Journal of Industrial and Systems Engineering
Vol. 10, No. 2, pp 53-74
Spring (April) 2017

A game theoretic approach to pricing, advertising and collection decisions' adjustment in a closed-loop supply chain

E. Mohagheghian[1*], R. Sahraeian[1]

[1]*College of Engineering, Industrial Engineering Department, Shahed University, Tehran, Iran*

e.mohagheghb89@yahoo.com, sahraeian@shahed.ac.ir

Abstract

This paper considers advertising, collection and pricing decisions simultaneously for a closed-loop supply chain (CLSC) with one manufacturer (he) and two retailers (she). A multiplicatively separable new demand function is proposed which influenced by pricing and advertising. In this paper, three well-known scenarios in the game theory including the Nash, Stackelberg and Cooperative games are exploited to study the effects of pricing, advertising and collection decisions on the CLSC. Using these scenarios, we identify optimal decisions in each case for the manufacture and retailers. Extending the Manufacturer-Stackelberg scenario, we introduce the manufacturer's risk-averse behavior in a leader–follower type move under asymmetric information, focusing specifically on how the risk-averse behavior of the manufacturer influences all of the optimal decisions and construct manufacturer-Stackelberg games in which each retailer has more information regarding the market size than the manufacturer and another retailer. Under the mean–variance decision framework, we develop a closed-loop supply chain model and obtain the optimal equilibrium results. In the situation of the stackelberg game, we find that whether utility of the manufacturer is better off or worse off depends on the manufacturer's return rate and the degree of risk aversion under asymmetric and symmetric information structures. Numerical experiments compare the outcomes of decisions and profits among the mentioned games in order to study the application of the models.

Keywords: Closed-loop supply chain (CLSC), game theory, advertising, pricing, asymmetric information, risk-averse behavior

1- INTRODUCTION

The field of closed-loop supply chain is recently receiving growing attention in the literature and in practice. In the definition of CLSC it is possible to find a number of issues like pricing, advertising and competitiveness behavior, remanufacturing and quality, as well as concepts such as responsiveness, lean, agile, and the environmental consciousness (green approach).Studies regarding its effects on closed-loop supply chain management still remain sparse to our knowledge. This paper considers a CLSC through pricing, remanufacturing and advertising strategies; in addition due to the harmony of green products with environment, manufacture provides two types of products namely, green and non-green in order to response different customer needs.

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*Corresponding author.
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Thus, the supply chain may charge various prices for different products by retailers; also each product may have its advertising expenditure. The appropriate pricing helps the firms to gain an income.

The advertising can persuade customers to choose a good among many brands. Sometimes the manufacturer agrees to pay part of the retailer's local advertising costs in order to make more promotional initiatives aimed at increasing immediate sales. This type of advertising is called cooperative (co-op) advertising. A common approach adopted for investigating the role of advertising, pricing and remanufacturing models in the supply chain is the game theory.

In this paper, the manufacturer takes on the responsibility for collecting used products, i.e., he directly collects used products from the market but sometimes he cannot remanufactured successfully all that collected products. So Under this assumption, the manufacture may exhibit risk-averse behavior. On the other hand, although the manufacturer can obtain information regarding market size after along period of time in their whole sale market, the retailers have particular advantages in collecting the market data compared with manufacturers, such as interacting with customers directly and long-term experience in predicting market demands.

Thus, retailers can more accurately analyze the fluctuations and changes in market demand than manufacturers (Li, Gilbert, and Lai, 2013). Therefore, asymmetric information between members in the closed-loop supply chains appears.

In fact, in many situations, some information is only privet to one party of a closed-loop supply chain and the other party makes decisions with limited available information on the remanufacturing cost, the quantity, the market size, the collecting scale, and so on, of used products, and is unwilling to release the sensitive information to other members; thus information asymmetry commonly occurs.

Although remanufacturing has been extensively investigated in the literature, an important marketing tool that should be taken into account, i.e., advertising, has not been yet extensively studied in the CLSC models. Cooperative advertising has been widely considered since the 1970s in the forward supply chains. Among these studies, Berger, (1973) is the first paper to address a primary cooperative advertising model. Afterwards, (Chintagunta and Jain, 1992; Yue et al., 2006; Yan, 2010; Zhang and Liu, 2013) and many others extend Berger's work to different aspects.

Many researchers have also found that cooperative advertising can effectively coordinate forward supply chain (see, for example (Wang et al.,2011; Yue, 2006)). In contrast, Hong et al., (2015) expressed that cooperative advertising cannot coordinate the CLSC, but the CLSC can be coordinated by using a two-part tariff contract. They stated the terms of advertising expenditures through a linear integration.

There are studies on supply chains that assume agents' risk-averse behavior. Tsay, (2002) pointed out that various players should be allowed to have different attitudes towards risk sensitivity and found an informational motive to affect the use of return policies by a risk averse manufacturer and a risk-averse retailer in a supply chain. Recent empirical findings provide further support for the importance of incorporating risk preferences in business practices.

Motivated by these results, research on risk-averse models with different objective functions to reflect risk preferences has become an important stream, which greatly influence the decisions and revenues in supply chains. Expected utility, mean-variance, and VaR/CVaR are the three main research streams of modeling risk averseness in inventory problems. The framework of mean-variance introduced by Markowitz, (1959) is to address the trade-off between the expected return (mean) and the variation of return (variance).Choi, Li, and Yan, (2008) pointed out the CVaR criterion has been widely applied both in theoretical study and in practice due to its advantages.

To the best of our knowledge, the majority of the research results on forward supply chain management under the asymmetric information environment have been established. For instance, Lau (2005) assumed a model of a manufacturer and retailer in a supply chain as a non cooperative game with symmetric and asymmetric information in which the market demand is unknown to both manufacturer and retailer faced a price-sensitive demand. However, there are only a few studies that examine the asymmetric information structure in a closed-loop supply chain. According to the review, non-linear advertising expenditures are not yet considered in the CLSC models. In this paper, a form of non-linear advertising expenditures is particularly inspired by Wang et al., (2011), in which two-tier advertising is studied with a monopolistic manufacturer and two competing retailers in a supply chain. In addition, they all assumed that the channel members' risk preference is risk neutral, and none considered the impact of the members' risk preference on the optimal decisions in supply chains. Because of demand uncertainty, risk sensitivity plays an important role in the decisions of the supply chain members (Tsay, 2002). Differing from those of prior studies, the key points of this research is to consider pricing, advertising and collecting decisions at the same time, whitin a manufacturing and remanufacturing closed-loop supply chain with one risk-neutral manufacturer and two risk-neutral retailers. In this situation, we consider two game models called Nash and cooperative games. After that, we consider stackleberg games with a risk-averse manufacturer and two risk-neutral retailers under symmetric and asymmetric information environments.

The main contributions of this paper can be listed as follows: First, a multiplicatively separable new demand function is adopted by taking pricing, advertising and remanufacturing decisions into account. Second, the stackelberg game model is formulated and analyzed under symmetric and asymmetric information environments. Third, the information asymmetric structure is formulated such that each retailer is better informed about her market size and they have no reason to reveal this information to each other as well as the manufacture. Fourth, under the mean–variance decision framework, the impacts of retailer's risk-averse behavior in the model of Stackelberg are explored. In addition, we study how the retailer's optimal decisions influence the manufacturers expected utility and the retailer's expected profit. Fifth, the optimal decisions cannot be expressed in closed-form for the manufacturer and retailers under each kind of information structure, so we propose numerical approach. The total aim of the present study is to investigate the optimal decisions of channel members in a CLSC consist of one manufacturer and two retailers with one cooperative and two non-cooperative games, including the Stackelberg and the Nash games. So we can be one step closer to the better management and optimization of the channel. The rest of the paper is organized as follows. In the Section2, we briefly describe the basic problem and design the assumptions. Section 3 introduces algorithm to gain Nash, Manufacturer-Stackelberg and cooperative equilibriums. Section 4 describes numerical examples. Section 5 summarizes our main findings and concludes the paper by providing some directions for future research.

2- Problem Description

In this paper, two substitutable products are considered: the environmental (or green) and the traditional products, named product 1 and 2; respectively. Assume each product has two attributes, price (denoted as p and green degree (denoted asθ_l, θ_h; respectively) influencing consumer demand. Both price and green degree of the green product are greater than those of the traditional product (i.e.$p_1 > p_2$, $\theta_l > \theta_h$).

In our two-echelon closed-loop supply chain there is a bilateral monopoly between a single manufacturer and two retailers where the retailers carry out retail, and The manufacturer takes charge of both new manufacturing and remanufacturing as well as collection of each kind of product which are different in quality and sold in the different market, in other words, manufacturer supplies green product to retailer 1, and also supplies non-green product at a lower price to retailer 2, satisfying their demand. Figure 1 depicts the particular structure.

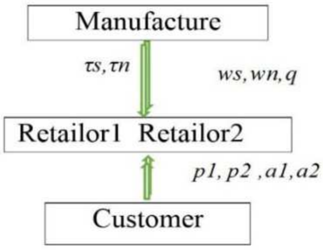

Figure1. Closed loop supply chain model

2-1- Model notations and assumptions

The following notations throughout the paper are shown in table 1:

Table1. Parameters and Decision variables

c	The unit cost of manufacturing each kind of product (weather green or not)
c_r	The unit cost of remanufacturing a returned product into a new one (weather green or not)
d	The retailer's unit cost incurred
θ_l	Green degree of the non-green product
θ_h	Green degree of the green product
g_1	The demand for the green product in the market
g_2	The demand for the non-green product in the market
S_i	Expected advertisings' function of retailer i(i = 1, 2)
v	The additional amount that consumers are willing to pay for each unit increased in the green degree of their desired product
δ	Manufacturer's effectiveness of national advertising
y	Retailer- j 's effectiveness of local advertising
γ	Retailer- i 's effectiveness of local advertising
η	Advertising sensitivity
A_i	The maximal potential demand of retailer$i$(i = 1, 2)
π_i	Retailer's Profit function
π_m	Manufacturer's profit function
π_s	Supply chain's profit function
m, r	Denotes the manufacturer and the retailer; respectively
w_s	(Manufacturer Decision variable)The unit wholesale price of a green product
w_n	(Manufacturer Decision variable)The unit wholesale price of a non-green product
τ_s	(Manufacturer Decision variable)The collection rate of a green product
τ_n	(Manufacturer Decision variable)The collection rate of a non-green product
p_1	(Retailer *1* Decision variable)Retailer's selling price of the green product
p_2	(Retailer *2* Decision variable)Retailer's selling price of the non-green product
a_i	(Retailer i Decision variable)The local advertising expenditures invested by retailer i (i = 1, 2)
q	(Manufacturer Decision variable)The national advertising investment of the manufacturer
'*'	Superscript '*' denotes optimal value.

