

Model and Solution Approach for Multi objective-multi commodity Capacitated Arc Routing Problem with Fuzzy Demand

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ABSTRACT

The capacitated arc routing problem (CARP) is one of the most important routing problems with many applications in real world situations. In some real applications such as urban waste collection and etc., decision makers have to consider more than one objective and investigate the problem under uncertain situations where required edges have demand for more than one type of commodity. So, in this research, a new fuzzy chance constrained programming model based on credibility measure for CARP with two objectives: minimizing the number of vehicle and minimizing the total travel cost is formulated. In this model each required edge has demand for more than one type of commodity and also all demands for each commodity are supposed to be triangular fuzzy numbers. Then we develop a multi-objective genetic algorithm using the Pareto ranking technique and hybrid it with stochastic simulation to design an intelligent algorithm to solve the fuzzy chance constrained model. In order to improve the quality of final solutions, we also propose a new heuristic method to generate a good initial solution in initial population of genetic algorithm. Some data sets with fuzzy demand generated randomly are used to evaluate and investigate key characteristics of the new proposed model and solution approach.

Keywords: Multi objective capacitated arc routing problem; Fuzzy chance constrained programming; Stochastic simulation; Genetic algorithm; Pareto ranking; Heuristic.

1. INTRODUCTION

The capacitated arc routing problem (CARP) is one of the most important routing problems in literature, and attracted interest of many researchers. It has numerous applications in real world situations such as urban waste collection (Dijkgraaf and Gradus, 2007), winter gritting (Eglese and Li, 1992), snow removing (Labelle *et al.*, 2002), inspection of gas pipeline (Han *et al.*, 2004), street sweeping (Tobin and Brinkmann, 2002), and electric meter reading (Dror and Stern, 1979). The CARP was first introduced by Golden and Wong (1981) and deals with connected and undirected graph $G = (V, E)$, where “ V ” is the set of vertices (nodes) and “ E ” is the set of edges. Each edge of “ E ” has a definite travel cost and some edges have positive demand, called required edges ($E_R \subseteq$

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E), which must be serviced by some vehicles with limited capacity. All vehicles are identical and located at a single depot. The aim of CARP is to design a set of vehicle tours of minimum total routing cost such that each tour starts and ends at the depot, each required edge is serviced by exactly one vehicle, and the total demand serviced by any vehicle must not exceed the vehicle's capacity. Some instances (in small and medium size) of CARP can be solved by exact methods. But CARP is a NP-hard problem (Golden and Wong, 1981) and exact methods are not able to solve the large-scale instances in reasonable time. So, due to the computational complexity of the problem, the remarkable attempts have been done by researchers in developing heuristics and metaheuristics algorithms to solve it. Among known heuristics, one can refer to Augment-Merge (Golden and Wong, 1981), Path-Scanning (Golden *et al.*, 1983), Double Outer Scan (Wöhlk, 2005), and Ulusoy's splitting heuristic (Ulusoy, 1985). Some of metaheuristics are also proposed such as tabu search (Hertz *et al.*, 2000), guided local search (Beullens *et al.*, 2003), variable neighborhood search (Hertz and Mittaz, 2001), tabu scatter search (Greistorfer, 2003), and memetic algorithms (Lacomme *et al.*, 2004).

The multi-objective CARP (MOCARP) has received sparse attention in the literature compared to the single-objective. In some real world applications, more than one objective must be taken into account. For example, a waste collection company is interested in to minimize the number of vehicles used to service the required edges, moreover minimizing the total distance cost. Minimizing the number of vehicles affects vehicle and manpower costs, while minimizing distance affects time and fuel resource. Therefore, CARP is a multi-objective problem naturally. The multi-objective CARP was first studied by Lacomme *et al.* (2006). While considering two objectives, minimizing the total distance cost and minimizing the makespan, they combine nondominated sorting genetic algorithm II (NSGA-II) (Deb *et al.*, 2002), and their proposed approach in Lacomme *et al.* (2004) to solve the problem. Mei *et al.* (2009) investigated MOCARP for same objectives in Lacomme *et al.* (2006), and proposed a hybrid algorithm by combining the memetic algorithm with extended neighborhood search (MAENS) (Tang *et al.*, 2009) with a decomposition-based method. Minimizing three objectives including the total distance cost, the longest route (makespan) and the number of vehicles simultaneously, were studied by Grandinetti *et al.* (2010), and they also provided an optimization-based heuristic procedure. Fleury *et al.* (2005) proposed a memetic algorithm for the MOCARP with two objectives (minimizing total travel cost and minimizing makespan) and stochastic demands.

In most of previous researches about CARP, all of the problem information including demand values, travel times, etc were assumed deterministic. But in real world situations such as urban waste collection, some of parameters are often non-deterministic and not precise enough. The non-deterministic parameters can be stated as either stochastic or fuzzy. A stochastic case occurs whenever some parameters are random and have a known distribution while fuzzy case occurs whenever some parameters are uncertain, subjective and ambiguous and do not have known distribution. Stochastic CARP was considered in few literatures but to the author's knowledge, fuzzy CARP (FCARP) has not yet been studied. Fleury *et al.* (2004) considered stochastic CARP with Normal distributed demands. Their solution approach is based on hybrid genetic algorithm that is modified version of the memetic algorithm earlier studied in Lacomme *et al.* (2004). Stochastic CARP with Poisson distribution was studied by Christian *et al.* (2009). They developed an exact method for solving the problem and formulated the stochastic CARP as a Set Partitioning Problem and developed a Branch-and-Price algorithm, in which the pricing is done by dynamic programming. Also, Mei *et al.* (2010) investigated uncertain CARP with stochastic demands. They design robust solutions for uncertain environments.

More recently Muyltermans and Pang (2010) introduced the multi-compartment capacitated arc

routing problem where the required edges have a demand for different products, and multi-compartment vehicles are available to co-distribute these commodities. They developed a guided local search algorithm to solve it.

In this paper in order to handle more realistic situations, we investigate the multi-objective capacitated arc routing problem with multi-commodities in assumption that the demands are uncertain and cannot be determined specifically and are regarded as fuzzy variables. The assumed objectives are minimizing the total distance cost and minimizing the number of vehicles. We will present a fuzzy chance constrained program (fuzzy CCP) based on credibility measure and then for building efficient solution method, we will provide a multi-objective genetic algorithm (a powerful algorithm) solution using the Pareto ranking technique, and hybrid it with stochastic simulation to design an intelligent algorithm to solve the fuzzy chance constrained model. Finally, the designed model is a generic and applicable in refuse collection, snow removal, etc.

The remainder of this paper is organized as follows: In section 2, we will give an overview of multi-objective optimization search, and some basic concept on fuzzy set theory. In section 3, we will present a fuzzy CCP model to deal with the problem and develop a mathematical model of multi objective-multi commodity CARP with fuzzy demands (MOMC-CARPFDP). The stochastic simulation and integrating it with genetic algorithm to design an intelligent algorithm to solve the proposed model is described in section 4. The computational results are devoted in section 5 and followed by concluding remarks and directions for future research in section 6.

