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# Continuous Review Perishable Inventory System with OneSupplier, One Retailer andPositive Lead Time

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**ABSTRACT**

We consider a two-echelon single commodity inventory system with one warehouse(supplier) at the higher echelon and one retailer at the lower echelon. The retailer stocks individual items of a commodity and satisfies unit demands which occur according to a Poisson process. The supplier stocks these items in packets and uses these packetsto: (*i*) satisfy the demands that occur for single packet and that form an independent Poisson process and (*ii*) replenish the retailer's stock. The supplier implements (*s,S*)ordering policy to replenish the stock of packets and the lead time is assumed to have exponential distribution. Though the retailer's stock is replenished instantaneously by the supplier if packets are available, and a random stock out period may occur at the retailer node when the supplier has zero stock. It is assumed that because of better stocking facility that is usually available at the warehouse, the items do not perish at the warehouse, but they do perish at the retailer node. It is also assumed that the items have exponential life time distribution at the lower echelon. The joint probability distribution of the inventory levels on both nodes is obtained in the steady state. Various system performance measures are calculated. The long run total expected cost per unit time is derived. These results are illustrated with numerical examples. Some special cases are discussed in detail.

***Keywords*:** Stochastic inventory, Supply chain management, Poissondemand, Perishable items,Positive lead time.

## 1.INTRODUCTION

The mathematical models for the Supply Chain Management (SCM) are increasingly being considered by many researchers during the last few decades. The maintenance of a supply chain is a complex one as it consists of a broad network of activities such as production/procurement of raw materials, production units, logistics, suppliers, retailers and customers with stocking of items of processed or pre processed units between any two activities. To get a better understanding of SCM, one can think of several stages where each stage may signify the start or end of a specific process. These stages are also referred to as `echelons'. Moreover most inventory systems encountered in

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real life situations are multi-echelon in nature. Clark and Scarf (1960) considered the problem of determining optimal purchasing quantities in a multi-echelon model.

An informal survey of multi-echelon stochastic inventory systems was given by Clark (1972). A wide range of systems with deterministic demand (Roundy, 1985) and the optimal policies for periodic review were obtained during 1980's. Graves (1985) developed a multi-echelon inventory system with stochastic demands. Simon (1971) and Axsäter (1990) provided algorithms for finding stockage policies to two-echelon systems.

Beamon (1998) provided a review of literature on multi-stage supply-chain modeling. More specifically, this paper walked through various models of deterministic, analytical, stochastic analytical, economic and simulation.

A model with multiple raw materials, single stage, and stochastic demands was presented by Powell and Pyke (1998) by providing a heuristic solution. Wong et al. (2007) provided the optimal policies, heuristics and algorithms for a multi-echelon system.

One supplier and one retailer two-echelon model has been considered by many researchers. Andersson and Melchiors (2001) developed a two-echelon model with lost sales. Recently, Olsson and Hill(2007) considered a model with one warehouse and several retailers in which the suppliers follow($S,S$-1) ordering policy and the individual retailers adopts($Q,R$) ordering policy. Kogan and Herbon (2008), Szmerkorsky and Zhang (2009), Duc et al. (2008), etc. considered this type of systems. A common assumption is that the items are sent to the retailer from the supplier. The supplier can be considered as either a manufacturer (Hill et al. (2007), Bykadorov et al. (2009)) or a distribution centre (Axsäter and Marklund, 2008) or who sells the items to the retailer only. There are different kinds of demand processes such as stochastic demands, periodic demands, stock dependent demands (Goyal and Tao, 2009). Authors aim at enriching the efficiency in terms of expected cost minimization or profit improvement. In particular, some have used Stackelberg equilibrium (Szmerekovsky and Zhang, 2009), mixed auto regression coefficient (Duc et al., 2008), Mean Variance(MV) (Ming et al., 2008) analysis etc. to analyze these models.

We consider three factors namely lead time, perishable nature of the items and ordering policy.
In most of earlier works, the ordering policy was assumed as ($S,S$-1) inventory control policy in all installations. Axsäter (1990), Anderson and Melchiors (2001) proposed ($S,S$-1) policy in all installations and Olssonand Hill (2007) have presented a model with ($S,S$-1) ordering policy at the higher level and ($Q,R$) control policy at the lower level.

According to the ordering policy, researchers assumed that the demands which occurred during the stock out period are either lost or not. A common assumption is that end-customer demand is backordered. It is technically easier to analyze the end-customer was itself an organisation or an operating unit. Andersson and Melchiors (2001) and Hill et al. (2007) concerned lost sales cases in two-echelon models.

Our second consideration is the lead time. It may be a constant or not. A common assumption which runs through almost all of earlier research is that the lead time is constant. Axsäter (1990) and Hill et al. (2007) followed that the lead time is a fixed transportation time in all installations.

At last, we consider the perishable nature of items. The items may either perish or not. The lead time plays a seminal role when the items are perishable in nature. Before the replenishment for the ordered items, though there is no demand occur at the system due to the perishable nature of the items, the inventory level depletes. The supply delay(leadtime) affects the good running of organization when the items are perishable.

In this work we consider a two-echelon inventory system with one supplier at the higher echelon and one retailer at the lower echelon. The supplier stocks packets of a commodity and the retailer sells single item of this commodity. The supplier meets the demands for packets from the retailer who places the order when the stock is depleted and from an external source. The supplier implements $(s,S)$ordering policy to replenish the stock with random lead time. At the time of stock depletion at the retailer node, the inventory of the retailer is replenished immediately from the supplier. The retailer adopts $(0,R)$policy according to which a reorder is placed for$R$items when the inventory level is zero. We assume that the order is delivered instantaneously.

Though the stock at the retailer node is replenished instantaneously when the supplier has the requested stock, the retailer may experience the stock out period when the supplier has already run out of stock. We also assume that the demands that occur during the stock out period at the retailer node is lost. As observed in practice, the suppliers usually have better stocking facilities than that of the retailer. Hence we assume that the items may perish over the time at the retailer node only. We assume further that the life time of an item at the retailer node has exponential distribution.

The rest of the paper is organized as follows. The Section 2describes the model and the assumptions. In Section 3the inventory process is modelled as a Markovian process. The Section 4 presents steady state distribution for the inventory levels both at supplier node and retailer node. The Section 5 gives various measures of system performance in the steady state. In the Section 6, the total expected cost function per unit time is constructed. We present a special case in Section 7. The Section 8 deals some numerical illustrations for this model. The conclusion is presented in the last section.

## 2. THE MODEL

We consider a two-echelon inventory system with a supplier at the higher echelon and one retailer at the lower echelon. The retailer can stock a maximum of$R$units which is consumed one at a time by a stream of Poisson demands which occur at the rate of$\lambda_1$. On depletion of stock, it is replenished instantaneously from the stock of the supplier, if available. We assume that the supplier maintains a stock of these items in packets and each packet contains$R$items. The supplier can have a stock of maximum$S$ packets. The supplier also receives demand requests for single packet from a separate demand source. The time points of these demand sequences form a Poisson process with a rate$\lambda_1$. As the supplier receives request for single packet whenever the stock is depleted at the retailer node, the demand process at the supplier node is a superposition of Erlang process E($R$, $\lambda_1$) and Poisson process (P($\lambda_0$). We assume that the supplier implements $(s,S)$policy for ordering items with a random lead time that is distributed as exponential with parameter$\mu$. The items are assumed to have random life time only at the retailer node and it is assumed that the life time is distributed as exponential with parameterv(≥0).

We use the following notation in the rest of the paper.

$E_1 = \{0,1,2,\ldots,S\}$ where $S$ denotes the maximum number of packets stocked at the supplier node

$E_2 = \{1,2,\ldots,R\}$ where $R$ denotes the maximum inventory level at the retailer node

$E = \{E_1 \times E_2\} \cup \{(0,0)\}$

$e^T = (1,1,\ldots,1)$ of appropriate dimension

$I_{n\times n}$ denotes an identity matrix of order $n$

$A_{ij}$ denotes the $(i,j)$th entry/sub matrix of the matrix **A**

## 3. ANALYSIS

We assume that $X(t)$ denotes the inventory level at the warehouse and $Y(t)$ denotes the inventory level at the retailer node. Then $L= \{X(t),Y(t);t\geq 0\}$ is a stochastic process with state space $E$. Using the assumptions made on the input and output processes, it can be easily shown that the process $L$ is a Markov process. The intensities of transitions $a((i,j),(k,l))$ for this process can be obtained by using the following arguments.

