), we assume η is uniformly distributed, i.e.$\eta \sim U[\bar{\eta} - \varepsilon, \bar{\eta} + \varepsilon]$. Also, η is known only for the traditional retailer and the other players know the probability distribution function of η and can estimate$\bar{\eta}$ and ε. With these assumptions, the manufacturer's, traditional retailer's and online retailer's profits are respectively determined by:

$$\pi_M = [w - c](d_t + d_e) - \theta a_t - A \quad (8)$$

$$\pi_{R_t} = [p_t - w - C(s)]d_t - (1 - \theta)a_t \quad (9)$$

$$\pi_{R_e} = [p_e - w]d_e - a_e \quad (10)$$

As mentioned before, only the traditional retailer provides some services to customers and has private information about his service cost information. Therefore the manufacturer only cooperates with the traditional retailer and share a certain percentage of the traditional retailer's advertising cost (θ) to motivate the traditional retailer sharing his information.

3-Stackelberg game

We model the problem as a three-stage game, where the manufacturer acts as the Stackelberg leader and the retailers are the followers. The manufacturer first maximizes his profit and sets his optimal global advertising expenditure and wholesale price by using the retailers' decisions and the information obtained from the traditional retailer. In the next step, based on the manufacturer's decisions, the traditional retailer sets his service level, retail price and local advertising expenditure and the online retailer sets his retail price and local advertising expenditure. We solve this three-stage game with the backward induction approach. Therefore, we assume the manufacturer's global advertising expenditure and wholesale price is given and first solve the decision problem of the traditional and online retailers as below

$$\begin{aligned} \underset{p_t,s,a_t}{Max}\, E[\pi_{R_t}|w, A] \\ = \underset{p_t,s,a_t}{max}\, E\big[(p_t - w - C(v))(\alpha_t - \beta_1 p_t + (\beta_2 - \mu)s + \gamma_1(p_e - p_t))(k_1\sqrt{a_t} \\ + k_2\sqrt{a_e} + k_3\sqrt{A}) - (1 - \theta)a_t|w, A\big] \end{aligned} \quad (11)$$

$$\underset{p_e,a_e}{Max}\, E\left[\pi_{R_e}|w,A\right] = \underset{p_e,a_e}{max}\, E\left[(p_e-w)\left(\alpha_e-\beta_1 p_e+\mu s+\gamma_1(p_t-p_e)\right)\left(k_1\sqrt{a_e}+k_2\sqrt{a_t}+k_3\sqrt{A}\right)|w,A\right] \quad (12)$$

Proposition 3.1*Optimal response function of the traditional retailer is*

$$\begin{cases} s^* = \frac{(\beta_2-\mu)}{\eta(\beta_1+\gamma_1)} \\ p_t^* = B_t + \frac{(\beta_2-\mu)(3\beta_2(\beta_1+\gamma_1)-\mu(3\beta_1+2\gamma_1))}{\eta(\beta_1+\gamma_1)(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)} \\ a_t^* = \frac{{k_1}^2}{1024\,\eta^4(\beta_1+\gamma_1)^6(1-\theta)^2}\left[2\eta(\beta_1+\gamma_1)(E_t+\frac{F\gamma_1}{4(\beta_1+\gamma_1)})+(\beta_2-\mu)^2\right]^4 \end{cases}$$

And the optimal response function of the online retailer is

$$\begin{cases} p_e^* = B_e + \frac{(\beta_2-\mu)(3\beta_2\gamma_1+\mu(4\beta_1+\gamma_1))}{4\varepsilon(\beta_1+\gamma_1)(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)} ln\left(\frac{\bar{\eta}+\varepsilon}{\bar{\eta}-\varepsilon}\right) \\ a_e^* = \frac{{k_1}^2}{64\,(\beta_1+\gamma_1)^6}(E_e+\frac{F}{2})^4 \end{cases}$$

where B_t, B_e, E_t, $E_e$and $F$are defined as following

$$B_t = \frac{2\alpha_t(\beta_1+\gamma_1)+\alpha_e\gamma_1+w(\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}{(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}$$

$$B_e = \frac{2\alpha_e(\beta_1+\gamma_1)+\alpha_t\gamma_1+w(\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}{(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}$$

$$E_t = \frac{4\alpha_t(\beta_1+\gamma_1)^2+2\alpha_e\gamma_1(\beta_1+\gamma_1)-2w\beta_1(\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}{(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}$$

$$E_e = \frac{4\alpha_e(\beta_1+\gamma_1)^2+2\alpha_t\gamma_1(\beta_1+\gamma_1)-2w\beta_1(\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}{(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}$$

$$F = \frac{(\beta_2-\mu)(3\beta_2\gamma_1+\mu(4\beta_1+\gamma_1))}{\varepsilon(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)} ln\left(\frac{\bar{\eta}+\varepsilon}{\bar{\eta}-\varepsilon}\right)$$

Proof: *See Appendix A.*

Retailers' variables are now used as constraints in the manufacturer's decision problem:

$$\underset{w,\ A}{Max}\, E[\pi_M] = \underset{w,\ A}{Max}\, E[(d_t+d_e)(w-c)-\theta a_t-A] \quad (13)$$

We will solve the manufacturer's decision problem and obtain the optimal global advertising expenditure and wholesale price for the manufacturer in the full information case and asymmetric information case, respectively.

