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Simultaneous Lot Sizing and Scheduling in a Flexible Flow Line

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ABSTRACT

This paper breaks new ground by modelling lot sizing and scheduling in a flexible flow line (FFL) simultaneously instead of separately. This problem, called the '*General Lot sizing and Scheduling Problem in a Flexible Flow Line*' (GLSP-FFL), optimizes the lot sizing and scheduling of multiple products at multiple stages, each stage having multiple machines in parallel. The objective is to satisfy varying demand over a finite planning horizon with minimal inventory, backorder and production setup costs. The problem is complex as any product can be processed on any machine but with different process rates and sequence-dependent setup times & costs. The efficiency of two alternative models is assessed and evaluated using numerical tests.

***Keywords*:** Lot sizing, Scheduling, Flexible Flow Line, Mathematical Modelling.

1. INTRODUCTION

The Flexible Flow Line (FFL) is a very prevalent production system and found in many industries, especially automotive, chemical, electronics, steel making, food and textile (Linn and Zhang, 1999) It consists of several production stages in series with parallel machines at each stage. The decisions to be taken are the determination of production quantities (lots), machine assignments and production sequences (schedules) on each machine at each stage in a FFL. Lot sizing and scheduling problems are closely interrelated. However, it can be difficult and complex to combine both problems. As a result, they are often modelled and solved independently in spite of their interdependency. Our work in this paper builds on integrated models developed by other researchers for simpler contexts. Fleischmann and Meyr (1997) first integrated the lot sizing and scheduling of several products on a single capacitated machine, calling their model the *General Lot sizing and Scheduling Problem* (GLSP). Meyr (2000) included sequence-dependent *Setup Times*, resulting in the GLSPST model.

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Meyr (2002) then extended GLSPST to parallel machines or *Parallel Lines* (GLSPPL). In an alternative approach, Clark and Clark (2000) designed a mixed integer programming (MIP) model for simultaneous sequencing and lot sizing production lots on a set of parallel machines in the presence of sequence-dependent setup times.

The survey by Linn and Zhang (1999) reviewed the state of FFL scheduling research ten years ago and described a variety of different configurations. They noted the lack of research on FFLs with more than two stages and the extensive using of dispatching rules in practice. Their survey did not include any research or mention of lot sizing and scheduling within FFLs. Six years later, Quadt and Kuhn (2005) explicitly identified a lack of literature for lot sizing and scheduling in FFLs and went on to describe a hierarchical 3-phase approach for integrative lot-sizing and scheduling. The second phase consisted of capacitated lot-sizing problem (CLSP) model (Bitran and Yanasse, 1982) generalised to the sequencing of lots of product families lot , the possibility of back-orders and parallel machines. While more general than needed for FFLs, the approach of Quadt and Kuhn (2005) is limited partly due to its aggregation of products into families, but primarily because of the necessity of bottleneck stage identification and stability during the planning run. In our FFL models below, these two restrictive assumptions are not considered and the whole problem is modelled in its entirety. Subsequently, in Quadt and Kuhn (2007), they gathered a wide range of literature on the FFL scheduling problem and built a taxonomy for FFL scheduling procedures (excluding lot-sizing), classifying them by general solution approach. They concluded by noting again that very little research has been published combining both lot sizing and scheduling in FFL, although in the same year Quadt and Kuhn (2007) did deal with batch scheduling. Relevant to the sequential stages of FFLs, Fandel and Stammen-Hegene (2006) formulated the *Multi Level General Lot sizing and Scheduling Problem with Multiple Machines* (MLGLSP-MM), based on the GLSP for single level production and parallel machines. However, the job shop structure in the MLGLSP-MM model is more general than needed for a flowshop. Moreover the paper contains only a mathematical model for the MLGLSP-MM without any numerical tests or solution procedure, possibly because the authors themselves recognized that the model's complexity limits optimal solutions to just small instances. Responding to the challenge, this paper's research contribution is to propose three mathematical models for *General Lotsizing and Scheduling Problem in FFL* (GLSP-FFL) and to obtain initial insight into their comparative computational performance via experimental tests. The paper is organized as follows. Section 2 develops mathematical formulations of the GLSP-FFL. Numerical results are discussed in Section 3 and finally conclusions and suggestions for further research are contained in Section 4.