- The arrival of a demand for the supplier makes the transition from $(i,j)$ to $(i\text{-}1,j)$ with transition intensity $\lambda_0$ where $i =1,2,\ldots,S$ and $j =1,2,\ldots,R$.
- A transition from $(i,j)$ to $(i+Q,j)$, $i = 0,1,2,\ldots,s$, $j =1,2,\ldots,R$ occurs at the time of the receipt of the order made by the supplier. For this the rate is $\mu$ .
- A transition from $(0,0)$ to $(Q\text{-}1,R)$ takes place when a reorder is received at the supplier node. Hence the transition rate for this is given by $\mu$ .
- A transition from $(i,j)$ to $(i,j\text{-}1)$, $i = 0,1,2,\ldots,s$, $j =2,3,\ldots,R$ will take place only when either a demand occurs at the retailer node or one of the $j$ items perish; for this the rate is $\lambda_1+jv$. We denote this quantity by $\beta_j$.
- When the state of the process is $(i,j)$ with $i = 1,2,\ldots,s$, $j = 1$, either a demand or perishing of an item at the retailer node takes the state to $(i,0)$. This forces the retailer to make a request for supply of one pack from the warehouse. As it is instantaneously delivered, the state becomes $(i\text{-}1,R)$. Hence the rate of transition from $(i,1)$ to $(i\text{-}1,R)$ is $\lambda_1+v$.
- The other transition, those are not considered in the above cases and with $(i,j) \neq (k,l)$ are not possible and the rate of these transitions are zero.
- We obtain the intensity of transitions for the cases $(i,j) = (k,l)$ by using the following equation

$$a((i,j),(i,j)) = -\sum_{(k,l)\neq(i,j)}\sum a((i,j),(k,l)).$$

Finally we write down the transition rates as follows:

$$
a((i,j),(k,l)) = \begin{cases}
\lambda_0 & k = i-1; \quad i = 1,2,\ldots,S; \\
 & l = j; \quad j = 1,2,\ldots,R, \\
\mu & k = i+Q; \quad i = 0,1,2,\ldots,s; \\
 & j = 1,2,\ldots,R; \quad l = j, \\
\mu & k = Q-1; \quad i = 0; \\
 & l = R; \quad j = 0, \\
\beta_1 & k = i-1; \quad i = 1,2,\ldots,S; \\
 & l = R; \quad j = 1, \\
\beta_1 & k = 0; \quad i = 0; \\
 & l = 0; \quad j = 1, \\
\beta_j & k = i; \quad i = 0,1,2,\ldots,S; \\
 & l = j-1; \quad j = 2,3,\ldots,R, \\
-\mu & k = i; \quad i = 0; \\
 & l = j; \quad j = 0, \\
-(\lambda_0 + \beta_j) & k = i; \quad i = s+1, s+2,\ldots,S; \\
 & l = j; \quad j = 1,2,\ldots,R, \\
-(\lambda_0 + \beta_j + \mu) & k = i \quad i = 1,2,\ldots,s, \\
 & l = j; \quad j = 1,2,\ldots,R, \\
-(\lambda_0 + \beta_j) & k = i; \quad i = 0; \\
 & l = j; \quad j = 1,2,\ldots,R, \\
0 & \textit{otherwise.}
\end{cases}
$$

We order the states of $E$ as shown below

$$E = (\underline{S}, \underline{S-1}, \underline{S-2}, \ldots, \underline{2}, \underline{1}, \underline{0}, \overline{0})$$

where $\underline{q} = ((q,R),(q,R-1),\ldots,(q,2),(q,1))$ for $q = 0,1,2,\ldots,S$ and $\overline{0} = (0,0)$.

Using the above ordering of states of $E$, we write down the rate matrix A = $a((i,j),(k,l))$ in block-partitioned form

$$A = ((A_{ik}))_{i,k \in E_1}$$

where

$$A_{ik} = \begin{cases} A_1 & k = i; \quad i = s+1, s+2, \ldots, S, \\ A_2 & k = i; \quad i = 1,2,\ldots,s, \\ A_3 & k = i; \quad i = 0, \\ A_4 & k = i; \quad i = \bar{0}, \\ B & k = i-1; \quad i = 1,2,\ldots,S, \\ B_1 & k = \bar{0}; \quad i = 0, \\ C & k = i+Q; \quad i = 1,2,\ldots,s, \\ C_1 & k = Q-1; \quad i = \bar{0}, \\ 0 & \textit{otherwise.} \end{cases}$$

The sub matrices are given by

$$A_1 = \begin{bmatrix} -(\lambda_0+\beta_R) & \beta_R & \cdots & 0 & 0 \\ 0 & -(\lambda_0+\beta_{R-1}) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -(\lambda_0+\beta_2) & \beta_2 \\ 0 & 0 & \cdots & 0 & -(\lambda_0+\beta_1) \end{bmatrix}_{R\times R},$$

$$A_2 = \begin{bmatrix} -(\lambda_0+\beta_R+\mu) & \beta_R & \cdots & 0 \\ 0 & -(\lambda_0+\beta_{R-1}+\mu) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta_2 \\ 0 & 0 & \cdots & -(\lambda_0+\beta_1+\mu) \end{bmatrix}_{R\times R},$$

$$A_3 = \begin{bmatrix} -(\mu+\beta_R) & \beta_R & \cdots & 0 & 0 \\ 0 & -(\mu+\beta_{R-1}) & \cdots & 0 & \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -(\mu+\beta_2) & \beta_2 \\ 0 & 0 & \cdots & 0 & -(\mu+\beta_1) \end{bmatrix}_{R\times R},$$

$$A_4 = [-\mu]_{1\times 1}, \; C = \mu I_{R\times R}, \; C_1 = \begin{bmatrix} \mu & 0 & \cdots & 0 \end{bmatrix}_{1\times R},$$

$$B = \begin{bmatrix} \lambda_0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \lambda_0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \lambda_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \\ 0 & 0 & 0 & \cdots & \lambda_0 & 0 \\ \beta_1 & 0 & 0 & \cdots & 0 & \lambda_0 \end{bmatrix}_{R\times R} \text{ and } B_1 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \beta_1 \end{bmatrix}_{R\times 1}.$$

It may be noted that the rate matrix $A$ is almost upper triangular in block-partitioned form with non-zero sub matrices along the diagonals specified by $j = i, j = i-1$ and $j = i+Q$ only. Because of this simple structure obtained by grouping the states into blocks as indicated above, it is possible to get simple algorithmic solutions to the model considered in this work.

## 4. STEADY STATE ANALYSIS

From the structure of the infinitesimal generator $A$, it can be shown that the homogeneous Markov process $\{X(t), Y(t); t \geq 0\}$ on the finite state space $E$ is irreducible. Hence the steady state probabilities $\pi_{(i,j)}$, $(i,j) \in E$ exist. These probabilities satisfy

$$\sum_{(i,j)} \pi_{(i,j)} a((i,j),(k,l)) = 0 \quad and$$

$$\sum_{(i,j)} \pi_{(i,j)} = 1.$$

These system of equations can be written in matrix form

$$\Pi A = 0$$

and

$$\Pi e = 1, \tag{1}$$

where

$$\Pi = (\Pi_{\underline{S}}, \Pi_{\underline{S-1}}, \ldots, \Pi_{\underline{1}}, \Pi_{\underline{0}}, \Pi_{\bar{0}})$$

and

$$\Pi_{\underline{i}} = (\pi_{(i,R)}, \pi_{(i,R-1)}, \ldots, \pi_{(i,1)}),\ i = 0, 1, 2, \ldots, S \text{ , and}$$

$$\Pi_{\bar{0}} = \pi_{(0,0)}.$$

We define the matrices $D_i^{'}$s as follows:

$$D_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -\dfrac{\beta_1}{\mu} \end{bmatrix}_{R\times 1}$$

$$D_1 = \begin{bmatrix} \dfrac{-(\lambda_0+\beta_R)}{\lambda_0} & \dfrac{\beta_R}{\lambda_0} & \cdots & 0 & 0 \\ 0 & \dfrac{-(\lambda_0+\beta_{R-1})}{\lambda_0} & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dfrac{-\beta_1\beta_2}{\lambda_0^2} & 0 & \cdots & \dfrac{-(\lambda_0+\beta_2)}{\lambda_0} & \dfrac{\beta_2}{\lambda_0} \\ \dfrac{\beta_1(\lambda_0+\beta_1)}{\lambda_0^2} & 0 & \cdots & 0 & \dfrac{-(\lambda_0+\beta_1)}{\lambda_0} \end{bmatrix}_{R\times R}$$

$$D_2 = \begin{bmatrix} \dfrac{-(\lambda_0+\beta_R+\mu)}{\lambda_0} & \dfrac{\beta_R}{\lambda_0} & \cdots & 0 & 0 \\ 0 & \dfrac{-(\lambda_0+\beta_{R-1}+\mu)}{\lambda_0} & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dfrac{-\beta_1\beta_2}{\lambda_0^2} & 0 & \cdots & \dfrac{-(\lambda_0+\beta_2+\mu)}{\lambda_0} & \dfrac{\beta_2}{\lambda_0} \\ \dfrac{\beta_1(\lambda_0+\beta_1+\mu)}{\lambda_0^2} & 0 & \cdots & 0 & \dfrac{-(\lambda_0+\beta_1+\mu)}{\lambda_0} \end{bmatrix}_{R\times R}$$

$$D_3 = \begin{bmatrix} \dfrac{-(\beta_R+\mu)}{\lambda_0} & \dfrac{\beta_R}{\lambda_0} & \cdots & 0 & 0 \\ 0 & \dfrac{-(\beta_{R-1}+\mu)}{\lambda_0} & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dfrac{-\beta_1\beta_2}{\lambda_0^2} & 0 & \cdots & \dfrac{-(\beta_2+\mu)}{\lambda_0} & \dfrac{\beta_2}{\lambda_0} \\ \dfrac{\beta_1(\beta_1+\mu)}{\lambda_0^2} & 0 & \cdots & 0 & \dfrac{-(\beta_1+\mu)}{\lambda_0} \end{bmatrix}_{R\times R}$$

$$D_4 = \begin{bmatrix} \dfrac{\mu}{\lambda_0} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{1\times R}$$

and

$$D_5 = \begin{bmatrix} \frac{\mu}{\lambda_0} & 0 & \cdots & 0 & 0 \\ 0 & \frac{\mu}{\lambda_0} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{\mu}{\lambda_0} & 0 \\ \frac{-\mu\beta_1}{\lambda_0^2} & 0 & \cdots & 0 & \frac{\mu}{\lambda_0} \end{bmatrix}_{R\times R}.$$