3-1- Full information case (FI)

Suppose that the traditional retailer decides to share his private information with the manufacturer. Therefore, the manufacturer has complete information about the traditional retailer's service level cost information$\eta$and the manufacturer's decision problem is:

$$\underset{w,A}{Max}\, \pi_M = \Big(\left(\alpha_t-\beta_1 p_t+(\beta_2-\mu)s+\gamma_1(p_e-p_t)\right)\left(k_1\sqrt{a_t}+k_2\sqrt{a_e}+k_3\sqrt{A}\right) + \left(\alpha_e-\beta_1 p_e+\mu s+\gamma_1(p_t-p_e)\right)\left(k_1\sqrt{a_e}+k_2\sqrt{a_t}+k_3\sqrt{A}\right)\Big)(w-c) - \theta a_t - A \quad (14)$$

In order to solve this problem, the traditional retailer's and online retailer's variables first have to be substituted into (14). Then, the optimal global advertising expenditure and wholesale price are identified by setting the partial first order derivatives$\partial\pi_M/\partial A$ and$\partial\pi_M/\partial w$to zero and solving the resulting system of equations (all mathematical calculations are given in Appendix B). But, due to complexity of the problem, the first order condition $\partial\pi_M/\partial A = 0$ and$\partial\pi_M/\partial w = 0$ cannot be solved analytically. Therefore, the optimal global advertising expenditure and wholesale price are determined through illustrative example.

3-2- Asymmetric information case (AI)

Now, we assume that there is no information exchange between the manufacturer and traditional retailer. Therefore, η is unknown to the manufacturer and the manufacturer's decision problem is:

$$\underset{w,\ A}{Max}\,\pi_M = \int_{\bar{\eta}-\varepsilon}^{\bar{\eta}+\varepsilon} \Big[\Big(\big(\alpha_t - \beta_1 p_t + (\beta_2-\mu)s + \gamma_1(p_e - p_t)\big)\big(k_1\sqrt{a_t} + k_2\sqrt{a_e} + k_3\sqrt{A}\big) + \big(\alpha_e - \beta_1 p_e + \mu s + \gamma_1(p_t - p_e)\big)\big(k_1\sqrt{a_e} + k_2\sqrt{a_t} + k_3\sqrt{A}\big)\Big)(w-c) - \theta a_t - A\Big] f(\eta)d\eta \quad (15)$$

Similar to the previous case, the optimal global advertising expenditure and wholesale price can be determined by substituting the traditional retailer's and online retailer's variables into (15) and setting the first order conditions to zero.

4-Illustrative example

In this section, we present an example in order to investigate effects of the free riding, service cost efficiency and participation rate on information sharing and supply chain coordination. Firstly, we perform a comparison between full information (FI) and asymmetric information (AI) cases. Then, we study the effect of information sharing between manufacturer and traditional retailer on supply chain coordination. We consider the following default values for parameters:

$\alpha_t = \alpha_e = 10$, $\beta_1 = 1$, $\beta_2 = 0.8$, $\gamma_1 = 0.8$ $k_1 = 1$, $k_2 = 0.25$, $c = 2$, , $\bar{\eta} = 0.5$, $\varepsilon = 0.4$,$\eta \in \{0.1, 0.5, 0.9\}$, $\mu \in \{0, 0.1, 0.2, \ldots, 0.8\}$ and $\theta \in (0,1)$.

4-1- Comparison between FI and AI cases

This section provides a comparison of the manufacturer's and traditional retailer's profit under full information and asymmetric information cases. Then, the conditions in which the traditional retailer has an incentive to vertically share his private information with the manufacturer, are determined.

The results of comparison of two cases for different values of μ are demonstrated in figure 1 which specifies the region in which cooperative advertising program makes FI better than AI case with the manufacturer's and traditional retailer's point of view. For example, for $\eta = 0.5$and $\mu = 0.4$ as shown in figure 1-(b), information sharing is beneficial for the manufacturer and traditional retailer, if $\theta \in (0.16, 0.78)$. In other words, proposing cooperative advertising contract with$\theta \in (0.16, 0.78)$ leads to better profit for the manufacturer and traditional retailer with vertical information sharing (FI) rather than no information sharing (AI). From figure 1-(b), it is figure out that, if $\eta = 0.5$ and $\theta \in (0.3, 0.74)$, for all values of free riding parameter (η) vertical information sharing is beneficial for the manufacturer and the traditional retailer. In other words, with $\theta \in (0.3, 0.74)$, the win-win situation is achieved for the manufacturer and traditional retailer. For other values of η and μ the manufacturer's participation interval is shown in table 1. For all values of η and μin table 1,$\pi_M^{FI} > \pi_M^{AI}$and $\pi_{R_t}^{FI} > \pi_{R_t}^{AI}$, as long as the manufacturer's participation rate is in the specified range.

Considering figure 1 and table 1, it can be implied that for all values of η and μ, there is a feasible cooperative advertising program which makes vertical information sharing beneficial for the manufacturer and traditional retailer. Therefore, for every η and μ, the traditional retailer has enough incentives to reveal his information to the manufacturer.

