

Efficiency Analysis of Public Universities in Iran Using DEA Approach: Importance of Stakeholder's Perspective

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ABSTRACT

Our primary aim, in this paper, is to propose and investigate, for the first time, the application of Data Envelopment Analysis (DEA) technique in assessing Iranian public universities. We provide an analysis on the importance of the stakeholder's perspective on the structure of DEA and the variations of efficiency results. For illustrations, we perform efficiency analysis on a sample of public universities in Iran from three different perspectives of importance, i.e., teaching quality, research productivity, and cost efficiency by using available data.

Keywords: Data envelopment analysis; Efficiency; Academic performance.

1. INTRODUCTION

Efficiency may mean different things to different stakeholders. There are a variety of stakeholders in higher education, including students, employers, faculty members, staff, government and its funding agencies, accreditors, validators, auditors, and assessors including professional bodies. Each of these stakeholders has a different view on efficiency. For example, to the committed scholar the efficiency of higher education is its ability to produce a steady flow of people with high intelligence and commitment to learning that will continue the process of transmission and advancement of knowledge. To the government a highly efficient system is one that produces trained scientists, engineers, architects, doctors and so on in numbers judged to be required by society. To an industrialist an efficient educational institution may be one that turns out graduates with wide-ranging, flexible minds, readily able to acquire skills, and adapt to new methods and needs. Each of these views represents a valid expectation of higher education and its efficiency. The measurements thus required and the standards to be applied will surely be different for each of these perspectives as they present different notions of efficiency (Tam, 2001). It is for this change of perspective that a ranking results published by an agency or an independent assessor may produce different reactions, both in favor and against and hence efficiency analysis in academia has

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remained a controversial issue from the beginning.

Indeed, the league tables and ranking lists of universities have become, over the past few decades, increasingly popular. These rankings have certainly serious impacts on universities' prestige and the number and quality of applicants. The methods used in these league tables (The League, 2009) to rank universities have been criticized by many as they use simple weighted sum of performances, which are not rigorous and has some methodological problems and limitations. First, the weights are often determined subjectively with very little justification of their value. Second, as each performance indicator is measured in a different unit, they need to be normalized in order to be aggregated together (Giannoulis and Ishizaka, 2010). To alleviate these problems more sophisticated methods like total quality management (TQM), statistical methods, cost-benefit, econometric methods, regression, multi criteria decision making (MCDM) and DEA are used. The focus was on cost efficiency, teaching quality, research productivity, or aggregate performance.

Among these methods it has been DEA, a non-parametric linear programming technique, which has become a reasonable tool for evaluating efficiencies. For a more recent and in-depth introductory treatment to DEA and areas of applications see Cooper *et al.*(2006), Cook *et al.* (2009,2009) and Mehregan(2009). The DEA in particular has been widely used to evaluate performance and rank universities or schools. In fact, education and higher education has been one of the five most important fields that are under scrutiny by DEA researchers (Liu et al, 2012).

Following earlier works using MCDM techniques on quality assessment of Iranian universities (see Monfared *et al.*, 2004 and 2006) we now apply the DEA approach to provide a novel analysis of academic efficiency in Iran. In this paper, the efficiency of the academic universities in utilizing the scarce resources in teaching students and producing research results is measured using a DEA model. This investigation is focused on the importance of stakeholders' perspective on efficiency results of academic performance and the utilization of resources.

This paper is organized as follows. In section 2, some backgrounds on the assessment of public funded universities in Iran are considered. Also, the literature on assessing higher education institutes worldwide is briefly considered and the importance of stakeholders' perspective on efficiency analysis is discussed. In Section 3 we briefly review the mathematics of DEA models. Then, in Section 4 we consider three different perspectives for assessing Iranian Universities, develop appropriate DEA structures, and report the results. Finally, a conclusion is drawn in Section 5.

2. BACKGROUND

In the last 30 years of 1979 to 2012 as families more seriously encouraged their children to receive a university degree and the population has begun to doubled and then tripled, an ever increasing demand has forced higher education sector to expand: most current universities to expand their capacities almost 2 to 3 times, some new universities have been established (e.g., in Ghom, Kerman, and Shahre-kord provinces), many junior colleges have been promoted into full scale universities (e.g., College of economy, College of petroleum, and College of insurance), and very recently virtual university of Payam-Noor has been expanded to a capacity compatible to normal universities. Add to this the satellite branches of public funded universities (e.g., Kish Pardis of Sharif university), and joint established programs with international universities. However, this was only the contribution of so called state run universities as semi state run university of Azad (open universities), and Non-profit universities have doubled the overall capacity responding to ever

increasing demands for higher education.

In the past, public-funded universities in Iran received all of their budgets from the government. At the same time, they followed the rules and regulations of the government completely. Although the budget allocated from the government still account for an important portion of a university's budget, income from external sources including tuition fees, private donations, grants and contracts, etc. must be sought. State run universities nowadays enjoy more freedom in self-governance and management. This makes academic performance evaluation and efficiency analysis a more vital element for university top administrators, policy makers and Government. In Monfared et al. (2006) we proposed a mathematical programming method to minimize the inconsistencies between the judgmental based rankings of Iranian Universities and those obtained using available data to produce more quantitative based rankings. In the following subsection we review works which have been undertaken to analyze the efficiency of higher education institutes using DEA.

2.1. University modeling using DEA

DEA modeling in higher education analysis is under influence of the study scope or the extent of DMUs which is considered. For example, we may build a DEA model using a study scope where DMUs are the public funded universities or a DEA model using a different study scope where all Iranian universities are included. In Table 1 we propose some of the important alternative study scopes or perspectives for DEA modeling of Iranian universities.