**Theorem 1** *The steady state probabilities* $\mathbf{\Pi}_i$*s are given by*

$$\Pi_i = \begin{cases} (-1)\Pi_{\underline{0}}(D_0), & i = \bar{0} \\ (-1)\Pi_{\underline{0}}(D_3), & i = \underline{1} \\ (-1)^i\Pi_{\underline{0}}(D_3)(D_2)^{i-1} & i = \underline{2}, \underline{3}, \ldots, \underline{s+1} \\ (-1)^i\Pi_{\underline{0}}(D_3)(D_2)^s(D_1)^{i-(s+1)} & i = \underline{s+2}, \underline{s+3}, \ldots, \underline{Q-1} \\ (-1)^i\Pi_{\underline{0}}(D_3)(D_2)^s D_1^{i-(s+1)} + (-1)^2\Pi_{\underline{0}}(D_0)(D_4), & i = \underline{Q} \\ (-1)^i\Pi_{\underline{0}}(D_3)(D_2)^s(D_1)^{i-(s+1)} & \\ \quad + (-1)^{i-Q+2}\Pi_{\underline{0}}(D_0)(D_4)(D_1)^{i-Q} & \\ \quad + (-1)^{i-Q}\Pi_{\underline{0}}(D_5)(D_1)^{i-Q+1} & \\ \quad + \sum_{j=0}^{i-Q-2}(-1)^{i-Q}\Pi_{\underline{0}}(D_3)(D_2)^j(D_5)(D_1)^{i-j-Q-2} & i = \underline{Q+1}, \underline{Q+2}, \ldots, \underline{S} \end{cases} \tag{2}$$

and $\Pi_{\underline{0}}$ is determined by solving the following

$$\begin{aligned} \Pi_{\underline{0}} \Big[ \; & (-1)D_0 + I + (-1)D_3 + \sum_{i=2}^{s+1}(-1)^i(D_3)(D_2)^{i-1} \\ & + \sum_{i=s+2}^{Q-1}(-1)^i(D_3)(D_2)^s(D_1)^{i-s-1} \\ & + (-1)^Q(D_3)(D_2)^s(D_1)^{Q-(s+1)} + (-1)^2(D_0)(D_4) \\ & + \sum_{i=Q+1}^{S}(-1)^i\{(D_3)(D_2)^s(D_1)^{i-s-1} + (-1)^{i-Q}(D_5)(D_1)^{i-Q-1} \\ & + \sum_{j=0}^{i-Q-2}(-1)^{i-Q}(D_3)(D_2)^j(D_5)(D_1)^{i-j-Q-2}\} \Big] = 0. \end{aligned} \tag{3}$$

and

$$
\begin{aligned}
\Pi_{\underline{0}} [ \quad & (-1)D_0 + I + (-1)D_3 + \sum_{i=2}^{s+1}(-1)^i (D_3)(D_2)^{i-1} \\
& + \sum_{i=s+2}^{Q-1}(-1)^i (D_3)(D_2)^s (D_1)^{i-s-1} \\
& + (-1)^Q (D_3)(D_2)^s (D_1)^{Q-(s+1)} + (-1)^2 (D_0)(D_4) \\
& + \sum_{i=Q+1}^{S}(-1)^i \{(D_3)(D_2)^s (D_1)^{i-s-1} + (-1)^{i-Q}(D_5)(D_1)^{i-Q-1} \\
& + \sum_{j=0}^{i-Q-2}(-1)^{i-Q}(D_3)(D_2)^j (D_5)(D_1)^{i-j-Q-2}\} \quad ] e = 1
\end{aligned} \tag{4}
$$

where

$$D_0 = -\frac{1}{\mu} B_1, D_1 = A_1 B^{-1}, D_2 = A_2 B^{-1}, D_3 = A_3 B^{-1}, D_4 = C_1 B^{-1} \ \ and \ \ D_5 = CB^{-1}. \tag{5}$$

**Proof.** The first equation of (1) gives

$$\Pi_{\underline{S}} A_1 + \pi_{\underline{s}} C = 0,$$

$$\Pi_{\underline{i}} B + \Pi_{\underline{i-1}} A_1 + \Pi_{\underline{i-(Q+1)}} C = 0, \quad i = Q+1, Q+2, \cdots, S,$$

$$\Pi_{\underline{i}} B + \Pi_{\underline{i-1}} A_1 + \Pi_{\bar{0}} C_1 = 0, \quad i = Q,$$

$$\Pi_{\underline{i}} B + \Pi_{\underline{i-1}} A_1 = 0, \quad i = s+2, s+3, \cdots, Q-1,$$

$$\Pi_{\underline{i}} B + \Pi_{\underline{i-1}} A_2 = 0, \quad i = 2, 3, \cdots, s+1,$$

$$\Pi_{\underline{1}} B + \Pi_{\underline{0}} A_3 = 0, \quad and$$

$$\Pi_{\underline{0}} B_1 + \Pi_{\bar{0}} A_4 = 0.$$

Starting from the last equation solving the above equations except the first one, we get the following recursive equations

$$\Pi_{\bar{0}} = -\Pi_{\underline{0}} B_1 (A_4^{-1}),$$

$$\Pi_{\underline{1}} = -\Pi_{\underline{0}} A_3 (B^{-1}),$$

$$\Pi_{\underline{i}} = -\Pi_{\underline{i-1}} A_2 (B^{-1}), \qquad i = 2, 3, \ldots, s+1,$$

$$\Pi_{\underline{i}} = -\Pi_{\underline{i-1}} A_1 (B^{-1}), \qquad i = s+2, s+3, \ldots, Q-1, \ \ and$$

$$\Pi_{\underline{i}} = -(\Pi_{\underline{i-1}} A_1 + \Pi_{\underline{i-Q+1}} C) B^{-1}, \quad i = Q+1, Q+2, \ldots, S.$$

By using (5), the recursive solution of the above gives (2).Also, by using the above solution in $\Pi_{\underline{S}} A_1 + \Pi_{\underline{s}} C = 0$ and $\Pi e = 1$, we get respectively (3) and (4).

## 5. SYSTEM PERFORMANCE MEASURES

In this section we derive some performance measures of the system in the steady state. Using these measures, we can construct the total expected cost per unit time.

### 5.1.Mean reorder rates

The supplier places an order to replenish stock when the inventory level at the supplier node drops from$s$+1 to$s$by a demand either from the retailer or the external source. The retailer's request arises when stock moves from 1to 0 due to an occurrence of a demand or perishing of an item. Hence the mean rate of reorder made at supplier node is given by

$$\eta_{R_S} = \big(\lambda_0 + (\lambda_1 + \nu)\big)\pi_{(s+1,1)} + \lambda_0 \sum_{j=2}^{R} \pi_{(s+1,j)} .$$

The retailer orders for a pack of$R$ items when inventory level drops from 1 to 0either a demand occurs or an item perishes. Then, the rate for reorder at retailer node in the steady state is given by

$$\eta_{R_R} = (\lambda_1 + \nu) \sum_{i=0}^{S} \pi_{(i,1)} .$$

### 5.2.Mean inventory levels

The mean inventory level at the supplier node in the steady state of the system is

$$\eta_{I_S} = \sum_{i=1}^{S} \sum_{j=1}^{R} i\pi_{(i,j)} .$$

Also, the mean inventory level at the retailer node in the steady state of the system is

$$\eta_{I_R} = \sum_{j=1}^{R} \sum_{i=0}^{S} j\pi_{(i,j)} .$$

### 5.3.Shortage rates

When a demand occurs during the stock out period at the supplier node, this demand is lost if it is from the external source and this demand is backordered if it is to replenish the stock of the retailer. In either case the pending reorder is not received when the demand occurs. Hence the shortage rate at the supplier node is given by

$$\eta_{S_S} = \lambda_0 \sum_{j=0}^{R} \pi_{(0,j)} .$$

A forced stock out at the retailer node occurs when both retailer and supplier run out of stock. A demand occurs at the retailer node during this stock out period is lost. The rate of shortage in the steady state is given by

$$\eta_{S_R} = \lambda_1 \pi_{(0,0)} .$$

### 5.4.Mean perishable rate

The expected perishable rate at the retailer node in the steady state of the system is given by

$$\eta_P = \nu \sum_{j=1}^{R} j \sum_{i=0}^{S} \pi_{(i,j)} .$$

## 6.COST FUNCTION

The expected total cost per unit time in the steady state for this model is defined as

$$TC(S,R) = K_S \eta_{R_S} + K_R \eta_{R_R} + h_S \eta_{I_S} + h_R \eta_{I_R} + \gamma_S \eta_{S_S} + \gamma_R \eta_{S_R} + c_P \eta_{P_R}$$

where

$K_S$ Setup cost per order to the supplier

$K_R$ Setup cost per order to the retailer

$h_S$ Holding cost per unit per unit time to the supplier

$h_R$ Holding cost per unit per unit time to the retailer

$\gamma_S$ Stock out cost per customer per unit time at the supplier node

$\gamma_R$ Stock out cost per customer per unit time at the retailer node

$c_P$ Perishable cost per item per unit time to the retailer.