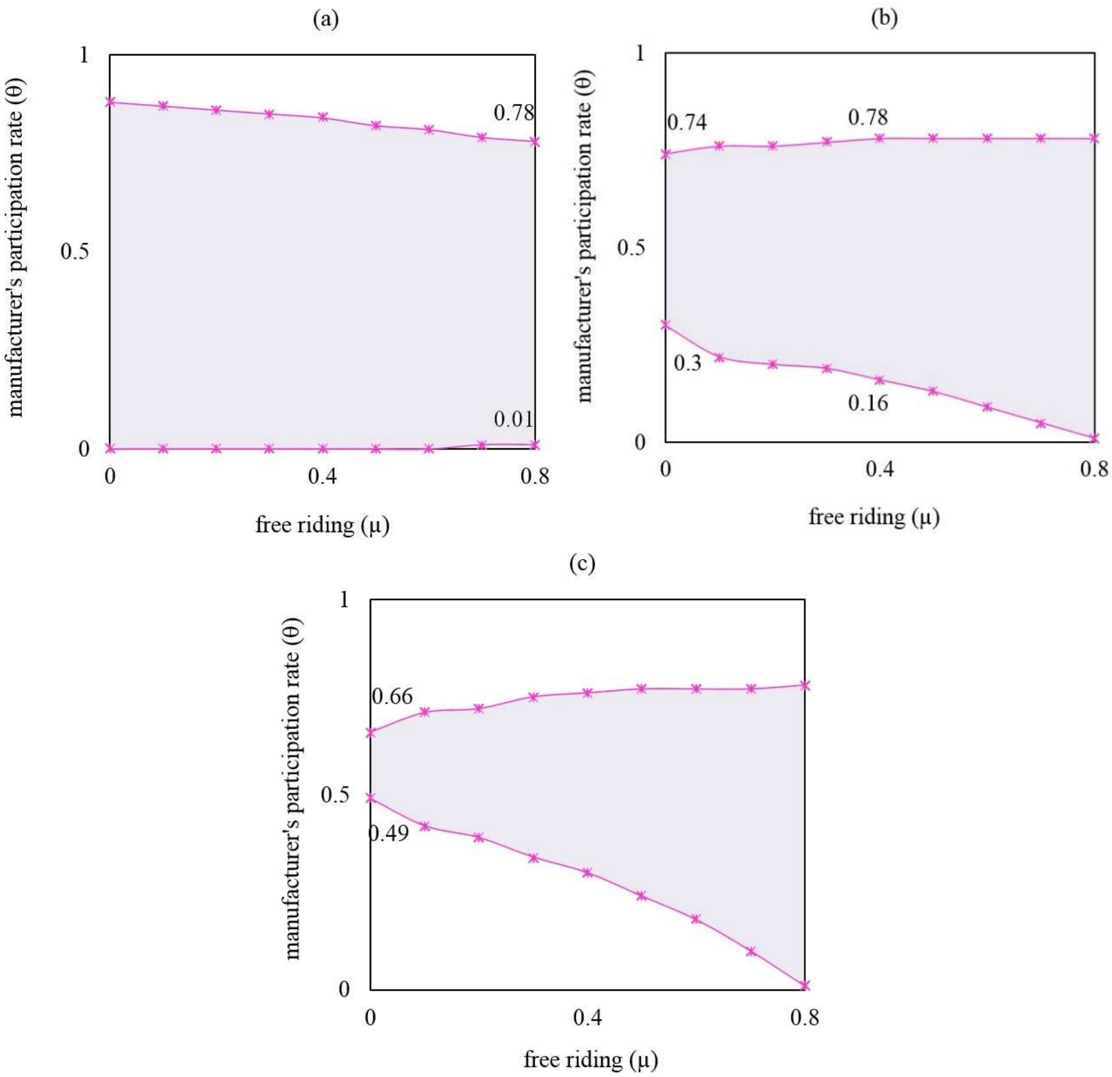

Figure 1. The region in which cooperative advertising program makes FI better than AI for the manufacturer and the traditional retailer (a) $\eta = 0.1$(b) $\eta = 0.5$ (c) $\eta = 0.9$

Table 1. Ranges of θ for which FI is better than AI for the manufacturer and traditional retailer

	0.1	0.5	0.9	$0.1 \le \eta \le 0.9$
0	(0,0.88)	(0.3,0.74)	(0.49,0.66)	(0.49,0.66)
0.1	(0,0.87)	(0.22,0.76)	(0.42,0.71)	(0.42,0.71)
0.2	(0,0.86)	(0.2,0.76)	(0.39,0.72)	(0.39,0.72)
0.3	(0,0.85)	(0.19,0.77)	(0.34,0.75)	(0.34,0.75)
0.4	(0,0.84)	(0.16.0.78)	(0.3,0.76)	(0.3,0.76)
0.5	(0,0.82)	(0.13,0.78)	(0.24,0.77)	(0.24,0.77)
0.6	(0,0.81)	(0.09,0.78)	(0.18,0.77)	(0.18,0.77)
0.7	(0.01,0.79)	(0.05,0.78)	(0.1,0.77)	(0.1,0.77)
0.8	(0.01,0.78)	(0.01,0.78)	(0.01,0.78)	(0.01,0.78)

4-2-Impact of information sharing on supply chain coordination

In this section we study the effect of information sharing between the manufacturer and traditional retailer on supply chain coordination and determine the condition in which information sharing can coordinate the entire supply chain. To achieve this purpose, we compare profits of all supply chain members under FI and AI cases. Figure 2 demonstrates the comparison results of two cases for different values of η. In this figure, the regions where cooperative advertising program makes FI better than AI case for all supply chain members are highlighted. For example, figure 2-(b) illustrates that, if $\eta = 0.5$ and $\theta \in (0.3,0.61)$, for all values of μ vertical information sharing between the manufacturer and traditional retailer is beneficial for all the supply chain members. In other words, for all values of μ with $\theta \in (0.3,0.61)$, vertical information sharing between the manufacturer and traditional retailer creates a win–win situation for all supply chain members. For other values of $\eta$and μ, the manufacturer's participation interval is shown in table 2. For specific values of η and μin table 2, $\pi_M^{FI} > \pi_M^{AI}$and $\pi_{R_i}^{FI} > \pi_{R_i}^{AI}$, $i \in \{t, e\}$, as long as the manufacturer's participation rate is in the specified range. Note that, "dashes (-)" for some values of $\eta$and μin table 2 implies that there is no feasible range of participation rate in cooperative advertising contract. It can be implied that information sharing through the cooperative advertising program is beneficial for all the supply chain members and therefore the entire supply chain is coordinated except when the traditional retailer is not efficient (the traditional retailer's efficiency parameter is large) and the degree of free riding is relatively small. Furthermore, figure 3 specifies the region in which information sharing between the manufacturer and traditional retailer is beneficial for all the supply chain members through the cooperative advertising program. In other words, for any values of μ and ηin the highlighted region, there exists a feasible range of participation rate in the cooperative advertising contract.