2. PROBLEM DEFINITION AND MATHEMATICAL FORMULATIONS

A flexible flow line (FFL) or hybrid flow shop can be considered as an extension of two classical systems, namely the flow shop and the parallel shop. The production line consists of several processing stages in series, separated by finite intermediate buffers, where each stage has one or more parallel identical machines (Pinedo, 1995). The layout of FFL is shown in Figure 1.

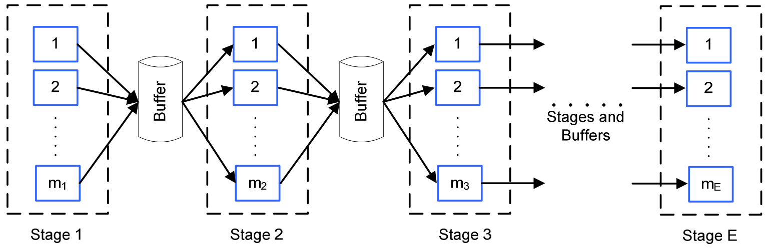

Figure 1 Flexible flow line

The GLSP-FFL was developed from the single-level GLSP of Fleischmann and Meyr (1997) and the multi-stage capacitated lot sizing and loading problem (MCLSLP) of Özdamar and Barbarosoglu (1999). According to different formulations, three distinct models are introduced for GLSP-FFL, all of which are based on the following assumptions: Multiple products can be produced at stages in the flexible flow shop. Production at each stage involves unrelated parallel machines with different production rates. All machines can produce any product. The available capacity of each machine is limited and can vary between periods and stages. The finite planning horizon is divided into T macro-periods. The independent demand for all products is felt at the final stage at the end of each macro-period. It is known with certainty, but varies dynamically over the planning horizon.

The main assumptions of the problem were described in the following

- Demand for items in other stages is dependent on the production of the next stage.
- Backlog shortages are permitted for products at the final stage but are upper-bounded by a given percentage of demand in each macro-period.This is the practiced assumption in flow shop manufacturing systems (Özdamar and Barbarosoglu, 1999).
- The products may be manufactured in lots of varying size on any one of the parallel machines in each stage.
- The production rate can vary between products and machines, but is constant over the planning horizon.
- A changeover from one product to another requires a setup time during which the machine is unproductive. Setup times and costs are sequence dependent and can vary between machines.
- The setup state is conserved when no product is being processed.
- At the beginning of the planning horizon, each machine is setup for a specified product.
- A two-level time structure is assumed. Each macro-period consists of a variable number of micro-periods with variable length. Each machine has its own micro-period segmentation, i.e., the number of micro-period can differ between machines. Micro-periods do not have to be of equal durations on the same machine.
- At the start of a micro-period, a machine is setup and then produces just one product until the end of the micro-period.
- If setup costs and times are triangular, then it is not economical to produce a product in more than one lot on the same machine in the same micro-period. Thus there will be at most one setup per product per macro-period on each machine and so the number of micro-periods on a machine will be at most the number of products.
- Lot-splitting is permitted at any stage, i.e., each product can be simultaneously produced on more than one machine at any given stage.
- In order to obtain viable schedules, we assume that there is the lead time of one period between different production stages. In this case, a product which is produced at a stage is available for production at the next stage only in the next period. However in some industries, assuming a lead time of period may be unrealistic and lead to inferior model solutions.

The parameters and indices of the GLSP-FFL for all the models are

J	Number of total products i, j, k
E	Number of different stages e
M_e	Number of different machines m_e available for production at stage e (so that the total number of machines over all stages is $M = \sum_e M_e$)
T	Number of macro-periods t in the planning horizon
F_{max}	Maximum number of micro-periods *f* in each macro-period *t*
F_{mt}	The number of micro-periods *f* in macro-period *t* on machine m_e, $F_{mt} \leq F_{max}$

Note that in the definition of F_{mt} above, to avoid notational clutter such as $F_{m_e t}$, the simple index *m* is used when strictly speaking the subscripted index m_e should have been used. Similarly, the simple index *f* will be used when strictly speaking the subscripted index $f_{m_e t}$ should be used. From now on, this convention will be used so that the subscripts *e* and *t* are implied wherever the indices m and *f* are used. Figure 2 illustrates the segmentation of macro-periods into micro-periods on a machine *m* at any stage *e*. Note how the varying lengths of macro-periods differ between macro-periods.