As the analytical solution of the cost function has complex form, it is difficult to obtain explicitly. We present some numerical illustrations in Section 8 to show the computability of the results obtained in this work and to illustrate the existence of local optima when the total cost function is treated as a function of two variables by keeping the other values fixed. Also in the Section 8, we discuss the optimum(local) values of*S*and*R*by numerical illustrations.

## 7. SPECIAL CASE

In this case, we assume that $\mu = 0$. That is, there is no lead time at the supplier node. The state space of this process is $\widetilde{E} = \widetilde{E}_1 \times \widetilde{E}_2, \widetilde{E}_1 = \{1,2,\ldots,S\}$ and $\widetilde{E}_2 = \{1,2,\cdots,R\}$. In this case taking $A_3 = 0, A_4 = 0, B_1 = 0, C = 0, C_1 = 0, A_1 = A_2,$ the infinitesimal generator $M$ takes the form

$$M = \begin{bmatrix} A & B & \cdots & 0 & 0 \\ 0 & A & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & A & B \\ B & 0 & \cdots & 0 & A \end{bmatrix}_{R\times R}$$

where

$$A = \begin{bmatrix} -(\lambda_0+\beta_R) & \beta_R & \cdots & 0 & 0 \\ 0 & -(\lambda_0+\beta_{R-1}) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -(\lambda_0+\beta_2) & \beta_2 \\ 0 & 0 & \cdots & 0 & -(\lambda_0+\beta_1) \end{bmatrix}_{R\times R},$$

$$B = \begin{bmatrix} \lambda_0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \lambda_0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \lambda_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_0 & 0 \\ \beta_1 & 0 & 0 & \cdots & 0 & \lambda_0 \end{bmatrix}_{R\times R}, \quad \beta_j = \lambda_1 + j\beta.$$

**Theorem 2** *The steady-state probability vector* $\Pi$ *is given by*

$$\Pi = \frac{1}{S\delta} e^T \otimes \left( \frac{1}{\beta_R}, \frac{1}{\beta_{R-1}}, \dots, \frac{1}{\beta_1} \right) \tag{6}$$

where

$$\delta = \sum_{k=1}^{R} \frac{1}{\beta_k}.$$

**Proof.** We first, observe that the generator$A$ is a circulant matrix in block structure. If$A$ and$B$ are scalars, then the stationary probability distribution is uniform on the set $\{1,2,\dots,S\}$. We proceed to obtain the stationary distribution in the non-scalar case.

By rewriting$\mathbf{\Pi}M$= 0we get

$$\Pi_{\underline{1}} A + \Pi_{\underline{S}} B = 0,$$

$$\Pi_{\underline{i}} B + \Pi_{\underline{i+1}} A = 0, \; i = 1,2,\dots,S-1.$$

By taking$\mathbf{\Pi}_i$= $\omega_1$, $i$ = 0,1,2,…,$s$, all the above equations take the form $\omega_1(A+B) = 0$. Hence $\omega_1$can be thought of as an invariant measure of a Markov process with generator$A+B$.

Since$A+B$ is given by

$$\begin{bmatrix} -\beta_R & \beta_R & \cdots & 0 & 0 \\ 0 & -\beta_{R-1} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\beta_2 & \beta_2 \\ \beta_1 & 0 & \cdots & 0 & -\beta_1 \end{bmatrix},$$

it can be easily verified that $\left(\frac{1}{\beta_R}, \frac{1}{\beta_{R-1}}, \ldots, \frac{1}{\beta_1}\right)$ satisfies $\omega_1(A+B) = 0$.

By using the normalizing condition, we get

$$\omega = \frac{1}{\delta}\left(\frac{1}{\beta_R}, \frac{1}{\beta_{R-1}}, \ldots, \frac{1}{\beta_1}\right) = \frac{\omega_1}{\delta}.$$

The solution**Π**can be expressed as $\Pi = e^T \otimes \omega$. By using the normalizing condition for the generator $A$, we get

$$\Pi = \frac{1}{S\delta} e^T \otimes \left(\frac{1}{\beta_R}, \frac{1}{\beta_{R-1}}, \ldots, \frac{1}{\beta_1}\right). \tag{7}$$

The expected total cost per unit time for this special case is

$$TC(S,R) = \frac{K_S}{S}(\lambda_0 + \frac{1}{\delta}) + \frac{K_R}{\delta} + h_S \frac{S+1}{2} + (h_R + c_P \nu)\frac{\sum_{k=1}^{R} \frac{k}{\beta_k}}{\delta}.$$

To study the convexity of the cost function$TC(S,R)$, we prove the following lemma.

**Lemma 1** *The function* $f(x) = \frac{ax^2 + bx + c}{dx}$ *(where a,b,c and d are constants that do not involve* $x$*) is a convex function. Also the minimum point* $x^*$ *of f(x)satisfies the set of inequalities* $(x^* - 1)x^* \le \frac{c}{a} < x^*(x^* + 1)$. *If the equality holds good on left side of the expression, then both*$x^*$ *and* $x^*$-1 *are the minimum points of f(x).*

**Proof.** By writing $\Delta(f(x)) = f(x+1) - f(x)$, we obtain

$$\Delta f(x) = \left\{\frac{a(x+1)^2 + b(x+1) + c}{d(x+1)}\right\} - \left\{\frac{ax^2 + bx + c}{dx}\right\}$$
$$= \frac{a}{d} - \frac{c}{dx(x+1)}.$$

It can be easily seen that as $x$ increases $\Delta f(x)$ increases, which implies that the function $f(x)$ is convex. Hence $\Delta f(x)$ changes the sign at most once. Therefore, if $f(x)$ has $x^*$ as the minimal point, then $\Delta f(x)$ changes sign at $x^*$. Hence we get $\Delta f(x^*-1\leq 0)$ and $\Delta f(x^*>0)$.
From the first inequality we obtain

$$x^*(x^*-1)\leq \frac{c}{a}.$$

Similarly, the second inequality yields $x^*(x^*+1) > \frac{c}{a}$.

Combining these two inequalities we get $(x^*-1)x^* \leq \frac{c}{a} < x^*(x^*+1)$.

Now, we consider the expected cost function $TC(S,R)$ as a function of any one variable by keeping the other as constant. Let $U(S) = TC(S,R)$, for a fixed $R$, we have

$$U(S) = \frac{l_1}{S} + m_1 + n_1(S+1) + o_1$$

$$= \frac{n_1 S^2 + (m_1 + n_1 + o_1)S + l_1}{S}$$

where

$$l_1 = K_S\left[\lambda_0 + \frac{1}{\delta}\right],\ m_1 = \frac{K_R}{\delta},\ n_1 = \frac{h_S}{2},\ and\ \ o_1 = (h_R + c_P\nu)\frac{\sum_{k=1}^{R}\frac{k}{\beta_k}}{\delta}.$$

Since the function $U(S)$ is similar to $f(x)$ of the above lemma, we conclude that the function $U(S)$ is convex. The minimum point $S^*$ of $U(S)$ satisfies the set of inequalities

$$S^*(S^*-1)\leq \frac{l_1}{n_1} < S^*(S^*+1).$$

That is,

$$S^*(S^*-1)\leq \frac{2K_S\left[\lambda_0 + \frac{1}{\delta}\right]}{h_S} < S^*(S^*+1). \tag{8}$$