2. BACKGROUND

2.1. Multi-objective optimization and Pareto ranking

A multi-objective optimization problem (MOP) is one in which two or more objectives contribute to the overall result. These objectives are conflict with each other, and optimizing a particular solution concerning a single objective can result in unacceptable results concerning the other objectives. The challenge is to find a set of solutions for them, which yield an optimization of the overall problem close at hand. Without loss of commonality, a multi-objective optimization problem can be briefly described as a minimization problem with n decision variables x and k objectives of the form:

$$\begin{aligned} \min f(x) &= (f_1(x_1, x_2, \dots, x_n), \dots, f_k(x_1, x_2, \dots, x_n)) \\ \text{s.t.} & \\ & x = (x_1, x_2, \dots, x_n) \in X \\ & f = (f_1, f_2, \dots, f_k) \in F \end{aligned}$$

Here x is decision vector, X is decision space, f is objective vector, and F is objective space. A decision vector $x \in X$ is said to dominate a decision vector $y \in X$, if and only if:

$$\forall i \in \{1, 2, \dots, k\} : f_i(x) \leq f_i(y) \wedge \exists j \in \{1, 2, \dots, k\} : f_j(x) < f_j(y)$$

Also, the decision vector $x \in X$ is said to be nondominated, if and only if: $\nexists y \in X : y > x$.

A decision vector $x \in X$ is Pareto-optimal if there is no other decision vector $y \in X$ that dominate x . The set of all Pareto-optimal solutions is called the Pareto-optimal front. Many optimization techniques such as evolutionary algorithm (EA) have been proposed in the literature to solve MOP (Veldhuizen, 1999; Coello, 1999; Veldhuizen and Lamont, 2000). To find Pareto optimal front,

often has been used Pareto ranking technique based genetic algorithm. This technique was first proposed by Goldberg (1989), as a means of assigning equal probability of procreation to all non-dominated individuals (solutions) in the population. The method comprise of assigning rank 1 to the non-dominated individuals and removing them from competition, then finding a new set of non-dominated individuals, ranked 2, and so on. The individuals within rank 1 are the best in the current population. The individuals in each rank set represent solutions that are in some sense incomparable with one another. Individuals that are clearly superior to others in all dimensions of the problem are characterized by Pareto ranking. Note that the objective function itself no more qualifies as fitness function since it is vector valued and fitness has to be a scalar value. Thus, Pareto ranking scheme is combined into a genetic algorithm by replacing the chromosome fitnesses with Pareto ranks.

2.2. Fuzzy sets and credibility measure

The concept of fuzzy set was introduced by Zadeh (1965), and then it has been well developed and applied in a wide variety of real problems. Fuzzy set theory generalizes 0 and 1 membership values of a crisp set to a membership function of a fuzzy set. In order to measure a fuzzy event, possibility measure theory of fuzzy variable was suggested by Zadeh (1978). Lately, Liu (2006) developed credibility theory.

Definition: Let q be a fuzzy variable with membership function μ , and let u and r be real numbers. The possibility of a fuzzy event, characterized by $q \leq r$, is defined by

$$Pos\{q \leq r\} = \sup_{u \leq r} \mu(u), \tag{1}$$

While the necessity of $q \leq r$ is defined as follow:

$$Nec\{q \leq r\} = 1 - Pos\{q > r\} = 1 - \sup_{u > r} \mu(u). \tag{2}$$

Then, the credibility measure Cr is an average of the possibility measure and the necessity measure (see Liu (2006)), i.e.

$$Cr\{q \leq r\} = \frac{1}{2}(Pos\{q \leq r\} + Nec\{q \leq r\}) \tag{3}$$

Clearly, the fuzzy event must hold if its credibility is 1, and fail if its credibility is 0. Credibility measure is self-dual while possibility measure is not. However, for a measure, the self-dual property is very important. When the credibility value of a fuzzy event obtains 1, the fuzzy event will certainly happen; but when the possibility value of a fuzzy event obtains 1, the fuzzy event can fail to happen.

For example, consider with triangular fuzzy variable $\tilde{q} = (q_1, q_2, q_3)$ as edge demand whose membership function is given as follow:

$$\mu(r) = \begin{cases} \frac{r-q_1}{q_2-q_1} & \text{if } q_1 \leq r \leq q_2 \\ \frac{r-q_3}{q_2-q_3} & \text{if } q_2 \leq r \leq q_3 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

From the definitions of possibility, necessity and credibility it is easy to obtain:

$$Pos\{q \leq r\} = \begin{cases} 0 & \text{if } r \leq q_1 \\ \frac{r-q_1}{q_2-q_1} & \text{if } q_1 \leq r \leq q_2 \\ 1 & \text{if } r \geq q_2 \end{cases} \quad (5)$$

$$Nec\{q \leq r\} = \begin{cases} 0 & \text{if } r \leq q_2 \\ \frac{r-q_2}{q_3-q_2} & \text{if } q_2 \leq r \leq q_3 \\ 1 & \text{if } r \geq q_3 \end{cases} \quad (6)$$

$$Cr\{q \leq r\} = \begin{cases} 0 & \text{if } r \leq q_1 \\ \frac{r-q_1}{2(q_2-q_1)} & \text{if } q_1 \leq r \leq q_2 \\ \frac{q_3-2q_2+r}{2(q_3-q_2)} & \text{if } q_2 \leq r \leq q_3 \\ 1 & \text{if } r \geq q_3 \end{cases} \quad (7)$$

3. MODELING

3.1. Fuzzy CCP (FCCP) model

In fuzzy mathematical programming, an optimization problem is defined as follows:

$$\begin{aligned} & \min_x f(x, \xi) \\ & \text{s.t.} \\ & \quad g_i(x, \xi), \quad i = 1, 2, \dots, p \end{aligned} \quad (8)$$

Where x is the decision variable set, ξ is the set of uncertain fuzzy parameters, g_i is i th constraint of the model and f is the objective function. Similarly, a chance constrained optimization problem can be defined here as

$$\begin{aligned} & \min_x f(x) \\ & \text{s.t.} \\ & \quad \Pr(g_i(x, \xi) \leq 0) \geq \alpha_i, \\ & \quad i = 1, 2, \dots, p \end{aligned} \quad (9)$$

Where $f(x)$ is the objective function, x is the set of decision variables, ξ is the set of random parameters, \Pr is the probability measure of the given probability space of the uncertain parameter and $\alpha \in (0, 1)$ is the probability level with which each of the p constraints $g_i(x, \xi) \leq 0$ of the entire constraint set $g(x, \xi) \leq 0$ needs to be satisfied.

The development of the chance constrained programming in a fuzzy environment can be defined as follows:

$$\begin{aligned} & \min_x f(x) \\ & \text{s.t.} \\ & \quad Cr\{\xi | g_i(x, \xi) \leq 0\} \geq \alpha_i, \end{aligned}$$

$$i = 1, 2, \dots, p \tag{10}$$

Where α_i are premeditated confidence levels to the respective constraints, $Cr\{\cdot\}$ denotes the credibility of the event in $\{\cdot\}$. This means a point x is feasible if and only if the credibility measure of the set $\{\xi | g_i(x, \xi) \leq 0\}$ is at least α . (Mitra, 2009)

3.2. Assumptions and mathematical model of MOMC-CARPFD

Let $G = (V, E)$ be a connected and undirected graph, where $V = \{1, 2, \dots, n\}$ is the set of nodes and $E = \{(i, j) | i, j \in V, i < j\}$ is the set of edges. Each edge is represented in two directions; positive (from i to j) and negative (from j to i) directions that so called arcs. Each edge (i, j) has a nonnegative cost or length c_{ij} , and a fuzzy demand represented by triangular fuzzy number $\tilde{q}_{ij}^m = (q_{ij}^{1m}, q_{ij}^{2m}, q_{ij}^{3m})$ for each commodity $m (m = 1, 2, \dots, M)$. The edges with positive demand (for at least one commodity) are called the required edges or tasks, $E_R \subseteq E$. Node 1 denotes the depot. A fleet of identical vehicles are located at the depot, also each vehicle has a dedicated compartments with limited and fixed capacity $Q^m (Q^m \geq q_{ij}^m)$ for each commodity m . Each vehicle tour starts and ends at the depot, and each required edge is serviced exactly once i.e. just in one of its directions. The total vehicle load for commodity m at any time should not exceed the definite capacity Q^m . The objective of MOMC-CARPFD is to determine a set of tours of all required edges such that the following objectives linked to economic consideration are met:

- Minimize the total distance cost
- Minimize the total number of vehicles used to service

The capacity of vehicle k can also be represented as a triangular fuzzy number $\tilde{Q}_k^m = (Q_k^m, Q_k^m, Q_k^m)$. Let the fuzzy number presenting demand at the r th required edge for commodity m , denoted by q_{rk}^m . After serving the first l tasks (required edges), the available capacity of the vehicle k for each commodity, will equal:

$$D_{lk}^m = Q_k^m - \sum_{r=1}^l q_{rk}^m \tag{11}$$

Using the rules of fuzzy arithmetic proposed by Kaufmann and Gupta (1985) we can easily show that D_{lk}^m is also a triangular fuzzy number, i.e.

$$D_{lk}^m = (Q_k^m - \sum_{r=1}^l q_{rk}^{3m}, Q_k^m - \sum_{r=1}^l q_{rk}^{2m}, Q_k^m - \sum_{r=1}^l q_{rk}^{1m}) = (d_{lk}^{1m}, d_{lk}^{2m}, d_{lk}^{3m}) \tag{12}$$

Therefore, the credibility or chance that the next required edge demand does not surpass the remaining capacity of the vehicle is founded as following:

$$Cr = cr\{q_{(l+1)k}^m \leq D_{lk}^m\} = Cr\{(q_{(l+1)k}^{1m} - d_{lk}^{3m}, q_{(l+1)k}^{2m} - d_{lk}^{2m}, q_{(l+1)k}^{3m} - d_{lk}^{1m}) \leq 0\}$$

$$= \begin{cases} 0 & \text{if } q_{(l+1)k}^{1m} \geq d_{lk}^{3m} \\ \frac{d_{lk}^{3m} - q_{(l+1)k}^{1m}}{2*(d_{lk}^{3m} - q_{(l+1)k}^{1m} + q_{(l+1)k}^{2m} - d_{lk}^{2m})} & \text{if } q_{(l+1)k}^{1m} \leq d_{lk}^{3m}, q_{(l+1)k}^{2m} \geq d_{lk}^{2m} \\ \frac{q_{(l+1)k}^{3m} - d_{lk}^{1m} - 2*(q_{(l+1)k}^{2m} - d_{lk}^{2m})}{2*(d_{lk}^{2m} - q_{(l+1)k}^{2m} + q_{(l+1)k}^{3m} - d_{lk}^{1m})} & \text{if } q_{(l+1)k}^{2m} \leq d_{lk}^{2m}, q_{(l+1)k}^{3m} \geq d_{lk}^{1m} \\ 1 & \text{if } q_{(l+1)k}^{3m} \leq d_{lk}^{1m} \end{cases} \tag{13}$$

It is obvious that $Cr \in [0,1]$.

The vehicle tours are designed in advance by using the proposed algorithm and the actual demand becomes known upon arrival at the edge. In ordering to formulation (13), deciding whether the vehicle services the next edge or not, depends on dispatcher preference size, denoted by Cr^* . Parameter Cr^* states the dispatcher's policy toward risk. Lower values of Cr^* show the dispatcher preference to use vehicle's capacity as much as possible, consequently the number of cases in which the vehicle fails to service the next required edge due to small remaining capacity (for at least one commodity), are increased. But then if dispatcher be risk averse, he/she chooses a greater Cr^* , hence the failure risk decreases; however the vehicle's loading becomes smaller. Hence, according to the credibility value, a decision must be made as to whether to send vehicle to service the next required edge or comeback it to depot. In other words if $Cr \geq Cr^*$, then the vehicle should be dispatch to the next edge; else, it should be comeback to the depot, and dispatch other vehicle to the next edge. In this paper we assume that once the vehicle arrives at the required edge and its remaining capacity is not enough to service that edge (even for one commodity), it should makes a back trip to the depot to deplete and returns to the edge where it had failure and continues service along the rest of its planned tour. It is obvious there is additional cost (distance) due to tour failure.

Let x_{ijk} and y_{ijk} be two decision variables that can be described as following:

$$x_{ijk} = \begin{cases} 1 & \text{If vehicle } k \text{ services arc}(i, j) \\ 0 & \text{Otherwise} \end{cases}$$

$$y_{ijk} = \begin{cases} 1 & \text{If arc}(i, j) \text{ is deadheaded by vehicle } k \\ 0 & \text{Otherwise} \end{cases}$$

It is supposing that there is no limitation on the number of vehicles that can be used, but to facilitate the model formulation, the maximum possible size of the fleet is symbolized by K . The actual number of vehicles will be found after solving the model that it would be equal to the number of routes.

The related fuzzy chance constrained model of MOMC-CARPF based on credibility measure can be mathematically formulated as shown below:

$$\min \sum_{k \in K} (\sum_{(i,j) \in E_R} c_{ij} x_{ijk} + \sum_{(i,j) \in E} c_{ij} y_{ijk}) + c' \quad (14)$$

$$\min K \quad (15)$$

s.t.

$$\sum_{k \in K} (x_{ijk} + x_{jik}) = 1, \quad \forall (i, j) \in E_R \quad (16)$$

$$Cr(\sum_{i \in V} \sum_{j \in V} q_{ij}^m x_{ijk} \leq Q_k^m) \geq Cr^*, \quad \forall k \in K, \forall m \in M \quad (17)$$

$$\sum_{k \in K} x_{ijk} = |E_R|, \quad \forall (i, j) \in E_R \quad (18)$$

$$\sum_{i \in V} y_{ijk} = \sum_{i \in V} y_{jik}, \quad \forall j \in V, \forall k \in K \quad (19)$$

$$\sum_{i \in V} y_{1ik} + \sum_{i \in V} y_{i1k} = 2, \quad \forall k \in K \quad (20)$$

$$\sum_{(i,j) \in \delta_R(S)} x_{ijk} + \sum_{(i,j) \in \delta(S)} y_{ijk} \geq 2x_{(p,q)k}, \quad \forall S \subseteq V \setminus \{1\}, (p, q) \in E_R(S), k \in K \quad (21)$$

$$x_{ijk} \in \{0, 1\}, \forall (i, j) \in E_R, k \in K \quad (22)$$

$$y_{ijk} \in Z_0^+, \forall (i, j) \in E, k \in K \quad (23)$$

The objective function (14) states that total routing costs and total fail cost should be minimized. The objective function (15) shows that the number of vehicles or routes should be minimized. Constraints (16) ensure that each task is served only by one vehicle. Chance constraints (17) assure that all required edges are visited within vehicle capacity for each commodity with a predefined confidence level. All required edges must be serviced. This constraint is met by (18). Constraints (19) are balance constraints. Constraints (20) assure that each vehicle tour start from and end at the depot. Sub-tours elimination, is satisfied by constraints (21), where $\delta(S) = \{(i, j) \in E : i \in S, j \notin S\}$, $\delta_R(S) = \{(i, j) \in E_R : i \in S, j \notin S\}$ and $E_R(S) = \{(i, j) \in E_R : i \in S, j \in S\}$. Constraints (22) and (23) define the domain of the decision variables.