Table 1 Different study scopes or perspectives

Levels	Study scope
International level	a- World wide
	b- Middle-eastern universities
	c- Asian universities
	d- OECD countries
National level	a- All Iranian universities
	b- Public funded universities
	c- Large scale universities
	d- Comprehensive universities
Discipline and Department level	a- Medical schools
	b- Engineering schools
	c- Human science schools
	e- Economies schools
Human level	a- Faculties
	b- Students
	c- Staff

DEA modeling in higher education analysis is also under influence of the study level. As shown in Table 2 we have reported 32 different DEA works which have been performed so far to analyze university efficiency at four different levels. What is interesting to see is that each study is unique and there could hardly be an agreed upon model including set of inputs and outputs by which any academic entity can be evaluated. We think the reason is that the analysis is difficult if we do not consider a certain perspective.

Table 2 Studies using DEA

Level	Examples
Universities	Katharaki and Katharakis(2010); Lukman, <i>et al.</i> (2010), Meng <i>et al.</i> (2008); Johnes and Yu (2008);Johnes(2006); Glass <i>et al.</i> (2006); Abbott and Doucouliagos (2003); Sarrico and Dyson (2003); Avkiran (2001); Johnes(1996); Johnes and Johnes (1995); Ahnet <i>al.</i> (1988); Glass <i>et al.</i> (1998).
Colleges and Departments	Kong and Fu (2012); Kao and Hung (2008); Beasley (1995); Sinuany-Stern <i>et al.</i> (1994); Johnes and Johnes (1995).
Programs	Celik and Ecer (2009); Johns and Johns (1995).

To represent taste of variations of models proposed in the literature, we consider two different DEA models from university performance studies in United Kingdom, and China. Glass *et al.* (1998) modeled 98 non-specialist UK Universities as producing three outputs (undergraduates, postgraduates and research) and four inputs (academic faculties, staff, capital expenditure and research grants). The UK's DEA model is shown in Table 3.

Table 3 UK's DEA model

Inputs:	Outputs:
1.Faculty to students ratio	1.Full time equivalent undergrad students
2.Staff to students ratio	2.Full time equivalent graduate students
3.Capital expenditure per student	3.Research rating per assessment unit
4.Research grants per academic staff	

Johnes and Yu (2008) studied the relative efficiency in the production of research of 109 Chinese regular universities in 2003 and 2004 using DEA. This model includes six inputs and three outputs as shown in Table 4. It is not surprising to see that the Chinese model is different from the UK's one as in the latter, the combined teaching and research performances was studied while in the former model only the research dimension was of concern, i.e., two different perspectives.

Table 4 Chinese Research's DEA model

Inputs:	Outputs:
1. Percentage of associate professor positions or higher	1. Total number of research publications
2. Staff to student ratio	2. Research publications per academic staff
3. Proportion of postgraduate students	3. Reputation measure or the opinions of peers
4. Research expenditure	
5. Library books	
6. Area of buildings	

In order to assess the working quality of higher education institutes one need to define the stakeholder's perspective. This is further pursued in Section 4.

3. THE MATHEMATICS OF DEA

Here, we briefly present an introduction into the basic DEA mathematics and models (Cooper *et al.*, 2006; Mehregan, 2009). DEA is an approach for identifying best practices of peer decision making units (DMUs) in the presence of multiple inputs and outputs. DEA computes a comparative ratio of

weighted outputs to weighted inputs for each unit, which is reported as the relative efficiency score. The efficiency score is usually expressed as either a number between 0 and 1, or as a percentage. A DMU with a score less than one is deemed inefficient relative to other units. Now, consider we have n Decision Making Units (DMUs) where each has m inputs denotes as $y_{10}, y_{20}, \dots, y_{m0}$ and s outputs denoted as $y_{10}, y_{20}, \dots, y_{m0}$. Weights assigned to outputs are as u_1, u_2, \dots, u_s and weights assigned to inputs are v_1, v_2, \dots, v_m . The efficiency is defined as a fraction (1) in Table 5. Then the basic model of DEA is called *CCR ratio model* which is defined as (2) in Table 5 and is referred to