Macro-period *t* = 1				...	Macro-period *t* = *T*			
$f = 1$	$f = 2$	...	$f = F_{m1}$	...	$f = 1$	$f = 2$	...	$f = F_{mT}$
Micro-periods *f*				...	Micro-periods *f*			

Figure 2 Micro-period segmentation on a machine differs between macro-periods

The data required in all the models are

d_{it}	Demand for product *i* realised at the end of macro-period *t*
C_{mt}	Available capacity of machine *m* in macro-period *t*
st_{ijm}	Time needed to setup on machine *m* from product *i* to product *j*
sc_{ijm}	Cost needed to setup on machine *m* from product *i* to product *j*
b_{im}	Capacity (processing time) on machine *m* required to produce a unit of product *i*
h_{ite}	Cost of holding a unit of product *i* from period *t* to *t*+1 at stage *e*
g_{it}	Cost of backordering a unit of end-item demand for product *i* from period *t* to *t*+1
BP	Maximum permitted percentage of total end-item demand that can be backordered
i_{0m}	The product setup on machine *m* at the end of period 0, i.e., the starting setup configuration
P_{im}	Cost of producing one unit of product *i* on machine *m*
UB_{imt}	Upper bound C_{mt}/b_{im} on the amount of product *i* produced in macro-period *t* on machine *m*
LB_{imt}	Lower bound on the amount of product *i* produced in macro-period *t* on machine *m*

The objective of all three models presented below is to minimise backorders, inventory and setup costs of producing the J products over the T macro-periods in the planning horizon.

2.1. Model FFL-FS

In the first model, the number F_{mt} of micro-periods in macro-period *t* is itself a variable to be minimised, as in Fandel and Stammen-Hegene's (2006) MLGLSP-MM. The setup constraints are based on Clark and Clark (2000). The decision variables are

I_{iet}	Inventory level of product *i* in stage *e* at the end of macro-period *t*.
B_{iEt}	Backordered amount of end-product *i* at the last stage *E* at the end of macro-period *t*.
x_{imf}	Production quantity of product *i* on machine *m* in micro-period *f*.
y_{ijmf}	Binary variable, = 1 if there is a changeover from product *i* to product *j* on machine *m* at the start of micro-period *f* , = 0 otherwise.
F_{mt}	Final micro-period on machine *m* at the end of macro-period *t*.

The objective function minimises backorders, inventory and setup costs

$$\sum_{ijemtf} sc_{ijm}\, y_{ijmf} + \sum_{ite} h_{ite}\, I_{iet} + \sum_{it} g_{it}\, B_{iEt} \tag{1}$$

Note how the implied summation limits and indices *e* and *t* avoid notational clutter in the first term in expression (1). The full cluttered version would be:

$$\sum_{i=1}^{J} \ \sum_{j=1,i\neq j}^{J} \ \sum_{e=1}^{E} \ \sum_{m_e=1}^{M_e} \ \sum_{t=1}^{T} \ \sum_{f_{m_e t}=1}^{F_{mt}} sc_{ijm_e}\, y_{ijm_e f_{m_e t}} \qquad \text{(cluttered 1)}$$

From now on, expressions will similarly be kept as concise as possible without sacrificing precision. Just occasionally, some clutter will be unavoidable, for example in constraints (3) and (9) below. If need be, production costs can be included in the objective function by appending the term $\sum_{iemtf} P_{im}\, x_{imf}$. Figure 3 shows the flow of production, inventory and backorders over different periods and stages.

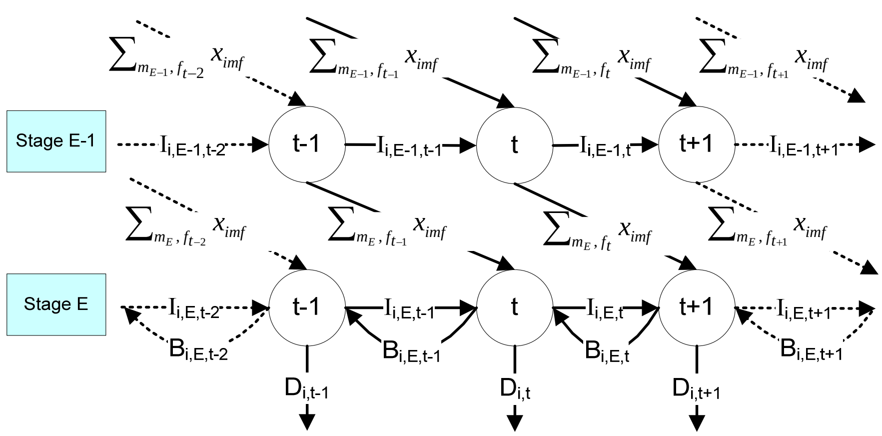

Figure 3 Flow diagram of GLSP-FFL

Constraints (2) and (3) follow from Figure 3.