Now we shall treat $TC(S,R)$ as a function of $R$ by keeping $S$ at a constant level. Let $V(R) = TC(S,R)$ for a fixed $S$. Then

$$V(R) = K_S\eta_{R_S} + K_R\eta_{R_R} + h_S\eta_{I_S} + (h_R + c_p\nu)\eta_{I_R}$$

$$= \left[ \frac{K_S \lambda_0}{S} + \frac{h_S}{2}(S+1) \right] + \left[ \frac{K_S}{S} + K_R \right] \frac{1}{a(R)} + (h_R + c_P \nu) \frac{b(R)}{a(R)}$$

$$= a_1 + \frac{b_1}{a(R)} + c_1 \frac{b(R)}{a(R)}$$

where

$$a(R) = \sum_{k=1}^{R} \frac{1}{\beta_k}, \quad b(R) = \sum_{k=1}^{R} \frac{k}{\beta_k},$$

$$a_1 = \left[ \frac{K_S \lambda_0}{S} + \frac{h_S}{2}(S+1) \right],$$

$$b_1 = \left[ \frac{K_S}{S} + K_R \right] \text{ and}$$

$$c_1 = (h_R + c_P \nu).$$

Consider

$$\Delta V(R) = V(R+1) - V(R) = \frac{W(R)}{a(R)a(R+1)}$$

where

$$W(R) = b_1[a(R) - a(R+1)] + c_1[a(R)b(R+1) - a(R+1)b(R)].$$

It can be verified that

$$a(R) - a(R+1) = \frac{-1}{\beta_{R+1}}, \text{ and} \tag{9}$$

$$a(R)b(R+1) - a(R+1)b(R) = \frac{1}{\beta_{R+1}} \sum_{k=1}^{R} \frac{R+1-k}{\beta_k}. \tag{10}$$

Using (9) and (10) we obtain

$$\Delta V(R) = \frac{1}{a(R)a(R+1)} \left\{ b_1 \left[ \frac{-1}{\beta_{R+1}} \right] + c_1 D(R) \right\} \tag{11}$$

where

$$D(R) = \left[ \frac{1}{\beta_{R+1}} \sum_{k=1}^{R} \frac{R+1-k}{\beta_k} \right].$$

We observe that the first term on right hand side of (11) is non-decreasing with$R$. The second term is also non decreasing as

$$D(R+1)-D(R)=\frac{1+R}{\beta_{R+1}\beta_{R+2}}>0.$$

Thus implies$V(R+1)-V(R)$is non decreasing as$a(R)a(R+1)$ is positive. Hence we conclude that $V(R)$is a convex function.

$$V(R^*+1)-V(R^*)>0$$

and

$$V(R^*)-V(R^*-1)\leq 0.$$

From the first inequality, we obtain

$$\frac{\frac{K_S}{S}+K_R}{h_R+c_P\nu}<\sum_{k=1}^{R^*}\frac{R^*+1-k}{\beta_k}. \tag{12}$$

Similarly, the second inequality yields

$$\sum_{k=1}^{R^*-1}\frac{R^*-k}{\beta_k}\leq\frac{\frac{K_S}{S}+K_R}{h_R+c_P\nu}. \tag{13}$$

Combining the inequalities(12)and (13) we obtain

$$L(R^*-1,\nu)\leq\frac{\frac{K_S}{S}+K_R}{h_R+c_P\nu} < L(R^*,\nu) \tag{14}$$

where

$$L(R^*,\nu)=\sum_{k=1}^{R^*}\frac{R^*+1-k}{\beta_k}$$

and$L(0,\nu)=0$. This result agrees with Kalpakam and Arivarignan (1988).

Suppose equality holds on the left hand side of the double inequality (14) it means that both$R^*$ and $R^*$-1are optimal.

**Note:** We shall consider a special case of above by taking zero lead time at the supplier node and the items do not perish at the retailer node, that is,$\mu$=0andv=0. In this case there are no shortage for packets or unit items. The state space of this process is $E' = E_1' \times E_2', E_1' = \{1,2,\ldots,S\}$ and $E_2' = \{1,2,\ldots,R\}$. The limiting distribution for this case is obtained from (7) as

$$\pi_{(i,j)} = \frac{1}{SR}, i = 1,2,\ldots,S, j = 1,2,\ldots,R .$$

The optimal decision rules for $S$ obtained from (8)

$$(S^* - 1)S^* \le \frac{2K_S\{\lambda_0 + \frac{\lambda_1}{R}\}}{h_S} < S^*(S^* + 1)$$

which agrees with Sivazilian's(1974) result when the pooled rate of external demand and that of retailer $\lambda = \lambda_0 + \frac{\lambda_1}{R}$. We note that the result is independent of$K_R$and$h_R$.

Similarly, the optimal decision rule for$R$ is obtained from (14)

$$(R^* - 1)R^* \le \frac{2\lambda_1\{K_R + \frac{K_S}{S}\}}{h_R} < R^*(R^* + 1)$$

which agrees with Sivazilian's (1974) result with $K = K_R + \frac{K_S}{S}$. We note that the result is independent of$K_S$, $H_S$ and $\lambda_0$ (the arrival rate of the external customers at the supplier node).

## 8. NUMERICAL ILLUSTRATION

In this section, we present some numerical illustrations, for the model and its special cases. Since we have not shown the convexity of the expected total cost function by analytical methods, we explore the behaviour of the function by varying any two parameters and keeping the others at a fixed level.

In the following tables, the shadedvalues are optimal in the respective column and the values that are shown *bold* are optimal in that row.

### 8.1.Cost analysis

Table 1 presents the total expected cost per unit time for various combinations of $S$ and $R$ where the other parameters and costs are fixed as $\mu = 1.75$ ,$\lambda_0 = .5$, $\lambda_1 = 2$, $\nu = 2$, $K_S = 5$, $K_R = 1$, $h_S = 0.01$, $h_R = 0.01$, $\gamma_S = 4$, $\gamma_R = 2$, and$\beta_R$= 0.2.

Table 1 Total expected cost as a function of $S$ and $R$

| $R$ / $S$ | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|
| 28 | 2.730678 | 2.664092 | **2.656439** | 2.681648 | 2.726817 | 2.784853 |
| 29 | 2.727680 | 2.662194 | **2.655232** | 2.680906 | 2.726399 | 2.784668 |
| 30 | 2.725456 | 2.660969 | **2.654636** | 2.680732 | 2.726521 | 2.785000 |
| 31 | 2.723897 | 2.660324 | **2.654566** | 2.681047 | 2.727104 | 2.785777 |
| 32 | 2.722916 | 2.660181 | **2.654950** | 2.681784 | 2.728088 | 2.786936 |
| 33 | 2.722439 | 2.660477 | **2.655729** | 2.682888 | 2.729419 | 2.788429 |
| 34 | 2.722406 | 2.661157 | **2.656856** | 2.684314 | 2.731054 | 2.790214 |
| 35 | 2.722765 | 2.662176 | **2.658289** | 2.686024 | 2.732957 | 2.792256 |
| 36 | 2.723471 | 2.663496 | **2.659993** | 2.687985 | 2.735098 | 2.794525 |
| 37 | 2.724487 | 2.665083 | **2.661938** | 2.690170 | 2.737449 | 2.796996 |
| 38 | 2.725781 | 2.666911 | **2.664099** | 2.692554 | 2.739989 | 2.799648 |

In Table 2, the total expected costs per unit time for various combinations of $S$ and $s$ are given by keeping the other parameters and costs as constants

Table 2 Total expected cost as a function of $S$ and $s$

| $s$ / $S$ | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|
| 80 | 3.173324 | 3.167891 | **3.165888** | 3.166707 | 3.169871 | 3.175005 |
| 81 | 3.172725 | 3.167295 | **3.165244** | 3.165971 | 3.169000 | 3.173962 |
| 82 | 3.172289 | 3.166866 | **3.164773** | 3.165413 | 3.168316 | 3.173116 |
| 83 | 3.172009 | 3.166596 | **3.164465** | 3.165025 | 3.167810 | 3.172457 |
| 84 | 3.171879 | 3.166479 | **3.164314** | 3.164799 | 3.167473 | 3.171975 |
| 85 | 3.171892 | 3.166507 | **3.164312** | 3.164729 | 3.167299 | 3.171664 |
| 86 | 3.172043 | 3.166675 | **3.164454** | 3.164806 | 3.167278 | 3.171515 |
| 87 | 3.172325 | 3.166977 | **3.164734** | 3.165026 | 3.167406 | 3.171521 |
| 88 | 3.172734 | 3.167407 | **3.165145** | 3.165382 | 3.167675 | 3.171675 |
| 89 | 3.173264 | 3.167961 | **3.165682** | 3.165868 | 3.168079 | 3.171970 |
| 90 | 3.173911 | 3.168633 | **3.166340** | 3.166479 | 3.168613 | 3.172400 |

For selected combinations of $R$ and $s$, Table 3 gives the expected total cost per unit time and the other parameters and costs are assumed to be $S = 65$, $\mu = 1.75$, $\lambda_0 = 4$, $\lambda_1 = 20$, $\nu = 2$, $K_S = .55$, $K_R = 8.75$, $h_S = 0.01$, $h_R = .01$, $\gamma_S = 3.4$, $\gamma_R = 2$ and $\beta_R = 0.2$.

