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An Integrated Model for a Two-supplier Supply Chain with Uncertainty in the Supply

Mehdi Sajadifar[1]*,Behrooz Pourghannad[2]

[1,2]Department of Industrial Engineering, University of Science and Culture, Tehran, Iran
[1]sajadifar@usc.ac.ir, [2]b.pourghanad@usc.ac.ir

ABSTRACT

The objective of this paper is to study an integrated two-supplier supply chain whose suppliers are unreliable. An unreliable supplier is alternative between available (*ON*) and unavailable (*OFF*) states which are considered to be independent exponential variables. The suppliers apply a continuous review policy and the retailer uses an adapted continuous review base on an(R,Q) policy. Transportation times are constant and lead times are non-zero random variables. The retailer faces independent Poisson demands. Using the idea of the one-for-one ordering policy, we implicitly incorporate the distribution function of the random delay for obtaining the value of the expected costs of system. Finally, resorting to a dozen of sample problems, we show that the average cost reduction in our inventory system is at least 3.69% and at most 36.95% comparing to the one with only one supplier.

***Keywords*:** Supply Chain Management, Unreliable Supply, Continuous Review Policy, Poisson Demand, Non-Zero Lead-time.

1. INTRODUCTION

One assumption that is frequently used in the inventory literature is the reliable supplier. This assumption indicates that each supplier be continuously available at any time an order is placed. Gurler and Parlar (1997) stated that this assumption is one of the unstated assumptions in almost every inventory model. An integrated two-supplier supply chain with unreliable suppliers is studied in this paper. Balcıog̃lu and Gürler (2011) described an unreliable supplier as the one that is alternative between available (*ON*) and unavailable (*OFF*) states. These two durations are considered to be independent exponential variables, like some other studies in the supply interruption literature. The non-zero random lead time is the sum of a constant transportation time and a random delay occurs due to the availability of stock at the supplier. As far as we know, for the first time in the literature, this paper aims at deriving the cost function for an integrated two-supplier supply chain with uncertainty in the supply.

There are several reasons for considering uncertainty of the supply process, as are mentioned in the

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*Corresponding Author

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literature; machine breakdowns, material shortages, labor strikes, capacity constraints, and political crises to name but a few. Each of these factors can result in changing the status of suppliers from available (*ON*) to unavailable (*OFF*) state and hence to supply interruption (Mohebbi, 2004). As some practical examples, OPEC oil embargoes and Canada's automobile industry supply interruption problems have been mentioned in the supply interruption literature (for more details see Parlar and Perry (1996), Mohebbi (2004), and Tajbakhsh*et al.* (2007)).

Some early studies in inventory models with an unreliable supplier use Economic Order Quantity (*EOQ*) assumptions. These studies analyze problems under various characterization probability distributions describing the *ON*/OFF periods (Mohebbi, 2004). Examples of works belonging to *EOQ* category are Parlar and Berkin (1991), Weiss and Rosenthal (1992), Parlar and Perry (1995), Parlar and Perry (1996), Gurler and Parlar (1997), and Parlar (2000). In these studies demands are deterministic, replenishments are instantaneous, and the lead time is zero. Although the *EOQ* assumptions do not correctly present the real world conditions, these studies provided a base for later studies.

In the late 1990s and after that, some researchers tried to relax *EOQ* assumptions. A number of studies including Parlar et al. (1995), Arreola-Risa and DeCroix (1998), and Ozekici and Parlar (1999) considered the problem in the context of an inventory system with random demands, zero lead times and an unreliable supply process. In 2006, Mohebbi and Hao (2006) indicated that analytical treatment of inventory systems with random supply interruption and non-zero lead time remains largely unexplored and there are just a few existing models in the inventory control literature in this area.

Parlar (1997) considered an unreliable supplier and used a continuous review inventory system with stochastic demands, random lead times and backorders. He extended Hadly and Within's (1963) approximation for his problem with the assumption that at any time at most one order can be outstanding. Gupta (1996) presented an exact cost minimization model for a continuous review inventory system with unit-sized Poisson demands, constant lead times and lost sales, in which the supplier's *ON* and *OFF* periods, are exponentially distributed and at any time, only one single order is outstanding.

Mohebbi (2003) developed an exact cost-minimization model for a lost sale, continuous review inventory system with compound Poisson demands and Erlang lead times under an (s,Q)-type control policy with at most one outstanding order at any time. Later, Mohebbi (2004) presented the exact treatment of a related problem, assuming that the supplier's *ON* and *OFF* periods constitute an alternating renewal process and lead times follow a Hyper-exponential distribution. Mohebbi and Hao (2006) studied the former problem which is a dyadic supply chain with random demands, random lead times and lost sales, assuming that lead times follow an Erlang distribution.

A review of the literature on the supply interruption reveals some gaps. First of all, as mentioned, purchasing managers are afraid of dependency on a single source because of high risk and uncertainty. As a result, one approach which is used frequently in the literature to overcome uncertainty in the supply is the diversification or the multiple sourcing. However, much of the existing literature on the supply uncertainty studies the dyadic supply chain. Although there are some researches considering a multiple sourcing problem, like Parlar and Perry (1996) and Gurler and Parlar (1997), most of them have used *EOQ* assumptions. Parlar and Perry (1996) investigated an approximation for a multi-suppliers model under *EOQ* assumptions and assumed that both *ON* and *OFF* periods are independent exponential distributions. Gurler and Parlar (1997) studied an inventory model with two randomly available suppliers modeling the availability of suppliers as a semi-Markov process. In their study, the *ON* periods have an Erlang distribution and the *OFF* periods have a general distribution for each supplier. In addition, their study includes *EOQ* assumptions, i.e. deterministic demand rate, zero lead times and no planned shortages. In case of the multi-supplier models with uncertainty in the supply, as far as we know, there are no studies considering stochastic demands and non-zero lead times.

As mentioned above, although inventory models with supply interruption and non-zero lead times present the most of real world conditions, the existing literature on them is scarce (Mohebbi and Hao, 2006). This fact illustrates the second gap in the supply interruption literature. Finally, the literature on the supply interruption problems reveals another gap. In the modern global competitive market, the supplier and the retailer should be treated as strategic partners in the supply chain with a long-term cooperative relationship in order to be beneficial for both of them. Nevertheless, to the best of our knowledge, there is no integrated model considering supply interruption in the literature. Previous studies only aimed at determining the optimum solutions that minimized the cost from the retailer's side, so the literature on the integrated uncertain supplier remains largely unexplored. Although there are some studies that investigate the integrated supplier systems like Sajadifar et al. (2008), and Haji and Sajadifar (2008), they have assumed that the supplier is available whenever an order is placed.

This paper deals with an integrated solution for a two-supplier supply chain with supply interruption, i.e. each supplier has available (*ON*) or unavailable (*OFF*) state and it is assumed that the *ON* and *OFF* durations are independent exponential random variables. This paper also considers an unreliable inventory system with Poisson demand, non-zero random lead time and backorder. The non-zero random lead time is the sum of a constant transportation time and a random delay occurs due to availability of stock at the supplier. The suppliers apply a continuous review policy and the retailer uses an adapted continuous review based on an (R,Q) policy. To the best of our knowledge, for the first time in the literature, this paper aims at deriving the cost function for an integrated two-supplier supply chain with uncertainty in the supply. This paper develops previous studies in the supply interruption literature, based on introduced gaps, by considering both the integrity and the uncertainty in a two-supplier supply chain with stochastic demands and non-zero lead times.