Following assumptions are considered for the proposed model of this paper:

Assumption1:$c_r < c$ producing a new product by using a used product of each kind is less costly than manufacturing a new one, thus we denote unit cost saving from reuse by Δ, i.e$\Delta = c - c_r$.

Assumption 2: The closed-loop supply chain decision plays as Stackelberg game in a single-period setting, and the manufacturer is the leader. Similar assumptions have been widely used in supply chain literatures (Savaskan, Bhattacharya, and VanWassenhove, 2004; Tayur, Ganeshan, and Magazine 2012).

Assumption 3: There is no differentiation between new product and remanufacturing product.

Assumption 4: Similar to (Zhang and Liu, 2013), we assume the two type of the green degree (those are $\theta_l, \theta_h$and$\theta_l > \theta_h$). Further, the green degree of the product it wants to buy is random variable ε, which assumed $\varepsilon \sim U[\theta_l, \theta_u]$ for the green product with higher cost, taking the consumers' preference for product green degree into consideration, the unit acceptance level of the consumers for purchasing the product can be denoted by

$$A = p_2 + v(\varepsilon - \theta_l) - p_1 \tag{1}$$

Obviously, only when consumers' acceptance level satisfies $A>0$, shall they purchase green product. Or else they purchase non-green. That is to say, when
$\varepsilon > \frac{p_1 - p_2}{v} + \theta_l$, the market demand for green products is

$$g_1 = \int_{\frac{p_1 - p_2}{v} + \theta_l}^{\theta_h} \frac{1}{\theta_h - \theta_l} d\varepsilon = \frac{v(\theta_h - \theta_l) - (p_1 - p_2)}{v(\theta_h - \theta_l)} \tag{2}$$

And also when $\varepsilon < \frac{p_1 - p_2}{v} + \theta_l$; the market demand for green products is

$$g_2 = \left(\int_{\theta_l}^{\theta_h} \frac{1}{\theta_h - \theta_l} d\varepsilon + \int_{\theta_l}^{\frac{p_1 - p_2}{v} + \theta_l} \frac{1}{\theta_h - \theta_l} d\varepsilon \right) = \frac{v(\theta_h - \theta_l) - (p_2 - p_1)}{v(\theta_h - \theta_l)} \tag{3}$$

Therefore, different to our references and according to our problem description, we assume two terminal consumer groups in the market whose same market capacity 1 and the demand functions related to our mentioned attributes for products 1 and 2, denoted by g_1 andg_2.

Assumption5:the recycling process incurs a total collection cost, which is characterized as a function of the collection rate τ and is given by $(c(\tau) = k\tau^2)$, where$k$is a scaling parameter(Savaskan, Bhattacharya, and Van Wassenhove, 2004).

Assumption6: In order to show the impact of advertising investments on the product sales, we consider advertising function S_i of retailer-$i$as another part of demand function which the manufacturer's national investment and the retailer-i's local advertising both have positive effects on the product sales of retailer-i. However, due to the competitive relationship between two retailers, retailer j's local advertising effort will have a negative impact on the demand retailer-$i$faces (i.e. sales volume; i-j). This kind of advertisement is referred to a paper presented by(Wang et al., 2011). Therefore, we assume the expected advertisings function $S_i$of retailer-i to be determined by

$$S(a_i, a_j, q) = A_i - \eta a_i^{-\gamma} a_j^{y} q^{-\delta}, i = 1,2 \quad j = 3 - i \tag{4}$$

Where A_i, η, y, γ and δ are positive constants. Similar to their model,A_i is the maximal potential demand faced by retailer-i and $y(\gamma)$ denotes the measure of sensitivity of retailer-i's sales related to changes of retailer-i's (retailer-j's) local advertising expenditures. δ denotes the measure of sensitivity of manufacturer's sales related to changes of manufacturer's national advertising expenditure. They also assumed that$y + \gamma > 1$and $y < \gamma$.The former is required to assure the existence of the equilibrium solution. The latter means that the demand each retailer faces is more sensitive to his own local advertising efforts than to his rival's. Otherwise, no one would be interested in spending money on the local advertising.

2-2-The retailers' model formulation

This paper uses a multiplicative non-linear form of demand function including both functions related to retail prices (g_i), and a function related to expected advertising expenditures (S_i). So demand's Equation is as follows (Wang 2011; Zhang and Liu 2013).

$$D_i(p_i, p_j, a_i, a_j, q) = S_i(a_i, a_j, q).\, g_i(p_i, p_j) \quad i = 1,2\,, j = 3 - i \tag{5}$$

The demand of each product at retailer-*i*, is a joint non-linear function of the retail prices and advertising expenditures in real world. Each retailer's main objective is to maximize his net profit by optimizing his decision variables including retail price, and advertising expenditure. These set of decision variables are known strategyX_{r_i}. Thus, the retailer-*i*'s net profit can be calculated as the total sales revenue of its product minus the purchasing cost from the manufacturer, advertising cost for each product, given as follows.

$$\pi_{r_2} = (p_2 - w_n - d) \times \left(\frac{v(\theta_h - \theta_l) - (p_2 - p_1)}{v(\theta_h - \theta_l)}\right) \times \left(A_2 - \eta {a_1}^{y} {a_2}^{-\gamma} q^{-\delta}\right) - a_2 \tag{6}$$

$$\pi_{r_1} = (p_1 - w_s - d) \times \left(\frac{v(\theta_h - \theta_l) - (p_1 - p_2)}{v(\theta_h - \theta_l)}\right) \times \left(A_1 - \eta {a_1}^{-\gamma} {a_2}^{y} q^{-\delta}\right) - a_1 \tag{7}$$

In order to avoid negativity of demand functions, the following conditions should be met:

$$D_i(a_i, a_j, p_i, p_j, q) > 0 \rightarrow A_i - \eta\, {a_i}^{-\gamma} {a_j}^{y} q^{-\delta} > 0, (p_i - p_j) < v(\theta_h - \theta_l); \tag{8}$$

$$(p_i - p_j) < v(\theta_h - \theta_l) \rightarrow \frac{p_i - p_j}{v(\theta_h - \theta_l)} - \frac{c + d}{v(\theta_h - \theta_l)} < 1 - \frac{c + d}{v(\theta_h - \theta_l)} \rightarrow \frac{\frac{p_i - p_j}{v(\theta_h - \theta_l)} - \frac{c+d}{v(\theta_h - \theta_l)}}{1 - \frac{c+d}{v(\theta_h - \theta_l)}} < 1 \rightarrow p_i' - p_j' < 1\,, i = 1,2, j = 3 - i \tag{9}$$

In order to avoid negativity of profit function, the following conditions should be met:

$$\pi_{r_1} > 0 \rightarrow p_1 > w_s + d; \tag{10}$$

$$\pi_{r_2} > 0 \rightarrow p_2 > w_n + d; \tag{11}$$

$$\pi_m > 0 \rightarrow w_s > c + \Delta.\tau_s\,; \;\; w_n > c + \Delta.\tau_n; \tag{12}$$

Following Xie and Neyret (2009), an appropriate change of variables will be employed to handle the problem in an equivalent but more convenient way shown in Table 2. According to Table 2, we will express the manufacturers and the retailers' profits as follows:

$$\pi_m'(w_s', w_n', \tau_s', \tau_n', q) = \\ = (w_s' + \tau_s')(1 - p_1' + p_2')\left(\frac{A_1'}{\eta^{\frac{1}{\gamma+\delta-y+1}}} - {a_1'}^{-\gamma} {a_2'}^{y} {q'}^{-\delta}\right) + (w_n' + \\ - p_2')\left(\frac{A_2'}{\eta^{\frac{1}{\gamma+\delta-y+1}}} - {a_2'}^{-\gamma} {a_1'}^{y} {q'}^{-\delta}\right) - q' - \frac{k'}{\eta'^{\frac{1}{\gamma+\delta-y+1}}}({\tau_s'}^2 + {\tau_n'}^2)$$

$$\pi_{r_1}'(p_1', a_1') = (p_1' - w_s')(1 - p_1' + p_2'')\left(\frac{A_1'}{\eta^{\frac{1}{\gamma+\delta-y+1}}} - a_1'^{-\gamma} a_2'^{y} q'^{-\delta}\right) - a_1'$$

$$\pi_{r_2}'(p_2', a_2') = (p_2' - w_n')(1 - p_2' + p_1')\left(\frac{A_2'}{\eta^{\frac{1}{\gamma+\delta-y+1}}} - a_2'^{-\gamma} a_1'^{y} q'^{-\delta}\right) - a_2'$$

Henceforth, the superscript ($'$) will be eliminated for the sake of simplicity.