$$I_{jE,t-1} - B_{jE,t-1} + \sum_{m_E, f_{t-1}} x_{jmf} - I_{jEt} + B_{jEt} = d_{jt} \qquad \forall\, j,t \tag{2}$$

Constraint (2) expresses the material balance for end items, including backorders. Some clutter is required in order to be clear that the term $\sum_{m_E f} x_{jmf}$ refers only to the final stage E. However, note again how the implied use of the index $f_{m_e t}$ in $\sum_{m_E f} x_{jmf}$ avoids further notational clutter. The context ($\forall\, t$) of (2) makes it reasonable to assume that the values of f apply respectively to just the micro-periods within the specific macro-period t. The fully cluttered version would be:

$$\sum_{m_E=1}^{M_E} \sum_{f_{m_E(t-1)}=1}^{F_{m(t-1)}} x_{j m_E f_{m_E(t-1)}} \, .$$

Constraint (3) expresses the material balance for work in process. Again, some clutter is required in order to be clear that the right-hand side refers to the successor stage $e+1$ of the left-hand side's stage e

$$I_{je,t-1} + \sum_{m_e, f_t} x_{jmf} - I_{jet} = \sum_{m_{e+1}, f_{t+1}} x_{jmf} \qquad \forall\, j,t \text{ and } e = 1,\dots,E-1. \tag{3}$$

Constraint (4) bounds backorders of end items in any macro-period to be within a specified proportion of demand

$$B_{itE} \le BP \cdot d_{it} \qquad \forall\, i,t. \tag{4}$$

Constraint (5) represents the limited capacity

$$\sum_{ijf} st_{ijm}\, y_{ijmf} + \sum_{if} b_{im}\, x_{imf} \le C_{mt} \qquad \forall\, e,m,t. \tag{5}$$

Constraints (6) and (7) specify the initial setup configuration.

$$y_{ijm1} = 0 \qquad \forall\, i \ne i_{om}, j, e, m \tag{6}$$

$$\sum_j y_{i_{om} jm1} = 1 \qquad \forall\; e,m \tag{7}$$

Constraints (6) to (9) ensure that a setup on a machine in each micro-period may only occur between a single pair of different products.

$$\sum_i y_{ijmf} = \sum_k y_{jkm,f+1} \qquad \forall\; j,e,m,t \text{ and } f = 1,\dots,F_{mt}-1 \tag{8}$$

$$\sum_i y_{ijmF_{m,t-1}} = \sum_k y_{jkm1} \qquad \forall\, j,e,m \text{ and } t = 2,\dots,T \tag{9}$$

Constraint (10) enforces the appropriate setup before production

$$x_{jmf} \le UB_{jmt} \sum_i y_{ijmf} \qquad \forall\, j,e,m,t,f \tag{10}$$

Constraint (11) enforces minimum lot sizes or specified lower bounds in order to avoid a setup change without subsequent production. If set-up costs or times do not satisfy the triangle inequality ($sc_{ijm} + sc_{jkm} \ge sc_{ikm}$ $\forall i,j,k,e,m$), then (11) prohibits that a setup from i to k passes through a third product j without minimal production of j. Non-triangular setups occur in many industries.

$$x_{jmf} \geq LB_{jmt} \sum_{i} y_{ijmf} \qquad \forall\, j, e, m, t, f \tag{11}$$

However, when some setups are non-triangular, an optimal solution can feature multiple lots of a product on the same machine in the same period. Constraint (12) simplifies the model by ensuring that a product cannot be produced in more than one lot on a machine in a macro-period

$$\sum_{jf} y_{ijmf} \leq 1 \qquad \forall\, i, e, m, t \tag{12}$$

Constraint (13) minimises the number F_{mt} of micro-periods in a macro-period to those actually required and thus prevents micro-periods with a length of zero time units. The right side of the constraint has two terms. The first term is F_{max} if there is production in micro-period *f*, or zero if not. The second term adds up the number of micro-periods with production.

$$F_{mt} \leq F_{max} \sum_{ij} y_{ijmf} + \sum_{ij} \sum_{s=1}^{F_{max}} y_{ijms} \qquad \forall\; e, m, t, f \tag{13}$$

Constraint (14) limits the number F_{mt} of micro-periods in a macro-period.

$$F_{mt} \leq F_{max} \qquad \forall\, e, m, t \tag{14}$$

Similarly to Fandel and Hegene (2006), the above model is not a MIP as it features F_{mt} as both a variable and an index upper limit in many constraints, for example, implicitly in the summation in (5). Fixing the values of F_{mt} will transform the model into a MIP, as shown in the next section.