Table 3 Total expected cost as a function of $R$ and $s$

| $s$ / $R$ | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|
| 40 | 18.153659 | 18.147043 | **18.144950** | 18.146470 | 18.150930 | 18.157831 |
| 41 | 18.145640 | 18.139238 | **18.137329** | 18.139009 | 18.143609 | 18.150636 |
| 42 | 18.142935 | 18.136741 | **18.135011** | 18.136847 | 18.141585 | 18.148735 |
| 43 | 18.145163 | 18.139173 | **18.137617** | 18.139605 | 18.144477 | 18.151749 |
| 44 | 18.151980 | 18.146187 | **18.144801** | 18.146938 | 18.151942 | 18.159333 |
| 45 | 18.163069 | 18.157470 | **18.156251** | 18.158533 | 18.163666 | 18.171174 |

Table 4 presents the total expected cost for the special case in Section 7 for various combinations of *S* and *R* where the other parameters and costs are assumed to be $\lambda_0 = 5$, $\lambda_1 = 2$, $h_S = .01$, $h_R = 0.01$, $K_S = 5$, $K_R = 2$, $c_P = 0.2$, and $v = 2$.

Table 4 Total expected cost as a function of *S* and *R* for the special case

| R / S | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| 72 | 4.413135 | 4.361038 | **4.346114** | 4.355787 | 4.382501 |
| 73 | 4.412621 | 4.360465 | **4.345488** | 4.355113 | 4.381784 |
| 74 | 4.412256 | 4.360042 | **4.345014** | 4.354593 | 4.381221 |
| 75 | 4.412034 | 4.359764 | **4.344685** | 4.354219 | 4.380807 |
| 76 | 4.411949 | 4.359624 | **4.344497** | 4.353987 | 4.380535 |
| 77 | 4.411997 | 4.359618 | **4.344444** | 4.353891 | 4.380400 |
| 78 | 4.412171 | 4.359741 | **4.344520** | 4.353926 | 4.380397 |
| 79 | 4.412468 | 4.359987 | **4.344721** | 4.354086 | 4.380520 |

Table 5 shows the total expected cost for various combinations of *S* and *R* for non perishable items where the other parameters and costs are kept as $\lambda_0 = 5$, $\lambda_1 = 2.15$, $h_S = .01$, $h_R = 0.01$, $K_S = 5$, $K_R = 5$ and $c_P = 0.2$.

Table 5 Total expected cost as a function of *S* and *R* for non perishable items

| R / S | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|
| 118 | 1.847112 | 1.843531 | **1.842644** | 1.844084 | 1.847547 |
| 119 | 1.847185 | 1.843438 | **1.842386** | 1.843660 | 1.846957 |
| 120 | 1.847340 | 1.843430 | **1.842215** | 1.843326 | 1.846461 |
| 121 | 1.847575 | 1.843505 | **1.842130** | 1.843081 | 1.846056 |
| 122 | 1.847888 | 1.843661 | **1.842128** | 1.842922 | 1.845739 |
| 123 | 1.848278 | 1.843895 | **1.842207** | 1.842846 | 1.845508 |
| 124 | 1.848741 | 1.844207 | **1.842366** | 1.842852 | 1.845362 |
| 125 | 1.849278 | 1.844593 | **1.842602** | 1.842939 | 1.845298 |

In Tables 1 to 5, the total cost function appears to have a convex form for the selected combinations of values for fixed two parameters.

## 8.2.Sensitivity analysis

In this section we show the changes that are observed in the expected total cost rate and optimal values($S^*$ and $s^*$) and for calculating these values, we restrict $S$ to take values from 80 to 90 and $s$ to take values from 9 to 14. The values of $S^*$ are given in the upper part of the cell and optimal cost rate is given in the lower part.

Table 6 gives the expected total cost rate for selected combinations of $\mu$ and $\nu$ where the other parameters and costs are assumed to be $\lambda_0 = 5$, $\lambda_1 = 2$, $K_S = 5$, $K_R = 1$, $h_S = 0.01$, $h_R = 0.01$, $\gamma_S = 4$, $\gamma_R = 2$ and $\beta_R = 0.2$. It may be observed that the optimum expected total cost rate increases with $\nu$.

But this trend is reversed with $\mu$. Further the optimal value, $S^*$ is formed to be increasing with $\nu$ but increasing with $\mu$. The same trend is observed for $s^*$ also.

Table 6 Effect of $\mu$ and $\nu$ on the optimal cost function for given($S^*$ and $s^*$) values

| $\nu$ / $\mu$ | 2 | 2.2 | 2.4 | 2.6 | 2.8 | 3 |
|---|---|---|---|---|---|---|
| 1.75 | 85 11 | 85 11 | 86 12 | 87 12 | 88 12 | 88 12 |
| | 3.164312 | 3.353342 | 3.541499 | 3.728839 | 3.915641 | 4.101965 |
| 2.00 | 81 10 | 82 10 | 82 10 | 83 10 | 84 10 | 84 10 |
| | 3.131557 | 3.320295 | 3.508248 | 3.695556 | 3.882327 | 4.068607 |
| 2.50 | 80 9 | 80 9 | 80 9 | 80 9 | 80 9 | 80 9 |
| | 3.103739 | 3.292062 | 3.479691 | 3.666745 | 3.853317 | 4.039403 |
| 2.75 | 80 9 | 80 9 | 80 9 | 80 9 | 80 9 | 80 9 |
| | 3.064130 | 3.251429 | 3.438034 | 3.624063 | 3.809610 | 3.994748 |
| 3.00 | 80 9 | 80 9 | 80 9 | 80 9 | 80 9 | 80 9 |
| | 3.049426 | 3.236378 | 3.422636 | 3.608317 | 3.793515 | 3.978303 |

In Table 7, we give the expected total cost for selected combinations of $\lambda_0$ and $\lambda_1$ where the other parameters and costs are assumed to be$R$ = 5, $\mu$ = 1.75, $\nu$ = 2, $K_s$ = 5, $K_R$ = 1, $h_S$ = 0.01, $h_R$ = 0.01 and $\gamma_S$ = 4, $\gamma_R$ = 2 and $\beta_R$ = 0.2. The optimal cost function is found to be increasing with $\lambda_0$ as well as with $\lambda_1$. Both optimal values $S^*$ and $s^*$ also increase with $\lambda_0$ and with $\lambda_1$.

Table 7 Impact of $\lambda_0$ and $\lambda_1$ on the optimal cost function for given ($S^*$ and $s^*$) values

| $\lambda_1$ / $\lambda_0$ | 1.4 | 1.6 | 1.8 | 2 | 2.2 | 2.4 |
|---|---|---|---|---|---|---|
| 4.4 | 80 10 | 80 10 | 80 10 | 80 10 | 80 10 | 80 10 |
| | 2.927974 | 2.988719 | 3.048302 | 3.106858 | 3.164500 | 3.221325 |
| 4.6 | 80 10 | 80 10 | 80 10 | 80 10 | 81 11 | 82 11 |
| | 2.946814 | 3.007755 | 3.067538 | 3.126296 | 3.184089 | 3.240800 |
| 4.8 | 81 11 | 82 11 | 82 11 | 83 11 | 83 11 | 84 11 |
| | 2.966278 | 3.027094 | 3.086715 | 3.145286 | 3.202893 | 3.259679 |
| 5.00 | 83 11 | 84 11 | 84 11 | 85 11 | 85 11 | 86 12 |
| | 2.985108 | 3.045995 | 3.105668 | 3.164312 | 3.221977 | 3.278776 |
| 5.2 | 85 11 | 86 12 | 86 12 | 87 12 | 87 12 | 88 12 |
| | 3.004223 | 3.064971 | 3.124490 | 3.182939 | 3.240444 | 3.297110 |
| 5.4 | 87 12 | 88 12 | 88 12 | 89 12 | 89 12 | 89 12 |
| | 3.022566 | 3.083329 | 3.142896 | 3.201420 | 3.258978 | 3.315717 |

Table 8 presents the local optimum values for various combinations of $\lambda_0$ and $\mu$ by assuming $R$ = 5, $\lambda_1$ = 2, $\nu$ = 2, $K_s$ = 5, $K_R$ = 1, $h_S$ = 0.01, $h_R$ = 0.01, $\gamma_S$ = 4, $\gamma_R$ = 2 and $\beta_R$ = 0.2. The optimal total cost increases with $\lambda_0$ and with $\lambda_1$. Both $S^*$ and $s^*$ increase with $\lambda_0$ and with $\lambda_1$.

In Table 9, we consider the total expected cost per unit time for various combinations of $\lambda_1$ and $\mu$ and the other parameters and costs are assumed to be$R$ = 5, $\lambda_0$ = 5, $\nu$= 2, $K_s$ = 5, $K_R$ = 1, $h_S$ = 0.01, $h_R$

= 0.01 and $\gamma_S = 4$, $\gamma_R = 2$ and $\beta_R = 0.2$. The optimal total cost increases with $\lambda_1$ with $\mu$. The optimal $S^*$ and $s^*$ are non decreasing with $\lambda_1$ but are non-increasing with $\mu$.