4. DESCRIPTION OR THE SOLUTION METHODOLOGY

Because of uncertainty extension in constraints of the proposed model, it would be difficult to solve this challenging problem in traditional ways. In this section, we design a hybrid intelligent algorithm integrating stochastic simulation and genetic algorithm (GA) to solve the above proposed fuzzy CCP model. GA has been discussed in many literatures, such as Gen and Cheng (2000). It is a stochastic search method based on the principle of survival of the fittest. Also GA is highly robust and avoids getting stuck at a local optimal solution. In this study, the technique of stochastic simulation is applied to simulate the actual value of demand of required edges for each commodity in order to compute the planned route costs and additional costs due to tour failure. Then stochastic simulation and genetic algorithm based on Pareto ranking are integrated to produce a hybrid intelligent algorithm for solving the model. In continuance we will describe the proposed solution methodology for solving the MOMC-CARPF, and present a new heuristic for generating the initial good solution and a solution similarity measure that enables the algorithm to provide a better estimation to the full Pareto front.

4.1. Design of stochastic simulation

As mentioned before, the demands of the required edges are considered as triangular fuzzy number, and the actual demand of each edge becomes known upon arrival at it. Therefore we use a stochastic simulation algorithm for each feasible planned tour in order to simulate the actual demand of each required edge and to compute approximate additional costs (c') due to tour failure. The stochastic simulation algorithm can be summarized as follows:

Step 1: the actual demand of each required edge for each commodity is generated by following processes: (a) randomly generate a real number α in the interval between the left and right bounds of the triangular fuzzy number representing demand of the edge, and calculate its membership $\mu(\alpha)$ by eq.(4); (b) generate a random number β , $\beta \in [0, 1]$ and compare β with $\mu(\alpha)$, if $\beta \leq \mu(\alpha)$, then actual demand of the edge is equal to α ; otherwise, if $\beta > \mu(\alpha)$, α is not accepted as the demand of edge. In this case, repeat the generating random numbers α and β until random number α and β are found that satisfy relation $\beta \leq \mu(\alpha)$; (c) check and repeat (a) and (b), and terminate the process when the actual demand quantity of all required edges for each commodity is simulated.

Step 2: Move forward the route designed by credibility measure, cumulate the amounts picked up from each required edge, and calculate the additional cost due to tours failure in terms of

the actual demand.

Step 3: Repeat Step 1 and Step 2, M times.

Step 4: Calculate the average value of additional costs by M times simulation, and report it as the additional costs c' .

4.2. Genetic algorithm procedure

The general process of the proposed GA is introduced in figure 1, and the particular components are represented below.

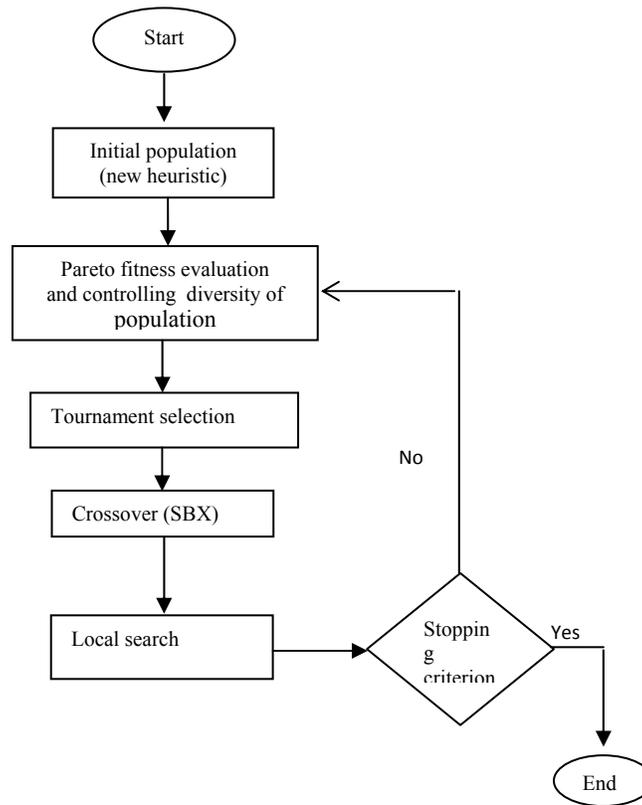


Figure 1 Flowchart of the proposed GA

4.2.1. Solution representation

Like Lacomme *et al.* (2006), we represent a model solution as a chromosome, including ordered list of the tasks (required edges), denoted by $R = (r_1, r_2, \dots, r_t)$. Figure 2 shows an example of solution representation. Thick arrows show the required edges and dotted arrows show the edges with least cost. There are two numbers beside each required edge. Those outside of the parentheses indicate the positive direction and those in the parentheses indicate the negative direction of each required edge. This solution (in figure 2) includes two routes, and each starts from and ends at the depot. In this representation each gene with index 0 represents in the chromosome a separator between two different routes. The corresponding routing plan can be achieved by connecting every two succeeding edges with the shortest path between them (i.e., finding a shortest path from the tail

node of the former edge to the head node of the succeeding edge), which can be obtained by Dijkstra’s algorithm (Dijkstra, 1959).

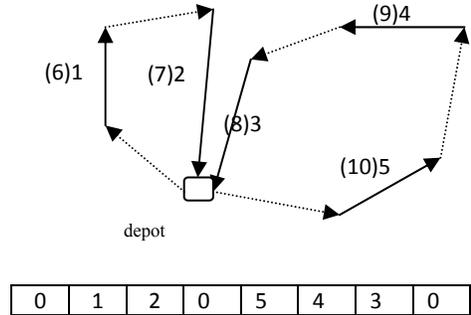


Figure 2 Illustration of solution representation

4.2.2. Initial population

Generating permutations randomly is a classical technique, but adding high-quality solutions always accelerates the convergence of the algorithm. For this mean while checking the capacity chance constraint (17), we used two classical CARP heuristic, namely Path-Scanning (Golden *et al.*, 1983) and Ulusoy’s heuristic (Ulusoy, 1985) and a new proposed heuristic that is presented in the next subsection. The rest of the population is generated randomly. Note that the population size is kept constant overall the algorithm.

4.2.2.1. The new heuristic algorithm

In this section we present our new heuristic algorithm. By employing some features of “Double Outer Scan heuristic (DOS)” (Wøhlk, 2005), and “Ellipse Rule based Path-Scanning (RSE-ER)” (see Santos *et al.*, 2009) with some differences, we present a new robust heuristic. Like DOS, we extend the current partial tour by its both of ends, but here, we use four new criteria, and like RSE-ER, we force the vehicle back to the depot when its load is near its capacity, but without servicing any edge between the last serviced edge and the depot. Finally, we merge the obtained tours in order to reduce the total cost. Our algorithm can be described as follows:

As mentioned before, we represent each edge in two directions; positive (from v_i to v_j) and negative (from v_j to v_i) directions that so called arcs. In order to facilitate, the arcs with positive direction and negative direction is denoted by p and n , respectively. Each arc has a tail node t and a head node h . Further, the opposite direction of arc is denoted by inv . Hence, the following features are notable:

$$h(p) = t(n) = v_i; t(p) = h(n) = v_j; c(p) = c(n); \tilde{q}^m(p) = \tilde{q}^m(n); inv(p) = n; inv(n) = p;$$

Let $td = \sum^{|E_R|} q^{3m}$ be the total demand of right boundary of triangular fuzzy number \tilde{q}^m , nre be the number of required edges, and α be a real parameter.

Step 1. Select one unserviced arc at random, and remove its inverse from unserviced arc set. This arc forms the current partial tour.