To aim the cost function of an integrated two-supplier supply chain with uncertainty in the supply, with mentioned assumptions, firstly, the simple integrated two-supplier model should be developed under continuous review policy for both suppliers and the retailer. Secondly, the integrated two-supplier supply chain with uncertainty in the supply is presented. The rest of the paper is organized as follows. In section 2, the notations and assumptions, which are used in the problem formulation, are introduced. Section 3 presents the mathematical model that is investigated in this study. Numerical examples and simulation studies are presented in section 4. Finally, section 5 summarizes the paper and presents future researches.

2. PROBLEM NOTATIONS AND ASSUMTIONS

The following notations are used in this paper:

S_r The inventory position for the retailer in the one-for-one ordering policy

S_s The inventory position for each supplier in the one-for-one ordering policy

L_r^i The transportation time from the supplier to the retailer where the i^{th} supplier is in the

ON state, i = 1, 2

EL_r^i The effective transportation time from the i^{th} supplier to the retailer, that is the lead time from the supplier to the retailer plus the expected value of the interruption incurs when the supplier is in the *OFF* state, i = 1, 2

X_r^i Lead time which the retailer experiences when she receives a batch from i^{th} supplier, i = 1, 2

w_r^i random delay occurs due to the availability of stock at the i^{th} supplier, i = 1, 2

t_k The arrival time of k^{th} customer after time zero at the retailer

L_s^i Transportation time from the outside source to the i^{th} supplier, i = 1, 2

λ_r Demand intensity at the retailer

R_r The retailer's reorder point

Q_r The retailer's order quantity

h_r The holding cost per unit per unit time at the retailer

R_s The suppliers' reorder point (in units of the retailer batches)

Q_s The suppliers' batch size (in units of the retailer batches)

h_s^i The holding cost per unit per unit time at the i^{th} supplier, i = 1, 2

β The shortage cost per unit per unit time at the retailer

C The expected total holding and shortage costs for understudy inventory system when the suppliers are available

C' The expected total holding and shortage costs for understudy inventory system, when at least one of the suppliers is unavailable, i = 1, 2

ζ^i The i^{th} supplier's exponential distribution parameter for the ON duration, i = 1, 2

ψ^i The i^{th} supplier's exponential distribution parameter for the *OFF* duration, i = 1, 2

K The expected total holding and shortage costs for a unit demand in the inventory system

$P_{12}^j(EL_r^1, EL_r^2)$ The probability that path 1 is shorter than path 2, when suppliers send j^{th} sub-batch to the retailer, and $EL_r^1$and EL_r^2 are the suppliers effective transportation time; j= 1, 2, …, Q_s ; i = 1, 2

$P_{21}^j(EL_r^1, EL_r^2)$ The probability that path 2 is shorter than path 1, when suppliers send j sub-batch to the retailer, and $EL_r^1$and EL_r^2 are the suppliers effective transportation time; j= 1, 2, …, Q_s; i = 1, 2

$c_i(S_s, S_r)$ The expected total holding and shortage costs for a unit demand in an inventory system with a one-for-one ordering policy when i^{th} supplier supply the retailer, and the i^{th} supplier is in the ON state, i = 1, 2

$c_i'(S_s, S_r|EL_r^1, EL_r^2)$ The expected total holding and shortage costs for a unit demand in an inventory system with a one-for-one ordering policy when i^{th} supplier supply the retailer, and at least one of the suppliers is unavailable,

$i = 1, 2$

TC_{total} The total expected cost in the inventory system.

To find $c(S_s, S_r)$ and $c_i'(S_s, S_r | EL_r^1, EL_r^2)$ we express them as a weighted mean of costs for the one-for-one ordering policies. As one shall see, with this approach we do not need to consider the parameters $L_r^i, L_s^i, h_r, h_s^i, \beta$ and λ_r explicitly, but these parameters will, of course, affect the costs implicitly through the one-for-one ordering policy costs. To derive the one-for-one carrying and shortage costs, the recursive method in Axsäter (1990) is suggested. A summary of Axsäter (1990), which is adapted for our inventory system, is presented in Appendix A. In 1993, using the procedure introduced by Axsäter (1990), Axsäter (1993) expressed exact cost function for a unit demand in a dyadic supply chain, when both the supplier and the retailer use the continuous review policy, according to (1). It is worth mentioning, Axsäter (1990) and Axsäter (1993) assume that the supplier is always available.

$$KK = \frac{1}{Q_s . Q_r} \sum_{j=R_s+1}^{R_s+Q_s} \sum_{k=R_r+1}^{R_r+Q_r} c(jQ_r, k) \tag{1}$$

KK is the expected value of the exact cost function for a unit demand in a dyadic supply chain with an available supplier. In our model, the inventory control policy at the retailer is an adapted continuous review policy, based on (*R,Q*) policy in the case that each supplier independently alternates between *ON* and *OFF* intervals. The inventory policy is to order Q_r units from two suppliers when the inventory position drops to the reorder point of R_r units. The order placed by the retailer splits equivalently between two suppliers. It means that each supplier receives an order with size $Q_r/2$ units from the retailer. Also, the suppliers use continuous review (R_s,Q_s) policy. In addition the following assumptions are considered here:

1- $Q_r/2$is an integer value, so the retailer's batch size, Q_r, is assumed to be even.
2- We assume that the orders do not cross each other.
3- Each customer demands only one unit of the product.
4- Delayed retailer orders are satisfied on a first-come, first-served base.

3. MATHEMATICAL MODEL

This section aims at deriving the cost function for the described system. To achieve this purpose, firstly, the exact cost function for a two-supplier system (without the supply interruption), in which the suppliers use continuous review (R_s,Q_s) policy and the retailer uses continuous review (R_r,Q_r) policy, is derived in subsection 3.1. Secondly, in subsection 3.2 the results of subsection 3.1 are used to present the proposed cost function for an integrated two-supplier supply chain with supply interruption.

3.1. Two-supplier model without interruption

This subsection derives the exact cost function for two-supplier supply chain without uncertainty in the supply. We will use the results of this subsection to derive the proposed expected cost function for the two-supplier supply chain with supply interruption.

In the case of an integrated two-supplier without supply interruption, the inventory control policy is as it follows. The retailer uses (R_r,Q_r) policy, so when the retailer's inventory position reaches R_r, the retailer places an order with batch size $Q_r/2$ to each supplier. Also, both suppliers use (R_s,Q_s) control policy. As mentioned, the suppliers' batch size is in the units of the retailer's batch size, therefore, based on introduced control policy, each supplier's batch size is multiple of $Q_r/2$.

We use the idea of tracking one unit demand's cost and weighted mean of cost for one-for-one ordering policy to derive the exact cost function for two-supplier supply chain model under continuous review policy for the retailer and both suppliers. One should note that, in each multi echelon inventory system with stochastic demands, the retailer's lead time consists of a constant transportation time and a random delay occurs due to availability of stock at the supplier. The method, which we use to derive the total expected cost function, is similar to the method that Sajadifar et al. (2008) used. In addition, it's worth mentioning that in this subsection, because we assume there are not any supply interruption, the EL_r^1 is equal to L_r^1 , and EL_r^2 is equal to L_r^2.

Let us consider the time that inventory position of suppliers reachesR_s. We designate this time as time zero. At this time, each supplier immediately places an order consisting of Q_s sub-batches with size $Q_r/2$ to the outside sources. We denote these batches by Q_s^0. At this time, the retailer's inventory position is exactly$R_r + Q_r$ and suppliers' inventory positions will just reach $R_s + Q_s$. Since it is assumed that the orders do not cross, the $(R_s + Q_s)Q_r{}^{th}$ order at the retailer will release the orders Q_s^0 at the suppliers. It can be easily seen that the $(R_s + Q_s)Q_r{}^{th}$ customer at the retailer will be caused an order placement at the retailer and the one which has been already assigned to this order at suppliers are the batches Q_s^0. This means that the batches Q_s^0 at the suppliers, are released from the suppliers when $(R_s + Q_s)Q_r{}^{th}$ system demand has occurred after time zero i.e. at time $t_{(R_s+ Q_s)Q_r}$.

