

A Fuzzy Based Mathematical Model for Vendor Selection and Procurement Planning with Multiple Discounts in the Presence of Supply Uncertainty

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ABSTRACT

Supplier selection and material procurement planning are the most important issues in supply chain management. This decision is complicated when the buyers face with discount price schemes. In real situation, each supplier may apply different methods such as different types of discount schedules and various types of payment in order to increase market share. In this situation, buyers try to select the best supplier/s by considering all tangible and intangible factors which may be included in this decision problem. Mohammad Ebrahim and Razmi (2009) introduced a Meta heuristic model in order to select the best suppliers and determine the procurement plan under two types of discount offers. In this paper, a fuzzy bi-objective model is proposed for single item single period supplier selection and purchasing problem under capacity constraint, supply uncertainty and budget limitation. This model includes different kinds of discount (all-unit discount, incremental discount). In addition, different methods for payments which ordinary may be proposed by each supplier are considered in this model. Finally, an interactive fuzzy programming approach (TH method), ϵ -constraint method and reservation level driven Tchebycheff procedure (RLTP) are applied to solve this bi-objective model. The efficiency of each method is evaluated by using an additive utility function which is offered by decision maker.

Keywords: Supplier selection, Price discount, Order lot sizing, Different method of payments, Budget limitation, Supply uncertainty, Fuzzy bi-objective model.

1. INTRODUCTION

Nowadays, due to drastic global competition, manufactures try to improve product quality and decrease the product costs. Therefore, supplier selection and procurement planning are important decision where there is a great competition among suppliers. Supplier selection problem is declared as a multi-objective model, when criteria such as price, quality, supplier's reliability and delivery performances are considered simultaneously. In this circumstance, the trade-off between tangible and intangible criteria in order to make an optimal decision is complicated.

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In real world situation, there are many ways to encourage buyers to procure large quantities of materials. One of them is discount schemes which ordinary are offered by suppliers. Various types of discount schedules (all-unit discount and/or incremental discount) may be offered by each supplier in each period simultaneously, Ebrahim and Razmi (2009). In the same way, different kinds of payment may be offered by suppliers in order to encourage buyers to procure large quantities when they faced with a limited budget to purchase materials in each period. Furthermore, several features of products such as quality, delivery performance, price and capacity may be considered in supplier selection decision simultaneously. Hence, supplier selection and order quantities based on buyer's budget conditions can be marked as a complex multiple criteria decision making problem.

Furthermore, supply uncertainty may be considered as a critical issue for supplier selection decision in risky environment. When the requested materials cannot be delivered in time, production slowdown or shutdown will be occurred for a manufacture, and it results in customer unsatisfaction, higher cost and lower efficiencies.

Due to recent development in globalization, where suppliers are appeared as global players, supply uncertainty is considered as an important issue in global business in new emerging markets. In this market, the vulnerability is increased when supply uncertainty is compounded by additional problems like scarcity of materials, inadequate transportation and distribution infrastructure, and irrational allocation of available supply capacity by local manufactures, Dornier et al. (1998).

Due to reducing supply uncertainty, buyer may increase the number of suppliers, or tries to build close relationship with suppliers or outsourcing partner.

In a real situation most of information is not known precisely in the planning horizon such as market demands, delivery time and cost/time coefficients and therefore assigning a set of crisp values for such ambiguous parameters is not appropriate. Due to incompleteness and/or unavailability of required data over the mid-term decision horizon, critical parameters (such as market demand and capacity levels) are assumed as imprecise in nature.

In this paper, a mathematical model for single-item, multiple suppliers, and single-period purchasing problem is developed under condition of two discount schemes such as all-unit discount and incremental discount simultaneously. In addition different types of payments which may be offered by suppliers can be evaluated by the proposed model. When buyers are faced with limited budget, different kinds of payment schemes, may be encourage them to buy materials from the supplier with the best offer. Also, supply uncertainty is considered as a major issue in this supplier selection problem. Razmi and Maghool (2010), proposed a fuzzy bi-objective model for multiple item, multiple period supplier selection and purchasing problem under capacity constraint and budget limitation. They considered different kinds of discount (all-unit discount, incremental discount and total business volume discount) and different methods of payments. We developed their model to a new model by considering supply uncertainty.

Two major issues are considered in this study: first: Determining an appropriate set of supplier, and second: Determining the procurement quantities placed upon a given set of supplier, which may differ in their reliability levels, capacities, yield rates, discount schemes, kinds of payment and unit cost.