Suppose that the selected arc is in positive direction and is shown as p_{sel} , so let hp_{sel} and tp_{sel} be

the first and last arcs of the current partial tour, respectively.

Step 2. Set $\{p_i\}$ as those tail-neighbor and head-neighbor required edges with p_{sel} such that head-neighbor = $\{p_i | t(p_i) = h(hp_{sel}) \neq 1\}$ and tail-neighbor = $\{p_i | h(p_i) = t(tp_{sel}) \neq 1\}$ (1 denotes the depot). In order to convenience, we abbreviate tail-neighbor and head-neighbor by $tngb$ and $hngb$, respectively. Figure 3.a shows this step of the algorithm.

Step 3. Randomly choose one of following four criteria with equal probability. Then based on the result, sort the arcs in $tngh$ and $hngb$ sets.

- 1) Minimum distance from $h(p_i), p_i \in hngb$ to depot and Minimum distance from $t(p_i), p_i \in tngb$ to depot;
- 2) Maximum distance from $h(p_i), p_i \in hngb$ to depot and Maximum distance from $t(p_i), p_i \in tngb$ to depot;
- 3) Minimum distance from $h(p_i), p_i \in hngb$ to depot and Maximum distance from $t(p_i), p_i \in tngb$ to depot;
- 4) Maximum distance from $h(p_i), p_i \in hngb$ to depot and Minimum distance from $t(p_i), p_i \in tngb$ to depot;

Step 4. If both $hngb$ set and $tngb$ set are not empty, choose the first arc with smaller demand to service and add it to current partial tour, and then if the remaining left boundary capacity of vehicle, d_i^{1m} , is greater than $\alpha \times td/nre$, add another arc in another set with greater demand to current partial tour, else back to the depot (see figure 3.b). By this idea, we force the vehicle to services just those unserved edges that are incident to the current partial tour. Consequently, more of the required edges are serviced by the vehicle in its tour. Note that when one arc is selected to service, its inverse must be deleted from unserved arcs.

If $hngb(tngb)$ set is empty, in other words, there are no unserved edges incident to current partial tour by its head, and the remaining capacity of vehicle, d_i^{1m} , is greater than $\alpha \times td/nre$, vehicle services the first arc in obtained $tngb(hngh)$ set in step 2, otherwise it should return to the depot.

Step 5. Update $hngb$ set and $tngb$ set.

Step 6. Repeat step 2 to step 4 until vehicle load approaches its capacity or both $hngb$ and $tngb$, becoming empty. Then connect the $h(hp_{sel})$ and $t(tp_{sel})$ to the depot by using the shortest path.

Step 7. Repeat step 1 to step 5 until all required edges to be serviced.

Step 8. Merge the constructed tours into less cost tours Subject to vehicle capacity.

Step 9. Repeat steps 1 to 8 for maximum iteration (stopping criterion) that is determine by decision maker; finally the best solution is selected.

This algorithm is same as explained before if the selected arc in step 1 be in negative direction, and just p in all notations is replaced with n (e.g. p_{sel} is replaced with n_{sel}).

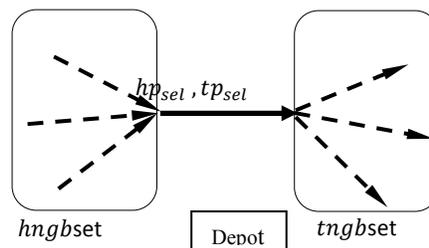


Figure 3a Illustration of forming the $hngb$ set and $tngb$ set

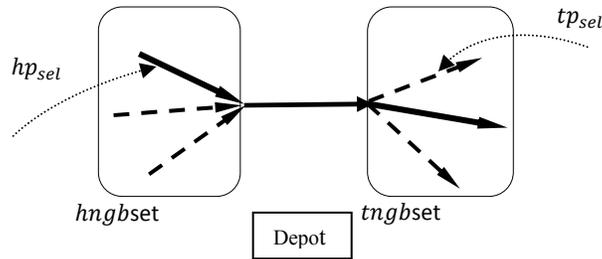


Figure 3b Illustration of determining the hp_{sel} and tp_{sel}

4.2.3. Pareto fitness evaluation and objective tradeoffs

In a multi-objective GA (MOGA), however, because of the Pareto optimality of a multi-objective optimization problem, the fitness of each chromosome is not as simple as that of a conventional GA for a single-objective problem. When solving a single-objective problem, fitness is easily assigned to an individual according to its single- objective function evaluation. In the multi-objective case, there is naturally more than one possible objective that is conflict with each other, and need to be minimized (or maximized) simultaneously. In other words a whole set of solutions is required that reflects the trade-offs between objectives. Non-dominance sorting criterion of Deb *et al.* (2002) is an efficient approach to assigning fitness to solutions that fulfills the current requirements. All non-dominated individuals in the current population are assigned rank one, while individuals dominated by one or more others are assigned rank two or higher. These ranks are then used by the GA as fitnesses for generating the next population. In this case the lower the front, the fitter the solution. Also here in all objectives are assumed to be minimized.

4.2.4. Controlling the diversity of population

One of the important issues in MOGA is population diversity in order to obtained solutions uniformly distributed over Pareto front. There are several methods such as crowding distance method and niching technique, for diversity preservation. The basic idea behind them is preventing the appearance of multiple solutions that are close to each other in the population. Here we use Jaccard's similarity coefficient, which measures the similarity between two sets as ratio intersection and union of them. Garcia-Najera and Bullinaria (2009, 2011) applied this criterion for vehicle routing problem.

The Jaccard's similarity between two sets U and V are measured using follow relation:

$$J(U, V) = \frac{|U \cap V|}{|U \cup V|} \tag{24}$$

Subject to equation (24), the similarity is 1, if all elements in both sets are same and the similarity is 0, if both sets have no same elements. We can adapt the equation (24) in order to apply it for the MOMC-CARPFD. For this purpose we consider each solution A as a set including required edges. And so the similarity of two solutions equals the ratio between the number of two successive connections of required edges that are common to both solutions and the total number connections used by them. Let $x_{ijA} = 1$, if arc (i,j) is serviced by any vehicle in solution A, and 0 otherwise. Arcs (i,j) and (j,i) are considered to be different, even if their cost is the same. So the similarity between solutions A and B is

$$J(A, B) = \frac{\sum_{i,j,k \in V} x_{ijA} \cdot x_{jkA} \cdot x_{ijB} \cdot x_{jkB}}{\sum_{i,j,k \in V} x_{ijA} \cdot x_{jkA} + \sum_{i,j,k \in V} x_{ijB} \cdot x_{jkB} - \sum_{i,j,k \in V} x_{ijA} \cdot x_{jkA} \cdot x_{ijB} \cdot x_{jkB}} \quad (25)$$

Suppose P is the population of solutions, D is the population size; the similarity of solution $A \in P$ with the rest of the GA population will be given by the average similarity of A with every other solution $B \in P$, that is

$$simi(A) = \frac{\sum_{B \in P \setminus \{A\}} J(A, B)}{D-1} \quad (26)$$

Thus the population diversity (τ) is given by equation (27):

$$\tau(A) = 1 - \frac{\sum_{A \in P} simi(A)}{D} \quad (27)$$

4.2.5. Parent selection

In order to mating or reproduction, it is necessary to select parents to create an offspring. For this mean, we choose randomly two solutions among the intermediate population by using a binary tournament selection strategy (Goldberg and Deb, 1991). A solution i is preferred to a solution j and is selected as first parent if it has a better rank or if they have the same rank and $\tau(i) > \tau(j)$. The second parent is selected by the same way.