Now, we consider the case that the batch Q_s^0 at the first supplier will be received earlier than batch Q_s^0 at the second supplier. The first sub-batch in the batch Q_s^0 will be received from the first supplier earlier than the batch Q_s^0 from the second supplier with the probability $P_{12}^1(L_r^1, L_r^2)$. Therefore, the first unit in the first sub-batch in the batchQ_s^0, which will be received from the first supplier, will be used in the same way to fill the $(R_r + 1)^{th}$retailer's demand after the retailer's order. Then the first unit in the first sub-batch in the batch Q_s^0, which will be received from first supplier, will have the same expected retailer and supplier costs as a unit in a base stock system with $S_s = (R_s + 1)Q_r/2$ and $S_r = R_r + 1$. Hence, the corresponding expected holding and shortage costs will be equal to $c_1((R_s + 1)Q_r/2\,, R_r + 1\,)$.

In the same way, it can be seen that the *i*^th^ unit in the first sub-batch in the batch Q_s^0, which will be received from the first supplier with probability $P_{12}^1(L_r^1, L_r^2)$, will be used to fill the $(R_r + i)$th retailer demand after the retailer's order. Then the ith unit in the first sub-batch in the batch Q_s^0 will have the same expected retailer and supplier costs as a unit in a base stock system with$S_s = (R_s + 1)Q_r/2$ and$S_r = R_r + i$ Therefore, the expected holding and shortage costs for the i^{th} unit in the first sub-batch in the batch Q_s^0 will be equal to $c_1((R_s + 1)Q_r/2\,, R_r + i\,)$. Similarly, i^{th} unit in the j^{th} sub-batch in the batch Q_s^0, which will be received from the first supplier with probability $P_{12}^j(L_r^1, L_r^2)$, will have expected holding and shortage costs as equal to $c_1((R_s + j)Q_r/2\,, R_r + i\,)$.

On the other hand, one can easily see that the i^{th} unit in the j^{th} sub-batch in the batch Q_s^0, which will be received from the second supplier with probability $P_{21}^j(L_r^1, L_r^2)$, will be used to fill the$(R_r + Q_r/2 + i)$retailer's demand after the retailer's order. Then this unit will have the same expected retailer and warehouse costs like a unit in a base stock system with$S_s = (R_s + j)Q_r/2$ and$S_r =$

$R_r + Q_r/2 + i$ and the expected holding and shortage costs for this unit will be equal to $c_2((R_s + j)Q_r/2, R_r + Q_r/2 + i)$, i=1,…,$Q_r/2$,j=1,2,…,Q_s .

It should be noted that each customer demands only one unit of a batch. In addition, considering the fact that inventory positions at the retailer and the suppliers are uniformly distributed (Hadley and Whitin, 1963), the average cost per time unit is determined by averaging over $Q_r \times Q_s$ individual unites as follows.

$$K = \frac{1}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)} P_{12}^{j}(L_r^1, L_r^2).\,c_1\left(\frac{(R_s + j)Q_r}{2}, k\right) + \\ \sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r} P_{12}^{j}(L_r^1, L_r^2).\,c_2\left(\frac{(R_s + j)Q_r}{2}, k\right)\end{array}\right] + \frac{1}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r} P_{21}^{j}(L_r^1, L_r^2).\,c_1\left(\frac{(R_s + j)Q_r}{2}, k\right) \\ + \sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)} P_{21}^{j}(L_r^1, L_r^2).\,c_2\left(\frac{(R_s + j)Q_r}{2}, k\right)\end{array}\right] \quad (2)$$

Since the average demand per unit of time is equal toλ_r, the expected total cost of the system per time unit can then be written as (3).

$$C = \frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)} P_{12}^{j}(L_r^1, L_r^2).\,c_1\left(\frac{(R_s + j)Q_r}{2}, k\right) + \\ \sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r} P_{12}^{j}(L_r^1, L_r^2).\,c_2\left(\frac{(R_s + j)Q_r}{2}, k\right)\end{array}\right] + \frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r} P_{21}^{j}(L_r^1, L_r^2).\,c_1\left(\frac{(R_s + j)Q_r}{2}, k\right) \\ + \sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)} P_{21}^{j}(L_r^1, L_r^2).\,c_2\left(\frac{(R_s + j)Q_r}{2}, k\right)\end{array}\right] \quad (3)$$

Lemma 1: The probabilities $P_{12}^{j}(L_r^1, L_r^2)$ and $P_{21}^{j}(L_r^1, L_r^2)$, that $P_{12}^{j}(L_r^1, L_r^2) + P_{21}^{j}(L_r^1, L_r^2) = 1$ and j=1,2,…, Q_s, are computed as follows:

1. If $L_r^1 > L_r^2$ and $L_s^1 > L_s^2$ then $P_{12}^{j}(L_r^1, L_r^2) = 0$ and $P_{21}^{j}(L_r^1, L_r^2) = 1$.
2. If $L_r^1 > L_r^2$, $L_s^1 < L_s^2$, and $L_r^1 + L_s^1 < L_r^2 + L_s^2$ then $P_{12}^{j}(L_r^1, L_r^2) = G^{Q_r(R_s+j)}(L_r^2 + L_s^2 - L_r^1)$ and $P_{21}^{j}(L_r^1, L_r^2) = 1 - G^{Q_r(R_s+j)}(L_r^2 + L_s^2 - L_r^1)$.
3. If $L_r^1 > L_r^2$, $L_s^1 < L_s^2$, and $L_r^1 + L_s^1 > L_r^2 + L_s^2$ then $P_{12}^{j}(L_r^1, L_r^2) = 0$ and $P_{21}^{j}(L_r^1, L_r^2) = 1$.

4. If $L_r^1 < L_r^2$ and $L_s^1 < L_s^2$ then $P_{12}^j(L_r^1, L_r^2) = 1$ and $P_{21}^j(L_r^1, L_r^2) = 0$.
5. If $L_r^1 < L_r^2$, $L_s^1 > L_s^2$, and $L_r^1 + L_s^1 > L_r^2 + L_s^2$ then $P_{12}^j(L_r^1, L_r^2) = 1 - G^{Q_r(R_w + j)}(L_r^1 + L_s^1 - L_r^2)$ and $P_{21}^j(L_r^1, L_r^2) = G^{Q_r(R_w + j)}(L_r^1 + L_s^1 - L_r^2)$.
6. If $L_r^1 < L_r^2$, $L_s^1 > L_s^2$, and $L_r^1 + L_s^1 < L_r^2 + L_s^2$ then $P_{12}^j(L_r^1, L_r^2) = 1$ and $P_{21}^j(L_r^1, L_r^2) = 0$.

Proof: See Appendix B.

3.2. Two-supplier model with supply interruption

This subsection considers a generalization of the previous two-supplier model, which has been presented in subsection 3.1, and assumes that the decision maker deals with two-supplier which may randomly be *ON* or *OFF*. As mentioned, the duration of *ON* and *OFF* periods are two independent exponential distributions with parameters ζ^i and ψ^i, i=1,2, respectively. The inventory policy is to split the orders equivalently between suppliers, i.e. order $Q_r/2$ units from each of the two suppliers. In this model, the states are according to Table 1. These four states are important because the states of each supplier can affect the expected total cost. Therefore, we will partition the expected total cost, based on these four states, into four partitions.

