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A Comparison of Four Multi-Objective Meta-Heuristics for a Capacitated Location-Routing Problem

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ABSTRACT

In this paper, we study an integrated logistic system where the optimal location of depots and vehicles routing are considered simultaneously. This paper presents a new mathematical model for a multi-objective capacitated location-routing problem with a new set of objectives consisting of the summation of economic costs, summation of social risks and demand satisfaction score. A new multi-objective adaptative simulated annealing (MOASA) is proposed to obtain the Pareto solution set of the presented model according to the previous studies. We also apply three multi-objective meta-heuristic algorithms, namely MOSA, MOTS and MOAMP, on the simulated data in order to compare the proposed procedure performance. The computational results show that our proposed MOASA outperforms the three foregoing algorithms.

***Keywords*:** Location-routing problem, Demand satisfaction score, Multi-objective meta-heuristic algorithms, Pareto solution set.

1. INTRODUCTION

The location routing problem (LRP) is a combination of the facility location problem (FLP) and the vehicle routing problem (VRP) that involves three inter-related, fundamental decisions: 1) where to locate the facilities, 2) how to allocate customers to facilities, and 3) how to route the vehicles to serve customers (Perl & Daskin, 1984).These problems belong to the class of NP-hard ones (Min *et al*., 1998). Due to its complexity, exact algorithms for solving the LRP have been used very limited and multi-objective exact algorithms have never been used. In this paper, we propose a modified multi-objective meta-heuristic algorithm based on the multi-objective meta-heuristic algorithm using an adaptative memory procedure (MOAMP) algorithm (Caballero *et al.*, 2007). However, we propose a multi-objective adaptative simulated annealing (MOASA) to solve the multi-objective

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capacitated location-routing problem. The remainder of this paper is structured as follows.

In the next section, we summarize the related literature. Section 3 defines a modified multi-objective location-routing model. Section 4 details a brief overview of three algorithms. Section 5 deals with the proposed algorithm. Section 6 gives a discussion about the performance metrics to assess the quality of algorithms. The computational experiments are presented in Section 7. Finally, a discussion of the results is explained in the last section.

2. LITERATURE REVIEW

The LRP addressed in this paper includes a few exact solutions and many heuristic and meta-heuristic algorithms in order to find a near-optimal solution in a reasonable computational time; most of them and LRP applications are summarized in survey papers (Min *et al*., 1998; Nagy & Salhi, 2007).

Min *et al.* (1998) showed that most early studies considered either capacitated routes or capacitated depots, but not both of them. Prins *et al*. (2006a) called "general LRP" for the case with capacities on both depots and routes. They developed a memetic algorithm with population management (MA|PM). Tuzun and Burke (1999) developed a two-phase tabu search for the LRP with capacitated routes and uncapacitated depots. The two-phase algorithm, introduced for the first time in such paper and they dedicated to routing and location. Wu *et al*. (2002) studied the general LRP with homogeneous or heterogeneous fleet types and with a limited number of vehicles. They designed a simulated annealing (SA) algorithm with a tabu list to avoid cycling. Lin *et al*. (2002) allow vehicles to take multiple trips. The problem is divided into three phases: 1) facility location phase, 2) routing phase and 3) loading phase. They used meta-heuristics for solving this type of problem. The applicability of these meta-heuristics is limited as it relies on evaluating how large the number of depot configurations is.

Albareda-Sambola *et al*. (2005) proposed another two-phase tabu search (TS) for the general LRP with one single route per open depot. The method has been tested on small instances for at most 30 customers. Yu *et al*. (2010) proposed the SA algorithm for solving the general LRP, which features a specially designed solution representation scheme for the LRP. It outperforms all other algorithms in terms of the computational speed and solution quality by testing it on benchmark datasets. Belenguer *et al*. (2011) proposed an exact algorithm based on a branch-and-cut algorithm for solving the general LRP. The algorithm is based on a zero-one linear model strengthened by new families of valid inequalities. It can solve just medium instances for at most 50 customers. Other popular meta-heuristics have been applied to the LRP as well. Table 1 summarizes related works about meta-heuristics for the LRP. All mentioned papers have a single objective in such a way that a linear combination of costs is minimized. These costs can be regarded as depot installation and operating; routes design; and vehicle fixed costs. Only three papers study multi-objectives.