Table2. Change of variables

$p_1' = \frac{1}{\nu(\theta_h - \theta_l)}(p_1 - \mathrm{c} - \mathrm{d})$	$p_2' = \frac{1}{\nu(\theta_h - \theta_l)}(p_2 - \mathrm{c} - \mathrm{d})$	$w_s' = \frac{1}{\nu(\theta_h - \theta_l)}(w_s - \mathrm{c})$
$w_n' = \frac{1}{\nu(\theta_h - \theta_l)}(w_n - \mathrm{c})$	$\eta' = \frac{1}{\nu(\theta_h - \theta_l)}(\eta)$	$A' = \frac{1}{\nu(\theta_h - \theta_l)}(A)$
$\tau_s' = \frac{1}{\nu(\theta_h - \theta_l)}\Delta\tau_s$	$q = q'\eta'^{\frac{1}{\gamma+\delta-y+1}}$	$a_1 = a_1'\eta'^{\frac{1}{\gamma+\delta-y+1}}$
$a_2 = a_2'\eta'^{\frac{1}{\gamma+\delta-y+1}}$	$k = \frac{(\nu(\theta_h - \theta_l))^2}{\Delta^2}k'$	$\pi' = \pi.\eta'^{\frac{1}{\gamma+\delta-y+1}}$

3- The game structures of the channel members

In this section, four game-theoretic models based on a cooperative, Nash with one risk-neutral manufacturer and two risk- neutral retailers and a Stackelberg-Manufacturer with one risk-averse manufacturer and two risk-neutral retailers under symmetric and asymmetric information is discussed. Because of models difficulty parametric solution could not obtain, so we introduce algorithms to each game structure.

3-1- Nash equilibrium

Consider a situation in which the retailers as a bottom-level compete simultaneously with each other as well as with manufacturer. In other words, in our game framework we face with horizontal and vertical competition in bottom-level. Retailers simultaneously and non-cooperatively compete with manufacturer and have the same decision power. We employ Nash equilibrium concept to analyze the whole model (all players, i.e. manufacturer and retailers).

In game theory, Nash equilibrium is one of the most famous non-cooperative solution concepts (Basar and Olsder, 1999). To achieve Nash equilibrium point, each player determine his own decisions according to the other player's decisions as given input parameters and adapt his decision corresponding to the changing of the other player's strategies to maximize his objective function. This process proceeds until any players does not desire to change his strategy unilaterally, because any one-sided changing lead to loss to him and then Nash equilibrium point is obtained. This kind of competition mode is to see in the real world, for example, Standalone supply chains, where each entity makes its decisions so as to maximize its own profits according to its own objectives. Definitely, the manufacturer's decision problem is:

$$\pi_m(w_s, w_n, \tau_s, \tau_n, q) =$$
$$= (w_s + \tau_s)(1 - p_1 + p_2)(\frac{A_1}{\eta^{\frac{1}{\Upsilon+\delta-y+1}}} - a_1^{-\gamma} a_2^{y} q^{-\delta}) + (w_n + \tau_n)(1 + p_1 - p$$
$$- a_2^{-\gamma} a_1^{y} q^{-\delta}) - q - \frac{k}{\eta^{\frac{1}{\Upsilon+\delta-y+1}}}(\tau_s^2 + \tau_n^2) \quad (13)$$

St:

$q, w_s, w_n \geq 0, \quad 0 \leq \tau_s, \tau_n \leq 1$

and the retailers's decision problem is:

$$\pi_{r_1}(p_1, a_1) = (p_1 - w_s)(1 - p_1 + p_2)\left(\frac{A_1}{\eta^{\frac{1}{\Upsilon+\delta-y+1}}} - a_1^{-\gamma} a_2^{y} q^{-\delta}\right) - a_1 \quad (14)$$

$$\pi_{r_2}(p_2, a_2) = (p_2 - w_n)(1 - p_2 + p_1)\left(\frac{A_2}{\eta^{\frac{1}{\Upsilon+\delta-y+1}}} - a_2^{-\gamma} a_1^{y} q^{-\delta}\right) - a_2 \quad (15)$$

We solve manufacturer and retailers' decision problems separately in order to determine the Nash equilibrium. But unfortunately, we cannot solve this model parametrically, so we introduced a repetitive algorithm. In addition, $\pi_m$is increasing in line withw_s, w_n, which means the optimal value for$w_s$and$w_n$isp_1, p_2;respectively. However,$p_1 > w_s$and$p_2 > w_n$; w_s, w_ncannot be equal top_1, p_2; respectively, or in other words there would be no profit for both sides. We use a similar approach as previously suggested in Xie and Neyret (2009) to handle the problem; we assume that the retailer will not sell the product if she does not obtain a minimum unit margin. We take manufacturer's unit margin related to each kind of product (green or non-green) as such minimum level and replace the wholesale price constraint with:

$$\mu_{r_1} > \mu_m \rightarrow p_1 - w_s > w_s \rightarrow w_s \leq \frac{p_1}{2} \quad (16)$$

$$\mu_{r_2} > \mu_m \rightarrow p_2 - w_n > w_n \rightarrow w_n \leq \frac{p_2}{2} \quad (17)$$

Where $\mu_{r_1} = p_1 - w_s$, $\mu_{r_2} = p_2 - w_n$and $\mu_m$is equal tow_s, for green product andequal tow_n, for other kind of product and are retailer's and manufacturer's unit margins, respectively. Thus, the optimal values ofw_s , $w_n$are$\frac{p_1}{2}$,$\frac{p_2}{2}$; respectively.

The first-order conditions for the manufacturer and the retailers are as following:

$$\frac{\partial \pi_m}{\partial ws} = \left((1 - p_1 + p_2)\left(-a_1^{-\gamma} a_2^{y} q^{-\delta} + A_1 \eta^{\frac{1}{\Upsilon+\delta-y+1}}\right)\right) > 0 \rightarrow w_s^* = \frac{p_1}{2} \quad (18)$$

$$\frac{\partial \pi_m}{\partial wn} = \left((1 + p_1 - p_2)\left(-a_1^{y} a_2^{-\gamma} q^{-\delta} + A_2 \eta^{\frac{1}{\Upsilon+\delta-y+1}}\right)\right) > 0 \rightarrow w_n^* = \frac{p_2}{2} \quad (19)$$

$$\frac{\partial \pi_m}{\partial \tau_n} = (1 + p_1 - p_2)\left(-a_1^{y} a_2^{-\gamma} q^{-\delta} + A_2 \eta^{\frac{1}{\Upsilon+\delta-y+1}}\right) - 2k\eta^{\frac{1}{\Upsilon+\delta-y+1}}\tau_n = 0 \quad (20)$$

$$\frac{\partial \pi_m}{\partial \tau_s} = (1 - p_1 + p_2)\left(-a_1{}^{-\gamma} a_2{}^{y} q^{-\delta} + A_1 \eta^{\frac{1}{\Upsilon + \delta - y + 1}}\right) - 2\, k\eta^{\frac{1}{\Upsilon + \delta - y + 1}} \tau_s = 0 \tag{21}$$

$$\frac{\partial \pi_{r_1}}{\partial p_1} = (1 - p_1 + p_2)\left(-a_1{}^{-\gamma} a_2{}^{y} q^{-\delta} + A_1 \eta^{-\frac{1}{1 - y + \delta + \Upsilon}}\right) - (p_1 - w_s)\left(-a_1{}^{-\gamma} a_2{}^{y} q^{-\delta} + A \right. \tag{22}$$

$$\frac{\partial \pi_{r_1}}{\partial a_1} = -1 + a_1{}^{-1-\gamma} a_2{}^{y} q^{-\delta} (1 - p_1 + p_2) q^{-\delta} (p_1 - w_s)\gamma = 0 \tag{23}$$

$$\frac{\partial \pi_{r_2}}{\partial p_2} = (1 + p_1 - p_2)\left(-a_1{}^{y} a_2{}^{-\gamma} q^{-\delta} + A_2 \eta^{-\frac{1}{1 - y + \delta + \Upsilon}}\right) - (p_2 - w_n)\left(-a_1{}^{y} a_2{}^{-\gamma} q^{-\delta} + A_2 \eta^{-\frac{1}{1 - y + \delta}}\right. \\ = 0 \tag{24}$$

$$\frac{\partial \pi_{r_2}}{\partial a_2} = -1 + a_1{}^{y} a_2{}^{-\gamma - 1} (1 + p_1 - p_2) q^{-\delta} (p_2 - w_n)\gamma = 0 \tag{25}$$

Therefore, by equating the first-order partial derivative of the player's profits to zero, regarding to the relevant decision variables, and also by solving all the derived equations simultaneously, one can get the following results from the Nash equilibrium.

$$w_n{}^* = \frac{p_2}{2} w_s{}^* = \frac{p_1}{2} \tag{26}$$

$$q = \left(a_1{}^{-\gamma} a_2{}^{-\gamma} \delta\left(a_1{}^{y+\gamma}(1 + p_1 - p_2)(w_n + \tau_n) - a_2{}^{y+\gamma}(-1 + p_1 - p_2)(w_s + \tau_s)\right)\right)^{\frac{1}{1+\delta}} \tag{27}$$

$$\tau_s = -\frac{a_1{}^{-\gamma}(-1 + p_1 - p_2) q^{-\delta}\left(a_1{}^{\gamma} A_1 q^{\delta} - a_2{}^{y} \eta^{-\frac{1}{1 - y + \delta + \Upsilon}}\right)}{2k} \tag{28}$$

$$\tau_n = \frac{a_2{}^{-\gamma}(1 + p_1 - p_2) q^{-\delta}\left(a_2{}^{\gamma} A_2 q^{\delta} - a_1{}^{y} \eta^{\frac{1}{1 - y + \delta + \Upsilon}}\right)}{2k} \tag{29}$$

$$p_1 = \frac{1}{2}(1 + p_2 + w_s) \tag{30}$$

$$a_2 = \left(a_1(1 + p_1 - p_2) q^{-\delta} (p_2 - w_n)\gamma\right)^{\frac{1}{1+\gamma}} \tag{31}$$

$$a_1 = \left(-a_2{}^{y}(-1 + p_1 - p_2) q^{-\delta} (p_1 - w_s)\gamma\right)^{\frac{1}{1+\gamma}} \tag{32}$$

$$p_2 = \frac{1}{2}(1 + p_1 + w_n) \tag{33}$$

The equations (26-33) was failed to analytically solve. For calculating the Nash equilibrium of two-echelon closed-loop supply chain numerically, we present the following solution algorithm based on Gauss-Seidel decomposition presented by (Facchinei and Kanzow, 2007), where $X$is denoted as the strategy set of the supply chain member. Thus X_m and $X_{r_i}$are the strategy profile sets of the manufacturer and retailer *I* strategies; respectively. A measure for the completion of algorithm is introduced, if $|X_i^* - X_i^0|$is lower than ε, algorithm is accomplished and available solution is close enough to equations solution. We give the following repetitive algorithm for solving the Nash game model:

Step 0: Give the initial strategy profile for the manufacturer and retailers

$X_r^0 = ({p_1}^0, {a_1}^0, {p_2}^0, {a_2}^0)$ in the strategy profile set *X* such that

$X_{r1}^0 = ({p_1}^0, {a_1}^0)$, $X_{r2}^0 = ({p_2}^0, {a_2}^0)$ are the initial strategy profile for retailer1 and retailer2; respectively.