Table 1 States of the system

state \ supplier	1	2
0	*ON*	*ON*
1	*ON*	*OFF*
2	*OFF*	*ON*
3	*OFF*	*OFF*

Lemma 2: The long run probabilities $P_j = lim_{n\to\infty} P_{ij}(t)$ are as follows:

$$[P_0, P_1, P_2, P_3] = \frac{1}{(\zeta^1 + \psi^1)(\zeta^2 + \psi^2)}[\psi^1\psi^2, \zeta^2\psi^1, \zeta^1\psi^2, \zeta^1\zeta^2]. \tag{4}$$

Proof: See Appendix C.

Using the idea of conditional probability, this section derives the cost function for an integrated two-supplier supply chain with supply interruption. To achieve this purpose, one needs to calculate the cost function related to each mentioned state. Therefore, lemma 3 to 5 present the cost functions relates to states 0, 1, 2, and 3, and lemma 6 presents the expected total cost function for an integrated two-supplier supply chain with supply interruption.

Lemma 3: The cost function related to state 0, denoted by C_0, is calculated as follows;

$$C_0 = \frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)} P_{12}^{j}(L_r^1, L_r^2).\, c_1\left(\frac{(R_s+j)Q_r}{2}, k\right) + \\ \sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r} P_{12}^{j}(L_r^1, L_r^2).\, c_2\left(\frac{(R_s+j)Q_r}{2}, k\right)\end{array}\right] + \frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r} P_{21}^{j}(L_r^1, L_r^2).\, c_1\left(\frac{(R_s+j)Q_r}{2}, k\right) \\ + \sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)} P_{21}^{j}(L_r^1, L_r^2).\, c_2\left(\frac{(R_s+j)Q_r}{2}, k\right)\end{array}\right] \tag{5}$$

Proof: See section 3.1 which presents the cost function of two-supplier supply chain without interruption. Because in state 0 both suppliers are available, this state is equivalent to the case where there is no interruption and equation 3 exactly presents the cost function related to this state.

Lemma 4: Denoted by $C_1$and C_2, the cost functions related to state 1 and 2 are calculated according to (6) and (7), respectively;

$$C_1 = \frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)} P_{12}^{j}\left(L_r^1, L_r^2+\frac{1}{\psi^2}\right)\times c_1'\left(\frac{(R_s+j)Q_r}{2}, k \mid L_r^1, L_r^2+\frac{1}{\psi^2}\right) + \\ \sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r} P_{12}^{j}\left(L_r^1, L_r^2+\frac{1}{\psi^2}\right)\times c_2'\left(\frac{(R_s+j)Q_r}{2}, k \mid L_r^1, L_r^2+\frac{1}{\psi^2}\right)\end{array}\right] + \frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r} P_{12}^{j}\left(L_r^1, L_r^2+\frac{1}{\psi^2}\right)\times c_1'\left(\frac{(R_s+j)Q_r}{2}, k \mid L_r^1, L_r^2+\frac{1}{\psi^2}\right) \\ + \sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)} P_{12}^{j}\left(L_r^1, L_r^2+\frac{1}{\psi^2}\right)\times c_2'\left(\frac{(R_s+j)Q_r}{2}, k \mid L_r^1, L_r^2+\frac{1}{\psi^2}\right)\end{array}\right] \tag{6}$$

$$C_2 = \frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)} P_{12}^{j}\left(L_r^1+\frac{1}{\psi^1}, L_r^2\right)\times c_1'\left(\frac{(R_s+j)Q_r}{2}, k \mid L_r^1+\frac{1}{\psi^1}, L_r^2\right) + \\ \sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r} P_{12}^{j}\left(L_r^1+\frac{1}{\psi^1}, L_r^2\right)\times c_2'\left(\frac{(R_s+j)Q_r}{2}, k \mid L_r^1+\frac{1}{\psi^1}, L_r^2\right)\end{array}\right] \tag{7}$$

$$+\frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r} P_{12}^{j}\left(L_r^1+\frac{1}{\psi^1},L_r^2\right)\times c_1'\left(\frac{(R_s+\ j)Q_r}{2},k|\ L_r^1+\frac{1}{\psi^1},L_r^2\right)\\ +\sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)} P_{12}^{j}\left(L_r^1+\frac{1}{\psi^1},L_r^2\right)\times c_2'\left(\frac{(R_s+\ j)Q_r}{2},k|\ L_r^1+\frac{1}{\psi^1},L_r^2\right)\end{array}\right]$$

Proof: In state 1, according to the Table 1, the first supplier is in the *ON* state and the second supplier is in the *OFF* state. So, when the retailer supplied from the first supplier, the transportation time is L_r^1, and when the retailer supplied from the second supplier, that was in the *OFF* state, the effective transportation time is $L_r^2+(1/\psi^2)$. The L_r^2 is a deterministic transportation time, and the $1/\psi^2$ is the expected value of a delay which occurs due to second supplier's *OFF* state. So, where the effective transportation times are $L_r^1$and $L_r^2+(1/\psi^2)$, for the first and the second supplier respectively, the related cost function, C_1, is calculated, similar to (3), according to (6).

In state 2, the first supplier is not available and the second one is available. In this case, similar to the C_1, the C_2 is calculated, where the effective transportation times are $L_r^1+(1/\psi^1)$ and L_r^2, for the first and the second suppliers respectively.

Lemma 5: The relation (8) calculates the cost function related to state 3, denoted by C_3, as follows;

$$C_3=\frac{\psi^1}{\psi^1+\psi^2}\times C'_3+\frac{\psi^2}{\psi^1+\psi^2}\times C''_3 \tag{8}$$

where,

$$C'_3=\frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)}\left(\begin{array}{c}P_{12}^{j}\left(L_r^1+\frac{1}{\psi^1},L_r^2+\frac{\psi^1+\psi^2}{\psi^1\psi^2}\right)\times\\ c_1'\left(\frac{(R_s+\ j)Q_r}{2},k\ |L_r^1+\frac{1}{\psi^1},L_r^2+\frac{\psi^1+\psi^2}{\psi^1\psi^2}\right)\end{array}\right)+\\ \sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r}\left(\begin{array}{c}P_{12}^{j}\left(L_r^1+\frac{1}{\psi^1},L_r^2+\frac{\psi^1+\psi^2}{\psi^1\psi^2}\right)\times\\ c_2'\left(\frac{(R_s+\ j)Q_r}{2},k\ |\ L_r^1+\frac{1}{\psi^1},L_r^2+\frac{\psi^1+\psi^2}{\psi^1\psi^2}\right)\end{array}\right)\end{array}\right]$$
$$+\frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r}\left(\begin{array}{c}P_{12}^{j}\left(L_r^1+\frac{1}{\psi^1},L_r^2+\frac{\psi^1+\psi^2}{\psi^1\psi^2}\right)\times\\ c_1'\left(\frac{(R_s+\ j)Q_r}{2},k|\ L_r^1+\frac{1}{\psi^1},L_r^2+\frac{\psi^1+\psi^2}{\psi^1\psi^2}\right)\end{array}\right)\\ +\sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)}\left(\begin{array}{c}P_{12}^{j}\left(L_r^1+\frac{1}{\psi^1},L_r^2+\frac{\psi^1+\psi^2}{\psi^1\psi^2}\right)\times\\ c_2'\left(\frac{(R_s+\ j)Q_r}{2},k|\ L_r^1+\frac{1}{\psi^1},L_r^2+\frac{\psi^1+\psi^2}{\psi^1\psi^2}\right)\end{array}\right)\end{array}\right] \tag{9}$$