In previous studies, supplier reliability is considered as a random variable in stochastic environment. In this paper, we considered supplier reliability as a fuzzy number due to imprecision

of this parameter. Furthermore, by considering the vagueness of some parameters like market demand, supplier reliability and capacity levels, fuzzy constraints are proposed to instruct this multi-objective model. Two objective functions of proposed model are: minimizing the cost of purchasing item and maximizing the total value of purchasing item. The total value of purchasing takes into account intangible factors in purchasing decision. Ultimately, Fuzzy mathematical programming with ambiguous coefficients in objective functions and constraints which is called possibilistic programming is proposed to address this problem.

The augmented ϵ -constraint method, reservation level Tchebycheff procedure method and an interactive fuzzy programming approach (TH method) are applied for solving this model. Finally, the preference of each method is tested by applying an additive weighted utility function which is proposed by decision maker. This utility function is used to compute the satisfaction level of decision makers. In this article, we try to stimulate more conditions in proposed model which are consistent with real situation, such as considering different kinds of payment and discount schedules in supplier selection problem. Furthermore, we considered demand, supply uncertainty and supplier's capacity in fuzzy environment in order to model the situation next.

The remainder of this paper is organized as follows. In section 2, some previous studies and relevant literature are reviewed. In section 3, proposed model is presented and the algorithms of three proposed method for solving this problem are declared in section 4. In section 5, the result of a numerical example is presented and finally conclusions and future research directions are outlined in section 6.

2. LITERATURE REVIEW

Supplier selection problem under condition of different types of discount schemes can be formulated as a single objective or multi objective programming model. Purchasing cost can be considered as a major objective function in single objective models. Benton (1991) developed a nonlinear program for supplier selection problem with the object of minimizing the sum of purchasing cost, inventory costs and order costs, under condition of multiple items, resource limitation and quantity discount. A heuristic procedure using Lagrangian relaxation is applied for solving this problem. The objective of the model is to minimize aggregate price by considering both all-unit and incremental discounts. Rosenthal et al. (1995) studied supplier selection problem with bundling. They developed a mixed integer programming model under condition of limited capacity, different quality and delivery performances. The discounted prices are offered for bundled products for this problem. Burke et al. (2008) considered the optimal sourcing policy for a single buyer under a variety of supplier pricing schemes including linear discounts, incremental unit discounts, and all unit discounts and supplier capacity limitations. In their paper, a heuristic solution methodology is developed to identify a quantity allocation decision for the buyer. Recently, Razmi et al. (2008) considered supplier evaluation and order allocation problem. They applied a fuzzy TOPSIS model with a combination of two validated coefficients to evaluate suppliers. Besides, a fuzzy integer programming was formulated to assign optimal quantity of order to allocated supplier.

In real situation, many different criteria, such as quality and delivery performance under various types of discount schemes can be considered in supplier selection problem as a multi objective programming model.

Mohammad Ebrahim and Razmi (2009) considered a mathematical model for supplier selection and single item purchasing problem by considering different discount schemes (all-unit discount, incremental discount, and total business volume discount) simultaneously. They proposed a Scatter

Search algorithm to solve this problem. Dahel (2003) considered multi-objective supplier selection problem in a multiple product, multiple supplier competitive sourcing environment under capacity constraint. In this context, the vendors offer discounts on total amount of sales volumes. Price, delivery, and quality are considered as main criteria for purchasing decision in this model. Xia and Wu (2007) proposed an integration of analytical hierarchy process improved by rough set theory and multi-objective mixed integer programming to determine the number of suppliers to employ and the optimal order quantity which is allocated to these suppliers. The proposed model formulated in multiple sourcing environment, multiple products with multiple criteria and by considering supplier capacity constraint where each supplier offer price discount under total business volume policy. Ravindran and Wadhwa (2007) considered the vendor selection problem as a multi-objective optimization problem, where each buyer orders multiple products from different vendors in a multiple sourcing network. Three conflicting criteria such as price, lead –time and quality are defined for the problem. A pricing model under quantity discounts is used to represent the purchasing cost. They applied several methods such as weighted objective goal programming, and compromise programming for solving this vendor selection problem.