Step 1: For the manufacturer find the optimal strategy profile set

$X_m^* = ({w_s}^*, {w_n}^*, {\tau_n}^*, {\tau_s}^*, q^*)$based on$X_{r1}^0$, X_{r2}^0.

Step 2: For the retailer1 based on (X_{r2}^0, X_m^*) the optimal reaction is(X_{r1}^*), in the strategy profile set X.

Step 3: For the retailer2 based on (X_{r1}^*, X_m^*) the optimal reaction is (X_{r2}^*), in the strategy profile set X.

Step 4: For the whole supply chain, find out; respectively. If,
$|X_{r1}^* - X_{r1}^0| < \varepsilon, |X_{r2}^* - X_{r2}^0| < \varepsilon, |X_m^* - X_m^0| < \varepsilon.$ Equilibrium is obtained. Output the optimal results and stop. Else $X^0 = X$ and go to step 0. (ε is very small positive number).

Proposition1. To proof the optimality of these solutions, we calculate the Hessian matrix

$$H_{N_m} = \begin{bmatrix} \frac{\partial^2 \pi_m}{\partial {w_s}^2} & \frac{\partial^2 \pi_m}{\partial w_n \partial w_s} & \frac{\partial^2 \pi_m}{\partial \tau_s \partial w_s} & \frac{\partial^2 \pi_m}{\partial \tau_n \partial w_s} & \frac{\partial^2 \pi_m}{\partial q \partial w_s} \\ \frac{\partial^2 \pi_m}{\partial w_s \partial w_n} & \frac{\partial^2 \pi_m}{\partial {w_n}^2} & \frac{\partial^2 \pi_m}{\partial \tau_s \partial w_n} & \frac{\partial^2 \pi_m}{\partial \tau n \partial w_n} & \frac{\partial^2 \pi_m}{\partial q \partial wn} \\ \frac{\partial^2 \pi_m}{\partial w_s \partial \tau s} & \frac{\partial^2 \pi_m}{\partial w_n \partial \tau_s} & \frac{\partial^2 \pi_m}{\partial {\tau_s}^2} & \frac{\partial^2 \pi_m}{\partial \tau_n \partial \tau_s} & \frac{\partial^2 \pi_m}{\partial q \partial \tau_s} \\ \frac{\partial^2 \pi_m}{\partial w_s \partial \tau_n} & \frac{\partial^2 \pi_m}{\partial w_n \partial \tau_n} & \frac{\partial^2 \pi_m}{\partial \tau_s \partial \tau_n} & \frac{\partial^2 \pi_m}{\partial {\tau_n}^2} & \frac{\partial^2 \pi_m}{\partial q \partial \tau_n} \\ \frac{\partial^2 \pi_m}{\partial w_s \partial q} & \frac{\partial^2 \pi_m}{\partial w_n \partial q} & \frac{\partial^2 \pi_m}{\partial \tau_s \partial q} & \frac{\partial^2 \pi_m}{\partial \tau_n \partial q} & \frac{\partial^2 \pi_m}{\partial q^2} \end{bmatrix}$$

Proof 1. The second order partial derivatives are as follows:

$$\frac{\partial^2 \pi_m}{\partial {w_s}^2} = \frac{\partial^2 \pi_m}{\partial w_n \partial w_s} = \frac{\partial^2 \pi_m}{\partial w_s \partial w_n} = \frac{\partial^2 \pi_m}{\partial \tau_s \partial w_s} = \frac{\partial^2 \pi_m}{\partial w_s \partial \tau s} = \frac{\partial^2 \pi_m}{\partial \tau_n \partial w_s} =$$

$$\frac{\partial^2 \pi_m}{\partial w_s \partial \tau_n} = \frac{\partial^2 \pi_m}{\partial wn^2} = \frac{\partial^2 \pi_m}{\partial \tau_s \partial w_n} = \frac{\partial^2 \pi_m}{\partial w_n \partial \tau_s} = \frac{\partial^2 \pi_m}{\partial \tau_n \partial w_n} = \frac{\partial^2 \pi_m}{\partial w_n \partial \tau_n} = \frac{\partial^2 \pi_m}{\partial \tau_n \partial \tau_s} = \frac{\partial^2 \pi_m}{\partial \tau_s \partial \tau_n} = 0$$

$$\frac{\partial^2 \pi_m}{\partial q \partial w_s} = \frac{\partial^2 \pi_m}{\partial w_s \partial q} = a1^{-\gamma} a2^{y} (1 - p_1 + p_2) q^{-1-\delta} \delta$$

$$\frac{\partial^2 \pi_m}{\partial q \partial \tau_n} = \frac{\partial^2 \pi_m}{\partial \tau_n \partial q} = \frac{\partial^2 \pi_m}{\partial q \partial w_n} = \frac{\partial^2 \pi_m}{\partial w_n \partial q} = a1^{y} a2^{-\gamma} (1 + p_1 - p_2) q^{-1-\delta} \delta$$

$$\frac{\partial^2 \pi_m}{\partial q^2} = a1^{y} a2^{-\gamma} (1 + p_1 - p_2) q^{-2-\delta} (-1 - \delta) \delta (w_n + \tau_n) + a1^{-\gamma} a2^{y} (1 - p1 + p2) q^{-2-\delta} (-1 - \delta) \delta (ws + \tau_s)$$

$$\frac{\partial^2 \pi_m}{\partial q \partial \tau_n} = \frac{\partial^2 \pi_m}{\partial \tau_n \partial q} = a1^{y} a2^{-\gamma} (1 + p_1 - p_2) q^{-1-\delta} \delta$$

$$\frac{\partial^2 \pi_m}{\partial {\tau_s}^2} = \frac{\partial^2 \pi_m}{\partial {\tau_n}^2} = \frac{-2k}{\eta^{\frac{1}{\gamma + \delta - y + 1}}}$$

$$\frac{\partial^2 \pi_m}{\partial q \partial \tau_s} = \frac{\partial^2 \pi_m}{\partial \tau_s \partial q} = {a_1}^{-\gamma} {a_2}^{y} (1 - p_1 + p_2) q^{-1-\delta} \delta$$

Due to the complexness of the expression stated above, we are not able to prove analytically. Instead of that, we computed one numerical study with randomly generated sets of parameters and then the Hessian matrix stated above is checked for that. If it is not satisfied, we will further check the instance with the boundary conditions in order to tackle this problem. According to the assumption Hessian stated above is satisfied with our instance, hence proof is completed. To proof the optimality of the first retailer solutions, we have to calculate the below Hessian matrix:

$$H_{N_{r1}} = \begin{bmatrix} \frac{\partial^2 \pi_{r_1}}{\partial p_1} & \frac{\partial^2 \pi_{r_1}}{\partial a_1 \partial p_1} \\ \frac{\partial^2 \pi_{r_1}}{\partial p_1 \partial a_1} & \frac{\partial^2 \pi_{r_1}}{\partial {a_1}^2} \end{bmatrix}$$

The second order partial derivatives are as follows:

$$\frac{\partial^2 \pi_{r_1}}{\partial {p_1}^2} = -2(-{a_1}^{-\gamma} {a_2}^{y} q^{-\delta} + A_1 \eta^{-\frac{1}{1-y+\delta+\Upsilon}})$$

$$\frac{\partial^2 \pi_{r_1}}{\partial {a_1}^2} = {a_1}^{-2-\gamma} {a_2}^{y} (1 - p_1 + p_2) q^{-\delta} (p_1 - w_s)(-1-\gamma)\gamma$$

$$\frac{\partial^2 \pi_{r_1}}{\partial p_1 \partial a_1} = {a_1}^{-1-\gamma} {a_2}^{y} (1 - p_1 + p_2) q^{-\delta} \gamma - {a_1}^{-1-\gamma} {a_2}^{y} q^{-\delta} (p_1 - w_s)\gamma$$

The first principle minor of $H^1_{N_{r1}}$is$H^1_{N_{r1}} = -2\left(-{a_1}^{-\gamma} {a_2}^{y} q^{-\delta} + A_1 \eta^{-\frac{1}{1-y+\delta+\Upsilon}}\right)$which is always negative. The second principle minor of $H^2_{N_{r1}}$is as follows:

$$H^2_{N_{r1}} = {a_1}^{-2(1+\gamma)} {a_2}^{y} q^{-2\delta} \gamma(-{a_2}^{y}(1 - 2p_1 + p_2 + w_s)^2 \gamma + 2(-1 + p_1 - p_2)(p_1 - w_s)(1+\gamma)\eta^{-\frac{1}{1-y+\delta+\Upsilon}}(-{a_1}^{\gamma} A_1 q^{\delta} + {a_2}^{y} \eta^{\frac{1}{1-y+\delta+\Upsilon}}))$$

This is always positive. Therefore, we are able to prove the optimality of our solutions analytically. The same result for another retailer can be easily derived, so we omit it here.

3-2- Cooperative game

In this subsection, we focus on a cooperative game structure in which the manufacturer and the duopolistic retailers cooperate as integrated channel and agree to make decisions together that maximize the joint total profit which is the sum of the manufacturer's profit and the retailers' profit. Cooperative structures prove to be more beneficial. Thus, in the real world many companies are fundamentally changing their way of doing business by exceeding individual actions toward collective actions and cooperative strategies. A very recent survey on applications of cooperative game theory to supply chain management, the so called supply chain collaboration, is Meca and Timmer (2008).For theoretical issues and a framework for more general supply chain networks we refer to the book by Slikker and van den Nouweland (2001).