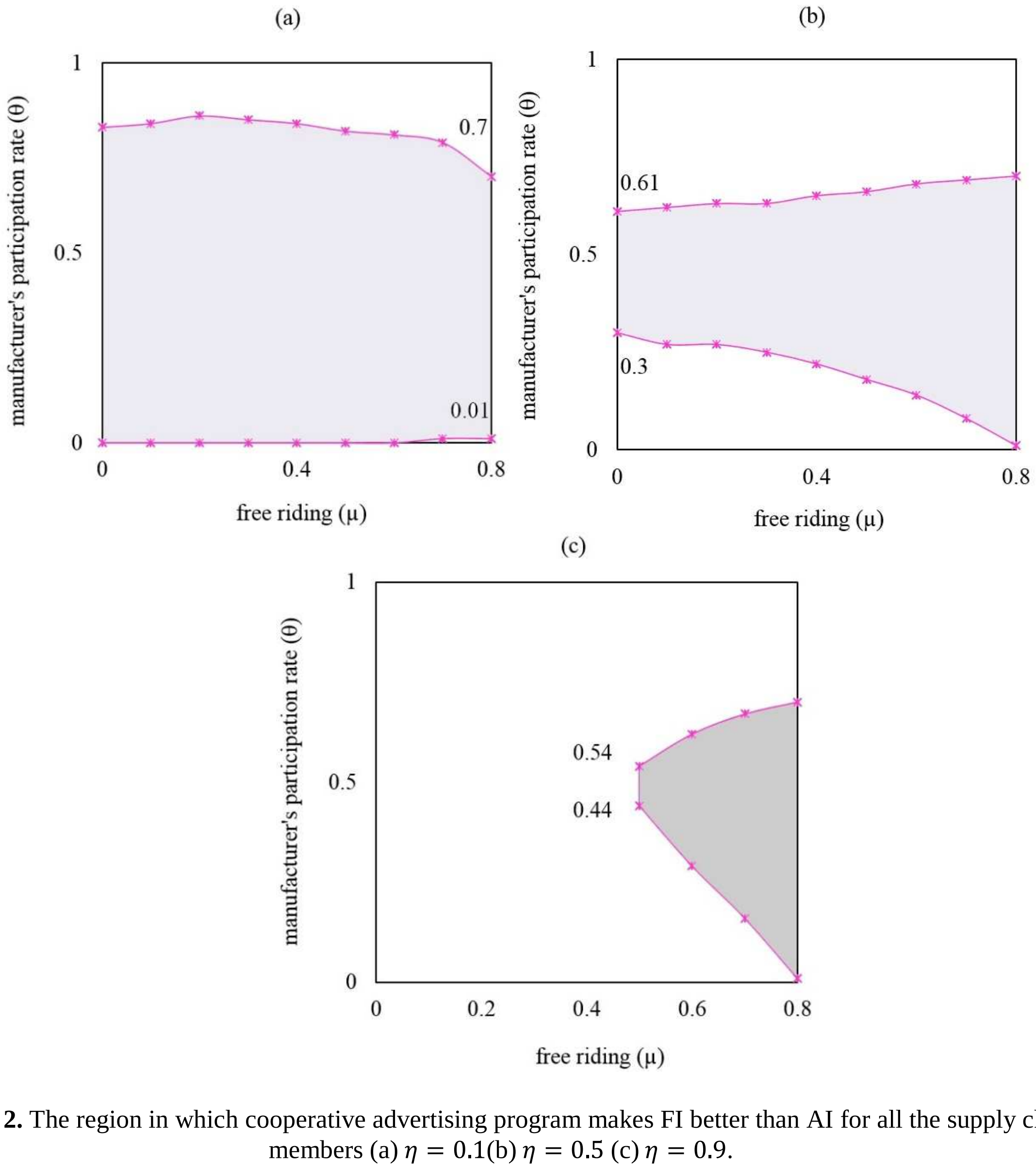

Figure 2. The region in which cooperative advertising program makes FI better than AI for all the supply chain members (a) $\eta = 0.1$(b) $\eta = 0.5$ (c) $\eta = 0.9$.

Table 2. Ranges of θ for which FI is better than AI for all channel members

	0.1	0.5	0.9	$0.1 \le \eta \le 0.9$
0	(0,0.83)	(0.3,0.61)	-	-
0.1	(0,0.84)	(0.27,0.62)	-	-
0.2	(0,0.86)	(0.27,0.63)	-	-
0.3	(0,0.85)	(0.25,0.63)	-	-
0.4	(0,0.84)	(0.22.0.65)	-	-
0.5	(0,0.82)	(0.18,0.66)	(0.44,0.54)	(0.44,0.54)
0.6	(0,0.81)	(0.14,0.68)	(0.29,0.62)	(0.29,0.62)
0.7	(0.01,0.79)	(0.08,0.69)	(0.16,0.67)	(0.16,0.67)
0.8	(0.01,0.7)	(0.01,0.7)	(0.01,0.7)	(0.01,0.7)