Table 5 Basic DEA models

	Oriented models	
	CCR	BCC
Efficiency ratio	$\frac{\sum_{r=1}^s u_r y_{r0}}{\sum_{i=1}^m v_i x_{i0}}$	(1) $\frac{\sum_{r=1}^s u_r y_{r0} + w}{\sum_{i=1}^m v_i x_{i0}}$ (1)'
Fractional model	(2) $\begin{aligned} Max Z_0 &= \frac{\sum_{r=1}^s u_r y_{r0}}{\sum_{i=1}^m v_i x_{i0}} \\ \text{s.t.} & \\ \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} &\leq 1, \text{ for } j = 0, 1, 2, \dots, n \\ u_r, v_i &\geq 0 \end{aligned}$	(2) $\begin{aligned} Max Z_0 &= \frac{\sum_{r=1}^s u_r y_{r0} + w}{\sum_{i=1}^m v_i x_{i0}} \\ \text{s.t.} & \\ \frac{\sum_{r=1}^s u_r y_{rj} + w}{\sum_{i=1}^m v_i x_{ij}} &\leq 1, \text{ for } j = 0, 1, 2, \dots, n \\ u_r, v_i &\geq 0, w \text{ unrestricted in sign} \end{aligned}$
Equivalent linear programming model (input oriented) in multiplicative form	(3) $\begin{aligned} Max Z_0 &= \sum_{r=1}^s u_r y_{r0} \\ \text{s.t.} & \\ \sum_{i=1}^m v_i x_{i0} &= 1, \\ \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} &\geq 0, \\ &\text{for } j = 0, 1, 2, \dots, n \\ u_r, v_i &\geq 0 \end{aligned}$	(3) $\begin{aligned} Max Z_0 &= \sum_{r=1}^s u_r y_{r0} + w \\ \text{s.t.} & \\ \sum_{i=1}^m v_i x_{i0} &= 1, \\ \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} + w &\geq 0, \\ &\text{for } j = 0, 1, 2, \dots, n \\ u_r, v_i &\geq 0 \quad w \text{ unrestricted in sign} \end{aligned}$
Equivalent linear programming model (output oriented) in multiplicative form	(4) $\begin{aligned} Min Z_0 &= \sum_{r=1}^s v_i x_{i0} \theta \\ \text{s.t.} & \\ \sum_{r=1}^s u_r y_{r0} &= 1 \\ \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} &\geq 0 \\ &\text{for } j = 0, 1, 2, \dots, n \\ u_r, v_i &\geq 0 \end{aligned}$	(4) $\begin{aligned} Min Z_0 &= \sum_{r=1}^s v_i x_{i0} + w \\ \text{s.t.} & \\ \sum_{r=1}^s u_r y_{r0} &= 1 \\ \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} + w &\geq 0, \\ &\text{for } j = 0, 1, 2, \dots, n \\ u_r, v_i &\geq 0 \end{aligned}$
The envelopment form for output oriented model (dual)	(5) $\begin{aligned} Max y_0 &= \theta \\ \text{s.t.} & \\ \sum_{j=1}^n \lambda_j x_{ij} &\leq x_{i0} \text{ for } i = 1, 2, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} &\geq \theta y_{r0} \\ &\text{for } r = 1, 2, \dots, s \\ \lambda_j &\geq 0 \text{ for } j = 1, 2, \dots, n \\ \theta &\text{ unrestricted in sign} \end{aligned}$	(5) $\begin{aligned} Max y_0 &= \theta \\ \text{s.t.} & \\ \sum_{j=1}^n \lambda_j x_{ij} &\leq x_{i0} \text{ for } i = 1, 2, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} &\geq \theta y_{r0} \text{ for } r = 1, 2, \dots, s \\ \sum_{j=1}^n \lambda_j &= 1, \text{ for } j = 1, 2, \dots, n \\ \lambda_j &\geq 0, \theta \text{ unrestricted in sign} \end{aligned}$
Non-oriented model		
	Primal	Multiplicative
Additive model	(6) $\begin{aligned} Max Z_0 &= -\sum_{r=1}^s S_r^+ - \sum_{i=1}^m S_i^- \\ \text{s.t.} & \\ \sum_{j=1}^m \lambda_j x_{ij} - S_i^- &= x_{i0} \\ &\text{for } i = 1, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} - S_r^+ &= y_{r0}, \\ &\text{for } r = 1, \dots, s \\ \sum_{j=1}^n \lambda_j &= 1, \text{ for } j = 1, \dots, n \\ \lambda_j, S_r^+, S_i^- &\geq 0, \text{ for } j = 1, \dots, n \end{aligned}$	(6) $\begin{aligned} Max y_0 &= \sum_{r=1}^s u_r y_{r0} - \sum_{i=1}^m v_i x_{i0} + w \\ \text{s.t.} & \\ \sum_{i=1}^m v_i &\geq 1 \\ \sum_{i=1}^s u_r &\geq 1 \\ -\sum_{i=1}^m v_i x_{ij} + \sum_{r=1}^s u_r y_{rj} + w &\leq 0, \\ &\text{for } j = 0, 1, 2, \dots, n \\ u_r, v_i &\geq 0 \quad w \text{ unrestricted in sign} \end{aligned}$

as constant returns to scale model. This fractional nonlinear program model of (2) can be converted into a linear program of (3) which is called, the *multiplicative form* of input oriented CCR model.

The multiplicative form for output oriented CCR model is defined as (4) and its dual which is called *envelopment form* of output oriented CCR model is presented as (5) in Table 5. Here, θ is the dual variable for constraint $\sum_{r=1}^s u_r y_{r0}$ and the dual variable for constraint $\sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0$ is λ_j .

In column 4 and 5 of Table 5 we present equivalent formulas for BCC models that are referred to as variable return to scale models. Here (1)' defines the efficiency ratio, (2)' the fractional model, (3)' converts (2)' into a linear program for input oriented case, (4)' defines output oriented model of BCC in multiplicative form and (5)' defines the envelopment form for BCC. We then move from oriented models to non-oriented model of so called additive model. Here, (6) is the primal model of a non-oriented additive model and (6)' is its envelopment form which is in the multiplicative form. It should be noted that the additive models, do not help us to specify the amount of inefficiencies, as they only determine whether a DMU is efficient or not. So, in order to have a view on the amount of inefficiencies, we have implemented the BCC and the CCR models as will be considered next.

4. DEA MODELS FOR IRAN FROM DIFFERENT PROSPECTIVES

Although teaching and research have been considered by most people as two major tasks of the university, they are difficult to measure. We need some metrics which are capable of representing the achievement of these two tasks. We also need metrics for the resources that the universities have consumed in performing those two tasks.

Each DEA structure in Table 6 represents a specific notion of efficiency which is specified by its input and output factors. The selection of input and output factors for evaluating the performance of university is an important issue in DEA analysis. Here, we employ the factors Taken from

Table 6 Three DEA structures representing three perspectives: Teaching quality model, Research quality model and Cost efficiency model

Teaching model	Inputs: 1. Staffs per students(max) 2. Faculty members per major(max) 3. Faculty member's positions(max) 4. Books per student(max) 5. Material procurement cost(max) 6. Capital expenditure(max)	Outputs: 1. Students per faculty(min) 2. Teaching load(min) 3. Faculty receivable from extra lecturing hours(min)
Research model	Inputs: 1. Staffs per students(max) 2. Faculty members per major(max) 3. Faculty member's positions(max) 4. Teaching load(min) 5. Books per student(max) 6. Material procurement cost(max) 7. Capital expenditure(max)	Outputs: 1. External contracts to total(max)
Cost model	Inputs: 1. Staffs per students(min) 2. Faculty members per major(min) 3. Faculty member's positions(max) 4. Books per student(min) 5. Material procurement cost(min) 6. Capital expenditure(min)	Outputs: 1. Students per faculty(max) 2. Teaching load(max) 3. External contracts(max)

Monfaredet *al.* (2006) to assess Iranian universities from different perspectives. The choices of input-output factors for each structure are based on our own understandings of effective factors and the availability of data and so it may vary from one analysis to another. For example, two types of teaching material procurement costs and capital expenditure costs are considered to be important in teaching quality model. It should be mentioned that the capital expenditures are used for constructing new buildings and buying new equipment while the teaching material procurements are spent for items such as whiteboard markers or laboratory materials.