2.2. Model FFL-CC

A generalisation of Clark and Clark (2000), model FFL-CC can be adapted from model FFL-FS by treating F_{mt} as a parameter, not a variable, thus eliminating constraints (12) to (14). If setup times and costs are triangular it is inefficient to produce more than one lot of a given a product on a machine in a given micro-period. Therefore F_{mt} is upper bounded by *J*, the number of products, and can be fixed at this value. The number of setups may be less than *J*, but the remaining ones are treated as phantom setups from a product i to itself ($y_{iimf} = 1$) with zero setup time ($st_{iim} = 0$) and no consequent production thus invalidating constraint (12). Constraint (15) replaces constraint (11) so as to exclude phantom setups when enforcing minimum lot-sizes.

$$x_{jmf} \geq LB_{jmt} \sum_{i \neq j} y_{ijmf} \qquad \forall\, e, m, t, j, f \tag{15}$$

2.3. Model FFL-FM

Fleischmann and Meyr (1997)'s adaptation of the General Lot sizing and Scheduling Problem (GLSP) to sequence-dependent setup times and parallel machines (Meyr, 2002) can be extended to the FFL. The parameters and continuous decision variables for this new model, denoted FFL-FM, are the same as for the FFL-CC model. However, to be consistent with Meyr's notation, the variable $\mathrm{y_{ijmf}}$ is renamed $\mathrm{z_{ijmf}}$, and *y* becomes a new setup-state variable as follows

y_{imf} = 1 if machine *m* is setup for product *i* in the micro-period *f*, otherwise = 0.

z_{ijmf} = 1 if there is a setup changeover from product *i* to product *j* on machine *m* at the start of micro-period *f*, otherwise = 0.

Note that there is no need to define z_{ijmf} a binary variable in the model since z_{ijmf} as a positive variable will take on the value 0 or 1 in any optimal solution (Fleischmann and Meyr, 1997). As in model FFL-CC, the number F_{mt} of micro-periods within a macro-period is fixed at the number *J* of products. Like models FFL-FS and FFL-CC, the objective function also minimises backorders, inventory and setup costs

$$\sum_{ijemtf} sc_{ijm}\, z_{ijmf} + \sum_{ite} h_{ite}\, I_{iet} + \sum_{it} g_{it}\, B_{iEt} \tag{16}$$

Constraints (17) - (19) are identical to (2) - (4) of models FFL-FS and FFL-CC .

$$I_{jE,t-1} - B_{jE,t-1} + \sum_{m_E, f_{t-1}} x_{jmf} - I_{jEt} + B_{jEt} = d_{jt} \qquad \forall\, j,t \tag{17}$$

$$I_{je,t-1} + \sum_{m_e, f_t} x_{jmf} - I_{jet} = \sum_{m_{e+1}, f_{t+1}} x_{jmf} \qquad \forall\, j,t \text{ and } e = 1,\dots,E-1 \tag{18}$$

$$B_{itE} \le BP \cdot d_{it}; \qquad \forall\, i,t \tag{19}$$

Constraints (20) and (21) are (5) and (7) adapted to the new variables y_{jmf} and z_{ijmf}

$$\sum_{ijf} st_{ijm}\, z_{ijmf} + \sum_{if} b_{im}\, x_{imf} \le C_{mt} \qquad \forall\, e,m,t \tag{20}$$

$$\sum_{j} z_{i_{om} jm1} = 1 \qquad \forall\; e,m \text{ and } t = 1 \tag{21}$$

Note that this formulation has no strict equivalent of constraint (6) which states that the first setup in a macro-period *t* cannot be from a product which is not i_{om}. However, constraint (22) prohibits the value of y_{imf} from indicating that the initial setup-state on a machine is any product which is not i_{om}

$$y_{im1} \le z_{i_{om} im1} \qquad \forall\; i,e,m \text{ and } t = 1 \tag{22}$$