and similarly,

$$C''_3 = \frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)}\left(\begin{array}{c}P_{12}^{j}\left(L_r^1+\frac{\psi^1+\psi^2}{\psi^1\psi^2},L_r^2+\frac{1}{\psi^2}\right)\times \\ c_1'\left(\frac{(R_s+j)Q_r}{2},k \mid L_r^1+\frac{\psi^1+\psi^2}{\psi^1\psi^2},L_r^2+\frac{1}{\psi^2}\right)\end{array}\right)+ \\ \sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r}\left(\begin{array}{c}P_{12}^{j}\left(L_r^1+\frac{\psi^1+\psi^2}{\psi^1\psi^2},L_r^2+\frac{1}{\psi^2}\right)\times \\ c_2'\left(\frac{(R_s+j)Q_r}{2},k \mid L_r^1+\frac{\psi^1+\psi^2}{\psi^1\psi^2},L_r^2+\frac{1}{\psi^2}\right)\end{array}\right)\end{array}\right]$$
$$+\frac{\lambda_r}{Q_s.Q_r}\left[\begin{array}{l}\sum_{j=1}^{Q_s}\sum_{k=R_r+(Q_r/2)+1}^{R_r+Q_r}\left(\begin{array}{c}P_{12}^{j}\left(L_r^1+\frac{\psi^1+\psi^2}{\psi^1\psi^2},L_r^2+\frac{1}{\psi^2}\right)\times \\ c_1'\left(\frac{(R_s+j)Q_r}{2},k \mid L_r^1+\frac{\psi^1+\psi^2}{\psi^1\psi^2},L_r^2+\frac{1}{\psi^2}\right)\end{array}\right) \\ +\sum_{j=1}^{Q_s}\sum_{k=R_r+1}^{R_r+(Q_r/2)}\left(\begin{array}{c}P_{12}^{j}\left(L_r^1+\frac{\psi^1+\psi^2}{\psi^1\psi^2},L_r^2+\frac{1}{\psi^2}\right)\times \\ c_2'\left(\frac{(R_s+j)Q_r}{2},k \mid L_r^1+\frac{\psi^1+\psi^2}{\psi^1\psi^2},L_r^2+\frac{1}{\psi^2}\right)\end{array}\right)\end{array}\right] \tag{10}$$

Proof: In state 3 when the retailer's inventory position reaches R_r, both suppliers are unavailable. To derive the related cost function, firstly, suppose that the first supplier becomes available before the second supplier. In this case, the effective transportation time for the first supplier is$L_r^1 + (1/\psi^1)$. The expected time when the second supplier is unavailable after the first supplier became available is $1/\psi^2$. It is because of the exponential *ON/OFF* periods and the fact that the exponential distribution is memory less. Therefore, the second supplier's effective transportation time is $L_r^2 + (1/\psi^1) + (1/\psi^2)$. Subsequently, the C'_3expresses the conditional expected cost where the first supplier becomes available before the second supplier.

Now, suppose that the second supplier becomes available first. Similarly, the effective transportation times are $L_r^1 + (1/\psi^1) + (1/\psi^2)$ and $L_r^2 + (1/\psi^2)$, for the first and the second supplier respectively. Therefore, the C''_3expresses the conditional expected cost where the second supplier becomes available before the first supplier. It is worth mentioning that the probability of the simultaneity of both suppliers become available is zero.

Now using the conditional probability, the expected cost related to the state 3, which is denoted by C_3, can be expressed as (11).

$$\begin{aligned} C_3 = {} & P[supplier\ 1\ becomes\ ON\ at\ first\,] \\ & \times E\left[\begin{array}{c} cost\ where\ EL_r^1\ and\ EL_r^2\ are \\ L_r^1+(1/\psi^1)\ and\ L_r^2+(1/\psi^1)+(1/\psi^2), respectively\end{array}\right] \\ & + P[supplier\ 2\ becomes\ ON\ at\ first\,] \\ & \times E\left[\begin{array}{c} cost\ where\ EL_r^1\ and\ EL_r^2\ are \\ L_r^1+(1/\psi^1)+(1/\psi^2)\ and\ L_r^2+(1/\psi^1), respectively\end{array}\right] \end{aligned} \tag{11}$$

As we know from Poisson Process, the probability that initially the supplier 1 changes its state to *ON* is $\psi^1/(\psi^1+\psi^2)$, and similarly this probability for the supplier 2 is $\psi^2/(\psi^1+\psi^2)$. So we can rewrite C_3 by substituting (9) and (10) in (11).

Lemma 6: The total cost function for an integrated two-supplier supply chain with supply interruption is as follows;

$$TC_{total} = P_0 \times C_0 + P_1 \times C_1 + P_2 \times C_2 + P_3 \times C_3 \tag{12}$$

Where C_0, C_1, C_2, and C_3 are calculated according to lemmas 3 to 5.

Proof: Using the idea of conditional probability, one can divide the two-supplier supply chain with supply interruption to four states. Lemmas 3 to 5 present cost function related to each state. Denoted by TC_{total}, the total cost function for an integrated two-supplier supply chain with supply interruption is calculated as follows ;

$$\begin{aligned} TC_{total} = {} & P[state\ 0] \times E\begin{bmatrix} cost\ function\ where \\ both\ suppliers\ are\ ON \end{bmatrix} + P[state\ 1] \\ & \times E\begin{bmatrix} cost\ function\ where \\ the\ first\ supplier\ is\ ON \quad and \\ the\ second\ supplier\ is\ OFF \end{bmatrix} + P[state\ 2] \\ & \times E\begin{bmatrix} cost\ function\ where \\ the\ first\ supplier\ is\ OFF\ \ and \\ the\ second\ supplier\ is\ ON \end{bmatrix} + P[state\ 3] \\ & \times E\begin{bmatrix} cost\ function\ where \\ both\ suppliers\ are\ OFF \end{bmatrix} \end{aligned} \tag{13}$$

By substituting the (4) to (8) in (13) the approximated total cost function of an integrated two-supplier supply chain with supply interruption is obtained according to (12).

4. NUMERICAL EXAMPLES

In this section, first, in the subsection 4.1 the effectiveness of proposed cost function is investigated. Subsequently, in subsection 4.2 the question "is it necessary to consider interruption through modeling the problem?" will be answered. Finally, in subsection 4.3, some numerical examples are resorted, and the effect of diversification strategy in cost reduction is analyzed.

4.1. Efficiency of the proposed cost function

In this subsection, numerical examples are presented to evaluate the effectiveness of the proposed approximation cost function. The problems are constructed by taking into account all possible combinations of following parameters;

λ_r	=	1, 3;	β	=	5, 10;	L_s^2	=	1;
Q_r	=	2, 6, 12;	Q_s	=	1, 3;	ζ^1	=	0.05, 0.2, 0.4;
h_r	=	1;	h_s^1	=	0.1;	ζ^2	=	0.05, 0.2, 0.4;
L_r^1	=	0.5, 2, 4;	h_s^2	=	0.1;	ψ^1	=	0.5;
L_r^2	=	1;	L_s^1	=	1;	ψ^2	=	0.5.

It is worth mentioning that to present the suppliers' availability, one can use $\psi^i/(\zeta^i + \psi^i)$ as a ratio that presents the long-run fraction of time in which the ith supplier is in the *ON* state. These problems reflect three levels of availability for each supplier that is 91%, 71%, and 56%. These three levels are similar to those availability levels that Mohebbi and Hao (2006) have considered. The combination of these three levels for both suppliers provides nine availability levels for sample problems. These nine levels cover most of the real world situations.