Our analysis for choice of appropriate model has led to the following characterizations (see Cooper *et al.*, 2006; Mehregan, 2009; Cook and Seiford, 2009; (Dyson, *et al.*2001):

1. All our input and output variables are *discretionary* or controllable, i.e., they can be managed and controlled by university administrators.
2. Input variables are by default of minimization type unless shown otherwise as in Table 6. For instance, in our teaching quality model, five out of six input variables are of maximization type. Also, output variables are by default of maximization type unless shown otherwise as in Table 6.
3. The nature of our inputs and outputs dictates us to adopt the *variable return to scale* (VRS) against *constant return to scale* (CRS) models. There is no guarantee that if we change these inputs, the outputs will also change proportionally. For example, if we double the number of students, this does not mean that the material procurement cost could also be doubled. Sometimes, suppliers provide discount on quantity of purchase and this will decrease the unit price, i.e. violates proportionality again. So, it is preferred to use VRS models rather than CRS models to compute the DMU's efficiencies.
4. As we need to minimize the input variables and at the same time maximize the output variable we take non-oriented models for all three models in Table 6. Non-oriented models are defined against input-oriented and output-oriented models. A modified version of additive models is called Slack-Based Models (SBM) which is mathematically richer. In this paper, we implement additive model of (6)' as illustrated in Table 5. The additive model, however, can only show whether a DMU is efficient or not and it does not measure the amount of inefficiencies. To remedy this deficit we have also implemented BCC and CCR models.
5. There is a suggested rule of thumb that was proposed by Charns, Cooper and Rohdes, which relate number of inputs and outputs to the number of DMUs. This rule states that the number of DMUs that are going to be evaluated should be greater than or equal to triple sum of inputs and outputs. So, we have to consider at least 27 DMUs or universities in our evaluations (i.e. $6inputs + 3outputs$, and $3 \times 9 = 27$).

Now, we use data reported in (Monfaredet *al.*, 2006) to produce efficiency scores for teaching, research and cost as illustrated in Tables 7 and 8 for different models. To save space we present details for our efficiency analysis for our teaching perspective using BCC and CCR models in details in Table 7.

We then report our total results for all three different perspectives using all three different models of additive, BCC and CCR in Table 8. See sample codes in the appendix. All results presented here can be reproduced using the models of Figure 3 and data from (Monfaredet *al.* 2006).

From the results in Table 8, the following can be stated:

1. We see that only three universities (Arak, Elmo-sanaat and Hormozgan) are fully efficient from all three different perspectives and with respect to all three different models of additive, BCC and CCR.
2. From teaching perspective there are 21 efficient universities if additive model is used. This number of efficient universities reduces to 19 and 15 if BCC and CCR models are adopted, respectively. model
3. We see efficient number of universities vary from one perspective to another depending on which model is used.

Table 7 Efficiency scores for BCC and CCR models from teaching perspective

Row	University	BCC model		CCR model	
		Objective value θ	Efficiency $Z=1/\theta$	Objective value θ	Efficiency $Z=1/\theta$
1	Allametatababaei	1.28283	0.7795265	1.373086	0.7282865
2	Alzahra	1.317242	0.7591619	1.424034	0.7022304
3	Arak	1	1	1	1
4	Beinolmelaliemam Khomeini	1	1	1	1
5	Elmo-sanaat	1	1	1	1
6	Esfahan	1	1	1.087859	0.9192368
7	Ferdosimashhad	1.086746	0.9201782	1.183053	0.8452707
8	Gilan	1	1	1	1
9	Gorgan	1	1	1	1
10	Hormozgan	1	1	1	1
11	Kashan	1	1	1	1
12	Mazandaran	1	1	1.105459	0.9046016
13	Oroomiye	1	1	1	1
14	Razi	1	1	1.054036	0.9487342
15	Sanaatiamirkabir	1	1	1.318638	0.7583582
16	Sanaati Esfahan	1.145294	0.8731383	1.210305	0.826238
17	Sanaatikhajenasir	1.163386	0.8595599	1.165629	0.8579059
18	SanaatiSahand	1	1	1	1
19	Sanaatisharif	1	1	1	1
20	Semnan	1	1	1	1
21	Shahidbeheshti	1.16654	0.8572359	1.247941	0.8013199
22	Shahidchamran	1	1	1.092741	0.9151299
23	Shahrekord	1	1	1	1
24	Shiraz	1	1	1	1
25	Tarbiyatmodares	1	1	1.40128	0.7136332
26	Tehran	1	1	1	1
27	Zanjan	1	1	1	1

Based on additive models, we see that the cost efficiency perspective shows that only 16 out of 27 universities are efficient. This might show that in Iranian public universities cost has not been a major concern. On other hand, it could be said that the teaching quality has been a major concern as 21 out of 27 are efficient. Research productivity with 18 efficient universities tangles around between higher teaching quality and lower cost efficacy.

It is important to note that DEA efficiency scores reported here are sensitive to our choice of DMUs as we have only considered 27 academic units out 47 academic units and any other combination of DMUs could produce different efficiency scores. We have considered only a sample to illustrate our point of view, i.e., our perspective is important in the structure of our DEA model. We have discussed earlier that there could be better set of input-output metrics to represent teaching quality, research productivity and cost efficiency (see e.g., Celik and Ecer, 2009; Kao and Hung, 2008).