Constraint (23) imposes a minimum initial lot-size except for i_{om}

$$x_{jm1} \ge LB_{jmt} \cdot z_{i_{om} jm1} \qquad \forall\; j \ne i_{om}\,, e,m \text{ and } t = 1 \tag{23}$$

Constraint (24) is requires that a product can only be processed on a machine if it is setup for that product

$$x_{jmf} \le UB_{jmt} \cdot y_{jmf} \qquad \forall\, e,m,j,t,f \tag{24}$$

Constraints (25) and (26) enforce minimum lot sizes, again avoiding intermediate non-zero production setups if set-up costs/times do not satisfy the triangle inequality

$$x_{jmf} \ge LB_{jmt}\left(y_{jmf} - y_{jm,f-1}\right) \qquad \forall\, e,m,j,t,f = 2,\dots,F_{mt} \tag{25}$$

$$x_{jm1} \ge LB_{jmt}\left(y_{jm1} - y_{jmF_{m,t-1}}\right) \qquad \forall\, e,m,j,t = 2,\dots,T \tag{26}$$

Constraint (27) ensures that only one setup state is defined in each micro period

$$\sum_j y_{jmf} = 1 \qquad \forall\ e, m, t, f \tag{27}$$

Constraint (28) ensures that only one setup changeover occurs in each micro period

$$\sum_{ij} z_{ijmf} = 1 \qquad \forall\ e, m, t, f \tag{28}$$

Constraints (29) and (30) relate the setup state variables and changeover variables

$$z_{ijmf} \geq y_{im,f-1} + y_{jmf} - 1 \qquad \forall\ e, m, i, j, t, f = 2, \dots, F_{max} \tag{29}$$

$$z_{ijm1} \geq y_{jm1} + y_{imF_{m,t-1}} - 1 \qquad \forall\ e, m, i, j, t = 2, \dots, T \tag{30}$$

2.4. Comparison of the FFL-CC and FFL-FM models

The main difference between models FFL-CC and FFL-FM is in the setup variables. As in Clark and Clark (2000), the FFL-CC setups are modelled with just one set of binary variables, y_{ijmf}, whereas in the FFL-FM model setups are formulated with one set of binary variables y_{imf} and one set of positive variables z_{ijmf}, similar to Fleischmann and Meyr (1997). Table 1 shows the number of variables and constraints in models FFL-CC and FFL-FM. Note that FFL-CC has a smaller number of continuous and total variables than FFL-FM. The total number of variables in FFL-CC equals to the number of continuous variable in FFL-FM, but the latter has fewer binary variables than FFL-CC, suggesting that it might be faster to solve in very large problem. However, the order of magnitude of the number of variables and constraints is the same in both models. The computational tests in the next section will provide some insights into the relative efficiencies of the two models.

Table 1 Number of Variables and Constraints in models FFL-CC and FFL-FM

Number of:	**Model FFL-CC**	**Model FFL-FM**
Continuous variables	$J^2TM + JT(E+1) + 1$	$J^3TM + J^2TM + JT(E+1) + 1$
Binary variables	J^3TM	J^2TM
Capacitated lot sizing constraints	$1 + JTE + JT + TM$	$1 + JTE + JT + TM$
Sequencing constraints	$3J^2TM + JM(J-2) + M$	$2J^2TM + JM$
Setup state and changeover connecting constraints	$-$	$J^2M(JT-1) + 2JTM$

3. COMPUTAIONAL RESULTS

The aim of this section is to explore the FFL-CC and FFL-FM models through initial computational tests on small and larger problems in order to gain insight into their relative speed and performance. Özdamar and Barbarasoglu (1999) designed test problems to solve the CLSP in FFLs. Later Quadt (2004) also used their testing method, as does this paper. The problem parameters outside the statistical experimental design are randomly generated as follows: Processing times b_{im} (in hours) are generated from uniform distribution U(1,5) for all products i and machines m. Setup costs sc_{ijm} are generated from U(300,500). Set-up times are related to the total processing time