Hence, the optimization problem under this model can be written as:

$$\pi_s = (p_1 + \tau_s)(1 - p_1 + p_2)\left(\frac{A_1}{\eta^{\frac{1}{\Upsilon+\delta-y+1}}} - {a_1}^{-\gamma} {a_2}^{y} q^{-\delta}\right) + (p_2 + \tau_n)(1 + p_1 - p_2)\left(\frac{A_2}{\eta^{\frac{1}{\Upsilon+\delta-y+1}}} - {a_2}^{-\gamma} {a_1}^{y} - a_1 - \frac{k}{\eta^{\frac{1}{\Upsilon+\delta-y+1}}}({\tau_s}^2 + {\tau_n}^2)\right. \quad (34)$$

$$0 < p_1, p_2 < 1, \quad 0 < \tau_n < 1, \ 0 < \tau_s < 1, \qquad q > 0$$

By solving the first order condition of$\pi_s$with respect to $p_1, a_1, p_2, a_2, \tau_n, \tau_s, q$ one has:

$\frac{\partial \pi_s}{\partial p_1} = (1 - p_1 + p_2)\left(-a_1{}^{-\gamma} a_2{}^{y} q^{-\delta} + A_1 \eta^{-\frac{1}{1-y+\delta+Y}}\right) + \left(-a_1{}^{y} a_2{}^{-\gamma} q^{-\delta} + A_2 \eta^{-\frac{1}{1-y+\delta+Y}}\right)(p_2 + \tau_n)$ -
$\left(-a_1{}^{-\gamma} a_2{}^{y} q^{-\delta} + A_1 \eta^{-\frac{1}{1-y+\delta+Y}}\right)(p_1 + \tau_s) = 0$ (35)

$\frac{\partial \pi_s}{\partial p_2} =$
$(1 + p_1 - p_2)\left(-a_1{}^{y} a_2{}^{-\gamma} q^{-\delta} + A_2 \eta^{-\frac{1}{1-y+\delta+Y}}\right) - \left(-a_1{}^{y} a_2{}^{-\gamma} q^{-\delta} + A_2 \eta^{-\frac{1}{1-y+\delta+Y}}\right)(p_2 + \tau_n) + \left(-a_1{}^{-\gamma} a_2{}^{y} q^{-\delta}\right.$
$\left. A_1 \eta^{-\frac{1}{1-y+\delta+Y}}\right)(p_1 + \tau_s) = 0$ (36)

$$\frac{\partial \pi_s}{\partial a_1} = -1 - a_1{}^{-1+y} a_2{}^{-\gamma}(1 + p_1 - p_2) q^{-\delta} y (p_2 + \tau_n) + a_1{}^{-1-\gamma} a_2{}^{y}(1 - p_1 + p_2) q^{-\delta} \gamma (p_1 + \tau_s) = 0 \quad (37)$$

$$\frac{\partial \pi_S}{\partial a_2} = -1 + a1^{y} a2^{-1-\gamma}(1 + p_1 - p_2) q^{-\delta} \gamma (p_2 + \tau_n) - a1^{-\gamma} a_2{}^{-1+y}(1 - p_1 + p_2) q^{-\delta} y (p1 + \tau_s) = 0 \quad (38)$$

$$\frac{\partial \pi_s}{\partial q} = -1 + a_1{}^{y} a_2{}^{-\gamma}(1 + p_1 - p_2) q^{-1-\delta} \delta (p2 + \tau_n) + a1^{-\gamma} a_2{}^{y}(1 - p_1 + p_2) q^{-1-\delta} \delta (p_1 + \tau_s) = 0 \quad (39)$$

$$\frac{\partial \pi_s}{\partial \tau_n} = (1 + p_1 - p_2)\left(-a_1{}^{y} a_2{}^{-\gamma} q^{-\delta} + A_2 \eta^{-\frac{1}{1-y+\delta+Y}}\right) - 2k\eta^{-\frac{1}{1-y+\delta+Y}} \tau_n = 0 \quad (40)$$

$$\frac{\partial \pi_S}{\partial \tau_s} = (1 - p_1 + p_2)\left(-a_1{}^{-\gamma} a_2{}^{y} q^{-\delta} + A_1 \eta^{-\frac{1}{1-y+\delta+Y}}\right) - 2k\eta^{-\frac{1}{1-y+\delta+Y}} \tau_s = 0 \quad (41)$$

Proposition2. To proof the optimality of the Cooperative solutions, we have to calculate the Hessian matrix.

$$H_C = \begin{bmatrix} \frac{\partial^2 \pi_S}{\partial p_1{}^2} & \frac{\partial^2 \pi_S}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_S}{\partial a_1 \partial p_1} & \frac{\partial^2 \pi_S}{\partial a_2 \partial p_1} & \frac{\partial^2 \pi_S}{\partial \tau_s \partial p_1} & \frac{\partial^2 \pi_S}{\partial \tau_n \partial p_1} & \frac{\partial^2 \pi_S}{\partial q \partial p_1} \\ \frac{\partial^2 \pi_S}{\partial p_1 \partial p_2} & \frac{\partial^2 \pi_S}{\partial p_2{}^2} & \frac{\partial^2 \pi_S}{\partial a_1 \partial p_2} & \frac{\partial^2 \pi_S}{\partial a_2 \partial p_2} & \frac{\partial^2 \pi_S}{\partial \tau_s \partial p_2} & \frac{\partial^2 \pi_S}{\partial \tau_n \partial p_2} & \frac{\partial^2 \pi_S}{\partial q \partial p_2} \\ \frac{\partial^2 \pi_S}{\partial p_1 \partial a_1} & \frac{\partial^2 \pi_S}{\partial p_2 \partial a_1} & \frac{\partial^2 \pi_S}{\partial a_1{}^2} & \frac{\partial^2 \pi_S}{\partial a_2 \partial a_1} & \frac{\partial^2 \pi_S}{\partial \tau_s \partial a_1} & \frac{\partial^2 \pi_S}{\partial \tau_n \partial a_1} & \frac{\partial^2 \pi_S}{\partial q \partial a_1} \\ \frac{\partial^2 \pi_S}{\partial p_1 \partial a_2} & \frac{\partial^2 \pi_S}{\partial p_2 \partial a_2} & \frac{\partial^2 \pi_S}{\partial a_1 \partial a_2} & \frac{\partial^2 \pi_S}{\partial a_2{}^2} & \frac{\partial^2 \pi_S}{\partial \tau_s \partial a_2} & \frac{\partial^2 \pi_S}{\partial \tau_n \partial a_2} & \frac{\partial^2 \pi_S}{\partial q \partial a_2} \\ \frac{\partial^2 \pi_S}{\partial p_1 \partial \tau_s} & \frac{\partial^2 \pi_S}{\partial p_2 \partial \tau_s} & \frac{\partial^2 \pi_S}{\partial a_1 \partial \tau_s} & \frac{\partial^2 \pi_S}{\partial a_2 \partial \tau_s} & \frac{\partial^2 \pi_S}{\partial \tau_s{}^2} & \frac{\partial^2 \pi_S}{\partial \tau_n \partial \tau_s} & \frac{\partial^2 \pi_S}{\partial q \partial \tau_s} \\ \frac{\partial^2 \pi_S}{\partial p_1 \partial \tau n} & \frac{\partial^2 \pi_S}{\partial p_2 \partial \tau_n} & \frac{\partial^2 \pi_S}{\partial a_1 \partial \tau_n} & \frac{\partial^2 \pi_S}{\partial a_2 \partial \tau_n} & \frac{\partial^2 \pi_S}{\partial \tau_s \partial \tau_n} & \frac{\partial^2 \pi_S}{\partial \tau_n{}^2} & \frac{\partial^2 \pi_S}{\partial q \partial \tau_n} \\ \frac{\partial^2 \pi_S}{\partial p_1 \partial q} & \frac{\partial^2 \pi_S}{\partial p_2 \partial q} & \frac{\partial^2 \pi_S}{\partial a_1 \partial q} & \frac{\partial^2 \pi_S}{\partial a_2 \partial q} & \frac{\partial^2 \pi_S}{\partial \tau_s \partial q} & \frac{\partial^2 \pi_S}{\partial \tau_n \partial q} & \frac{\partial^2 \pi_S}{\partial q^2} \end{bmatrix}$$

Proof 2. The first, second, third, fourth and fifth principle minors of H_C are as follows; respectively:

$H_C{}^1, H_C{}^2, H_C{}^3, H_C{}^4, H_C{}^5$

Due to the complexness of the above expressions, we are not able to prove the optimality of our solutions analytically. Instead of that, we computed a numerical study with randomly generated sets of parameters and then Hessian matrix is checked for that. If Hessian matrix is not negative define for our instance, we should check the boundary nodes. Since, Hessian matrix is negative define, so the objective function is concave for our instance.

Proposition 3. In this decision case, the whole system's optimal decsion variables satisfy the following equations:

$$\frac{\partial \pi_S}{\partial p_1} = 0, \frac{\partial \pi_S}{\partial a_1} = 0, \frac{\partial \pi_S}{\partial p_2} = 0, \frac{\partial \pi_S}{\partial a_2} = 0 \frac{\partial \pi_S}{\partial q} = 0, \frac{\partial \pi_S}{\partial \tau_s} = 0, \frac{\partial \pi_S}{\partial \tau_n} = 0 \quad (42)$$

Proof 3. Obtaining a closed-form analytical solution for equations (42) is less possible. Similar to (Esmaeili and Zeephongsekul, 2010), to solve equations (42), the Simpson Quadrature method could be initially used to find the value of the decision variables. Then, the problem changes to an equations system for our example, which will provide the Cooperative solutions for the whole system.

3-3- Stackelberg manufacture games

In this subsection, similar to previous literature (Savaskan et al., 2004; Hong et al., 2015), a channel members' decision process is modeled as a two-stage non cooperative game; with the manufacturer as the leader and the duopolistic retailers as the followers (the manufacturer has the greater power than the retailers). In the first stage, the manufacturer decides its national advertising expenditure, wholesale price and its collecting rate. In the second stage, the two competitive retailers decide, simultaneously and independently, their local advertising expenditures and their retail price. This game is solved by backward induction method and its solution is called Stackelberg equilibrium.