Table 8 Comparing efficiency scores from different perspectives on different models

Row	University name	Teaching			Research			Cost Efficiency		
		Additive	BCC	CCR	Additive	BCC	CCR	Additive	BCC	CCR
1	Allamatabatabaei	-11.18	0.77	0.72	-0.00	1	1	-2.44	1	1
2	Alzahra	0	0.75	0.70	0	1	0.88	-51.01	0.81	0.80
3	Arak	0	1	1	0	1	1	0	1	1
4	Beinolmelaliemam Khomeini	0	1	1	0	1	1	-154.95	0.55	0.55
5	Elmo-sanaat	0	1	1	0	1	1	0	1	1
6	Esfahan	-14.39	0.89	0.91	-33.01	0.72	0.67	0	0.91	0.91
7	Ferdosimashhad	-2.02	0.92	0.84	-47.40	0.69	0.67	0	1	1
8	Gilan	0	1	1	0	1	1	-88.95	0.73	0.73
9	Gorgan	0	1	1	0	1	1	-53.62	1	1
10	Hormozgan	0	1	1	0	1	1	0	1	1
11	Kashan	0	1	1	0	1	0.94	0	1	1
12	Mazandaran	0	1	0.90	0	1	1	0	1	1
13	Oroomiye	0	1	1	-3.00	0.85	0.85	-66.33	0.73	0.69
14	Razi	0	1	0.94	0	1	1	0	1	1
15	Sanaati Amir Kabir	0	1	0.75	0	1	0.53	0	1	1
16	Sanaati Esfahan	-2.95	0.87	0.82	-39.98	0.65	0.56	0	1	1
17	Sanaatikhajenasir	-12.73	0.85	0.85	-18.60	0.62	0.61	0	1	1
18	SanaatiSahand	0	1	1	0	1	0.35	-206.95	0.62	0.46
19	Sanaatisharif	0	1	1	0	1	0.57	-41.68	0.92	0.82
20	Semnan	0	1	1	-2.59	0.62	1	0	1	1
21	Shahidbeheshti	-5.77	0.85	0.80	0	1	0.95	-24.92	0.97	0.97
22	Shahidchamran	0	1	0.91	-24.65	0.48	0.48	-9.77	0.99	0.87
23	Shahrekor	0	1	1	0	1	0.24	0	1	0.59
24	Shiraz	0	1	1	0	1	0.83	0	1	1
25	Tarbiyatmodares	0	1	0.71	-22.70	0.83	0.69	0	1	1
26	Tehran	0	1	1	0	1	1	-97.16	0.72	0.72
27	Zanjan	0	0.91	1	0	1	0.25	0	1	1
Total efficient universities		21	19	15	18	19	11	16	17	16

5 CONCLUSION

Higher education institutes as complex entities are formed by an intermix of humans including policy makers, government, administrators, faculties, students, staffs, employers, society at large, and physical resources such as buildings, books, lab, etc. Hence, the stakeholders are different and they pursue different perspectives. Each perspective holds a different view on efficiency, influenced by own interests.

In this paper, we proposed that the DEA, as a standard technique of efficiency analysis, can only assess universities effectively if prior to assessment we explicitly take a clear and specific perspective. On the contrary, our investigations shows that any efficiency performance analysis that is based on the notion of unstructured aggregate performance measures may lead to controversial results, though this remains to be shown in future research. Upon this understanding, we then proposed different DEA structures to assess public universities in Iran using available data. For illustrations, we performed efficiency analysis on a sample of public universities in Iran from three different perspectives of importance, i.e., teaching quality, research productivity, and cost efficiency.

We note that, the data we have used in our experiments are more than 10 years old, while, in recent years, the universities in Iran have undergone dramatic changes in particular with respect to postgraduate programs (MSc and PhD degrees) and research orientations. A fresh set of data can lead to a different set of efficiency scores as reported in Table 8.

In our DEA models can hardly represent research productivity measures so to pursue further research in this area we propose to develop more able metrics We also propose that new researches could be directed toward aggregate efficiency analysis of academic units using newly developed Network DEAs to account for sub functional efficiencies such as teaching, research and cost as well as the overall efficiency in a unified framework.

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APPENDIX

Sample code for Model (5) in Table 5:

Max = θ ;
 $25.29 * \lambda_1 + 16.69 * \lambda_2 + 25.12 * \lambda_3 + 11.61 * \lambda_4 + 26.07 * \lambda_5 + 18.68 * \lambda_6 + 15.92 * \lambda_7 + 16.33 * \lambda_8 +$
 $16.97 * \lambda_9 + 15.83 * \lambda_{10} + 12.4 * \lambda_{11} + 24.87 * \lambda_{12} + 14.4 * \lambda_{13} + 22.23 * \lambda_{14} + 12.41 * \lambda_{15} +$
 $12.71 * \lambda_{16} + 16.05 * \lambda_{17} + 3.03 * \lambda_{18} + 10.97 * \lambda_{19} + 21.25 * \lambda_{20} + 19.91 * \lambda_{21} + 10.9 * \lambda_{22} +$
 $2.88 * \lambda_{23} + 19 * \lambda_{24} + 14.45 * \lambda_{25} + 18 * \lambda_{26} + 4.61 * \lambda_{27} - 10.97 * \theta \geq 0$;
 $0.2083333 * \lambda_1 + 0.1136364 * \lambda_2 + 0.030303 * \lambda_3 + 0.0210084 * \lambda_4 + 0.1041667 * \lambda_5 + 0.15625 * \lambda_6 +$
 $0.1020408 * \lambda_7 + 0.1052632 * \lambda_8 + 0.1785714 * \lambda_9 + 0.3571429 * \lambda_{10} + 0.2 * \lambda_{11} + 0.1333333 * \lambda_{12} +$
 $0.0826446 * \lambda_{13} + 0.1923077 * \lambda_{14} + 0.1123596 * \lambda_{15} + 0.0694444 * \lambda_{16} + 0.1010101 * \lambda_{17} +$
 $0.03003 * \lambda_{18} + 0.0833333 * \lambda_{19} + 0.18867 * \lambda_{20} + 0.1176471 * \lambda_{21} + 0.0952381 * \lambda_{22} +$
 $0.2777778 * \lambda_{23} + 0.0900901 * \lambda_{24} + 0.0833333 * \lambda_{25} + 0.0617284 * \lambda_{26} + 0.0925926 * \lambda_{27} \leq$
 0.0833333 ;
 $0.0917 * \lambda_1 + 0.0971 * \lambda_2 + 0.2326 * \lambda_3 + 0.119 * \lambda_4 + 0.0422 * \lambda_5 + 0.1493 * \lambda_6 + 0.1087 * \lambda_7 + 0.0694 * \lambda_8 +$
 $0.1389 * \lambda_9 + 0.1923 * \lambda_{10} + 0.2326 * \lambda_{11} + 0.125 * \lambda_{12} + 0.1429 * \lambda_{13} + 0.2 * \lambda_{14} + 0.125 * \lambda_{15} + 0.1235 * \lambda_{16}$
 $+ 0.0592 * \lambda_{17} + 0.1075 * \lambda_{18} + 0.1104 * \lambda_{19} + 0.1111 * \lambda_{20} + 0.0769 * \lambda_{21} + 0.1471 * \lambda_{22} + 0.1 * \lambda_{23} +$
 $0.0901 * \lambda_{24} + 0.5 * \lambda_{25} + 0.0847 * \lambda_{26} + 0.1667 * \lambda_{27} \leq 0.1104$;
 $0.5814 * \lambda_1 + 0.6452 * \lambda_2 + 0.7299 * \lambda_3 + 0.6711 * \lambda_4 + 0.5435 * \lambda_5 + 0.6024 * \lambda_6 + 0.5988 * \lambda_7 + 0.7299 * \lambda_8 +$
 $0.6944 * \lambda_9 + 0.7407 * \lambda_{10} + 0.9615 * \lambda_{11} + 0.641 * \lambda_{12} + 0.7407 * \lambda_{13} + 0.7463 * \lambda_{14} + 0.5495 * \lambda_{15} +$
 $0.5319 * \lambda_{16} + 0.6329 * \lambda_{17} + 0.7519 * \lambda_{18} + 0.4425 * \lambda_{19} + 0.8475 * \lambda_{20} + 0.5291 * \lambda_{21} + 0.6757 * \lambda_{22} +$
 $0.7194 * \lambda_{23} + 0.5348 * \lambda_{24} + 0.5181 * \lambda_{25} + 0.4878 * \lambda_{26} + 0.7407 * \lambda_{27} \leq 0.4425$;
 $32.4 * \lambda_1 + 38.22 * \lambda_2 + 15.75 * \lambda_3 + 15.6 * \lambda_4 + 32.86 * \lambda_5 + 43.1 * \lambda_6 + 58.51 * \lambda_7 + 10.79 * \lambda_8 + 8.2 * \lambda_9 +$
 $6 * \lambda_{10} + 45 * \lambda_{11} + 55.8 * \lambda_{12} + 12 * \lambda_{13} + 11.22 * \lambda_{14} + 68.76 * \lambda_{15} + 46 * \lambda_{16} + 32.5 * \lambda_{17} + 15.48 * \lambda_{18} +$
 $32.729 * \lambda_{19} + 18 * \lambda_{20} + 51 * \lambda_{21} + 37.9 * \lambda_{22} + 5.5 * \lambda_{23} + 64.41 * \lambda_{24} + 44.4 * \lambda_{25} + 10.95 * \lambda_{26} +$
 $25.55 * \lambda_{27} \leq 32.729$;
 $0.0438 * \lambda_1 + 0.0348 * \lambda_2 + 0.0635 * \lambda_3 + 0.0641 * \lambda_4 + 0.0596 * \lambda_5 + 0.0588 * \lambda_6 + 0.0446 * \lambda_7 + 0.0796 * \lambda_8 +$
 $0.122 * \lambda_9 + 0.1667 * \lambda_{10} + 0.0222 * \lambda_{11} + 0.0286 * \lambda_{12} + 0.0833 * \lambda_{13} + 0.0891 * \lambda_{14} + 0.0563 * \lambda_{15} +$
 $0.1096 * \lambda_{16} + 0.0839 * \lambda_{17} + 0.0646 * \lambda_{18} + 0.0438 * \lambda_{19} + 0.0556 * \lambda_{20} + 0.0319 * \lambda_{21} + 0.0385 * \lambda_{22} +$
 $0.1818 * \lambda_{23} + 0.0485 * \lambda_{24} + 0.0345 * \lambda_{25} + 0.0205 * \lambda_{26} + 0.0391 * \lambda_{27} \leq 0.0438$;
 $0.4785 * \lambda_1 + 0.1953 * \lambda_2 + 0.8475 * \lambda_3 + 0.369 * \lambda_4 + 0.1799 * \lambda_5 + 0.2488 * \lambda_6 + 0.289 * \lambda_7 +$
 $0.2688 * \lambda_8 + 0.578 * \lambda_9 + 0.5051 * \lambda_{10} + 0.1244 * \lambda_{11} + 0.4348 * \lambda_{12} + 0.2755 * \lambda_{13} + 0.3145 * \lambda_{14} +$
 $0.2874 * \lambda_{15} + 0.4274 * \lambda_{16} + 1.9231 * \lambda_{17} + 0.3731 * \lambda_{18} + 0.4739 * \lambda_{19} + 0.3802 * \lambda_{20} +$
 $0.2513 * \lambda_{21} + 0.3745 * \lambda_{22} + 0.5882 * \lambda_{23} + 0.2208 * \lambda_{24} + 0.5291 * \lambda_{25} + 0.2114 * \lambda_{26} +$
 $1.1111 * \lambda_{27} \leq 0.4739$;
 $0.0476 * \lambda_1 + 0.0323 * \lambda_2 + 0.0476 * \lambda_3 + 0.012 * \lambda_4 + 0.0526 * \lambda_5 + 0.0476 * \lambda_6 + 0.0345 * \lambda_7 +$
 $0.0192 * \lambda_8 + 0.02 * \lambda_9 + 0.0714 * \lambda_{10} + 0.0625 * \lambda_{11} + 0.0143 * \lambda_{12} + 0.0286 * \lambda_{13} + 0.0238 * \lambda_{14} +$
 $0.0299 * \lambda_{15} + 0.0435 * \lambda_{16} + 0.05 * \lambda_{17} + 0.0068 * \lambda_{18} + 0.0269 * \lambda_{19} + 0.0208 * \lambda_{20} + 0.0455 * \lambda_{21} +$
 $0.0303 * \lambda_{22} + 0.0182 * \lambda_{23} + 0.0435 * \lambda_{24} + 0.0333 * \lambda_{25} + 0.05 * \lambda_{26} + 0.0179 * \lambda_{27} \leq 0.0269$;