To verify the efficiency of the proposed cost function, 40 problems are randomly selected among 648 sample problems and the Table D1 is constructed (see Appendix D). For each problem the proposed cost function and the optimum values of $\mathrm{R_r}$ and $\mathrm{R_s}$ are calculated and presented in Table D1. One can use recursive process suggested by Axsäter (1990) for calculating cost function and the optimum values of R_r and R_s. Subsequently, one can easily compute the following statistical hypothesis test, where TC_{sim} and TC_{total} are the simulation and the proposed total cost, respectively:

$$\begin{cases} \text{H0:} \quad \mu_{\mathrm{TC_{sim}}} = \mu_{\mathrm{TC_{total}}} \\ \text{H1:} \quad \mu_{\mathrm{TC_{sim}}} \neq \mu_{\mathrm{TC_{total}}} \end{cases}$$

The result indicates a failure to reject the null hypothesis at the 5% significance level. Therefore, it is inferred that there is no significant difference between the proposed and the simulated cost functions. In addition, one can define the absolute error as $|TC_{sim} - TC_{total}|/TC_{sim}$; and observe that the mean of absolute error is 0.0295. This, also, means that our proposed cost function works with insignificant errors.

4.2. The effect of considering interruption

Obviously, considering the supply interruption complicates the calculations. In this subsection, the following question is answered: "is it necessary to consider interruption through modeling the problem?" It can be concluded from Table 2 that in most of the situations it is necessary to consider the effect of interruption.

To achieve the answer of the mentioned question, 16 sample problems, randomly selected among the 648 problems, are selected. For each sample problem the reorder points and total cost functions are calculated for two cases, namely with or without considering supply interruption. For each studied problem, our findings are expressed as % deviation between total cost function with or without supply interruption, denoted by $TC_{\mathrm{total\,|\,without\,interruption}}$ and $TC_{\mathrm{total\,|\,with\,interruption}}$ respectively, in which:

$$\%\ \text{deviation} = 100 \times \frac{(TC_{\mathrm{total\,|\,with\,interruption}} - TC_{\mathrm{total\,|\,without\,interruption}})}{TC_{\mathrm{total\,|\,with\,interruption}}}.$$

The results are depicted in Table 2. The minimum and the maximum of % deviation which are reported in Table 2 are 4.192 and 52.4779, respectively. Also, in these 16 sample problems, the mean of % deviation is 25.66. So when there is supply interruption, it is worth using the proposed method even though this method increases the complexity of calculations.

Table 2 The effect of ignoring supply interruption in the reorder points and total cost function*

No.	λ_r	L_r^1	Q_r	β	Q_s	ζ^1	ζ^2	With interruption			Without interruption			% deviation
								TC_{total}	R_s^*	R_r^*	TC_{total}	R_s^*	R_r^*	
1	3	2	2	5	3	0.2	0.4	7.9074	2	10	4.2285	4	5	46.5249
2	1	4	12	10	1	0.05	0.05	5.2729	-1	1	4.9856	-1	1	5.4486
3	1	4	6	5	1	0.2	0.4	4.0461	0	2	3.0314	0	1	25.0799
4	1	0.5	2	5	3	0.4	0.2	3.0945	0	2	1.6870	1	0	45.4835
5	3	2	2	5	3	0.4	0.4	8.6727	1	12	4.2285	4	5	51.2434
6	3	2	6	10	3	0.2	0.05	8.5128	0	8	5.0051	1	4	41.2053
7	1	2	6	5	1	0.05	0.2	3.1884	0	0	2.8115	0	0	11.8230
8	1	2	6	5	3	0.05	0.05	3.2385	0	0	3.1028	0	0	4.1920
9	1	2	12	10	1	0.4	0.4	6.2131	-1	2	5.7989	-1	1	6.6658
10	3	0.5	12	10	1	0.05	0.2	7.2358	0	2	5.6549	0	0	21.8487
11	1	4	6	10	1	0.4	0.4	5.3529	0	4	3.7416	0	1	30.1005
12	1	4	12	10	1	0.2	0.05	5.3573	-1	1	4.9856	-1	1	6.9391
13	3	0.5	2	5	3	0.4	0.05	6.0685	2	6	2.8778	4	2	52.5779
14	3	0.5	12	10	1	0.05	0.05	6.4042	0	1	5.6549	0	0	11.6999
15	3	2	2	10	3	0.2	0.05	9.0468	3	10	5.2060	3	7	42.4548
16	1	4	2	5	1	0.05	0.4	3.6648	1	4	3.3961	1	3	7.3324

*Other parameters are constant and are as follow: h_r=1, L_r^2=1, h_s^1=h_s^2=0.1, L_s^1=L_s^2=1, and ψ^1=ψ^2= 0.5

4.3. Two-supplier supply chain versus dyadic supply chain

As pointed earlier, one approach which has been frequently used in the literature to overcome uncertainty in the supply is diversification. Sajadifar and Pourghannad (2010) study an integrated dyadic supply chain with uncertainty in the supply, but their study, like most of the other studies in the supply interruption literature, focuses on a dyadic supply chain. In this subsection, we investigate the effect of diversification on integrated supply chain with uncertainty in the supply. To inquire the effect of diversification 16 sample problems, which are presented in Table 3, are randomly constructed. The optimal values of the suppliers' reorder point, the retailer's reorder point, and the total cost function are calculated for both dyadic and two-supplier supply chain. In the case of two-supplier supply chain all parameters for both suppliers are the same and equal to the case of the dyadic supply chain, i.e. $\psi = \psi^1 = \psi^2$, $\zeta = \zeta^1 = \zeta^2$,$L_r = L_r^1 = L_r^2$, $L_s = L_s^1 = L_s^2$, and h_s=h_s^1=h_s^2. The optimization process for integrated dyadic supply chain with uncertainty in the supply is similar to the process that used in this paper and is available at Sajadifar and Pourghannad (2012). For each problem, to compare the results, %cost reduction is proposed, in which:

$$\%\ \text{cost reduction} = 100 \times \frac{TC_{\text{total}\,|\,\text{dyadic}} - TC_{\text{total}\,|\,\text{two-supplier}}}{TC_{\text{total}\,|\,\text{dyadic}}}$$

Where the$TC_{\text{total}\,|\,\text{dyadic}}$ is the total cost in the dyadic supply chain, and the $TC_{\text{total}\,|\,\text{two-supplier}}$ is the total cost in the tow-supplier supply chain. As Table 3 shows, using diversification strategy results in cost reduction between %3.69 and %36.95. Our findings support former studies indicating that using multi-supplier can decrease the total inventory cost (for more details see Hill (1996) and Minner(2003)).

Table 3 The comparison between dyadic and two-supplier supply chain when there is supply interruption*

No.	λ_r	L_r	Q_r	β	Q_s	ζ	Dyadic SC			Two-supplier SC			% Cost Reduction
							TC_{total}	R_s^*	R_r^*	TC_{total}	R_s^*	R_r^*	
1	3	2	12	10	1	0.2	10.3430	-1	15	9.1957	0	8	11.09
2	1	4	6	10	3	0.2	6.4630	-1	7	5.5408	-1	5	14.27
3	3	4	6	5	3	0.2	8.7395	-1	19	8.2295	0	15	5.84
4	3	4	12	10	1	0.05	10.7678	-1	22	9.3297	0	13	13.36
5	1	0.5	6	5	3	0.4	4.3864	-1	1	3.6108	0	0	17.68
6	1	0.5	12	5	1	0.05	5.8341	-1	1	5.1705	-1	-1	11.37
7	1	0.5	2	5	3	0.05	3.1220	-1	3	1.9684	1	0	36.95
8	1	0.5	2	5	3	0.2	3.1576	0	2	2.7718	1	1	12.22
9	3	4	12	5	1	0.4	9.6035	-1	18	8.9650	0	14	6.65
10	3	0.5	12	10	1	0.05	8.8626	-1	10	6.6973	0	0	24.43
11	3	4	12	10	3	0.2	12.3538	-1	19	10.8661	0	14	12.04
12	3	2	6	5	3	0.05	7.1854	0	12	5.8227	0	6	18.96
13	1	4	2	10	3	0.05	5.2031	0	8	4.4829	0	6	13.84
14	3	4	12	5	3	0.2	10.4525	-1	17	8.9794	-1	14	14.09
15	1	2	12	5	3	0.4	7.2074	-1	1	5.8793	-1	0	18.43
16	1	2	2	10	1	0.2	4.3605	0	5	4.1994	1	4	3.69

*Other parameters are constant and are as follow: h_r = 1, h_s= 0.1, L_s = 1, and ψ = 0.5

5. CONCLUSION AND FUTURE RESEARCHES

In this paper, an integrated two-supplier supply chain with supply interruption has been studied. As mentioned, although the diversification strategy is used to overcome uncertainty in the supply, there are a few studies on it. In addition, the integration is another important factor that has been taken into account. The results show that our proposed expected cost function works well. Also, as it has been anticipated, the numerical examples demonstrate that using two suppliers can cause reduction in the expected total cost.