4.2.6. Crossover (Recombination) operation

One of the unique and important operators in genetic algorithm is crossover. After selection, the sequence based crossover (SBX) operator with specified crossover rate is applied to parents to generate a single offspring. SBX Originally proposed for VRP (Potvin and Bengio, 1996) to work on a sequence of vertices. But this operator is adaptable to work on sequence of edges, too (Tang *et al.*, 2009). SBX can be summarized as follow: considering two selected parents P_1 and P_2 , randomly two routes r_1 and r_2 are chosen from them, respectively. Then, by randomly splitting both r_1 and r_2 into two sub routes, say $r_1 = (r_{11}, r_{12})$ and $r_2 = (r_{21}, r_{22})$, a new route is obtained by replacing r_{12} with r_{22} . The next step is repair the resulted offspring. Because it may that some tasks appear more than once in the new solution, or some tasks in r_1 are removed from the new route. In the former case, the duplicated tasks are removed from the new solution and in the latter case, the best possible locations should be found to locate the removed tasks in the new solution. Hence, we delete delimiters in the obtained offspring, and evaluate it by split method.

4.2.7. Local search

Incorporation of a local search into the multi-Objective evolutionary optimization is important. A

local search with a given probability p_l is applied to improve the new solution generated during the crossover phase. It is noted that in this paper, we replace the mutation operator with a local search scheme because the proposed Jaccard's similarity coefficient satisfies the diversification requirement. Frequently, local search is directed by move operators. Hence, we use three traditional move operators that originally used in the vehicle routing problem (VRP) (Dror and Levy, 1986) and then were developed in CARP by researchers. These move operators are single relocation, double relocation and exchange. In the single relocation move, a required edge from its current position is transferred to another position of the current solution or a new empty route. Both directions of the selected edge should be considered when transferring the edge. In the double relocation move, two consecutive edges are moved instead of a single edge. Similar to the single insertion, both directions are considered for selected edges. In the exchange move, an edge exchanges position with another edge in another position. Here, also both directions are considered for selected edges.

There is no preference made among the local heuristics, and one of three aforementioned will be randomly executed at each generation for the offspring obtained by crossover to search for better local routing solutions.

4.2.8. Creating the next generation

In this stage to form the next generation, the individuals from the combined population (parent and offspring population) are compared using their Pareto ranks. The individuals with higher rank are selected to enter the next generation. If the Pareto ranks fasten, the Jaccard's similarity coefficient is compared to complete the population of the next generation.

4.2.9. Stopping criteria

The proposed GA can be stopped after a fixed number of generations without improving the current best solutions.

5. COMPUTATIONAL RESULTS

Since no standard benchmarks are available for evaluating the performance of the proposed model, two sets of problems are generated. The first set includes 23 two-commodity instances derived as follows from the 23 classical instances (*gdb* files) proposed by DeArmon (1981): the network is kept, with the same required edges and costs. It is assumed that all required edges have demand for both commodities, and all demands for each commodity is a triangular fuzzy number that generated in the interval 0.3 and 0.4 from the original demand in the corresponding *gdb* files, for commodity type one and two, respectively. All vehicles have two compartments, each compartment with a capacity equal to the original vehicle capacity in the corresponding *gdb* files.

The second set of problems contains 2 instances that randomly generated in a 7×7 , square grid network including 112 total number of edges. In the first instance in this set, all edges are required and in the second instance 50% of edges are required. We named these two instances A and B, respectively. Each required edge has demand for each commodity. The cost for each edge is randomly generated in $[1, 10]$; the triangular fuzzy demands for commodity type one and commodity type two are randomly generated in $[1, 5]$ and $[6, 10]$, respectively. In both instances the depot is located at the lower left corner point in the 7×7 square grid network. Vehicles have two compartments that the capacity of the compartment for commodity type two is double the capacity of the compartment for commodity type one.

To show our computational results, the MOMC-CARPF algorithm has been coded in C# language and run on a laptop computer with CPU clock frequency 2.66 GHz and 4Gbyte of RAM. The value of dispatcher preference index Cr^* varied with the interval of 0 to 1 with a step of 0.1. Using one single setting of parameters, the best combination of evolutionary parameters has been found during the preliminary testing. These settings are as follow:

Population size = 60;
 Number of generation = 150;
 Crossover rate = 1;
 Local search rate = 0.1;
 Number of iteration of new heuristic = 1000;
 The real parameter $\alpha = 3$, and
 Number of iteration of stochastic simulation = 100;

As mentioned earlier, this paper considers MOMC-CARP with two objectives including minimizing the number of vehicles and total traveling cost (forming of routing cost plus failure cost), concurrently. The results from experiments were analyzed from three aspects: First, to analyze the influence of the initial solution on the final solution quality by implementing the proposed multi-objective genetic algorithm on *gdb* files with proposed new heuristic (Alg1) and without the proposed new heuristic (Alg2). Second, to analyze the effect of dispatcher preference index Cr^* on the number of vehicles, routing cost, failure cost and total cost, and to provide an approximation value of Cr^* in which the total cost is minimum and the total vehicle used is acceptable. To show that the better results are obtained when the vehicles service the required edges by collecting both type of commodity simultaneously with when they service the required edges by collecting each commodity separately.

In order to evaluate the first and second aspects, we use the problem set 1(*gdb* files) and to evaluate the third aspect, we use the problem set 2.

5.1. Results for the data set1

In this section for each instance in *gdb* files, two of the best solutions in the resulting non-dominated set rank1, and for each value of Cr^* in [0, 1] with step 0.1, are taken over 10 runs. One solution has the minimal total cost and another solution has the minimal number of vehicle. Then, the average for each objective and also the average for routing cost and failure cost are calculated. Finally these are averaged over *gdb* files category.

Table 1 presents a comparison between the results found by Alg1 and Alg2 for *gdb* files. In this table the column labeled " Cr^* " shows the dispatcher preference index, column total cost is divided to three columns "alg1", "alg2" and "equal". The column named "alg1" gives the number of instances in which the obtained total cost by Alg1 is less than the obtained total cost by Alg2. The column named "alg2" gives the number of instances in which the obtained total cost by Alg2 is less than the obtained total cost by Alg1. The column named "equal" gives the number of instances in which the total cost obtained by both Alg1 and Alg2 is equal. These three columns below the column headed "#vehicle" have the same definition but for the number of vehicle obtained by Alg1 and Alg2. As shown in table 1, the Alg1 outperforms the alg2 with respect to total cost for all dispatcher preference index values. Note that in most of instances, both Alg1 and Alg2 have equal performance with respect to the number of vehicle. However the number of instances in which the number of vehicles obtained by Alg1 is less than the number of vehicles obtained by Alg2, is more. Fig 4.a and Fig 4.b respectively illustrate both the average of total cost and the average of number

of vehicles, Alg1 is superior to Alg2. So one can conclude that the proposed new heuristic has an important effect on the final solution quality and especially on the decreasing the total cost.