In many previous studies, supplier selection and purchasing problem was considered within long-term framework. Chang (2006) addressed the single-item, multi-supplier, multi-period problem with real world constraint by proposing an exact acquisition policy. The objective function of this problem is minimizing the periodic purchasing cost, ordering cost, and holding cost. Also, a variety of price quantity discount policies which is proposed by suppliers is considered in the paper. Tempelmeier (2001) proposed a simple heuristic for a supplier selection and purchase order sizing for a single item problem in dynamic demand environment offering all-unit or incremental quantity discounts. The object of this model is minimizing the ordering cost of purchasing item and holding cost. Stadtler (2006) considered a supplier selection problem by introducing a linear mixed integer programming model which represents both all-unit discount and incremental discount case. The objective of this model is to minimize the purchase cost of items bought at specific discount rates plus the fixed cost of ordering and delivery. For all cost components constant interest has been considered.

Recently, supplier selection problem has been considered in imprecise environment. Amid et al. (2006) developed a fuzzy multi-objective linear model to consider the vagueness of the information relates to the goals, constraints and parameters in supplier selection problem. They applied an asymmetric fuzzy-decision making technique to assign different weights to various criteria. Amid et al. (2008) considered a supplier selection problem under price breaks. They developed a fuzzy weighted additive and multi-objective mixed integer programming to determine the order quantities in imprecise environment. The model includes three objective functions: minimizing the net cost, minimizing the net rejected items and minimizing the net late deliveries under condition of capacity and demand requirement constraints. In order to solve the problem, the proposed model aggregates weighted membership functions of objectives to construct the relevant decision functions, in which objective have different relative importance. Hassanzadeh Amin and Razmi (2009) considered an internet service provider (ISP) selection, evaluation and development problem. They composed two proposed model in order to evaluate the best ISP based on qualitative criteria and quantitative metrics. Besides, they applied a novel algorithm by using fuzzy logic and triangular fuzzy numbers to evaluate selected ISPs.

Burke et al. (2004) considered supplier reliability on a firm's sourcing decisions in an environment with uniformly distributed demand. They implied conditions for choosing a single versus multiple sourcing strategies. They indicated that there are interactions between demand uncertainty and supply uncertainty. They showed when the mean demand is high, the firm diversifies its orders

amongst a subset of supplier, even if single low cost supplier can handle the complete demand volume and when demand uncertainty is low, diversifying firm's total orders amongst several suppliers is optimal. Wu et al. (2009) proposed a fuzzy multi-objective programming model by considering multiple criteria and risk factors for supplier selection problem. They applied a possibility approach for this fuzzy multi-objective programming model. They taken into account both quantitative risk factor, such as cost, quality, and logistics and qualitative risk factor which include economic environmental factors and vendor ratings using fuzzy data in their model. The proposed model can be applied as an appropriate tool to evaluate suppliers by considering cost, quality acceptance level, on-time delivery and risk factors simultaneously in an uncertain global market.

Determining the rational level of supply base is critical issue under condition of supplier unreliability. Sarkar and Mohapatra (2008) considered the problem of determining the optimal size of supply base under condition of supply risk due to unforeseen events such as earthquake, tsunami, flood, strikes, fire and terrorist attacks. Yang et al. (2004) developed two solution methodologies for supplier selection problem in doubled-layered supply chain. In their study, a buyer faces a single period demand for its product and multiple unreliable suppliers. They extended the newsvendor model to analyze this problem under condition of supply uncertainty. The efficiency of proposed two algorithms was tested in their paper.

All previous study under condition of supply uncertainty in supplier selection problem can be classified into random yields or variable capacities (Wang & Gerchak, 1996). Karlin (1958) proposed analytical models for applying uncertainty in the form of random yields. Parlar and Wang (1993) considered the optimal sourcing policy from two suppliers under condition of supply uncertainty. Yang et al (2007) proposed a solution method based on combination of the active set method and the Newton search procedure for supplier selection problem. In their study, a buyer faces with random demand which should be satisfied from a set of unreliable suppliers with different yields and prices. Furthermore, the efficiency of proposed models is tested in their study.

In all previous studies, some aspects of vendor selection problem have been considered and it seems that proposing a model for vendor selection by considering all aspects of this problem simultaneously, is essential. In practical situation, there is not a comprehensive model for vendor selection which considers all aspects such as supplier reliability, various types of discount schemes and different kind of payment simultaneously under condition of capacity constraint and budget limitation.

This paper introduces an integrated bi objective model for supplier selection and lot-sizing planning under different types of discount schemes and method of payments by considering resource limitations in fuzzy environment. It seems that proposed model can be applied as a decision support system in practical situation.