Table 8 The influence of $\mu$ and $\lambda_0$ on the optimal cost function for given ($S^*$ and $s^*$) values

| $\lambda_0$ / $\mu$ | 3 | | 4 | | 5 | | 6 | | 7 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.00 | 80 | 12 | 90 | 14 | 90 | 14 | 90 | 14 | 90 | 14 |
| | 3.082404 | | 3.204677 | | 3.356383 | | 3.561071 | | 3.826530 | |
| 1.25 | 80 | 10 | 83 | 13 | 90 | 14 | 90 | 14 | 90 | 14 |
| | 3.038851 | | 3.145514 | | 3.255507 | | 3.393713 | | 3.571433 | |
| 1.5 | 80 | 9 | 80 | 11 | 89 | 13 | 90 | 14 | 90 | 14 |
| | 3.008377 | | 3.102700 | | 3.202946 | | 3.307267 | | 3.436966 | |
| 1.75 | 80 | 9 | 80 | 9 | 85 | 11 | 90 | 13 | 90 | 14 |
| | 2.987540 | | 3.069684 | | 3.164312 | | 3.257304 | | 3.359540 | |
| 2 | 80 | 9 | 80 | 9 | 80 | 9 | 90 | 12 | 90 | 14 |
| | 2.971956 | | 3.044608 | | 3.131557 | | 3.220311 | | 3.310993 | |
| 2.25 | 87 | 12 | 88 | 12 | 88 | 12 | 89 | 12 | 89 | 12 |
| | 2.959352 | | 3.025807 | | 3.103739 | | 3.190108 | | 3.274350 | |

Table 9 Impact of $\mu$ and $\lambda_1$ on the optimal cost function for given ($S^*$ and $s^*$) values

| $\lambda_1$ / $\mu$ | 1.6 | | 1.8 | | 2 | | 2.2 | | 2.4 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.00 | 90 | 14 | 90 | 14 | 90 | 14 | 90 | 14 | 90 | 14 |
| | 3.230173 | | 3.293741 | | 3.356383 | | 3.418210 | | 3.479323 | |
| 1.25 | 90 | 14 | 90 | 14 | 90 | 14 | 90 | 14 | 90 | 14 |
| | 3.134845 | | 3.195672 | | 3.255507 | | 3.314464 | | 3.372641 | |
| 1.5 | 88 | 13 | 89 | 13 | 89 | 13 | 90 | 13 | 90 | 13 |
| | 3.084334 | | 3.144174 | | 3.202946 | | 3.260805 | | 3.317799 | |
| 1.75 | 84 | 11 | 84 | 11 | 85 | 11 | 85 | 11 | 86 | 12 |
| | 3.045995 | | 3.105668 | | 3.164312 | | 3.221977 | | 3.278776 | |
| 2 | 80 | 10 | 80 | 10 | 81 | 10 | 81 | 10 | 82 | 10 |
| | 3.013710 | | 3.073184 | | 3.131557 | | 3.189002 | | 3.245584 | |
| 2.25 | 80 | 9 | 80 | 9 | 80 | 9 | 80 | 9 | 80 | 9 |
| | 2.986484 | | 3.045634 | | 3.103739 | | 3.160914 | | 3.217254 | |

The effect of values values of $\lambda_1$ and $\nu$ on the optimal values is shown in Table 11. The other parameters and costs are assumed to be $R=5$, $\mu=1.75$, $\lambda_0=5$, $K_S=5$, $K_R=1$, $h_S=0.01$, $h_R=0.01$, $\gamma_S=4$, $\gamma_R=2$ and $\beta_R=0.2$. The optimal total cost rate increases with $\lambda_1$ and $\mu$. The value of $s^*$ is almost same for the selected combinations. On the other hand, $S^*$ increases as $\lambda_1$ and $\nu$ increase.

Table 12 gives the total expected cost for various combinations of $K_S$ and $K_R$ where the other parameters and costs are assumed to be fixed. When the ordering cost increase, the expected total cost per unit time increase. When $K_R$ is increased the optimal $S^*$ and $s^*$ remains constant. But when $K_s$ is increased $S^*$ increases but $s^*$ decreases.

Table 10 Effect of $\nu$ and $\lambda_0$ on the optimal cost function for given ($S^*$ and $s^*$) values

| $\lambda_0$ / $\nu$ | 3 | | 4 | | 5 | | 6 | | 7 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.6 | 80 | 9 | 80 | 9 | 83 | 11 | 90 | 13 | 90 | 14 |
| | 2.609740 | | 2.689470 | | 2.783238 | | 2.876093 | | 2.976348 | |
| 1.8 | 80 | 9 | 80 | 9 | 84 | 11 | 90 | 14 | 90 | 14 |
| | 2.799153 | | 2.880094 | | 2.974331 | | 3.067225 | | 3.168474 | |
| 2 | 80 | 9 | 80 | 9 | 85 | 11 | 90 | 13 | 90 | 14 |
| | 2.987540 | | 3.069684 | | 3.164312 | | 3.257304 | | 3.359540 | |
| 2.2 | 80 | 9 | 80 | 9 | 85 | 11 | 90 | 14 | 90 | 14 |
| | 3.175097 | | 3.258435 | | 3.353342 | | 3.446293 | | 3.549744 | |
| 2.4 | 80 | 9 | 80 | 9 | 86 | 12 | 90 | 14 | 90 | 14 |
| | 3.361972 | | 3.446497 | | 3.541499 | | 3.634573 | | 3.739240 | |

Table 11 Change of $\nu$ and $\lambda_1$ on the optimal cost function for given ($S^*$ and $s^*$) values

| $\lambda_1$ / $\nu$ | 3 | | 4 | | 5 | | 6 | | 7 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.6 | 82 | 11 | 83 | 11 | 83 | 11 | 83 | 11 | 83 | 11 |
| | 2.669131 | | 2.726718 | | 2.783238 | | 2.838866 | | 2.893645 | |
| 1.8 | 83 | 11 | 83 | 11 | 84 | 11 | 84 | 11 | 85 | 11 |
| | 2.858011 | | 2.916706 | | 2.974331 | | 3.031025 | | 3.086889 | |
| 2 | 84 | 11 | 84 | 11 | 85 | 11 | 85 | 11 | 86 | 12 |
| | 3.045995 | | 3.105668 | | 3.164312 | | 3.221977 | | 3.278776 | |
| 2.2 | 84 | 11 | 85 | 11 | 85 | 11 | 86 | 12 | 87 | 12 |
| | 3.233225 | | 3.293808 | | 3.353342 | | 3.411841 | | 3.469349 | |
| 2.4 | 85 | 11 | 85 | 11 | 86 | 12 | 87 | 12 | 87 | 12 |
| | 3.419840 | | 3.481263 | | 3.541499 | | 3.600668 | | 3.658985 | |

Table 12 Impact of $K_S$ and $K_R$ on the optimal value for given ($S^*$ and $s^*$) values

$R = 5; \mu = 1.75; \lambda_1 = 2; \nu = 2; \lambda_0 = 5; h_S = 0.01; h_R = 0.01; \gamma_S = 4; \gamma_R = 2; \beta_R = 0.2.$

| $K_R$ / $K_S$ | .6 | | .8 | | 1.0 | | 1.2 | | 1.4 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 80 | 12 | 80 | 12 | 80 | 12 | 80 | 12 | 80 | 10 |
| | 2.485656 | | 2.763562 | | 3.041467 | | 3.319373 | | 3.597279 | |
| 4 | 80 | 12 | 80 | 12 | 80 | 12 | 80 | 12 | 80 | 10 |
| | 2.548276 | | 2.826182 | | 3.104087 | | 3.381993 | | 3.659899 | |
| 5 | 84 | 11 | 84 | 11 | 85 | 11 | 85 | 11 | 85 | 11 |
| | 2.608955 | | 2.886635 | | 3.164312 | | 3.441965 | | 3.719618 | |
| 6 | 90 | 11 | 90 | 11 | 90 | 11 | 90 | 11 | 90 | 11 |
| | 2.664205 | | 2.941736 | | 3.219267 | | 3.496798 | | 3.774330 | |
| 7 | 90 | 11 | 90 | 11 | 90 | 11 | 90 | 11 | 90 | 11 |
| | 2.717132 | | 2.994663 | | 3.272194 | | 3.549726 | | 3.827257 | |

The effect of $h_S$ and $h_R$ on the optimal cost function is shown in Table 13. In this system, if the holding costs are increased, the total expected cost rate as well as $S^*$ and $s^*$ values increase.

Table 14 shows the influence of $\gamma_S$ and $\gamma_R$ on the optimal values. The optimal $S^*$ and $s^*$ and the associated expected total cost function increase along with $V_S$ and $V_R$.

We present the effect of $\beta_R$ and $\gamma_R$ on the optimal cost function in Table 15. In this system, if the perishable rate increases, the total expected cost rate increases. Also, the values of $S^*$ and $s^*$ remain same. The expected total cost rate is more sensitive for the increase of perishable rate than the reorder rate at the retailer node.