Table 1 Summary of the recent papers on meta-heuristics for a general LRP

Reference	Solution method
Bouhafs *et al*. (2006)	Combined simulated annealing and ant colony
Prins *et al*. (2006b)	Combined GRASP method with a learning process and a path relinking mechanism
Barreto *et al*. (2007)	Clustering-based method
Prins *et al*. (2007)	Cooperative Lagrangean relaxation and tabu search
Duhamel *et al*. (2010)	Combined GRASP method with evolutionary local search
Prodhon(2011)	Evolutionary local search

Lin and Kwok (2006) considered a multi-objective form of Lin *et al*. (2002). They applied the TS and SA algorithms on the real data and simulated data. Caballero *et al*. (2007) applied the MOAMP for a multi-objective LRP in a real case. All vehicles are identical and are constrained under capacity; however, depots do not have the capacity constraints. Tavakkoli-Moghaddam *et al*. (2010) proposed a new integrated mathematical model for a bi-objective capacitated LRP, in which demands could be unsatisfied. They developed a two-phase scatter search (SS) algorithm. The given problem divided into two phases, namely facility location and routing. These phases are tackled repeatedly for a set of facilities of the minimal size until the total costs justify the algorithm termination.

In this paper, we propose a modified version of the MOAMP algorithm for the multi-objective general LRP that features a solution representation proposed by Yu *et al*. (2010) because the model can be solved in a single phase to have higher efficiency. It is worth to mention that two new objectives not considered in the previous studies are considered in a three-objective LRP model.

Table 2 Multi-objective LRP

Paper	Solution method	Objectives
Lin and Kwok (2006)	Two phase with Hierarchical structure Designed based on Lin et al. (2002)	1- Minimization of total costs 2- Minimization of work time imbalance 3- Minimization of load imbalance
Caballero et al. (2007)	Two phase with Hierarchical structure Designed based on Albareda-Sambola et al. (2005)	1- Minimization of depot opening cost 2- Minimization of routing cost 3- Minimization of routing risk 4- Minimization of maximum routing risk 5- Minimization of maximum depot opening risk
Tavakkoli-Moghaddam et al. (2010)	Two phase with Hierarchical structure Designed based on Lin and Kwok (2006)	1- Minimization of total costs 2- Maximization of demand satisfaction
This study	One phase Designed based on Yu *et al*. (2010)	1- Minimization of total costs 2- Minimization of total risk 3- Maximization of demand satisfaction score

3. PROBLEM FORMULATION

The following formal mathematical model is modified model of Prins *et al*. (2006b). The model is defined on a complete, weighted and undirected network $G= (V, E, C)$. V is a set of nodes comprised of a subset I of m possible depot locations and a subset $J = V\backslash I$ of n customers. The traveling cost between any two nodes i and j is given by c_{ij} and the traveling risk given by r_{ij} and the traveling time given by u_{ij}. Capacity W_i, opening cost O_i and opening risk OR_i are associated with each depot site $i \in I$. Each customer $j \in J$ is characterized by a demand d_j, a service or dwell time s_j and a best score time window $[a_j, b_j]$, where a_j is the earliest time to begin service and b_j is the latest time, in which we acquire the highest customer satisfaction score. A set of identical vehicles of capacity Q is available, which is called K. In case of using each vehicle, fixed cost F is incurred depending the departing depot and performs a single route. The total number of vehicles used (or routes performed) is a decision variable. Each route should start and terminate at the same depot, and its total load should not exceed the vehicle capacity. The total load of the routes assigned to a depot should fit the capacity of the depot. For each vehicle $k \in K$ and each customer $j \in J$, continuous variable t_{kj} indicates the serving start time at customer j if it is served by vehicle k. When customer j

is not served by vehicle k, it should be zero. For each vehicle $k \in K$, t_{k0} is the departure time from the depot.

The single-objective of a general LRP is to determine which depots should be opened and which routes should be constructed to minimize the sum of depot opening costs and the travelling costs, including the routing and the vehicle fixed costs.