3. PROPOSED MODEL

In this paper, a new model formulation is developed for supplier selection and order sizing problem under dynamic demand condition, supply uncertainty and capacity constraint by considering different types of discount policies and different kinds of delay in payments with limited budget. The proposed model considers two objectives including minimizing the purchasing cost and maximizing the total value of purchasing which considers the impact of qualitative (intangible) performance criteria in purchasing decisions (such as after sale services, delivery performance,

business structure and technical capabilities of the suppliers) (Torabi & Hassini, 2008; Xia & Wu, 2007), subject to the following assumption:

- The demand is fuzzy possibilistic number;
- The purchased quantities are integer;
- The buyer is faced with limited budget in purchasing item;
- The buyer is faced with a set of independent unreliable suppliers. In this context, unreliable means that the marginal amount supplied (i.e., delivered) by a given supplier is less than or equal to the marginal amount ordered from the supplier;
- There are several suppliers, each one may offer time varying deterministic prices and time varying quantity discount schemes (all-unit and/or incremental discounts) are offered by each supplier;
- The specific delivery periods are proposed by each supplier;
- The specific delivery lead –times are offered by each supplier;
- Each supplier may offer different types of payment. Three kinds of payment are considered in this model;
 1. The total amount of purchased items is paid at the order point;
 2. The percentage of the total amount of purchased items is paid at the order point and the remaining amount is paid at the delivery point;
 3. The percentage of the total amount of purchased items is paid at the delivery point and the remaining amount is paid at the next months after delivery point (This delay of payment is determined with each supplier in each period);
- The imprecise capacity levels are considered for each supplier.

3.1. Notations

h	inventory holding cost per monetary unit per unit time
i	Index of suppliers
k	Index of discount intervals
x_{ik}	Purchased quantity from supplier i in discount interval k .
sx_i	Summation of purchased quantity from supplier i .
MI_i	A constant coefficient representing a known upper bound for sx_i
e_i	An indicator variable to distinguish between the state where $sx_i=0$ and the state where $sx_i>0$
y_{ik}	Binary variables; if the purchased quantity from supplier i falls on the interval k Corresponding to this variable then $y_{ik} = 1$, otherwise $y_{ik} = 0$.
S	Total number of suppliers.
n_1	Number of suppliers which offer all-unit.
m_{ab}	Number of suppliers which offer the discount scheme with type a ($a \in \{1,2\}$ where 1 is related with all unit discount, 2 is related with incremental discount) and followed the b^{th} method of payment. $b \in \{1,2,3\}$
l_{ik}	Lower bound of the discount interval k offered by supplier i .
u_{ik}	Upper bound of the discount interval k offered by supplier i .

- p_{ik} Discounted unit price of the discount interval k offered by supplier i .
- k_i Index of the last interval offered by supplier i .
- d Net requirements
- r_i The percentage of payment which is paid to supplier i .
- le_i The lead time of part which is supplied by supplier i
- θ_i The delay of payment which is declared by supplier i
- ir The interest rate which is used for payment
- B The total budget of buyer
- pt_i unit capacity requirement at supplier i
- c_i The capacity of supplier i
- f_i The reliability of supplier i
- TVP Total value of purchasing
- sc_i Overall score of supplier i considering qualitative performance factors.
- ρ The parameter which is used in RLTP method
- β The minimum acceptable degree of feasibility for fuzzy constraint
- ξ The scalar which is used in augmented ϵ -constraint method
- RL The reservation level which is used in RLTP method

3.2. Objective functions

Two objective functions are considered in proposed model. Purchasing cost is an important practical objective function which is used as the first objective function in this model. The total value of purchasing is the second objective function which is applied as a qualitative objective function in proposed model.