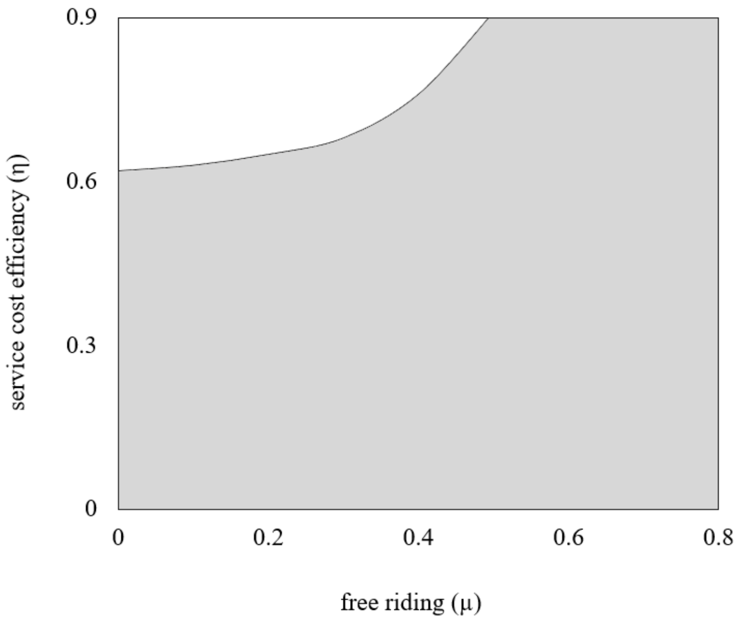

Figure 3. The region in which information sharing between the manufacturer and traditional retailer is beneficial for all the supply chain members through the cooperative advertising program

4-3-Effect of free riding on optimal decisions and profits

In this subsection, we examine the impact of free riding on the traditional retailer's service level, retailers' retail prices and members' profit. The results are depicted in figure 4 and figure 5. Figure 4-(a) indicates that the traditional retailer's service level is decreased by increasing free riding. Figure 4-(b) shows that the traditional retailer's and online retailer's retail prices are decreased by increasing free riding. Figure 5 also indicates that the manufacturer's and retailers' profits are decreased by increasing free riding.

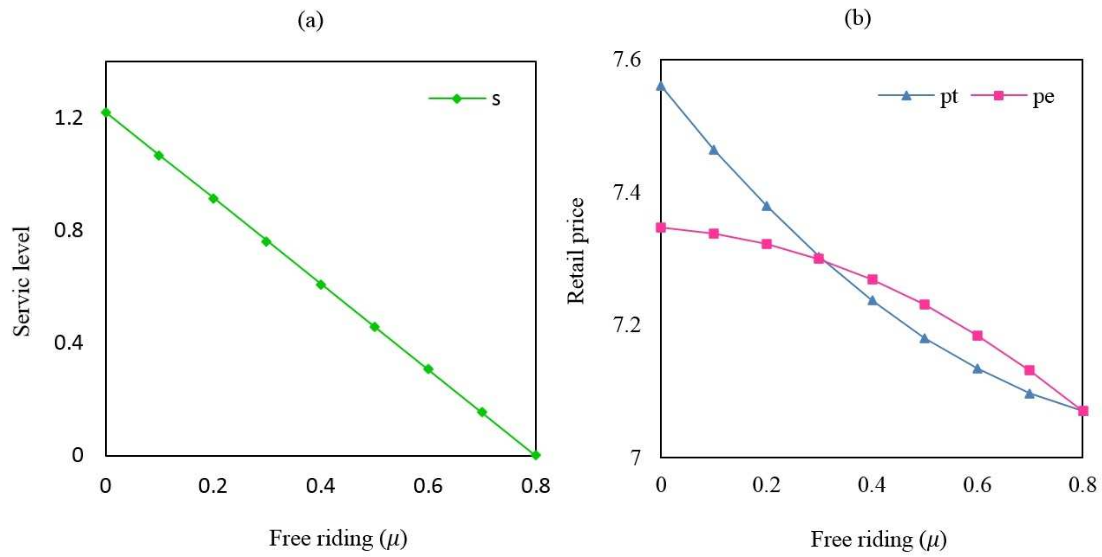

Figure 4. (a) The impact of free riding on the traditional retailer's service level. (b) The impact of free riding on the retailers' retail prices.

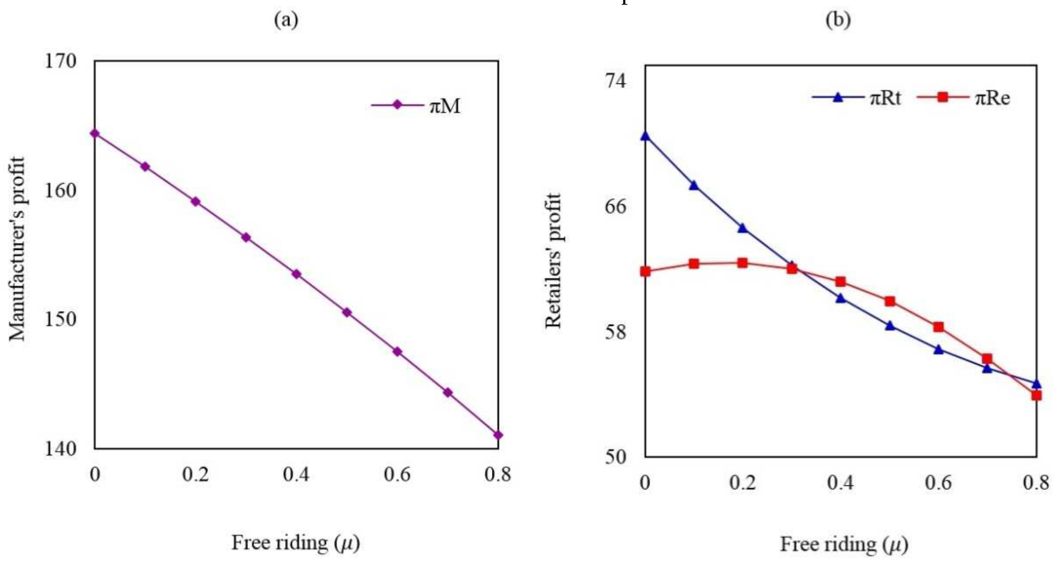

Figure 5. (a) The impact of free riding on the manufacturer's profit. (b) The impact of free riding on the retailers' profit.