The problem can be modeled as a zero-one linear programming. The following Boolean variables are used: $y_i = 1$ if depot i is opened, $f_{ij} = 1$ if customer j is assigned to depot i, and $x_{jlk} = 1$ if edge (j, l) is traversed from j to l in the route performed by vehicle $k \in K$.

$$\min z_1 = \sum_{i\in I} O_i y_i + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K} c_{ij}x_{ijk} + \sum_{k\in K}\sum_{i\in V}\sum_{j\in V} F_i x_{ijk} \tag{1}$$

$$\min z_2 = \sum_{i\in I} OR_i y_i + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K} r_{ij}x_{ijk} \tag{2}$$

$$\max z_3 = \sum_{i\in V}\sum_{j\in V}\sum_{k\in K} \left(\frac{t_{jk1}}{a_j} - \frac{t_{jk3}}{T-b_j}\right)x_{ijk} \tag{3}$$

$s.t.$

$$\sum_{k\in K}\sum_{i\in V} x_{ijk} = 1, \qquad \forall j \in J \tag{4}$$

$$\sum_{j\in J}\sum_{i\in V} u_{ij}x_{ijk} \le T, \qquad \forall k \in K \tag{5}$$

$$\sum_{j\in J}\sum_{i\in V} d_j x_{ijk} \le Q, \qquad \forall k \in K \tag{6}$$

$$\sum_{j\in J} d_j f_{ij} \le W_i y_i, \qquad \forall i \in I \tag{7}$$

$$\sum_{j\in V} x_{ijk} - \sum_{j\in V} x_{jik} = 0, \qquad \forall i \in V, \forall k \in K \tag{8}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le 1, \qquad \forall k \in K \tag{9}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ijk} \le |S| - 1, \qquad \forall S \subseteq J, \forall k \in K \tag{10}$$

$$\sum_{u\in J} x_{iuk} + \sum_{u\in V\{j\}} x_{ujk} \le 1 + f_{ij}, \qquad \forall i \in I, \forall j \in J, \forall k \in K \tag{11}$$

$$t_{ik} + s_i + u_{ij} - M(1 - x_{ijk}) \le t_{jk}, \qquad \forall i \in V, \forall j \in V, \forall k \in K \tag{12}$$

$$(a_j)z_{jk1} < t_{jk1} \tag{13}$$

$$t_{jk1} < a_j \tag{14}$$

$$(b_j - a_j)z_{jk2} < t_{jk2} \tag{15}$$

$$t_{jk2} < (b_j - a_j)z_{jk1} \tag{16}$$

$$0 < t_{jk3} \tag{17}$$

$$t_{jk3} < (T - b_j)z_{jk2} \tag{18}$$

$$t_{jk} = t_{jk1} + t_{jk2} + t_{jk3} \tag{19}$$

$$x_{ijk} \in \{0,1\}, \quad \forall i \in I, \forall j \in V, \forall k \in K \tag{20}$$

$$y_i \in \{0,1\}, \quad \forall i \in I \tag{21}$$

$$f_{ij} \in \{0,1\}, \quad \forall i \in I, \forall j \in V \tag{22}$$

$$z_{jl} \in \{0,1\}, \quad \forall j \in V, l \in \{1,2,3\} \tag{23}$$

$$t_{jk} \geq 0, \quad \forall j \in V, \forall k \in K \tag{24}$$

$$t_{jkl} \geq 0, \quad \forall j \in V, \forall k \in K, l \in \{1,2,3\} \tag{25}$$

The first objective function (1) minimizes the total cost consisting of the sum of depot opening costs, routing costs and vehicles using fixed costs. The second objective function (2) minimizes the total risk consisting of the sum of depot opening risks and the routing risks. The third objective function (3) maximizes the total customers' demand satisfaction score. The score is calculated by equation (26) which shows in Figure 1. Constraint (4) guarantees that every customer belongs to one and only one route and that each customer has only one predecessor in the tour. Constraint (5) ensures that the length of each route does not exceed maximum allowable distance. Constraints (6) and (7) are capacity constraints associated with depots and routes, respectively. Constraints (8) and (9) guarantee the continuity of each route, and that each route terminates at the departing. Constraints (10) are sub-tour elimination constraints. Constraint (11) ensures that a customer must be assigned to a depot if there is a route connecting them. Equation (12) relates departure and arrival time of each vehicle. Constraints (13)-(19) compute the third objective. These constraints are similar to Equation (26). Constraints (20)-(22) state the Boolean nature of the decision variables and Constraint (23) is also Boolean variable requires to calculating third objective. Constraint (24) is continuous variable indicates each customer servicing start time by each vehicle. Finally, Constraint (25) is a continuous variable needed to compute the third objective.