Sample code for Model (6) in Table 5/primal model:

Min = $-S_1^+ - S_2^+ - S_3^+ - S_1^- - S_2^- - S_3^- - S_4^- - S_5^- - S_6^-$;
 $4.8 * \lambda_1 + 8.8 * \lambda_2 + 33 * \lambda_3 + 47.6 * \lambda_4 + 9.6 * \lambda_5 + 6.4 * \lambda_6 + 9.8 * \lambda_7 + 9.5 * \lambda_8 + 5.6 * \lambda_9 + 2.8 * \lambda_{10} +$
 $5 * \lambda_{11} + 7.5 * \lambda_{12} + 12.1 * \lambda_{13} + 5.2 * \lambda_{14} + 8.9 * \lambda_{15} + 14.4 * \lambda_{16} + 9.9 * \lambda_{17} + 33.3 * \lambda_{18} + 12 * \lambda_{19} +$
 $5.3 * \lambda_{20} + 8.5 * \lambda_{21} + 10.5 * \lambda_{22} + 3.6 * \lambda_{23} + 11.1 * \lambda_{24} + 12 * \lambda_{25} + 16.2 * \lambda_{26} + 10.8 * \lambda_{27} + S_1^- =$
 2.8 ;
 $10.9 * \lambda_1 + 10.3 * \lambda_2 + 4.3 * \lambda_3 + 8.4 * \lambda_4 + 23.7 * \lambda_5 + 6.7 * \lambda_6 + 9.2 * \lambda_7 + 14.4 * \lambda_8 + 7.2 * \lambda_9 +$
 $5.2 * \lambda_{10} + 4.3 * \lambda_{11} + 8 * \lambda_{12} + 7 * \lambda_{13} + 5 * \lambda_{14} + 8 * \lambda_{15} + 8.1 * \lambda_{16} + 16.9 * \lambda_{17} + 9.3 * \lambda_{18} +$
 $9.056 * \lambda_{19} + 9 * \lambda_{20} + 13 * \lambda_{21} + 6.8 * \lambda_{22} + 10 * \lambda_{23} + 11.1 * \lambda_{24} + 2 * \lambda_{25} + 11.8 * \lambda_{26} + 6 * \lambda_{27} + S_2^-$
 $= 5.2$;
 $1.72 * \lambda_1 + 1.55 * \lambda_2 + 1.37 * \lambda_3 + 1.49 * \lambda_4 + 1.84 * \lambda_5 + 1.66 * \lambda_6 + 1.67 * \lambda_7 + 1.37 * \lambda_8 + 1.44 * \lambda_9 +$
 $1.35 * \lambda_{10} + 1.04 * \lambda_{11} + 1.56 * \lambda_{12} + 1.35 * \lambda_{13} + 1.34 * \lambda_{14} + 1.82 * \lambda_{15} + 1.88 * \lambda_{16} + 1.58 * \lambda_{17} +$
 $1.33 * \lambda_{18} + 2.26 * \lambda_{19} + 1.18 * \lambda_{20} + 1.89 * \lambda_{21} + 1.48 * \lambda_{22} + 1.39 * \lambda_{23} + 1.87 * \lambda_{24} + 1.93 * \lambda_{25} +$
 $2.05 * \lambda_{26} + 1.35 * \lambda_{27} + S_3^- = 1.35$;
 $22.8278 * \lambda_1 + 28.72 * \lambda_2 + 15.75 * \lambda_3 + 15.6 * \lambda_4 + 16.77 * \lambda_5 + 17 * \lambda_6 + 22.4 * \lambda_7 + 12.57 * \lambda_8 +$