5- Managerial implications

As noted before, cooperative advertising program can be treated as an incentive mechanism for motivating retailers to reveal their private information. The problem studied in this paper tries to determine optimal advertising, pricing and other decisions under free riding in a supply chain including a single manufacturer, a traditional retailer and an online retailer. It is assumed that the traditional retailer has private information about the cost of the service level. The manufacturer decides to offer a vertical cooperative advertising program to encourage the traditional retailer sharing his information. Our analysis in the illustrative example shows that:information sharing is always beneficial for the manufacturer and traditional retailer since there exist a feasible range for cooperative advertising participation rate. However, in some conditions, i.e. $\mu < 0.5$, the online retailer's profit is worse-off by proposing cooperative advertising contract.

The interesting result is that regardless of value ofη,the manufacturer can offer cooperative advertising contract which ensure improvement in his profit and traditional retailer's profit. For example, given μin table 1, for any$0.1 \leq \eta \leq 0.9$in the fifth column, there exists feasible range for participation rate. This result is very important since the manufacturer compares full information with asymmetric information case where he is unaware of value of η. However, with the whole supply chain members' point of view, regardless of value of η for any $0.1 \leq \eta \leq 0.9$in the fifth column of table 2, the manufacturer can offer cooperative advertising contract only when$\mu \geq 0.5$.

6- Conclusions

With the rapid growth of Internet shopping, more and more firms use the Internet to sell their products. Since the online shopping has a lower retail price, the free riding problem occurs. We presented a supply chain model in which a manufacturer sells a common generic product through two traditional and online retailers under free riding market. The market demand simultaneously affected by retail prices, value added services of the traditional retailer, local advertising of the retailers and global advertising of the manufacturer. We studied the cost information sharing between the manufacturer and traditional retailer and the cooperative advertising program is used as an incentive mechanism for information sharing. Also, we investigated how the free riding phenomenon affects the information sharing between the manufacturer and traditional retailer and also the supply chain coordination.

Through illustrative example, we achieved the following findings: (1) If the manufacturer offers a cooperative advertising program, information sharing between the manufacturer and traditional retailer is always beneficial for the manufacturer and traditional retailer and therefore, the traditional retailer always has an incentive to share his private information with the manufacturer. (2) If the manufacturer offers a cooperative advertising program, information sharing between the manufacturer and traditional retailer is beneficial for all the supply chain members and therefore the entire decentralized supply chain is coordinated except when the traditional retailer is not efficient and the degree of free riding is relatively small. (3) Free riding has a negative effect on the traditional retailer's service level and the supply chain members' profits.

There are various directions for extension of our study. First, we show that through the cooperative advertising program, information sharing between the manufacturer and traditional retailer cannot always coordinate the entire supply chain. Therefore, we can study how the manufacturer designs another contract in addition to cooperative advertising to coordinate the decentralized supply chain. Second, there is another kind of free riding that the customer's first search for information on the Internet and then buy the product from a traditional retail store. Therefore, a possible extension is to study the decentralized supply chain coordination when such a free riding behavior exists.

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Appendix A

Proof of proposition 1

Replacing the traditional retailer's demand function into (9), his profit function can be rewritten as following

$$\underset{p_t,s,a_t}{Max}\, E\left[\pi_{R_t}|w,A\right] = E\left[\left(p_t - w - C(v)\right)d_t - (1-\theta)a_t\right] \\ = \left(p_t - w - \frac{1}{2}\eta s^2\right)\left(\alpha_t - \beta_1 p_t + (\beta_2-\mu)s + \gamma_1(p_e - p_t)\right)\left(k_1\sqrt{a_t} + k_2\sqrt{a_e} + k_3\sqrt{A}\right) - (1-\theta)a_t \quad (A.1)$$

The first order partial derivatives of the traditional retailer's decision problem are calculated and set to zero:

$$\frac{\partial E[\pi_{R_t}]}{\partial s} = \left((\beta_2-\mu)(p_t - w) - \eta s\alpha_t + \eta s p_t(\beta_1+\gamma_1) - \eta s\gamma_1 p_e - \frac{3}{2}\eta s^2(\beta_2-\mu)\right)\left(k_1\sqrt{a_t} + k_2\sqrt{a_e} + k_3\sqrt{A}\right) = 0 \quad (A.2)$$

$$\frac{\partial E[\pi_{R_t}]}{\partial p_t} = \left(\alpha_t - 2(\beta_1+\gamma_1)p_t + (\beta_2-\mu)s + \gamma_1 p_e + w(\beta_1+\gamma_1) + \frac{1}{2}\eta(\beta_1+\gamma_1)s^2\right)\left(k_1\sqrt{a_t} + k_2\sqrt{a_e} + k_3\sqrt{A}\right) = 0 \quad (A.3)$$

$$\frac{\partial E[\pi_{R_t}]}{\partial a_t} = \frac{k_1}{2\sqrt{a_t}}\left(p_t - w - \frac{1}{2}\eta s^2\right)\left(\alpha_t - (\beta_1+\gamma_1)p_t + (\beta_2-\mu)s + \gamma_1 p_e\right) - (1-\theta) = 0 \quad (A.4)$$