$$st_{ijm} = \frac{S \sum_{jtm} b_{jm} \cdot d_{jt}}{T \cdot Maxfac} \tag{31}$$

where S is generated from $U(0.05,0.10)$ and $Maxfac = \max_e\{M_e\}$ is the maximum number of machines at any stage. In the other words setup times are proportional to the mean production time per machine-period**.** The parameter levels within the experimental design are randomly generated as follows Demand variability is either low, $\mathrm{d_{it}}$ being generated from U(90,110), or high, from U(50,150). Holding and backordering costs assume that successive stages add value, so that work-in-process holding costs will increase as material progresses along the line. To reflect this, a *value-added percentage* factor VAP is used, whose value is 1.1 (*low*) or 1.3 (*high*). Inventory costs are then generated consecutively as follows The first stage's unit holding cost h_{it1} for product *i* is generated from *U*(1,20). For subsequent stages, $\mathrm{h_{ite}} = \mathrm{VAP} \cdot \mathrm{h_{it,e-1}}$ for $e \geq 2$. The backordering cost for product i is $B_{it} = 1.25 \cdot h_{itE}$. Capacity tightness is measured by a factor *CAT* with value 1.2 (*tight*) or 1.6 (*loose*). The mean capacity requirement *C* per machine at each stage is calculated as

$$C = \max_e \left\{\frac{\sum_{jtm} b_{jm} \cdot d_{jt}}{T \cdot Maxfac}\right\} \tag{32}$$

which is the maximum, over all stages, of the mean production time per machine-period. The capacity $\mathrm{C_{mt}}$ on machine m in macro-period t is then given by $\mathrm{C_{mt}} = \mathrm{CAT} \cdot \mathrm{C}$. Özdamar and Barbarosoglu (1999) did not specify the permitted percentage BP of end item demand that can be backordered, but this paper considers values of 20% (low) and 80% (high) for BP. Considering the four experimental attributes above, $2^4 = 16$ combinations were generated for each of two sets of test problems (*small* and *large*) of very different dimensionality. Three replications were generated for small problems while, due to the long solutions computing times involved, just one replication was generated for the large problems. Thus in total 48 small and 16 large problems were generated. The attributes and dimensionality of small problems are $E = 2, M_e = 2, J = 4, T = 6$ and large problems are $E = 3, M_e = 3, J = 8, T = 6$. To obtain some insight into the dimensionality of the models and problems, Table 2 computes the number of continuous and binary variables of the Özdamar and Barbarosoglu (1999) model, and FFL-CC and FFL-FM models for the small and large problems attributes in the former's paper (OzBa 1999). Note how the modelling of sequence-dependent setups can generate a very large number of binary and continuous variables. The conspicuous features are the huge number of binary variables in FFL-CC and continuous variables in FFL-FM, some 400 and 280 times more respectively than in OzBa 1999 for the big instance.

Table 2 OzBa (1999) attributes and comparing with FFL-CC and FFL-FM

Model	Small problems $\boldsymbol{E = 3, M_e = 3, J = 5, T = 6}$			Large problems $\boldsymbol{E = 4, M_e = 5, J = 20, T = 6}$		
	Continuous Variables	Binary Variables	Total Variables	Continuous Variables	Binary Variables	Total Variables
OzBa 1999	534	270	804	3,600	2,400	6,000
FFL-CC	1,471	6,750	8,221	48,601	960,000	1,008,601
FFL-FM	8,221	1,350	9,571	1,008,601	48,000	1,056,601

Table 3 illustrates the number of constraints and variables of both models, calculated based on the problems attributes of GLSP-FFL. As shown in Table 3, FFL-CC has fewer constraints, continuous and total number of variables than FFL-FM for both small and large problems. However FFL-FM has a smaller number of binary variables which, nevertheless, are still very many in the large problems.

Table 3 Number of constraints and variables in models FFL-CC and FFL-FM

Number of:	Small problem $E = 2, M_e = 2, J = 4, T = 6$		Large problem $E = 3, M_e = 3, J = 8, T = 6$	
	FFL-CC model	FFL-FM model	FFL-CC model	FFL-FM model
Constraints	1,285	2,545	11,056	35,167
Continuous variables	457	1,993	3,649	31,297
Binary variables	1,536	384	27,648	3,456
Total variables	1,993	2,377	31,297	34,753

Both models were implemented in the optimisation modelling software GAMS and solved using the industrial-strength CPLEX 9.0 solver on a computer with 2.1 GHZ CPU and 2 GB RAM.