This kind of competition mode is not unusual to see in the real world, for example, Nike, as a core corporation of its supply chain, is usually considered to be the Stackelberg leader, while the other supply chain members (e.g., its suppliers, retailers and third party logistics providers) are followers (Lee and Ren, 2011).

This situation is considered so that the manufacture as a leader has risk-averse behavior regarding to his return rates (τ_s, τ_n). Thus, these variables are different from before and are random variables instead of decision variables. In Addition, we assume that the potential market size $(A_i)$for i=1,2 is a random variable. Here, based on the results of (Karray, 2015; Lai, Xiao and Yang, 2012; Feng, Lai, and Lu, 2015).These papers further suppose that the demand distributions are taken as the truncated normal distributions; that is, for all$A_i \epsilon R^+,\ A_i \sim N(\mu_i\ , \sigma^2), i = 1,2and\mu_{1>}\mu_2$.

Before each retailer sends her order to the manufacturer, the retailers will first predict the market demand more exactly; however, the manufacturer only has the prior knowledge regarding the market size. The manufacturer and the retailers know the prior distribution of the demand, but only the retailers can predict the true market size. The manufacturers will judge the market size. Besides, all parameters in the models are common knowledge for two parties. Here, the mean–variance method is introduced to measure the retailer's risk concerns.
According to the literature (Choi, Li, and Yan, 2008; Lau and La, 1999; Xu, Zhang, and Liu, 2014), the expected utility of the manufacturer is denoted by$E\big(U_M(\tau_s, \tau_n, q, w_s, w_n)\big) = E\big(\pi_m(\tau_s, \tau_n, q, w_s, w_n)\big) - \lambda . Var\big(\pi_m(\tau_s, \tau_n, q, w_s, w_n)\big)$. Where$\lambda$denotes the risk-averse degree of the manufacture,$\lambda > 0$, and$\lambda$is common knowledge. $E(\pi_m)$ and $Var(\pi_m)$ denote the mean and the variance of the manufacture's profit. We suppose that the return rates(τ_s, τ_n) are independent so that the mean and variance of them can be denoted by$\bar{\tau}, \sigma_\tau^2$; respectively.The timeline of the model is defined as follows. First, the manufacturer as a risk-averse leader sets his wholesale prices and national advertising expenditures. Second, after predicting the market size, each retailer-I as a follower decides simultaneously and independently her retail price p_i and local advertising $a_i$to maximize her own profit.

3-3-1- Manufacturer-Stackelberg game with symmetric information (MSSI)

Consider a CLSC in which the risk-averse manufacturer and the risk-neutral retailers have symmetric information; that is, they all know the true market size. We first need to solve the retailers' optimal problems when the wholesale prices and national advertising investment declared by the manufacturer are given. The best retailers' responses should be initially determined from equations (14, 15).Thus; the best retailers' responses are as follows:

$$p_1 = \frac{1}{2}(1 + p_2 + w_s) \quad (43)$$

$$a_1 = (-a_2{}^{y}(-1 + p_1 - p_2)\mathrm{q}^{-\delta}(p_1 - w_s)\gamma)^{\frac{1}{1+\gamma}} \quad (44)$$

$$p_2 = \frac{1}{2}(1 + p_1 + w_n) \quad (45)$$

$$a_2 = \left(a_1{}^{\mathrm{y}}(1 + p_1 - p_2)\mathrm{q}^{-\delta}(p_2 - w_n)\gamma\right)^{\frac{1}{1+\gamma}} \quad (46)$$

After substituting equations (43-46) $a_i, p_i, i = 1,2$ into $U_m$we can formulate the manufacturer decision problem as following:

$$max \ \ U_m(w_s, w_n, q) = (w_s + \bar{\tau})(1 - p_1 + p_2)\left(\frac{A_1}{\eta^{\frac{1}{\Upsilon+\delta-y+1}}} - a_1{}^{-\gamma} a_2{}^{y} q^{-\delta}\right) + (w_n + \bar{\tau})$$

$$(1 + p_1 - p_2)(\frac{A_2}{\eta^{\frac{1}{\Upsilon+\delta-y+1}}} - a_1{}^{y} a_2{}^{-\gamma} q^{-\delta}) - q - \frac{2k}{\eta^{\frac{1}{\Upsilon+\delta-y+1}}}(\bar{\tau}^2) - \lambda\sigma_\tau(D_1^2 + D_2^2) - \lambda(- \quad (47)$$

$$p_1 = \frac{1}{2}(3 + 2w_s + w_n) \quad (48)$$

$$a_1 = (\frac{1}{9}\gamma(\frac{3 - w_n + w_s}{3 + w_n - w_s})^{\frac{2y}{1+\gamma+y}}(3 + w_n - w_s)^2 q^{-\delta})^{\frac{1}{1+\gamma-y}} \quad (49)$$

$$p_2 = \frac{1}{3}(3 + 2w_n + w_s) \quad (50)$$

$$a_2 = (\frac{1}{9}\gamma(\frac{3 + w_n - w_s}{3 - w_n + w_s})^{\frac{2y}{1+\gamma+y}}(3 - w_n + w_s)^2 q^{-\delta})^{\frac{1}{1+\gamma-y}} \quad (51)$$

$0 \le w_s, w_n$

$\mathrm{q} \ge 0$, $0 \le \tau_s, \tau_n \le 1$ (52)

Proposition 4. To proof the optimality of the manufacture's solutions, we have to calculate the Hessian matrix

$$H_{S_m} = \begin{bmatrix} \frac{\partial^2 U_m}{\partial {w_s}^2} & \frac{\partial^2 U_m}{\partial w_n \partial w_s} & \frac{\partial^2 U_m}{\partial q \partial w_s} \\ \frac{\partial^2 U_m}{\partial w_s \partial w_n} & \frac{\partial^2 U_m}{\partial {w_n}^2} & \frac{\partial^2 U_m}{\partial q \partial wn} \\ \frac{\partial^2 U_m}{\partial w_s \partial q} & \frac{\partial^2 U_m}{\partial w_n \partial q} & \frac{\partial^2 U_m}{\partial q^2} \end{bmatrix}$$

Proof 4. The first, second, third and fourth principle minors of H_{S_m} are as follows; respectively:

$H^1_{S_m}$, $H^2_{S_m}$, $H^3_{S_m}$

Due to the complexness of the above expressions, we are not able to prove the optimality of our solutions analytically. Instead of that, we computed a numerical study with randomly generated sets of parameters and then Hessian matrix is checked for that. If Hessian matrix is not negative define for the instance, we should check the boundary nodes. Since, Hessian matrix is negative define for our instance, so the objective function is concave for this instance.

Proposition 5. In the MSSI decision case, the manufacturer's optimal wholesale prices and advertising expenditures (denoted asw_s, w_n, q) satisfies the following equations:

$$\frac{\partial U_m}{\partial w_s} = 0, \frac{\partial U_m}{\partial w_n} = 0, \frac{\partial U_m}{\partial q} = 0 \tag{53}$$

Proof 5. Obtaining a closed-form analytical solution for equations (47) is less possible. Similar to (Esmaeili and Zeephongsekul, 2010), to solve equations (47), the Simpson Quadrature method could be initially used to find the value of the decision variables. Then, the problem changes to an equations system for our example, which will provide the Stackelberg solutions for the manufacturer so is for the retailers through equations (48-52)

3-3-2- Manufacturer-Stackelberg game under asymmetric information (MSAI)

In this case, both the retailers will have their own private information, namely, the information of market size A_i is only known to the retailer-*i*. Strategic games with asymmetric information can be formalized along the line of standard games with complete information, and are based on the pioneering work described in (Harsanyi, 1968). We have a set of players S_p and, for each player; we also specify the set of actions and corresponding pay off available to that player. Similar to (Esmaeili and Zeephongsekul, 2010), two features are introduced into games with asymmetric information, namely, the type space containing private information pertaining to a player and a probability distribution expressing the uncertainty over a player's type by other players. In the following description, we explain the components of the two-person game with asymmetric information, which will be used to model our closed-loop supply chain models in the next section.

(1) $\mathrm{S_p} = \{M, R_i\}$wherethemanufacturerisrepresentedby M.

(2) The set of actions available to the players are their decision variables. For player$M$, the set of actions is $S_M = \{(\tau_s, \tau_n, q, w_s, w_n): (\tau_s, \tau_n, q, w_s, w_n) \epsilon\ R^{+5}\}$and for player$R_i$, the set of actions is $S_{R_i} = \{(p_i, a_i) : (p_i, a_i) \epsilon\ R^+ \times R^+\}$where$i = 1{,}2$, $R^+ \epsilon [0, +\infty)$.

(3) Each player's type is identified by the parameters that are only known to them. Therefore, from the above problem description,$Q_i = \{(A_i) : (A_i)\ \epsilon R^+\}$. A typical element of $Q_i$will be denoted byε_i.

(4) We assume that players types are independent. Therefore,R_i's uncertainty can be expressed through the probability density function (pdf) $f_{R_i}(\varepsilon_i)$. We denote the expected value with respect to $f_{R_i}(\varepsilon_i)$ by $E_{R_i}(\varepsilon)$.

(5) The payoff functions $U_j(0), j \in \{M, R_i\}$where $i = 1{,}2$,for playerR_i is $U_{R_i}(p_i, a_i, \varepsilon_i) = \pi_{r_i}(p_i, a_i, \varepsilon_i)$and for player$M$ thepayoff function is

$E\big(U_M(\tau_s, \tau_n, q, w_s, w_n)\big) = E\big(\pi_m(\tau_s, \tau_n, q, w_s, w_n)\big) - \lambda . Var\big(\pi_m(\tau_s, \tau_n, q, w_s, w_n)\big)$.