$$\text{Demand score} = \begin{cases} 1 - \dfrac{t_{jk}}{a_j} & t_{jk} < a_j \\ \dfrac{T - t_{jk}}{T - b_j} & t_{jk} > b_j \\ 1 & \text{Otherwise} \end{cases} \tag{26}$$

4. MULTI-OBJECTIVE ALGORITHMS

The proposed algorithm is applied to the generated multi-objective test problems and its performance compared with the three well-known meta-heuristics used in the LRP literature. These algorithms are named as multi-objective simulated annealing (MOSA), multi-objective tabu search (MOTS) and multi-objective meta-heuristic algorithm using an adaptative memory procedure (MOAMP), respectively.

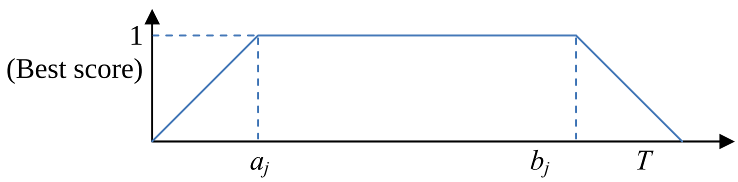

Figure1 Graph of the score function for customer *j*

4.1. Multi-objective simulated annealing

The simulated annealing (SA) algorithm is a well-known meta-heuristic algorithm for finding the optimal or near-optimal solution of a given function in a large search space. Ulungu*et al.* (1999) introduced the multi-objective simulated annealing (MOSA), which has some advantages over evolutionary algorithms because it does not need large memory to keep the population and can find a small group of Pareto solutions in a shorter time. Figure 2 shows the procedure of MOSA, where *S* represents the current search solution and *T* is the temperature parameter that is gradually decreased as time goes on. A new search solution *S'* is generated by the *N(s)* function, whose cost is evaluated and compared with the previous cost. When it is determined to be a good solution, the new state is accepted. Even when the new solution is not proper, it is accepted with an acceptance probability. When there is no superiority between the current and the next state, the new one is accepted to do search in spread search space and to escape from local optima.

The general transition rules, such as the Metropolis or logistic method, cannot be applied directly to the multi-objective problems because they support only a scalar cost function. The transition rule suggested in this paper is very similar to the Metropolis method as shown in Equation (27), where $C(i,j)$ is the cost criterion for transition from state i to j, and T is the annealing temperature. Equation (28) is the cost criterion, where $c_k(i)$ is the k-th cost value in the objective vector of the i-th solution.

$$P_t(i,j) = \exp\left(-\frac{C(i,j)}{T}\right) \tag{27}$$

$$C(i,j) = Min_k\ (\ C_k(j) - C_k(i)\ ) \tag{28}$$

S = Initial solution
T = Initial temperature
Repeat
  Generate a neighbor *S'* = *N*(*S*)
  IF *C*(*S'*) dominates *C*(*S*)
    Move to *S'*
  Else IF *C*(*S*) dominates *C*(*S'*)
    Move to *S'* with Transition Probability
  Else (C(S) and C(S') do not dominate each other**)**
    Move to *S'*
  End IF
  T = Annealing (*T*)
END Repeat (until the termination are satisfied)

Figure 2 Pseudo code of the MOSA

4.2. Multi-objective tabu search

Tabu search (TS) is a local search algorithm based meta-heuristic method, whose main idea is to avoid recently visited parts of the solution space and to guide the search towards new and promising areas (Gover & Kochenberger, 2003).