The first two rows of Table 4 show the result of both models for the small problems including the average of CPU time, percentage of optimality gap and RAM usage. Figure 4 illustrates the value of the objective function for all 16 small problems for one replication. Observe that both models exhausted the 2 GB of available RAM before terminating the CPLEX branch-&-cut search, leaving large optimality gaps. Figure 4 shows that model FFL-CC obtained better solutions in a shorter time compared to model FFL-FM for all 16 small problems considered, being a mean 32% faster and with solutions 12% better on average. Moreover overall small problems, the mean optimality gap for FFL-CC was 35% less than FFL-FM. Both models found the first feasible solution in the first second of running time.

Table 4 Mean CPLEX results for small and bigproblems

Models and problems		Average of CPU time (secs)	Average of Percentage of optimality gap	Average of RAM usage (MB)
Small problems	FFL-CC	6346.5	70.6%	1890
	FFL-FM	8361.8	95.5%	1939
Big problems	FFL-CC	21,628	89.9%	1867
	FFL-FM	23,919	99.9%	1756

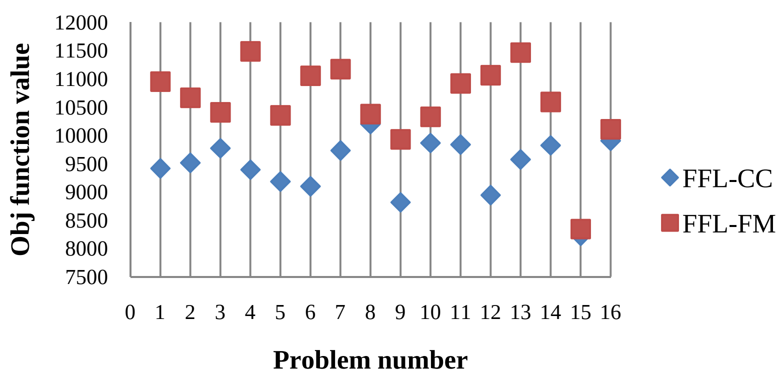

Figure 4 Result of CPLEX incumbent solution for the small problems

The last two rows of Table 4 show the results of both models for the large problems. Observe the large optimality gaps at termination after about 6 to 7 hours of running time. Similar to the small problems, FFL-CC obtained better results for all the large problems, on average, 10% shorter time, with 11% less optimality gap and with solutions 13% better. To provide more detailed insight into the large problems, the progression of the CPLEX solution is shown in Figure 5 for a sample of the large problems. Note that, on the one hand, at termination model FFL-CC obtained a better solution in a shorter time compared to model FFL-FM. On the other hand, FFL-FM found the first feasible solution more quickly (63 sec) than FFL-CC (3550 sec). To sum up, the test results above, although merely probing, and not conclusive, indicate that model FFL-CC finally obtains a better solution in a shorter time. However due to the importance of number of binary variables in large instances, FFL-FM finds the first feasible solution in much shorter time for over all the big problems compared with FFL-CC.

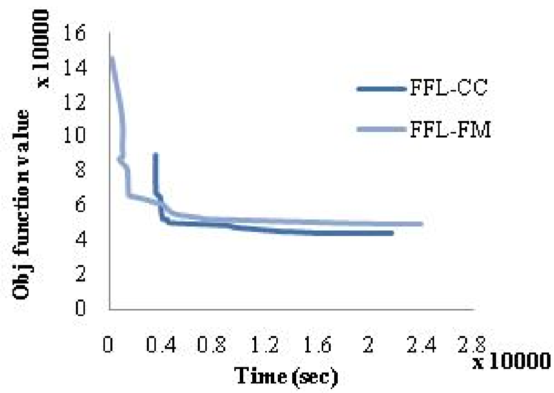

Figure 5 Result of CPLEX incumbent solution for a sample of the large problems

4. CONCLUSIONS

In this paper, three mathematical models were presented for the lot sizing and scheduling of flexible flow lines. The first model is dynamic so cannot be solved as a MIP, whereas the second and third can. Initial computational tests indicate that the second model (FFL-CC) is faster and more effective than the third (FFL-FM), but the third model (FFL-FM) can find an initial feasible solution much faster than second model (FFL-CC) for the big size problems. However, the complexity of the problem means that these tests to optimality could only be performed on small instances and so must be considered as just preliminary probing. Ongoing research is focusing on (i) more efficient formulations with ASTP subtour prohibitions constraints, that have already show promise in single-stage multi-machine problems (Clark et al. (2006), Almada-Lobo et al. (2007)) and (ii) heuristic solution approaches to solve larger problems within a reasonable time period.

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