In anticipation of the manufacturer's reactions, for *i* =1,2, each retailer-*i* chooses her own best response according to the following objectives:

$$E_{\varepsilon_2}(\pi_{r_1}\big(p_1, a_1, p_2(\varepsilon_2), a_2(\varepsilon_2)\big)) = \iint_{\varepsilon_2} \pi_{r_1}\big(p_1, a_1, p_2(\varepsilon_2), a_2(\varepsilon_2)\big) f_{R_2}(\varepsilon_2)\ )) \tag{54}$$

$$E_{\varepsilon_1}(\pi_{r_2}\big(p_2, a_2, p_1(\varepsilon_1), a_1(\varepsilon_1)\big)) = \iint_{\varepsilon_1} \pi_{r_2}\big(p_2, a_2, p_1(\varepsilon_1), a_1(\varepsilon_1)\big) f_{R_1}(\varepsilon_1) \tag{55}$$

Using the first order conditions on (6,7) by differentiating with respect to p_i anda_i for$i$=1,2, we can obtain each retailer's optimal strategies, for given (w_s , w_n , q), as follows:

$$p_1\,(\varepsilon_1) = \frac{1}{2}(1 + p_2\ + w_s\,) \tag{56}$$

$$a_1(\varepsilon_1) = (-a_2{}^{y}(-1 + p_1 - p_2)q^{-\delta}(p_1 - w_s)\gamma)^{\frac{1}{1+\gamma}} \tag{57}$$

$$a_2(\varepsilon_2) = \left(a_1{}^{y}(1 + p_1 - p_2)q^{-\delta}(p_2 - w_n)\gamma\right)^{\frac{1}{1+\gamma}} \tag{58}$$

$$p_2(\varepsilon_2) = \frac{1}{2}(1 + p_1 + w_n) \tag{59}$$

After substituting equations (56-59) $a_i, p_i, i = 1,2$ into U_m, we can formulate the manufacturer decision problem as following:

$$\begin{aligned} &max\ E_{(\varepsilon_1,\varepsilon_2)}(U_m\left(w_s, w_n, q, p_1(\varepsilon_1), a_1(\varepsilon_1), p_2(\varepsilon_2), a_2(\varepsilon_2)\right)) = \\ &E_{(\varepsilon_1,\varepsilon_2)}\left(\pi_m\left(w_s, w_n, q, p_1(\varepsilon_1), a_1(\varepsilon_1), p_2(\varepsilon_2), a_2(\varepsilon_2)\right)\right) \\ &\quad - \lambda.Var_{(\varepsilon_1,\varepsilon_2)}\left(\pi_m\left(w_s, w_n, q, p_1(\varepsilon_1), a_1(\varepsilon_1), p_2(\varepsilon_2), a_2(\varepsilon_2)\right)\right) \end{aligned} \tag{60}$$

$$p_1(\varepsilon_1) = \frac{1}{3}(3 + 2w_s + w_n) \tag{61}$$

$$a_1(\varepsilon_1) = \left(\frac{1}{9}\gamma\left(\frac{3 - w_n + w_s}{3 + w_n - w_s}\right)^{\frac{2y}{1+\gamma+y}}(3 + w_n - w_s)^2 q^{-\delta}\right)^{\frac{1}{1+\gamma-y}} \tag{62}$$

$$a_2(\varepsilon_2) = \left(\frac{1}{9}\gamma\left(\frac{3 + w_n - w_s}{3 - w_n + w_s}\right)^{\frac{2y}{1+\gamma+y}}(3 - w_n + w_s)^2 q^{-\delta}\right)^{\frac{1}{1+\gamma-y}} \tag{63}$$

$$p_2(\varepsilon_2) = \frac{1}{3}(3 + 2w_n + w_s) \tag{64}$$

Proposition 6. In the MSAI decision case, the manufacturer's optimal wholesale prices and advertising expenditures denoted as w_s, w_n, q) satisfies the following equations:

$$\frac{\partial E_{\varepsilon_1,\varepsilon_2}(U_m)}{\partial w_s} = 0, \frac{\partial E_{\varepsilon_1,\varepsilon_2}(U_m)}{\partial w_n} = 0, \frac{\partial E_{\varepsilon_1,\varepsilon_2}(U_m)}{\partial q} = 0 \tag{65}$$

Proof 6. Using equations (61-64), $E_{\varepsilon_1,\varepsilon_2}(U_m\left(w_s, w_n, q, p_1(\varepsilon_1), a_1(\varepsilon_1), p_2(\varepsilon_2), a_2(\varepsilon_2)\right))$ can be expressed as follows:

$$\begin{aligned} \int E_{(\varepsilon_1,\varepsilon_2)} &((w_s + \bar{\tau})(1 - p_1(\varepsilon_1) + p_2(\varepsilon_2))(\frac{A_1}{\eta^{\frac{1}{\gamma+\delta-y+1}}} - a_1(\varepsilon_1)^{-\gamma} a_2(\varepsilon_2)^{y} q^{-\delta}) + (w_n + \bar{\tau})(1 \\ &+ p_1(\varepsilon_1) - p_2(\varepsilon_2))(\frac{A_2}{\eta^{\frac{1}{\gamma+\delta-y+1}}} - a_1(\varepsilon_1)^{y} a_2(\varepsilon_2)^{-\gamma} q^{-\delta}) - q \\ &- \frac{2k}{\eta^{\frac{1}{\gamma+\delta-y+1}}}(\bar{\tau}^2) - \lambda\sigma_\tau(D_1^2 + D_2^2) - \lambda(\frac{2k}{\eta^{\frac{1}{\gamma+\delta-y+1}}})^2 \sigma_\tau{}^2) f_{R_1}(\varepsilon_1) f_{R_2}(\varepsilon_2) \end{aligned} \tag{66}$$

Differentiating $E_{\varepsilon_1,\varepsilon_2}(U_m)$ with respect to w_s, w_n and q yields first order conditions which satisfy the following equations:

$$\frac{\partial E_{\varepsilon_1,\varepsilon_2}(U_m)}{\partial w_s} = 0, \frac{\partial E_{\varepsilon_1,\varepsilon_2}(U_m)}{\partial w_n} = 0, \frac{\partial E_{\varepsilon_1,\varepsilon_2}(U_m)}{\partial q} = 0$$

Using equations (65), we can know that Proposition 6 holds. In proposition 6, we find that obtaining closed-form analytical solutions for w_s, w_n and q is less possible. Similar to (Esmaeili and Zeephongsekul, 2010), to solve equations (65), the Simpson Quadrature method could be initially used to find the values of two integrals that depend on the form of $f_{R_1}(\varepsilon_1)\ and\ f_{R_2}(\varepsilon_2)$. Then, the problem is changed to an equations system without any integrals which will easily yield the Stackelberg solution for the retailer.

4- Numerical example

In this section, we will interrogate the proposed game models and solution algorithms through numerical experiments in order to describe its application. The experiments are adopted in the following manner. First, for all parameter of the models, we extract randomly a value out of its given interval, which is shown in Table 3. We extract randomly more than 100 groups of values of the parameters in total in the experiment. Then we calculate the equilibrium solution of two models in the tree settings based on this group of extracted values of all parameters. Examples aimed at illustrating some important features of the model and managerial highlights of all groups.

In brief, two from all groups are randomly chosen, in which the values of parameters are listed in table 4 for duopolistic retailers' model; to illustrate our observations intuitively. All the instances have been solved using the proposed algorithm and the Simpson Quadrature method as mentioned before. Tables 5, 6, 7, 8 and 9, show Nash Equilibrium, cooperative equilibrium, MSSI, MSAI solutions.

Table3. The ranges of parameters

Parameter								
Ranges	[800-20000]	[800-20000]	[0-1]	[0.4 -2.5]	[0.4-2.5]	[0.2-1]	[10^4-10^7]	[0.5-10]

Table4. Two groups of values of parameters considered

Parameter	A_2	A_1	$\bar{\tau}$	σ_τ	μ_1	μ_2	λ	δ	γ	y	k	η
Example	900	900	0.02	0.001	1400	1400	0.04	1	1	0.5	10^5	5×10^6

For the duopolistic retailers model we use these two groups as shown in Table 4 and obtain the solution as reported in table 5-9

Table 5. Decisions and profits of players in Nash model

Two echelons	Two retailers	DecsionVariable	Example
Nash	Cournot	$p_1{}^*$	1.9
		$p_2{}^*$	1.85
		$a_1{}^*$	0.62
		$a_2{}^*$	0.56
		$w_s{}^*$	0.95
		$w_n{}^*$	0.92
		$\tau_s{}^*$	0.0021
		$\tau_n{}^*$	0.0051
		$q*$	1.45
		π_{r_1}	3.51
		π_{r_2}	3.77
		π_m	7.03
		π_s	13.17

Table 6. Decisions and profits of players in cooperative model

Two echelons	Two retailers	DecsionVariable	Example
Cooperation	……	$p1^*$	0.51
		$p2^*$	0.034
		$a_1{}^*$	0.03
		$a_2{}^*$	26.02
		$\tau_s{}^*$	0
		$\tau_n{}^*$	0.0023
		$q*$	8.22
		π_s	917.6

Table 7. Decisions and profits of players in a Stackelberg manufacture game with risk

Two echelons	Two retailers	DecsionVariable	Example
MSSI	Cournot	$p_1{}^*$	1.8
		$p_2{}^*$	1.39
		$a_1{}^*$	0.8
		$a_2{}^*$	1.5
		$w_s{}^*$	1.2
		$w_n{}^*$	0.02
		$q*$	0.916
		π_{r_1}	0.47
		π_{r_2}	7.74
		π_m	1.9
		π_s	10.11

Table 8. Decisions and profits of players in a Stackelberg manufacture game with risk

Two echelons	Two retailers	DecsionVariable	Example
M SAI	Cournot	${p_1}^*$	1.85
		${p_2}^*$	1.4
		${a_1}^*$	0.77
		${a_2}^*$	1.59
		${w_s}^*$	1.22
		${w_n}^*$	0.03
		q *	0.91
		π_{r_1}	0.49
		π_{r_2}	7.8
		π_m	1.6
		π_s	9.89

In order to understand the effect of risk behavior, we consider the stackelberg game models under another condition in which the manufacture has a risk-neutral behavior under both information structures (symmetric and asymmetric information) and obtain the solutions as reported in table 8.