The main obstacle for TS in multi-objective optimization is its inability to find multiple solutions. However by using the procedure mentioned in the previous section, MOTS can find a small group of Pareto solutions and then find more solutions by repeating the trials for detailed information about the Pareto frontier.

4.3. MOAMP algorithm

The multi-objective meta-heuristic algorithm using an adaptative memory procedure (MOAMP) introduced by Caballero *et al*. (2007) is for the resolution of multi-objective combinatorial optimization (MOCO) problems based on TS. The algorithm includes two different phases: (1) generating an initial set of efficient points through various tabu searches and (2) looking for efficient points with an intensification process around an initial set of efficient points. It is a well-known fact that the efficient points of MOCO problems are “connected”. In other words, any efficient point is close enough to another efficient point. This proximate optimality is the principle point in the MOAMP. Figure 3 depicts a pseudo code of the MOAMP.

```
S = Initial solution
Repeat
        Optimize 1st objective with TS
        Optimize 2nd objective with TS
        ⋮
        Optimize pth objective with TS
        Optimize 1st objective with TS
        Repeat
                Optimize Eq. (29) with TS
        END Repeat (N iterations)
END Repeat (until no change in the NDS list)
```

Figure 3 Pseudo code of the MOAMP

The first phase consists of linking p+1 tabu searches (i.e., the last point of one search becomes the initial point of the next search). The first TS starts from a random solution and attempts to find an optimal solution to the problem with the single objective $f_1(x)$. Let x^1 be the last point visited at the end of this search. Then, another tabu search is applied again to find the best solution to the problem with the second objective $f_2(x)$ using x^1 as its initial solution. This process is repeated until all the single-objective problems associated with the p objectives have been solved. At this point, we again solve the problem with the first objective $f_1(x)$ starting from x^p in order to complete a cycle. This phase yields the p points that approximate the best solutions to the single-objective problems, in which they are non-dominated solutions that result from ignoring all but one objective function. During this phase, we also collect other non-dominated solutions (NDS).

The second phase of the MOAMP explores the search space around the initial set of efficient points found in the first phase. In this phase N random weighting vectors $\lambda=(\lambda_1 , \dots , \lambda_p)$ generated and use these to make N tabu searches with the following objective function which should be minimized:

$$\min \quad F_\lambda(x) = max\left\{\lambda_i\left(\frac{f_i^{max} - f_i(x)}{f_i^{max} - f_i^{min}}\right);\ i = 1,2,\dots,p\right\} \tag{29}$$

where

f_i^{max} is the maximum value of the i-th objective over NDS obtained up to now.

f_i^{min} is the minimum value of the i-th objective over NDS obtained up to now.

N represents maximum number of tabu searches that could be carried out without any change in the NDS list.

Finally, at the end of these (p+1)+N tabu searches, the algorithm obtain a sample of the non-dominated solutions distributed by the areas where one of the objectives is predominant, as well as for those areas characterized by a balance among the different objectives. The process is repeated until this intensification no longer offers any new NDS.

5. PROPOSED ALGORITHM

The design of the proposed algorithm is based on the MOAMP. The algorithm is a meta-heuristic for the resolution of MOCO problems based on SA. We call the proposed algorithm as MOASA (i.e., multi-objective adaptative simulated annealing) that tries to adapt the SA search procedure to the structure of the efficient set of a multi-objective problem.

MOASA consists of linking p+2 simulated annealing searches. Like MOAMP, first search starts from a random solution and attempts to find the optimal solution to the problem with the single objective $f_1(x)$. Then, other SA algorithms are applied again to find the best solution of the p problems. At this point, we again solve the problem with the first objective, but using a final solution as an initial solution. After this cycle, a multi-objective SA algorithm is applied to explore the search space around the initial set of efficient points found in p+1 searches. The process is repeated until this intensification no longer offers any new NDS.