There are some directions one can use to extend this study, like derivation of exact or approximation cost function for an integrated multi-supplier supply chain, and sensitivity analyzing of the proposed cost function when each parameters change. Furthermore, some numerical examination, which is going to appear in future works, is required to understand the effect of interruption.

APPENDIX A. EVALUATION OF ONE-FOR-ONE ORDERING POLICIES

This appendix is a summary of Axsäter (1990), adapted for our inventory system. The following notations are defined:

$g^{S_s}(t)$ = Density function of the Erlang (λ, S_s)

and

$G^{S_s}(t)$ = The cumulative distribution function of $g^{S_s}(t)$.

Thus,

$$g^{S_s}(t) = \frac{\lambda^{S_s} t^{S_s-1}}{(S_s-1)!} e^{-\lambda t} \quad \text{(A.1)}$$

And,

$$G^{S_s}(t) = \sum_{k=S_s}^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad \text{(A.2)}$$

The average warehouse holding costs per unit is:

$$\gamma(S_s) = \frac{h_s S_s}{\lambda}\left(1 - G^{S_s+1}(L_s^i)\right) - h_s L_s^i \left(1 - G^{S_s}(L_s^i)\right), S_s > 0 \quad \text{(A.3)}$$

And for $S_s = 0$

$$\gamma(0) = 0 \quad \text{(A.4)}$$

Given that the value of the random delay at the warehouse is equal to t, the conditional expected costs per unit at the retailer is:

$$\pi^{S_r}(t) = e^{-\lambda(L_r^i + t)} \frac{h+\beta}{\lambda} \sum_{k=0}^{S_r-1} \frac{(S_r - k)}{k!} \left(L_r^i + t\right)^k \lambda^k + \beta\left(L_r^i + t - \frac{S_r}{\lambda}\right) \quad \text{(A.5)}$$

(0! = 1 by definition),

The expected retailer's inventory carrying and shortage cost to fill a unit of demand is:

$$\pi^{S_r}(t) = e^{-\lambda(L_r^i + t)} \frac{h+\beta}{\lambda} \sum_{k=0}^{S_r-1} \frac{(S_r - k)}{k!} \left(L_r^i + t\right)^k \lambda^k + \beta\left(L_r^i + t - \frac{S_r}{\lambda}\right) \quad \text{(A.6)}$$

And,

$$\Pi^{S_r}(S_s) = \int_0^{L_s^i} g^{S_s}\left(L_s^i - t\right) \pi^{S_r}(t) dt + \left(1 - G^{S_s}(L_s^i)\right) \pi^{S_r}(0). \quad \text{(A.7)}$$

Furthermore, for large value of $\mathrm{S_s}$, we have

$$\Pi^{\mathrm{S_r}}(\mathrm{S_s}) \approx \pi^{\mathrm{S_r}}(\mathrm{L_s^i}). \tag{A.8}$$

The procedure starts by determining $\overline{\mathrm{S_0}}$ such as

$$\mathrm{G}^{\overline{\mathrm{S_0}}}(\mathrm{L_s^i}) < \varepsilon, \tag{A.9}$$

where ε is a small positive number.

The recursive computational procedure is:

$$\Pi^{\mathrm{S_r}}(\mathrm{S_s}-1) = \Pi^{\mathrm{S_r}-1}(\mathrm{S_s}) + \left(1-\mathrm{G}^{\mathrm{S_s}}(\mathrm{L_s^i})\right) \times \left(\pi^{\mathrm{S_r}}(0) - \pi^{\mathrm{S_r}-1}(0)\right) \tag{A.10}$$

$$\Pi^{0}(\mathrm{S_s}) = \mathrm{G}^{\mathrm{S_s}}(\mathrm{L_s^i})\beta\mathrm{L_s^i} - \mathrm{G}^{\mathrm{S_s}+1}(\mathrm{L_s^i})\beta\frac{\mathrm{S_s}}{\lambda} + \beta\mathrm{L_r^i} \tag{A.11}$$

And finally the expected total holding and shortage cost for a unit demand in an inventory system with a one-for-one ordering policy is:

$$\mathrm{c}(\mathrm{S_s},\mathrm{S_r}) = \Pi^{\mathrm{S_r}}(\mathrm{S_s}) + \gamma(\mathrm{S_s}). \tag{A.12}$$

APPENDIX B. PROOF OF LEMMA 1

In this appendix, the proof of lemma 1 will be presented. First, the proof for part 2 of lemma 1 is developed. It is known that $X_r^i = L_r^i + w_r^i$, and $w_r^i = \max\{0, L_s^i - Q_r(R_s + k)\}$; $\underline{i}$ =1,2. In addition, consider the $t_{(R_s+j)Q_r}$ as the time when the $(R_s + j)Q_r$th demand occurred after time zero; *j* = 1, 2, *Qs*. In part 2 of lemma 1 it was assumed that $L_s^1 < L_s^2$. Now, the $P_{12}^j(L_r^1, L_r^2)$can be expressed, using the conditional probability, as follows;

$$\begin{aligned} P_{12}^j(L_r^1, L_r^2) &= P(X_r^1 < X_r^2) \\ &= P\left(t_{(R_s+j)Q_r} < L_s^1 < L_s^2\right) \times P\left(X_r^1 < X_r^2 \mid t_{(R_s+j)Q_r} < L_s^1 < L_s^2\right) \\ &\quad + P\left(L_s^1 < t_{(R_s+j)Q_r} < L_s^2\right) \times P\left(X_r^1 < X_r^2 \mid L_s^1 < t_{(R_s+j)Q_r} < L_s^2\right) \\ &\quad + P\left(L_s^1 < L_s^2 < t_{(R_s+j)Q_r}\right) \times P\left(X_r^1 < X_r^2 \mid L_s^1 < L_s^2 < t_{(R_s+j)Q_r}\right). \end{aligned}$$

$$\begin{aligned} P_{12}^j(L_r^1, L_r^2) &= P(X_r^1 < X_r^2) \\ &= P\left(t_{(R_s+j)Q_r} < L_s^1 < L_s^2\right) \times \\ &\quad P\left(L_r^1 + L_s^1 - t_{(R_s+j)Q_r} < L_r^2 + L_s^2 - t_{(R_s+j)Q_r} \mid t_{(R_s+j)Q_r} < L_s^1 < L_s^2\right) + \\ &\quad P\left(L_s^1 < t_{(R_s+j)Q_r} < L_s^2\right) \times P\left(L_r^1 < L_r^2 + L_s^2 - t_{(R_s+j)Q_r} \mid L_s^1 < t_{(R_s+j)Q_r} < \right. \\ &\quad \left. L_s^2\right) + P\left(L_s^1 < L_s^2 < t_{(R_s+j)Q_r}\right) \times P\left(L_r^1 < L_r^2 \mid L_s^1 < L_s^2 < t_{(R_s+j)Q_r}\right). \end{aligned}$$

$$P_{12}^j(L_r^1, L_r^2) = P(X_r^1 < X_r^2) = G^{Q_r(R_w+j)}(L_s^1) + G^{Q_r(R_w+j)}(L_r^2 + L_s^2 - L_r^1) - G^{Q_r(R_w+j)}(L_s^1) + 0$$

$$P_{12}^j(L_r^1, L_r^2) = G^{Q_r(R_w+j)}(L_r^2 + L_s^2 - L_r^1)$$

$$P_{21}^j(L_r^1, L_r^2) = 1 - P_{12}^j(L_r^1, L_r^2) = 1 - G^{Q_r(R_w+j)}(L_r^2 + L_s^2 - L_r^1)$$

All of the other parts of lemma 1 can be proved, easily, in the same way.