From these equations, the traditional retailer's optimal policies are derived as follows:

$$s^* = \frac{(\beta_2-\mu)}{\eta(\beta_1+\gamma_1)} \quad (A.5)$$

$$p_t^* = \frac{1}{4(\beta_1+\gamma_1)}\left[2\alpha_t + 2w(\beta_1+\gamma_1) + 2\gamma_1 p_e + \frac{3(\beta_2-\mu)^2}{\eta(\beta_1+\gamma_1)}\right] \quad (A.6)$$

$$a_t^* = \frac{{k_1}^2}{1024\,\eta^4(\beta_1+\gamma_1)^6(1-\theta)^2}\left[2\eta(\beta_1+\gamma_1)(\alpha_t - w(\beta_1+\gamma_1) + \gamma_1 p_e) + (\beta_2-\mu)^2\right]^4 \quad (A.7)$$

Similarly, the online retailer's profit function can be rewritten as following

$$\underset{p_e,a_e}{Max}\, E\left[\pi_{R_e}|w,A\right] = E[(p_e - w)d_e - a_e] \\ = (p_e - w)\left(\alpha_e - \beta_1 p_e + \mu s + \gamma_1(p_t - p_e)\right)\left(k_1\sqrt{a_e} + k_2\sqrt{a_t} + k_3\sqrt{A}\right) - a_e \quad (A.8)$$

The first order partial derivatives of the online retailer's decision problem are calculated and set to zero:

$$\frac{\partial E[\pi_{R_e}]}{\partial p_e} = \left(\alpha_e - 2(\beta_1+\gamma_1)p_e + \mu s + \gamma_1 p_t + w(\beta_1+\gamma_1)\right)\left(k_1\sqrt{a_e} + k_2\sqrt{a_t} + k_3\sqrt{A}\right) = 0 \quad (A.9)$$

$$\frac{\partial E[\pi_{R_e}]}{\partial a_e} = \frac{k_1}{2\sqrt{a_e}}(p_e - w)(\alpha_e - (\beta_1+\gamma_1)p_e + \mu s + \gamma_1 p_t) - 1 = 0 \quad (A.10)$$

From these equations, the online retailer's optimal policies are derived as follows:

$$p_e^* = \frac{1}{2(\beta_1+\gamma_1)}\left[\alpha_e + w(\beta_1+\gamma_1) + \gamma_1 p_t + \mu s\right] \quad (A.11)$$

$$a_e^* = \frac{{k_1}^2}{64\,(\beta_1+\gamma_1)}(\alpha_e - w(\beta_1+\gamma_1) + \gamma_1 p_t + \mu s)^4 \quad (A.12)$$

Since the online retailer has incomplete information of the traditional retailer's value added service cost, he has no complete knowledge about traditional retailer's service level $s$and price p_t. Thereby, similar to (Yao et al. (2008)), we assume that the online retailer finds the expected service level and retail price for the traditional retailer.

$$s=\int_{\bar{\eta}-\varepsilon}^{\bar{\eta}+\varepsilon}\frac{(\beta_2-\mu)}{\eta(\beta_1+\gamma_1)}f(\eta)d\eta=\frac{(\beta_2-\mu)\,ln\left(\frac{\bar{\eta}+\varepsilon}{\bar{\eta}-\varepsilon}\right)}{2\varepsilon(\beta_1+\gamma_1)} \tag{A.13}$$

$$\begin{aligned} p_t&=\int_{\bar{\eta}-\varepsilon}^{\bar{\eta}+\varepsilon}\frac{1}{4(\beta_1+\gamma_1)}\left[2\alpha_t+2w(\beta_1+\gamma_1)+2\gamma_1p_e+\frac{3(\beta_2-\mu)^2}{\eta(\beta_1+\gamma_1)}\right]f(\eta)d\eta\\ &=\frac{1}{4(\beta_1+\gamma_1)}\left[2\alpha_t+2w(\beta_1+\gamma_1)+2\gamma_1p_e+\frac{3(\beta_2-\mu)^2\,ln\left(\frac{\bar{\eta}+\varepsilon}{\bar{\eta}-\varepsilon}\right)}{2\varepsilon(\beta_1+\gamma_1)}\right] \end{aligned} \tag{A.14}$$

Equation (A.11) is substituted into Equations (A.6) and (A.7) and Equations (A.13) and (A.14) are substituted into Equations (A.11) and (A.12), and Equations (A.6), (A.7), (A.11) and (A.12) are solved simultaneously. Therefore, the traditional retailer's and the online retailer's optimal policies are as follows:

$$\begin{cases} s^*=\dfrac{(\beta_2-\mu)}{\eta(\beta_1+\gamma_1)}\\ p_t^*=B_t+\dfrac{(\beta_2-\mu)(3\beta_2(\beta_1+\gamma_1)-\mu(3\beta_1+2\gamma_1))}{\eta(\beta_1+\gamma_1)(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}\\ a_t^*=\dfrac{{k_1}^2}{1024\,\eta^4(\beta_1+\gamma_1)^6(1-\theta)^2}\left[2\eta(\beta_1+\gamma_1)(E_t+\dfrac{Fy_1}{4(\beta_1+\gamma_1)})+(\beta_2-\mu)^2\right]^4 \end{cases} \tag{A.15}$$

$$\begin{cases} p_e^*=B_e+\dfrac{(\beta_2-\mu)(3\beta_2y_1+\mu(4\beta_1+\gamma_1))}{4\varepsilon(\beta_1+\gamma_1)(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}ln\left(\dfrac{\bar{\eta}+\varepsilon}{\bar{\eta}-\varepsilon}\right)\\ a_e^*=\dfrac{{k_1}^2}{64\,(\beta_1+\gamma_1)^6}(E_e+\dfrac{F}{2})^4 \end{cases} \tag{A.16}$$

where B_t, B_e, E_t, E_e and F are defined as following

$$B_t=\frac{2\alpha_t(\beta_1+\gamma_1)+\alpha_e\gamma_1+w(\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}{(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}$$