MOASA applied N-1 less searches compare with MOAMP so it has less computational process. The procedure of MOASA is shown in Figure 4.

```
S = Initial solution
Repeat
        Optimize 1st objective with SA
        Optimize 2nd objective with SA
        ⋮
        Optimize pth objective with SA
        Optimize 1st objective with SA
        Apply MOSA
END Repeat (until no change in the NDS list)
```

Figure 4 Pseudo code of the MOASA

6. PERFORMANCE METRICS

In the recent years, many multi-objective optimization algorithms (MOOA) have been proposed and many metrics of the algorithm performances have been proposed as well.Deb (2001) classified the existing performance metrics into three classes for, namely convergence, diversity and both convergence and diversity.

Zitzler *et al*. (2000) suggested three goals for MOOA that can be identified and measured by:

1- The distance of the resulting NDS to the true Pareto front should be minimized.
2- A good distribution of the obtained NDS is desirable.
3- The size of the obtained NDS should be maximized (i.e., a wide range of values should be covered by NDS for each objective).

To compare the performance of the algorithms, two convergence metrics and two diversity metrics are applied.

6.1 Convergence metrics

The convergence metrics evaluate how obtained solutions are far from the true Pareto front. Many metrics for measuring the convergence of a set of approximation NDS towards the Pareto front have been proposed. We select two well-known of them. First one is the error ratio (ER) proposed by Veldhuizen (1999). Let A = $\{e_1, e_2, \ldots, e_n\}$ an approximation NDS set; $e_i = 0$ if solution i is in the true Pareto front, and $e_i = 1$ otherwise. The metric uses the true Pareto front as a reference set; so we assume that true Pareto front is the total Pareto-optimal solutions in a combined pool of all approximation NDS obtained from all runs of multi-objective algorithms. This metric is given by:

$$ER = \frac{\sum_{i=1}^{n} e_i}{n} \tag{30}$$

where n is the number of solutions in the approximation NDS. Lower values of the ER are preferable.

The second metric is the coverage of two sets (CS) introduced by Zitzler (1999). Using the metric CS, two sets of NDS can be compared to each other. Let P_1 and P_2 be the sets of approximation NDS obtained from one run of Algorithms **A** and **B**, respectively, and P be the union of the sets of approximation NDS (i.e., $P = P_1 \cup P_2$) so that it includes only NDS. The function CS maps the ordered pair (P_1, P_2) into the interval [0, 1]:

$$CS(P_1, P_2) = \frac{|\{p_2 \in P_2 / \exists p_1 \in P_1 : p_1 \succcurlyeq p_2\}|}{|P_2|} \tag{31}$$

where $P_1 \succcurlyeq P_2$means that the solutionP_1 is dominated by the solutionP_2.

The value CS(P_1, P_2) = 1 means that all the solutions in P_2 are dominated by P_1. The value CS(P_1, P_2) = 0 represents the situation when none of the points in P_2 are dominated by P_1. Note that CS(P_1, P_2) is not necessarily equal to 1–CS(P_2, P_1).

6.2 Diversity metrics

The diversity metrics evaluate the scatter of solutions in the final population on the Pareto front. Like the convergence metrics, many metrics for measuring the diversity of a set of approximation NDS towards the Pareto front have been proposed. We select the spacing metric (SM) and maximum spread metric (MS) to evaluate the applied algorithms. Schott (1995) introduced the spacing metric that provides a measure of uniformity of the spread of approximation NDS. This metric is given by:

$$SM = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\bar{d} - d_i)^2} \tag{32}$$

where

$$d_i = \min_{j \in NDS \wedge j \neq i} \sum_{k=1}^{K} \left| f_k^i - f_k^j \right| \tag{33}$$

and$\bar{d}$ is the mean of all d_i, n is the size of obtained NDS and f_k^i is the function value of the k-

thobjective function for solution *i*. The lower values of the SM are preferable. Note that the objective functions should be normalized.

Zitzler (1999) proposed the maximum spread metric that shows the distance between the boundary solutions in the obtained NDS. Deb (2001) suggested that maximum spread metric can be misleading if the objective functions have different ranges, so the normalized MS has been introduced, in which each function values is normalized with their range. This metric is given by:

$$MS = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\left[\left(\frac{\max_{i\in NDS} f_k^i - \min_{i\in NDS} f_k^i}{F_k^{max} - F_k^{min}}\right)\right]^2} \quad (34)$$

where n is the size of the obtained NDS, K is the number of objectives, f_k^i is the function value of the k-th objective function, F_k^{max} is the maximum value of the k-th objective in the true Pareto front and F_k^{min} is the minimum value of the k-th objective in the true Pareto front. The values closer to 1 are preferable.