APPENDIX C. PROOF OF LEMMA 2

Based on table 1, the states of suppliers are the continuous time Markov chain; and this continuous time Markov chain has a unique steady-state probability. It is given by the solution of the following equations;

$$\mathrm{P}_0\zeta^1 + \mathrm{P}_0\zeta^2 - \mathrm{P}_1\psi^2 - \mathrm{P}_2\psi^1 = 0 \quad \text{(C.1)}$$

$$\mathrm{P}_1\zeta^1 + \mathrm{P}_1\psi^2 - \mathrm{P}_0\zeta^2 - \mathrm{P}_3\psi^1 = 0 \quad \text{(C.2)}$$

$$\mathrm{P}_2\zeta^2 + \mathrm{P}_2\psi^1 - \mathrm{P}_0\zeta^1 - \mathrm{P}_3\psi^2 = 0 \quad \text{(C.3)}$$

$$\mathrm{P}_3\psi^2 + \mathrm{P}_3\psi^1 - \mathrm{P}_1\zeta^1 - \mathrm{P}_2\zeta^2 = 0 \quad \text{(C.4)}$$

$$\mathrm{P}_0 + \mathrm{P}_1 + \mathrm{P}_2 + \mathrm{P}_3 = 1 \quad \text{(C.5)}$$

The equations C.1 to C.4 are called balanced equations, and the equation C.5 is called normalized equation. By solving C.1 to C.5, the steady state probabilities can be obtained as follows;

$$P_0 = \frac{\psi^1\psi^2}{(\zeta^1+\psi^1)(\zeta^2+\psi^2)} \quad \text{(C.6)}$$

$$P_1 = \frac{\zeta^2\psi^1}{(\zeta^1+\psi^1)(\zeta^2+\psi^2)} \quad \text{(C.7)}$$

$$P_2 = \frac{\zeta^1\psi^2}{(\zeta^1+\psi^1)(\zeta^2+\psi^2)} \quad \text{(C.8)}$$

$$P_3 = \frac{\zeta^1\zeta^2}{(\zeta^1+\psi^1)(\zeta^2+\psi^2)} \quad \text{(C.9)}$$

APPENDIX D. TABLE D

Table D Sample problems with associated TC_{total}, TC_{sim} and absoluteerrors

No.	λ_r	L_r^1	Q_r	β	Q_s	ζ^1	ζ^2	R_s^*	R_r^*	TC_{total}	TC_{sim}	***|error|***
1	3	0.5	6	10	3	0.4	0.2	0	7	9.21071	9.81297	0.06137
2	1	4	12	10	3	0.4	0.2	-1	1	6.36758	6.45955	0.01424
3	3	0.5	12	5	1	0.05	0.05	0	0	5.37090	5.39581	0.00462
4	3	4	2	5	1	0.4	0.4	3	16	10.89594	11.65602	0.06521
5	3	0.5	6	10	1	0.05	0.2	0	5	7.49743	7.19284	0.04235
6	3	2	6	5	3	0.05	0.2	0	5	5.92968	5.81754	0.01928
7	3	4	12	5	1	0.05	0.05	0	5	7.17112	7.23899	0.00938
8	1	0.5	12	5	3	0.2	0.05	-1	-1	5.44354	5.52043	0.01393
9	1	2	6	5	3	0.2	0.2	-1	1	3.66058	3.77355	0.02994
10	3	0.5	12	10	1	0.2	0.05	0	2	7.10212	7.31937	0.02968
11	1	2	12	10	1	0.2	0.2	-1	1	5.99141	6.22019	0.03678
12	1	2	2	10	1	0.2	0.05	1	3	3.76641	3.90521	0.03554

Table D Sample problems with associated TC_{total}, TC_{sim} and absoluteerrors (Cont.)

No.	λ_r	L_r^1	Q_r	β	Q_s	ζ^1	ζ^2	R_s^*	R_r^*	TC_{total}	TC_{sim}	$\|error\|$
13	3	2	6	10	3	0.2	0.05	0	8	8.51284	8.54867	0.00419
14	1	4	2	10	3	0.4	0.05	0	5	5.50425	5.65550	0.02674
15	1	2	12	5	3	0.4	0.4	-1	0	5.62588	5.77219	0.02535
16	1	4	2	10	1	0.4	0.05	1	5	5.44998	5.24617	0.03885
17	1	0.5	2	10	3	0.2	0.05	1	1	3.19247	3.06099	0.04295
18	3	2	6	10	1	0.05	0.2	1	7	7.40563	7.32142	0.01150
19	1	0.5	2	5	1	0.4	0.4	1	2	3.28651	3.20450	0.02559
20	1	2	6	10	1	0.05	0.4	0	1	4.05997	4.17547	0.02766
21	1	4	2	5	3	0.4	0.05	0	4	4.38476	4.44185	0.01285
22	3	4	6	5	1	0.05	0.2	0	10	7.83154	8.02224	0.02377
23	1	4	12	5	1	0.2	0.05	-1	0	4.46051	4.58218	0.02655
24	1	4	12	10	3	0.2	0.05	-1	1	5.78474	5.89050	0.01795
25	1	2	2	10	1	0.05	0.05	2	2	3.22682	3.18618	0.01276
26	1	2	2	10	1	0.05	0.4	1	3	3.65160	3.78893	0.03624
27	3	2	2	5	2	0.2	0.05	3	8	7.01806	6.75052	0.03963
28	3	4	2	5	1	0.05	0.2	3	13	8.40043	8.73523	0.03833
29	3	0.5	2	10	1	0.05	0.05	4	5	6.19117	6.12086	0.01149
30	1	2	2	5	3	0.2	0.2	1	2	3.25704	3.33401	0.02309
31	1	2	6	10	1	0.2	0.4	0	2	4.45021	4.63957	0.04081
32	1	4	12	10	1	0.05	0.2	-1	2	5.78933	5.76064	0.00498
33	3	0.5	12	5	3	0.2	0.4	0	2	7.75414	8.11143	0.04405
34	3	4	12	5	3	0.05	0.05	0	5	7.75238	7.60421	0.01949
35	3	2	6	10	3	0.2	0.4	0	10	9.72501	10.43086	0.06767
36	3	2	2	5	3	0.05	0.2	3	7	6.06486	5.70380	0.06330
37	1	4	12	10	1	0.05	0.4	-1	2	6.03747	5.82513	0.03645
38	1	0.5	6	5	3	0.4	0.4	0	0	3.70361	3.83134	0.03334
39	1	2	6	10	3	0.4	0.2	-1	2	4.70324	4.89066	0.03832
40	1	4	2	5	1	0.2	0.4	1	4	4.13756	4.24181	0.02458
The mean absolute error = 0.0295												
*Other parameters are constant and are as follow: h_r=1, L_r^2=1, h_s^1=h_s^2= 0.1, L_s^1=L_s^2 =1, and ψ^1=ψ^2= 0.5												

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