$$B_e=\frac{2\alpha_e(\beta_1+\gamma_1)+\alpha_t\gamma_1+w(\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}{(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}$$

$$E_t=\frac{4\alpha_t(\beta_1+\gamma_1)^2+2\alpha_e\gamma_1(\beta_1+\gamma_1)-2w\beta_1(\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}{(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}$$

$$E_e=\frac{4\alpha_e(\beta_1+\gamma_1)^2+2\alpha_t\gamma_1(\beta_1+\gamma_1)-2w\beta_1(\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}{(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}$$

$$F=\frac{(\beta_2-\mu)(3\beta_2y_1+\mu(4\beta_1+\gamma_1))}{\varepsilon(2\beta_1+\gamma_1)(2\beta_1+3\gamma_1)}ln\left(\frac{\bar{\eta}+\varepsilon}{\bar{\eta}-\varepsilon}\right)$$

Appendix B

The profit function of the manufacturer is:

$$\underset{w}{Max}\,\pi_M=(d_t+d_e)(w-c)-\theta a_t-A \tag{B.1}$$

The first order partial derivatives of the manufacturer's decision problem are calculated and set to zero:

$$\frac{\partial\pi_M}{\partial w}=(d_t+d_e)+\left(\frac{\partial d_t}{\partial w}+\frac{\partial d_e}{\partial w}\right)(w-c)-\theta\frac{\partial a_t}{\partial w}=0 \tag{B.2}$$

$$\frac{\partial\pi_M}{\partial A}=\left(\frac{\partial d_t}{\partial A}+\frac{\partial d_e}{\partial A}\right)(w-c)-1=0 \tag{B.3}$$

where

$$\frac{\partial d_t}{\partial w} = \left(\gamma_1 \frac{\partial p_e}{\partial w} - (\beta_1+\gamma_1)\frac{\partial p_t}{\partial w}\right)\left(k_1\sqrt{a_t} + k_2\sqrt{a_e} + k_3\sqrt{A}\right) + \left(k_1\frac{\partial\sqrt{a_t}}{\partial w} + k_2\frac{\partial\sqrt{a_e}}{\partial w} + k_3\frac{\partial\sqrt{A}}{\partial w}\right)(\alpha_t - (\beta_1+\gamma_1)p_t + (\beta_2-\mu)s + \gamma_1 p_e)$$

$$\frac{\partial d_e}{\partial w} = \left(\gamma_1 \frac{\partial p_t}{\partial w} - (\beta_1+\gamma_1)\frac{\partial p_e}{\partial w}\right)\left(k_1\sqrt{a_e} + k_2\sqrt{a_e} + k_3\sqrt{A}\right) + \left(k_1\frac{\partial\sqrt{a_e}}{\partial w} + k_2\frac{\partial\sqrt{a_t}}{\partial w} + +k_3\frac{\partial\sqrt{A}}{\partial w}\right)(\alpha_e - (\beta_1+\gamma_1)p_e + \mu s + \gamma_1 p_e)$$

$$\frac{\partial d_t}{\partial A} = \frac{k_3}{2\sqrt{A}}(\alpha_t - (\beta_1+\gamma_1)p_t + (\beta_2-\mu)s + \gamma_1 p_e)$$

$$\frac{\partial d_t}{\partial A} = \frac{k_3}{2\sqrt{A}}(\alpha_t - (\beta_1+\gamma_1)p_t + (\beta_2-\mu)s + \gamma_1 p_e)$$

$$\frac{\partial a_t}{\partial w} = \frac{-{k_1}^2\beta_1}{64\,\eta^3(\beta_1+\gamma_1)^4(2\beta_1+\gamma_1)(1-\theta)^2}\left[2\eta(\beta_1+\gamma_1)(E_1 + \frac{Fy_1}{4(\beta_1+\gamma_1)}) + (\beta_2-\mu)^2\right]^3$$

where

$$\frac{\partial p_t}{\partial w} = \frac{(\beta_1+\gamma_1)}{(2\beta_1+\gamma_1)}$$

$$\frac{\partial p_e}{\partial w} = \frac{(\beta_1+\gamma_1)}{(2\beta_1+\gamma_1)}$$

$$\sqrt{a_t} = \frac{k_1}{32\,\eta^2(\beta_1+\gamma_1)^3(1-\theta)}\left[2\eta(\beta_1+\gamma_1)(E_1 + \frac{Fy_1}{4(\beta_1+\gamma_1)}) + (\beta_2-\mu)^2\right]^2$$

$$\frac{\partial\sqrt{a_t}}{\partial w} = \frac{-k_1\beta_1}{4\,\eta(\beta_1+\gamma_1)(2\beta_1+\gamma_1)(1-\theta)}\left[2\eta(\beta_1+\gamma_1)(E_1 + \frac{Fy_1}{4(\beta_1+\gamma_1)}) + (\beta_2-\mu)^2\right]$$

$$\sqrt{a_e} = \frac{k_1}{8\ (\beta_1+\gamma_1)^3}(E_2 + \frac{F}{2})^2$$

$$\frac{\partial\sqrt{a_e}}{\partial w} = \frac{-k_1\beta_1}{2\,\eta_2(\beta_1+\gamma_1)^2(2\beta_1+\gamma_1)}(E_2 + \frac{F}{2})$$

We set the first order conditions of the manufacturer's decision problem to zero and identify the optimal global advertising and wholesale price for the manufacturer. However, as previously mentioned, due to complexity of the resulting first order condition$\partial\pi_M/\partial A = 0$ and $\partial\pi_M/\partial w = 0$, we are not able to determine $A$and $w$analytically. Hence, we identify the optimal global advertising and wholesale price through illustrative example.