7. COMPUTATIONAL STUDY

The proposed algorithms have been coded in Matlab 7.1 and run on a Think Pad Lenovo computer with a 2.2 GHz Pentium(R) Dual-Core CPU with 2.00GB of RAM. Four test problems are generated as shown in Table 3. For each problem, a random initial solution is constructed and all the algorithms are applied.

Table 1 Test problems

Test Problem	**Customer**	**Depot**
1	8	2
2	12	2
3	15	3
4	20	3

7.1 Simulation results

Table 4 illustrates the average values of the ER, SM and MS metrics that outcome from 10 runs for each algorithm and test problem. Figure5 shows the stock chart based on the CS metric. This chart consists of a line, in which the upper and lower ends of the line are maximum and minimum values while the average value is marked on the line. In this figure, each rectangle box contains four stock plots representing the distribution of the CS metric for a certain ordered pair of the algorithms. The first stock plot of the left side relates to the test problem 1 and the right side relates to the test problem 4. The scale is 0 at down and 1 at top. Each rectangle box relates to CS(*A*,*B*), in which *A* and *B* refer to the algorithm corresponding to the row and column, respectively.

Table 2 Average of three performance metrics

Test Problem		1	2	3	4
ER	MOTS	0.038	0.899	0.763	0.904
	MOSA	0.031	0.592	0.729	0.934
	MOAMP	0.421	0.375	0.359	0.301
	MOASA	0.027	0.293	0.298	0.219
SM	MOTS	0.065	0.023	0.031	0.012
	MOSA	0.048	0.022	0.037	0.011
	MOAMP	0.035	0.026	0.013	0.006
	MOASA	0.046	0.020	0.011	0.005
MS	MOTS	0.931	0.897	0.886	0.827
	MOSA	0.987	0.911	0.927	0.899
	MOAMP	1.026	0.977	1.004	0.952
	MOASA	0.989	0.966	0.962	1.008