Table 13 Effect of $h_S$ and $h_R$ on the optimal values for given ($S^*$ and $s^*$) values
$R = 5; \mu = 1.75; \lambda_0 = 5; \lambda_1 = 2; \nu = 2; K_S = 5; K_R = 1; \gamma_S = 4; \gamma_R = 2; \beta_R = 0.2$

| $h_R$ / $h_S$ | .005 | | .007 | | .009 | | .011 | | .013 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 90 | 12 | 90 | 12 | 90 | 12 | 90 | 12 | 90 | 10 |
| | 2.918610 | | 2.923499 | | 2.928389 | | 2.933278 | | 2.938167 | |
| 4 | 90 | 12 | 90 | 12 | 90 | 12 | 90 | 12 | 90 | 12 |
| | 3.012868 | | 3.017758 | | 3.022647 | | 3.027536 | | 3.032425 | |
| 5 | 89 | 12 | 89 | 12 | 89 | 12 | 89 | 12 | 89 | 12 |
| | 3.107015 | | 3.111904 | | 3.116793 | | 3.121682 | | 3.126571 | |
| 6 | 81 | 11 | 81 | 11 | 81 | 11 | 81 | 11 | 81 | 11 |
| | 3.195341 | | 3.200229 | | 3.205116 | | 3.210003 | | 3.214890 | |
| 7 | 80 | 11 | 80 | 11 | 80 | 11 | 80 | 11 | 80 | 11 |
| | 3.279119 | | 3.284006 | | 3.288893 | | 3.293780 | | 3.298667 | |

Table 14 Effect of $\gamma_S$ and $\gamma_R$ on the optimal values for given ($S^*$ and $s^*$) values
$R = 5; \mu = 1.75; \lambda_0 = 5; \lambda_1 = 2; \nu = 2; h_S = 0.01; h_R = 0.01; K_S = 5; K_R = 1; \beta_R = 0.2$

| $\gamma_R$ / $\gamma_S$ | 1.5 | | 1.7 | | 1.9 | | 2.1 | | 2.3 | | 2.5 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 81 | 9 | 81 | 9 | 81 | 9 | 81 | 9 | 81 | 9 | 81 | 9 |
| | 3.135226 | | 3.135914 | | 3.136601 | | 3.137289 | | 3.137977 | | 3.138664 | |
| 3 | 83 | 10 | 83 | 10 | 83 | 10 | 83 | 10 | 83 | 10 | 83 | 10 |
| | 3.151403 | | 3.151935 | | 3.152468 | | 3.153001 | | 3.153534 | | 3.154067 | |
| 4 | 84 | 11 | 84 | 11 | 85 | 11 | 85 | 11 | 85 | 11 | 85 | 11 |
| | 3.163266 | | 3.163685 | | 3.164104 | | 3.164519 | | 3.164932 | | 3.165344 | |
| 5 | 86 | 12 | 86 | 12 | 86 | 12 | 86 | 12 | 86 | 12 | 86 | 12 |
| | 3.172437 | | 3.172761 | | 3.173086 | | 3.173410 | | 3.173735 | | 3.174059 | |
| 6 | 87 | 13 | 87 | 13 | 87 | 13 | 87 | 13 | 87 | 13 | 87 | 13 |
| | 3.180039 | | 3.180294 | | 3.180549 | | 3.180804 | | 3.181059 | | 3.181314 | |

We find the effect of $K_S$ and $h_S$ on the optimal cost function in Table 16. If the holding and reorder costs increase at the supplier node, the total expected cost rate increases and the values of $S^*$ and $s^*$ decrease.

Table 15 Effect of $\beta_R$ and $\gamma_R$ on the optimal values for given ($S^*$ and $s^*$) values
$R = 5; \mu = 1.75; \lambda_0 = 5; \lambda_1 = 2; \nu = 2; h_S = 0.01; h_R = 0.01; K_S = 5; K_R = 1; \gamma_S = 4$

| $\gamma_R$ / $\beta_R$ | 1.5 | 1.7 | 1.9 | 2.1 | 2.3 | 2.5 |
|---|---|---|---|---|---|---|
| .15 | 84 11 | 84 11 | 85 11 | 85 11 | 85 11 | 85 11 |
| | 2.918887 | 2.919306 | 2.919720 | 2.920133 | 2.920546 | 2.920958 |
| .17 | 84 11 | 84 11 | 85 11 | 85 11 | 85 11 | 85 11 |
| | 3.016639 | 3.017058 | 3.017474 | 3.017887 | 3.018300 | 3.018713 |
| .19 | 84 11 | 84 11 | 84 11 | 85 11 | 85 11 | 85 11 |
| | 3.114391 | 3.114809 | 3.115228 | 3.115642 | 3.116054 | 3.116467 |
| .21 | 84 11 | 84 11 | 84 11 | 85 11 | 85 11 | 85 11 |
| | 3.212142 | 3.212561 | 3.212980 | 3.213396 | 3.213809 | 3.214222 |
| .23 | 84 11 | 84 11 | 84 11 | 85 11 | 85 11 | 85 11 |
| | 3.309894 | 3.310313 | 3.310732 | 3.311150 | 3.311563 | 3.311976 |

Table 16 Effect of $K_S$ and $h_S$ on the optimal values for given ($S^*$ and $s^*$) values
$R = 5; \mu = 1.75; \lambda_0 = 5; \lambda_1 = 2; \nu = 2; K_R = 1; h_R = 0.01; \gamma_S = 4; \gamma_R = 2; \beta_R = 0.2.$

| $h_S$ / $K_S$ | .005 | .007 | .009 | 0.011 | .013 | 0.015 |
|---|---|---|---|---|---|---|
| 3 | 90 14 | 83 13 | 80 13 | 80 12 | 80 12 | 80 12 |
| | 2.821098 | 2.913754 | 2.999247 | 3.083613 | 3.167905 | 3.252197 |
| 4 | 90 13 | 90 12 | 82 12 | 80 12 | 80 11 | 80 11 |
| | 2.876271 | 2.971128 | 2.971128 | 3.146233 | 3.230108 | 3.313740 |
| 5 | 90 12 | 90 12 | 89 12 | 81 11 | 80 11 | 80 11 |
| | 2.930833 | 3.025092 | 3.119238 | 3.207559 | 3.291336 | 3.374969 |
| 6 | 90 12 | 90 11 | 90 11 | 87 11 | 81 10 | 80 10 |
| | 2.984797 | 3.078852 | 3.172462 | 3.265213 | 3.352265 | 3.435259 |
| 7 | 90 11 | 90 11 | 90 11 | 90 10 | 85 10 | 80 10 |
| | 3.038170 | 3.131780 | 3.225389 | 3.318970 | 3.409730 | 3.495149 |

The impact of$K_R$and $h_R$ on the optimal cost function is shown in Table 17. If the holding and reorder costs increase at the retailer node, the total expected cost rate increases and the values of$S^*$ and $s^*$ do not change.

## 9. CONCLUSION

A supply chain model is considered with one supplier who uses packets of a commodity to satisfy demands that may arise from a retailer who sells single units of the same commodity. Since the retailer gets replenishment from the supplier, the supply is instantaneous as long as the supplier has packets in the stock. The supplier replenishes stock by implementing $(s, S)$ ordering policy with random lead time for the delivery of reorders. It is also assumed that items are perishable at the retailer node and non-perishable at the supplier node due to better stocking environments available with the supplier. After obtaining the joint distribution of inventory levels both at the supplier and retailer nodes, the total expected cost per unit time is derived. Some special cases with numerical illustrations are provided. This model can also be applied to the situations involving one seller who stocks and sells the items in packets as well as single items of the commodity.

Table 17 Influence of $K_R$ and $h_R$ on the optimal values for given ($S^*$ and $s^*$) values
$R = 5; \mu = 1.75; \lambda_0 = 5; \lambda_1 = 2; \nu = 2; K_S = 5; h_S = 0.01; \gamma_S = 4; \gamma_R = 2; \beta_R = 0.2$

| $K_R$ / $h_R$ | .6 | | .8 | | 1 | | 1.2 | | 1.4 | |
|---|---|---|---|---|---|---|---|---|---|---|
| .005 | 84 | 11 | 84 | 11 | 85 | 11 | 85 | 11 | 85 | 11 |
| | 2.596737 | | 2.874416 | | 3.152093 | | 3.429746 | | 3.707398 | |
| .007 | 84 | 11 | 84 | 11 | 85 | 11 | 85 | 11 | 85 | 11 |
| | 2.601624 | | 2.879303 | | 3.156981 | | 3.434633 | | 3.712286 | |
| .009 | 84 | 11 | 84 | 11 | 85 | 11 | 85 | 11 | 85 | 11 |
| | 2.606512 | | 2.884191 | | 3.161869 | | 3.439521 | | 3.717174 | |
| .011 | 84 | 11 | 84 | 11 | 85 | 11 | 85 | 11 | 85 | 11 |
| | 2.611399 | | 2.889078 | | 3.166756 | | 3.444409 | | 3.722062 | |
| .013 | 84 | 11 | 84 | 11 | 85 | 11 | 85 | 11 | 85 | 11 |
| | 2.616287 | | 2.893966 | | 3.171644 | | 3.449297 | | 3.726949 | |
| .015 | 84 | 11 | 84 | 11 | 85 | 11 | 85 | 11 | 85 | 11 |
| | 2.621174 | | 2.898854 | | 3.176532 | | 3.454184 | | 3.454184 | |

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