Table 1 Comparison between Alg1 and Alg2 on *gdb* files

Cr*	total cost			#vehicle		
	alg1	alg2	equal	alg1	alg2	equal
0	20	3	0	0	9	14
0.1	18	5	0	8	1	15
0.2	17	6	0	5	1	17
0.3	17	6	0	3	4	16
0.4	13	10	0	6	1	16
0.5	18	3	2	2	1	20
0.6	12	4	7	2	2	19
0.7	12	2	9	8	0	15
0.8	9	4	10	5	3	15
0.9	10	0	13	3	0	20
1	13	1	9	3	1	19

The MOMC-CARPFDP before in this paper, the total cost may be reduced as the number of vehicles is increased or the total cost may be increased if the more vehicles are involved. However, the total cost includes the routing cost and the failure cost and increment or reduction of the number of vehicles effects on failure costs and consequently on routing cost. Varying values of Cr^* also influences failure cost. It is prospective that the failure costs are decreased and the number of vehicles are increased as Cr^* increased. It is clearly shown in table 2, that gives the results for the average of routing cost (RC), the average of failure cost (FC), the average of total cost (TC) and the average of the number of vehicles (#V) for two of the best non-dominated solutions on *gdb* files when the dispatcher preference index varied. The correlation between these results and Cr^* are illustrated in figure 5 to figure 7. Since the values of RC, FC, TC and #V in table 2 are not of the same scale, each of them has been normalized by equation (28):

$$Nr = \frac{(value^* - min)}{(max - min)} \quad (28)$$

Where Nr is the normalized value, $value^*$ is the before value, min and max are minimum and maximum values, respectively for per RC, FC, TC and #V. Regarding compression results in table 2 and due to figure 4.a. and figure 4.b, we propose two values for dispatcher preference index: $Cr^* = 0.3$ and $Cr^* = 0.6$ because the total cost for these values is less than other values. In order to determine the final value of Cr^* , the decision maker should consider some points. The lower values of Cr^* conduce to the use of a less vehicle and best utilization of vehicle capacity. On the other hand, the lower values of Cr^* increases the number of vehicles failing to service. Higher values of Cr^* decrease the utilization of vehicle capacity along the planned route and decrease the failure cost and on the other hand the number of vehicles is increased as Cr^* is increased. So the decision maker must evaluate and compare the vehicle and labor cost, vehicle capacity utilization, failure cost and total cost, then determine the value of Cr^* .

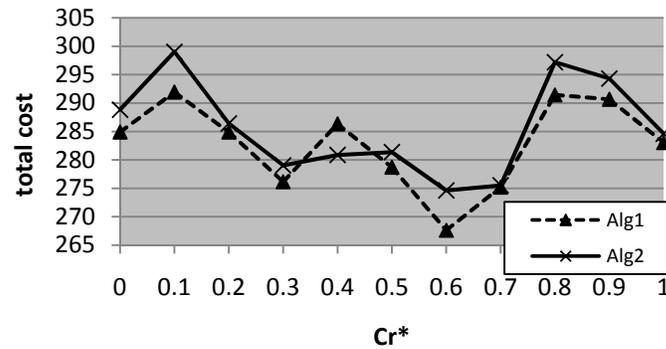


Figure 4a The average total cost obtained by Alg1 and Alg2 on *gdb* files

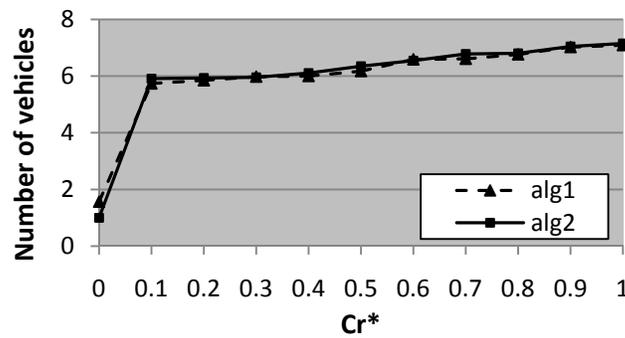


Figure 4b The average number of vehicle obtained by Alg1 and Alg2 on *gdb* files

Table 2 The results for average *RC*, *FC*, *TC* and *#V* for different values of *Cr** on *gdb* files

<i>Cr*</i>	<i>RC</i>	<i>FC</i>	<i>#V</i>	<i>TC</i>
0	237.85	47.04	1.57	284.89
0.1	272.65	23.15	5.74	295.8
0.2	270.17	14.7	5.85	284.87
0.3	258.2	13.6	5.98	271.8
0.4	272.59	13.76	6.00	286.35
0.5	270.46	8.18	6.17	278.64
0.6	266.39	1.21	6.59	267.6
0.7	274.87	0.52	6.61	275.39
0.8	291.89	0.11	6.76	292
0.9	290.65	0	7.02	290.65
1	283.02	0	7.09	283.02

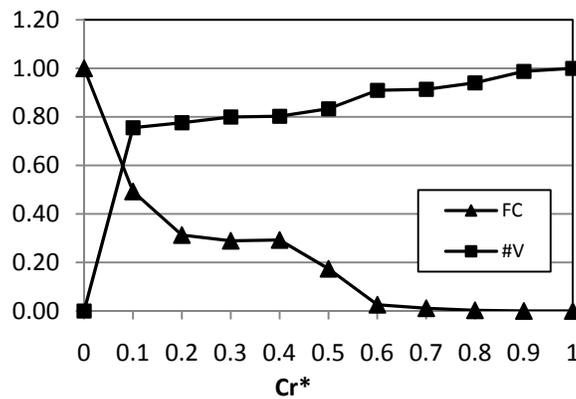


Figure 5 The correlation between Cr^* , FC and $\#V$

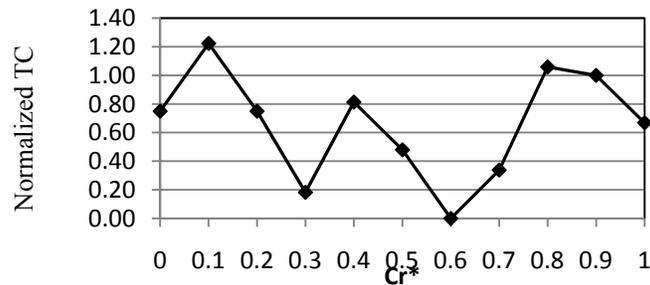


Figure 6 The correlation between Cr^* and TC

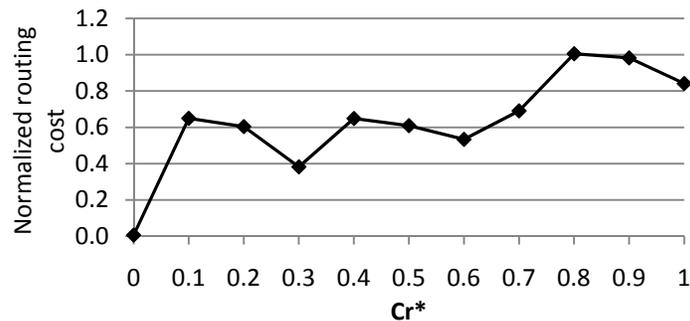


Figure 7 The correlation between Cr^* and RC

5.2. Results for the data set 2

In this section, we use data set 2 to investigate and compare the MOMC-CARPFDD for servicing the required edges for both commodity concurrently using multi-compartment vehicles and MOMC-CARPFDD for servicing the required edges for each commodity separately with single-compartment vehicles. We suppose the total vehicle capacity in both routing conditions is equal to Q and for concurrent servicing policy, Q is divided into Q_1 and Q_2 such that $Q_2 = 2Q_1$ and $Q = Q_1 + Q_2$. We solve both A and B instances 10 times by Alg2 for different values of Cr^* in $[0,1]$ with step 0.2. At the end of 10 runs, two of the best solutions of non-dominated set rank 1 for each objective are

selected. Then the average for each objective is calculated. We analyze both routing policy for six values of Q : 60, 75, 90, 105, 120 and 135. Figure 8.a shows an example of the average total cost curves for concurrent-servicing (CS) and for separate-servicing (SS) for “A” instance and figure 8.b shows the same results for “B” instance. These figures denote that the total cost is less when two-compartment vehicles are used to serve the required edges for both commodities simultaneously beside when the unpartitioned vehicles serve the required edges per commodity separately. In general, the average of the number of vehicles obtained by separate-servicing policy is less than the average of the number of vehicles obtained by concurrent-servicing policy. However, for higher dispatcher preference index and also for higher vehicle capacity, the number of vehicles used for both routing policy is close to each other and even in some cases is equal. The figure 9.a and figure 9.b demonstrate this fact. As it can see in figure 9.b, When $Cr^* = 0$, two vehicles are used in separate-servicing policy, each vehicle services the required edges for each commodity. But in concurrent-servicing policy, since the required edges are served for both commodities simultaneously, so when $Cr^* = 0$, one vehicle is used, but on the other hand, the failure cost is more compared with separate-servicing policy.