$$\begin{aligned}
& 8.2*\Lambda_9 + 6*\Lambda_{10} + 45*\Lambda_{11} + 35*\Lambda_{12} + 12*\Lambda_{13} + 11.22*\Lambda_{14} + 17.77*\Lambda_{15} + 9.127*\Lambda_{16} + \\
& 11.92*\Lambda_{17} + 15.48*\Lambda_{18} + 22.8278*\Lambda_{19} + 18*\Lambda_{20} + 31.38*\Lambda_{21} + 26*\Lambda_{22} + 5.5*\Lambda_{23} + 20.61*\Lambda_{24} \\
& + 29*\Lambda_{25} + 48.78*\Lambda_{26} + 25.55*\Lambda_{27} + S_4^- = 6; \\
& 2.09*\Lambda_1 + 5.12*\Lambda_2 + 1.18*\Lambda_3 + 2.71*\Lambda_4 + 5.56*\Lambda_5 + 4.02*\Lambda_6 + 3.46*\Lambda_7 + 3.72*\Lambda_8 + 1.73*\Lambda_9 + \\
& 1.98*\Lambda_{10} + 8.04*\Lambda_{11} + 2.3*\Lambda_{12} + 3.63*\Lambda_{13} + 3.18*\Lambda_{14} + 3.48*\Lambda_{15} + 2.34*\Lambda_{16} + 0.52*\Lambda_{17} + \\
& 2.68*\Lambda_{18} + 2.11*\Lambda_{19} + 2.63*\Lambda_{20} + 3.98*\Lambda_{21} + 2.67*\Lambda_{22} + 1.7*\Lambda_{23} + 4.53*\Lambda_{24} + 1.89*\Lambda_{25} + \\
& 4.73*\Lambda_{26} + 0.9*\Lambda_{27} + S_5^- = 1.98; \\
& 21*\Lambda_1 + 31*\Lambda_2 + 21*\Lambda_3 + 83*\Lambda_4 + 19*\Lambda_5 + 21*\Lambda_6 + 29*\Lambda_7 + 52.4*\Lambda_8 + 50*\Lambda_9 + 14*\Lambda_{10} + \\
& 16*\Lambda_{11} + 70*\Lambda_{12} + 35*\Lambda_{13} + 42*\Lambda_{14} + 33.4*\Lambda_{15} + 23*\Lambda_{16} + 20*\Lambda_{17} + 148*\Lambda_{18} + 37.136*\Lambda_{19} + \\
& 48*\Lambda_{20} + 22*\Lambda_{21} + 33*\Lambda_{22} + 55*\Lambda_{23} + 23*\Lambda_{24} + 30*\Lambda_{25} + 20*\Lambda_{26} + 56*\Lambda_{27} + S_6^- = 14; \\
& 20.058*\Lambda_1 + 12.576*\Lambda_2 + 24.463*\Lambda_3 + 16.792*\Lambda_4 + 22.142*\Lambda_5 + 23.716*\Lambda_6 + 19.396*\Lambda_7 + \\
& 15.347*\Lambda_8 + 30.888*\Lambda_9 + 50.673*\Lambda_{10} + 23.114*\Lambda_{11} + 13.578*\Lambda_{12} + 15.962*\Lambda_{13} + 18.824*\Lambda_{14} + \\
& 17.137*\Lambda_{15} + 19.517*\Lambda_{16} + 17.32072*\Lambda_{17} + 13.194*\Lambda_{18} + 27.149*\Lambda_{19} + 22.991*\Lambda_{20} + \\
& 15.065*\Lambda_{21} + 21.451*\Lambda_{22} + 23.447*\Lambda_{23} + 23.785*\Lambda_{24} + 10.461*\Lambda_{25} + 14.314*\Lambda_{26} + \\
& 15.962*\Lambda_{27} - S_1^+ = 50.673; \\
& 32.4*\Lambda_1 + 38.22*\Lambda_2 + 15.75*\Lambda_3 + 15.6*\Lambda_4 + 32.86*\Lambda_5 + 43.1*\Lambda_6 + 58.51*\Lambda_7 + 10.79*\Lambda_8 + \\
& 8.2*\Lambda_9 + 6*\Lambda_{10} + 45*\Lambda_{11} + 55.8*\Lambda_{12} + 12*\Lambda_{13} + 11.22*\Lambda_{14} + 68.76*\Lambda_{15} + 46*\Lambda_{16} + 32.5*\Lambda_{17} \\
& + 15.48*\Lambda_{18} + 32.729*\Lambda_{19} + 18*\Lambda_{20} + 51*\Lambda_{21} + 37.9*\Lambda_{22} + 5.5*\Lambda_{23} + 64.41*\Lambda_{24} + 44.4*\Lambda_{25} + \\
& 10.95*\Lambda_{26} + 25.55*\Lambda_{27} - S_2^+ = 6; \\
& 25.29*\Lambda_1 + 16.69*\Lambda_2 + 25.12*\Lambda_3 + 11.61*\Lambda_4 + 26.07*\Lambda_5 + 18.68*\Lambda_6 + 15.92*\Lambda_7 + 16.33*\Lambda_8 + \\
& 16.97*\Lambda_9 + 15.83*\Lambda_{10} + 12.4*\Lambda_{11} + 24.87*\Lambda_{12} + 14.4*\Lambda_{13} + 22.23*\Lambda_{14} + 12.41*\Lambda_{15} + \\
& 12.71*\Lambda_{16} + 16.05*\Lambda_{17} + 3.03*\Lambda_{18} + 10.97*\Lambda_{19} + 21.25*\Lambda_{20} + 19.91*\Lambda_{21} + 10.9*\Lambda_{22} + \\
& 2.88*\Lambda_{23} + 19*\Lambda_{24} + 14.45*\Lambda_{25} + 18*\Lambda_{26} + 4.61*\Lambda_{27} - S_3^+ = 15.83; \\
& \Lambda_1 + \Lambda_2 + \Lambda_3 + \Lambda_4 + \Lambda_5 + \Lambda_6 + \Lambda_7 + \Lambda_8 + \Lambda_9 + \Lambda_{10} + \Lambda_{11} + \Lambda_{12} + \Lambda_{13} + \Lambda_{14} + \Lambda_{15} + \Lambda_{16} + \\
& \Lambda_{17} + \Lambda_{18} + \Lambda_{19} + \Lambda_{20} + \Lambda_{21} + \Lambda_{22} + \Lambda_{23} + \Lambda_{24} + \Lambda_{25} + \Lambda_{26} + \Lambda_{27} = 1;
\end{aligned}$$