Table9. Decisions and profits of players in a Stackelberg manufacture game without risk

Two echelons	Two retailers	DecsionVariable	Example
MSSI	Cournot	${p_1}^*$	1.75
		${p_2}^*$	1.2
		${a_1}^*$	0.62
		${a_2}^*$	0.56
		${w_s}^*$	0.95
		${w_n}^*$	0.92
		${\tau_s}^*$	0.00219
		${\tau_n}^*$	0.00513
		q *	1.45
		π_{r_1}	3.41
		π_{r_2}	3.77
		π_m	7.03
		π_s	14.21
MSAI	Cournot	${p_1}^*$	1.78
		${p_2}^*$	1.32
		${a_1}^*$	0.86
		${a_2}^*$	1.514
		${w_s}^*$	1.22
		${w_n}^*$	1.03
		${\tau_s}^*$	0.001
		${\tau_n}^*$	0.005
		q *	0.9
		π_{r_1}	3.47
		π_{r_2}	4.73
		π_m	1.98
		π_s	10.18

Notably, figure 2 is provided to describe the scenarios under a different risk indicator λ

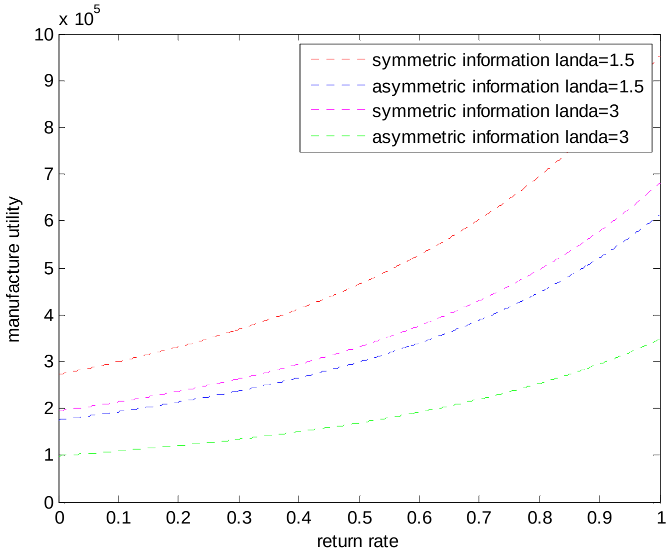

Figure 2. The impacts of the return rate $\bar{\tau}$and the risk-averse degree λ (landa) on the utilities of the manufacturer

Based on figure 2, we can observe that when the demand information is symmetric, the manufacturer's expected utility under $\lambda = 0.3$is lower than that under the case$\lambda = 1.5$. Similarly, under asymmetric information, the manufacturers expected utility will lower under $\lambda = 3$ than that under $\lambda = 1.5$. In other words, when the return rate $\bar{\tau}$is provided, the margin of the manufacturer's utility under asymmetric information is larger than that under symmetric information. When the value of$\bar{\tau}$is small, regardless of what value of $\lambda$is used the gap between the manufacturer utilities is particularly small. In figure 2, we find that the manufacturer's utility is increasing with the increasing of $\bar{\tau}$and decreasing with the manufacturer's risk indicator λ.

5-Managerial implication and conclusion

In this paper, we constitute a closed-loop supply chain that is composed of a manufacturer and two retailers. We followed a model recently published by(Zhang and Liu, 2013), in which a supply chain formed by one supplier one manufacturer and one retailer where demand is influenced by retail price. Unlike their research that mainly focused on channel members' pricing decisions and adopted an additive form of demand function, our paper presents a closed-loop supply chain model with one manufacture and duopolistic retailers' different competitive behaviors by adopting multiplicatively separable new demand function and taking advertising decisions into account. The channel members' advertising efforts is a multiplicative form as (Wang et al., 2011) model. So we introduced a new improved demand function of consumers for each retailer that depends on both advertising and pricing of retailers and manufacturer. By means of game theory, we have analyzed three different relationships within the closed-loop supply chain, i.e. Nash game, Manufacturer-Stackelberg game and cooperation games. The optimal solutions of the channel members were not analytically obtained; henceforth we presented some methods for solving the game models. In addition, in the setting of stackelberg, we investigate the influences of the manufacturer's risk aversion regarding his return rates under information asymmetry in which the retailers has more private information regarding their true market size. We show that in such a setting, based on the analysis of the model and results of numerical experiments, we obtain the following insights:

1. Cooperative structure improves the performance of the supply chain and they can gain more profits than non-cooperative situations in both models, i.e., Nash and Manufacturer-Stackelberg game.
2. The Nash game and Manufacturer-Stackelberg game solution are close and in some examples Nash game can gain more profit for the supply chain.
3. The lowest retailer price is made in the cooperative; hence it can bring a competitive advantage for the channel members.
4. The players' profits under the manufacturer's risk behavior are lowest.
5. The retailers' profits under the MSAI decision case are higher than that under the MSSI decision case; moreover, the manufacturer's profit under the MSAI decision case is lower than that under the MSSI decision case, from which we can see that the retailers with the private information take advantage of the other's response in the decision game.
6. The product's wholesale price under the MSAI decision case is higher than that under the MSSI game case, from which we can see that the manufacturer will charge a higher wholesale price when the retailer has private information on market size.

Our study leaves several unanswered questions for future research. Firstly, while our model focused on a game in a single-period setting, the same approach can be used to analyze a multi-period game and/or an infinitely repeated. Secondly, it would be interesting to incorporate some constraints in order to develop a modified version of the model. Thirdly, we could extend our work to a situation in which both the manufacturer and the retailer offer a price discount to the customer in order to obtain better insights into the underlying relationships of pricing and advertising.

References

Basar, T., & Olsder, G. J. (1999). Dynamic non cooperative game theory (Vol. 23). Siam.

Berger, P. D. (1973). Statistical decision analysis of cooperative advertising ventures. *Journal of the Operational Research Society*, *24*(2), 207-216.

Chintagunta, P. K., & Jain, D. (1992). A dynamic model of channel member strategies for marketing expenditures. *Marketing Science*, *11*(2), 168-188**.**

Choi, T. M., Li, D., & Yan, H. (2008). Mean–variance analysis of a single supplier and retailer supply chain under a returns policy. *European Journal of Operational Research*, *184*(1), 356-376.

Esmaeili, M., & Zeephongsekul, P. (2010). Seller–buyer models of supply chain management with an asymmetric information structure. *International Journal of Production Economics*, *123*(1), 146-154.

Facchinei, F., & Kanzow, C. (2007). Generalized Nash equilibrium problems. *4OR*, *5*(3), 173-210.

Feng, Q., Lai, G., & Lu, L. X. (2015). Dynamic Bargaining in a Supply Chain with Asymmetric Demand Information (With Online Appendices).

Harsanyi, J. C. (1968). Games with incomplete information played by "Bayesian" players part II. Bayesian equilibrium points. *Management Science*, *14*(5), 320-334.

Hong, X., Xu, L., Du, P., & Wang, W. (2015). Joint advertising, pricing and collection decisions in a closed-loop supply chain. *International Journal of Production Economics*, *167*, 12-22.

Karray, S. (2015). Modeling brand advertising with heterogeneous consumer response: channel implications. *Annals of Operations Research*, *233*(1), 181-199.

Lai, G., Xiao, W., & Yang, J. (2012). Supply chain performance under market valuation: An operational approach to restore efficiency. *Management Science, 58*(10), 1933-1951.

Lau, A. H. L., & Lau, H. S. (2005). Some two-echelon supply-chain games: Improving from deterministic-symmetric-information to stochastic-asymmetric-information models. *European Journal of Operational Research*, *161*(1), 203-223.

Lau, H. S., & Lau, A. H. L. (1999). Manufacturer's pricing strategy and return policy for a single-period commodity. *European Journal of Operational Research*, *116*(2), 291-304.

Lee, J. Y., & Ren, L. (2011). Vendor-managed inventory in a global environment with exchange rate uncertainty. *International Journal of Production Economics*, *130*(2), 169-174.

Li, Z., Gilbert, S. M., & Lai, G. (2013). Supplier encroachment under asymmetric information. *Management Science*, *60*(2), 449-462.

Markowitz, H. (1959). Portfolio selection: Efficient diversification of investments. Cowles Foundation monograph no. 16.

Meca, A., & Timmer, J. (2008). *Supply chain collaboration* (pp. 1-18). I-Tech Education and Publishing.

Savaskan, R. C., Bhattacharya, S., & Van Wassenhove, L. N. (2004). Closed-loop supply chain models with product remanufacturing. *Management science*, *50*(2), 239-252

Slikker, M., & Van den Nouweland, A. (2012). *Social and economic networks in cooperative game theory* (Vol. 27). Springer Science & Business Media.

Tayur, S., Ganeshan, R., & Magazine, M. (Eds.). (2012). *Quantitative models for supply chain management* (Vol. 17). Springer Science & Business Media.

Tsay, A. A. (2002). Managing retail channel overstock: Markdown money and return policies. *Journal of retailing*, *77*(4), 457-492.

Wang, S. D., Zhou, Y. W., Min, J., & Zhong, Y. G. (2011). Coordination of cooperative advertising models in a one-manufacturer two-retailer supply chain system. *Computers & Industrial Engineering*, *61*(4), 1053-1071.

Xie, J., & Neyret, A. (2009). Co-op advertising and pricing models in manufacturer–retailer supply chains. *Computers & Industrial Engineering, 56*(4), 1375-1385.

Xu, G., Dan, B., Zhang, X., & Liu, C. (2014). Coordinating a dual-channel supply chain with risk-averse under a two-way revenue sharing contract. *International Journal of Production Economics*, *147*, 171-179.

Yan, R. (2010). Cooperative advertising, pricing strategy and firm performance in the e-marketing age. *Journal of the Academy of Marketing Science*, *38*(4), 510-519.

Yue, J., Austin, J., Wang, M. C., & Huang, Z. (2006). Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount. *European Journal of Operational Research*, *168*(1), 65-85.

Zhang, C. T., & Liu, L. P. (2013). Research on coordination mechanism in three-level green supply chain under non-cooperative game. *Applied Mathematical Modelling*, *37*(5), 3369-3379.