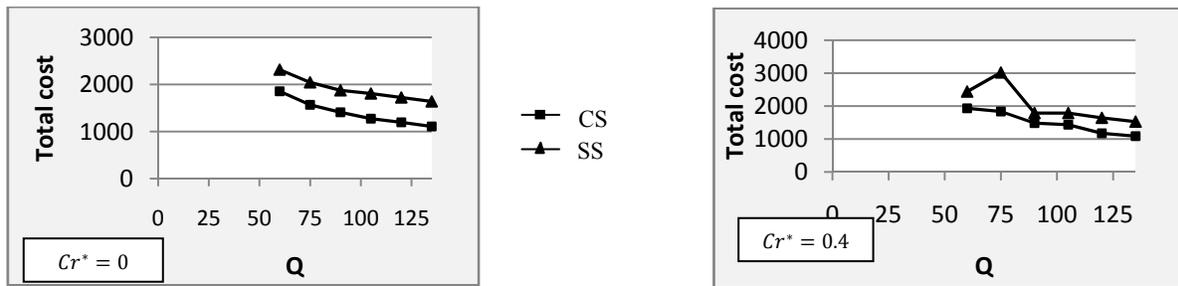


Figure 8a Total cost for CS and SS for instance “A”

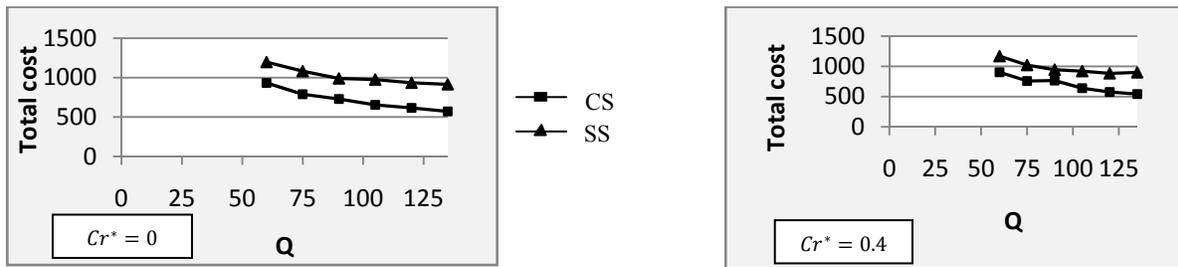


Figure 8b Total cost for CS and SS for instance “B”

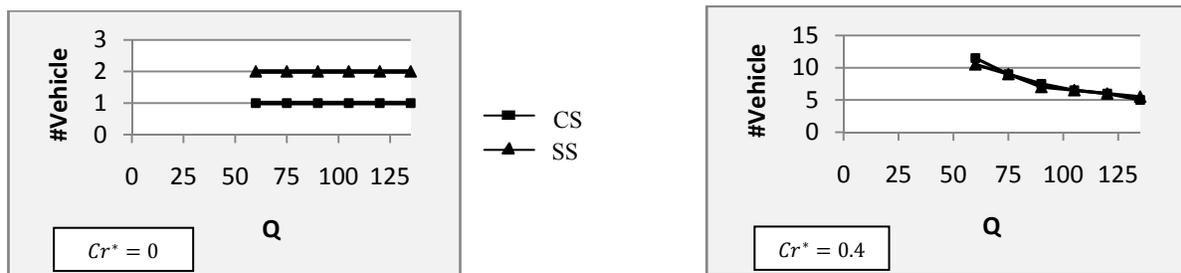


Figure 9a Number of vehicle for CS and SS for instance “A”

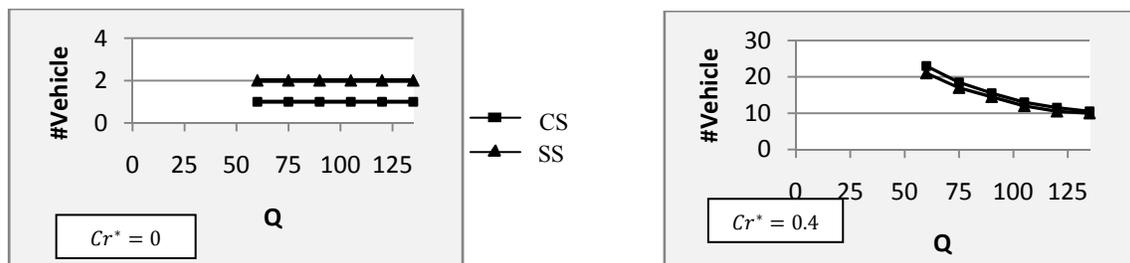


Figure 9b Number of vehicle for CS and SS for instance “B”

6. CONCLUSION

In this paper, we present a new model and solution approach for more realistic CARP that is multi-objective multi-commodity capacitated arc routing problem with fuzzy demand (MOMC-CARPFDD). A fuzzy chance constrained programming mathematical model was proposed based on credibility measure. In the proposed model, each required edge has demand for more than one type of commodity and also all demands for each commodity are supposed to be fuzzy numbers. Then a multi-objective genetic algorithm based on Pareto optimal set was developed to solve it, focusing on total travel cost minimization and total number of vehicle used minimization. Since the exact demand is unknown until vehicle reach to required edges, we use stochastic simulation and compute the total failure cost for planned routes. We also proposed a new heuristic to improve the initial solutions in initial population of genetic algorithm. To evaluate the new proposed model and solution approach, some data sets with fuzzy demand generated randomly are used. The computational results on data set 1 (*gdb* files) showed that the new heuristic has important influence on the quality of final solutions, especially on the total cost. Also the effectiveness of Cr^* on final results was shown by varying its value in interval $[0,1]$ by step 0.1. We proposed two values of dispatcher preference index; however, an advance research should be done to investigate whether these values are appropriate in other cases. In order to better analyze and compare the concurrent-servicing policy and separate-servicing policy, we solve the MOMC-CARPFDD on randomly generated instances in which the required edges have demand for two commodities (data set 2). The computational results showed that the total cost is less when two-compartment vehicle is used to servicing the required edges for two commodities concurrently. While applying separate-servicing, the number of vehicles in most cases is less compare with concurrent-servicing policy. Also the results showed that for both routing policies, the total cost and the number of vehicles is decreased as vehicle capacity increased.

In addition, four contributions of the paper can be highlighted: 1)To handle more realistic situations, we investigate the multi-objective capacitated arc routing with multi-commodities and demands uncertainty.2)We present fuzzy chance constrained model based on credibility with objectives: minimizing total travel cost and number of vehicles.3)We develop multi-objective genetic algorithm using Pareto ranking, and hybrid with stochastic simulation to design an efficient solution method.4)To improve quality of final solutions, we propose a new heuristic to generate a good initial solution for GA.

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