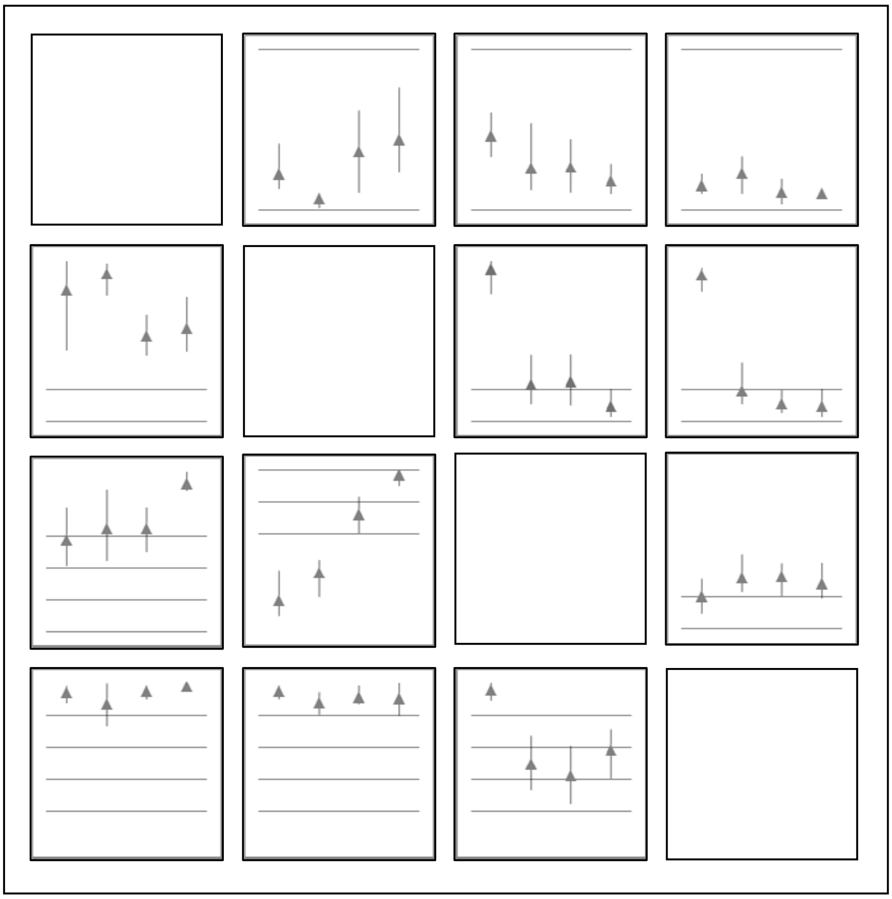

Figure 5 Stock plots based on the CS(*A*, *B*) metric:, where Row:*A* and Column:*B*

7.2 Comparative results

The ER metric shows that MOASA can truly achieve the NDS than other algorithms and MOAMP is superior to the other two algorithms. The stock plots show that MOASA outperforms the other algorithms. Furthermore, it can be observed that MOAMP performs well the other two algorithms. Considering the two convergence metrics show us our proposed MOASA algorithm is completely superior to MOAMP, and MOAMP is completely superior to MOSA and MOTS. It is remarkable that MOSA performs better comparing to MOTS.

The SM and MS metrics indicate all the algorithms find the NDS with good diversity. However, the results show that MOASA may be slightly superior to the other algorithms. Fig. 6 shows the difference value of the MS metric from a target value of one. Fig. 7 illustrates the SM values on the line chart and shows that there is not a significant difference between the index values between all the considered algorithms.

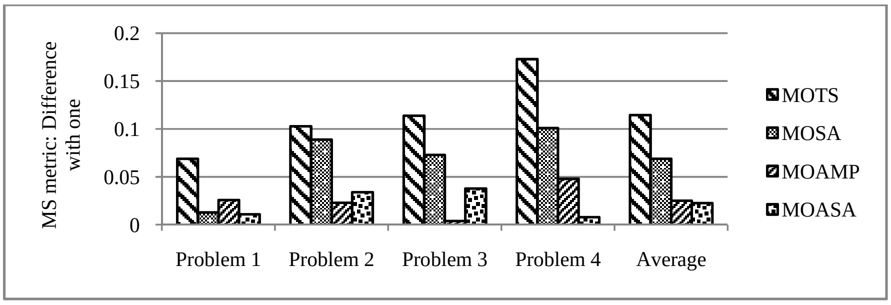

Figure 6 MS metric: Difference with one

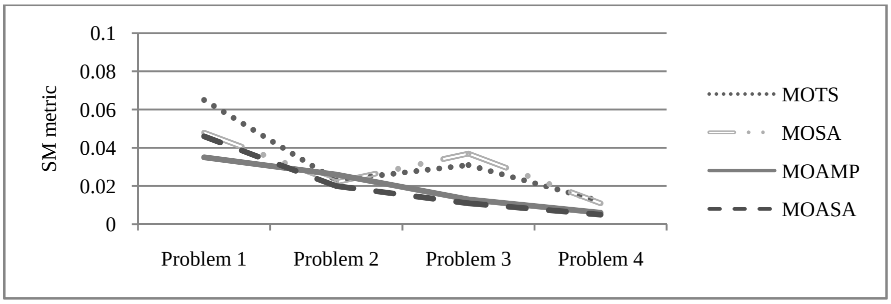

Figure 7 SM metric for all the considered algorithmes

8. CONCLUSION

In this paper, we have studied a multi-objective location-routing problem (LRP) in a general form and a new mathematical formulation has been presented. Moreover, a new multi-objective algorithm, called MOASA, has been proposed in this paper. The comparison of the computational results from four hypothetical test problems with other existing optimization algorithms, namely MOAMP, MOSA and MOTS, shows the efficiency of the proposed MOASA algorithm for multi-objective LRP. According to the measuring indices, we have realized that a hierarchy of the algorithms in a descending order of merit can be mentioned as the proposed MOASA, MOAMP, MOSA, and MOTS. Using the elitism strategy for the proposed MOASA algorithm in the LRP is recommended for future research.

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