Objective function 1: minimizing the total purchasing cost

$$\begin{aligned}
 & \left(\begin{aligned}
 & - \sum_{i=1}^{n_1} \sum_{k=1}^{k_i} p_{ik} \tilde{f}_i x_{ik} + \sum_{i=m_{11}+1}^{m_{12}} \sum_{k=1}^{k_i} p_{ik} \tilde{f}_i x_{ik} (1-r_i)(1+ir)^{le_i} \\
 & + \sum_{i=m_{12}+1}^{m_{13}} \sum_{k=1}^{k_i} p_{ik} \tilde{f}_i x_{ik} r_i (1+ir)^{le_i} + \sum_{i=m_{12}+1}^{m_{13}} \sum_{k=1}^{k_i} p_{ik} \tilde{f}_i x_{ik} (1-r_i)(1+ir)^{\theta_i+le_i} \\
 & - \sum_{i=n_1+1}^S \sum_{k=1}^{k_i} [p_{ik} (\tilde{f}_i x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] \\
 & + \sum_{i=m_{21}+1}^{m_{22}} \sum_{k=1}^{k_i} [p_{ik} (\tilde{f}_i x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] (1-r_i)(1+ir)^{le_i} \\
 & + \sum_{i=m_{22}+1}^{m_{23}} \sum_{k=1}^{k_i} [p_{ik} (\tilde{f}_i x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] r_i (1+ir)^{le_i} \\
 & + \sum_{i=m_{22}+1}^{m_{23}} \sum_{k=1}^{k_i} [p_{ik} (\tilde{f}_i x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] (1-r_i)(1+ir)^{\theta_i+le_i}
 \end{aligned} \right) \tag{1}
 \end{aligned}$$

The cost objective function consists of two major parts. The first part which is indicated by formula (1.1) is related to suppliers who offer all-unit discount. The first term of this part shows the payment at the order point. The remaining terms show delay of payments.

$$\begin{aligned}
& - \sum_{i=1}^{n_1} \sum_{k=1}^{k_i} p_{ik} \tilde{f}_i x_{ik} \\
& + \sum_{i=m_{11}+1}^{m_{12}} \sum_{k=1}^{k_i} p_{ik} \tilde{f}_i x_{ik} (1 - r_i)(1 + ir)^{le_i} \\
& + \sum_{i=m_{12}+1}^{m_{13}} \sum_{k=1}^{k_i} p_{ik} \tilde{f}_i x_{ik} r_i (1 + ir)^{le_i} \\
& + \sum_{i=m_{12}+1}^{m_{13}} \sum_{k=1}^{k_i} p_{ik} \tilde{f}_i x_{ik} (1 - r_i)(1 + ir)^{\theta_i + le_i}
\end{aligned} \tag{1.1}$$

Three types of payment were considered in this paper. The first part of formula (1.1), shows the total payment at order point. In the second type of payment, the percentage of the total amount of purchased items (r_i) is paid at the order point and the remaining amount ($1-r_i$) is paid at the delivery point. The second part of the formula (1.1), shows the future value of investing the remaining amount which is not paid at the order point with interest rate (ir) and period le_i .

In the third type of payment, the percentage of the total amount of purchased items is paid at the delivery point and the remaining amount is paid at the next months after delivery point. The third part of formula (1.1), shows the future value of investing the percentage of the total amount of purchased items (r_i) at the delivery point with interest rate (ir) and period le_i . The fourth part of formula (1.1), shows the future value of investing the remaining amount of purchased item ($1-r_i$) at the next month after delivery point with interest rate (ir) and period (θ_i+le_i) in which θ_i is the period of time after delivery point which is determined by each supplier for payment.

The second part of cost function which is indicated by formula (1.2) is related to suppliers who offer incremental discount. The first term of this part shows the payment at the order point. The remaining terms show the delay of payments.

$$\begin{aligned}
& - \sum_{i=n_1+1}^S \sum_{k=1}^{k_i} [p_{ik} (\tilde{f}_i x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] \\
& + \sum_{i=m_{21}+1}^{m_{22}} \sum_{k=1}^{k_i} [p_{ik} (\tilde{f}_i x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] (1 - r_i)(1 + ir)^{le_i} \\
& + \sum_{i=m_{22}+1}^{m_{23}} \sum_{k=1}^{k_i} [p_{ik} (\tilde{f}_i x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] r_i (1 + ir)^{le_i} \\
& + \sum_{i=m_{22}+1}^{m_{23}} \sum_{k=1}^{k_i} [p_{ik} (\tilde{f}_i x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] (1 - r_i)(1 + ir)^{\theta_i + le_i}
\end{aligned} \tag{1.2}$$

Objective function 2: maximizing the total value of purchasing (TVP). Total value of purchasing takes into account the impact of qualitative performance criteria in purchasing decision.

$$Max \left(\sum_{i \in S_1} sc_i \sum_{k=1}^{k_i} \tilde{f}_i x_{ik} \right) \quad (2)$$

Subject to:

Budget limitation:

$$\left(\begin{aligned} & \sum_{i=1}^{m_{11}} \sum_{k=1}^{k_i} p_{ik} \tilde{f}_i x_{ik} + \sum_{i=m_{11}+1}^{m_{12}} \sum_{k=1}^{k_i} p_{ik} \tilde{f}_i x_{ik} r_i \\ & + \sum_{i=n_1+1}^{m_{21}} \sum_{k=1}^{k_i} [p_{ik} (\tilde{f}_i x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] \\ & + \sum_{i=m_{21}+1}^{m_{22}} \sum_{k=1}^{k_i} [p_{ik} (\tilde{f}_i x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] r_i \end{aligned} \right) \leq B \quad (3)$$

Definition of order sizes:

$$\tilde{d} = \sum_{i \in S} \sum_{k=1}^{k_i} \tilde{f}_i x_{ik} \quad (4)$$

Definition of discount levels for the selected delivery period which is declared the lower and upper bound of discount level:

$$l_{ik} y_{ik} \leq \tilde{f}_i x_{ik} \leq u_{ik} y_{ik} \quad \forall i \in S \quad k = 1, \dots, k_i \quad (5)$$

$$\sum_{k=1}^{k_i} y_{ik} \leq 1 \quad \forall i \in S \quad (6)$$

$$x_{ik} \in Z^+ \cup \{0\} \quad \forall i \in S \quad k = 1, \dots, k_i \quad (7)$$

$$y_{ik} \in \{0,1\} \quad \forall i \in S \quad k = 1, \dots, k_i \quad (8)$$

Capacity constraint:

$$\sum_{k=1}^{k_i} p \tilde{t}_i x_{ik} \leq \tilde{C}_i \quad \forall i \in S \quad (9)$$

Constraints for determining the number of selected supplier:

$$\sum_{k=1}^{k_i} x_{ik} = sx_i \quad \forall i \in S \quad (10)$$

$$sx_i - M1_i e_i < 0 \quad \forall i \in S \quad (11)$$

$$sx_i \in Z^+ \cup \{0\} \quad \forall i \in S \quad (12)$$

$$e_i \in \{0,1\} \quad \forall i \in S \quad (13)$$

$$\sum_{i \in S} e_i \geq 2 \quad (14)$$

4. SOLUTION METHODOLOGY

The solution methodology which is applied to solve the proposed multiple objective possibilistic mixed integer linear programming model consist of two stages. In the first stage, the proposed multiple objective possibilistic mixed integer linear programming model is converted into auxiliary crisp multiple objective mixed integer linear programming model. In the second stage, the equivalent auxiliary crisp model can be solved by selecting appropriate multi objective programming methods. In this paper, an interactive fuzzy programming approach (TH method), the augmented ε -constraint method and the reservation level driven Tchebycheff procedure (RLTP) method, are applied to solve this multiple objective mixed integer programming model (Ustun and Aktar Demirtas, 2008; Haimes et al., 1971; Torabi and Hassini, 2008). There are various methods for solving multi objective programming. But based on Ustun and Aktar Demirtas (2008), some of these methods such as weighted sum method are not appropriate to find every non-dominated solution. Tchebycheff metric-based scalarization method can be applied as an efficient method for finding supported and unsupported nond-ominated solutions for multi objective programming with a non-convex feasible region. Unsupported non-dominated are solutions which dominated by convex combination of other non-dominated solutions. At the first stage, fuzzy constraints are converted into crisp constraint and fuzzy objective functions are treated.

Treating fuzzy constraint:

For constraint which has fuzzy parameters at the right hand side, the weighted average method (Lai & Hwang, 1992; Liang, 2006; Wang & Liang, 2005) is applied to defuzzify the imprecise parameter into a crisp number. Therefore, by considering the minimum acceptable degree of feasibility (β), the equivalent auxiliary crisp constraint can be obtained. After applying this method, the demand constraint can be represented as follows:

$$\tilde{f}_i = (f_i^p, f_i^m, f_i^o) \quad (15)$$

$$l_{ik} y_{ik} \leq (w_1 f_i^{p\beta} + w_2 f_i^{m\beta} + w_3 f_i^{o\beta}) x_{ik} \quad \forall i \in S \quad k = 1, \dots, k_i \quad (16)$$

$$w_1 = w_3 = \frac{1}{6}, w_2 = \frac{4}{6}$$

$$u_{ik} y_{ik} \geq (w_1 f_i^{p\beta} + w_2 f_i^{m\beta} + w_3 f_i^{o\beta}) x_{ik} \quad \forall i \in S \quad k = 1, \dots, k_i \quad (17)$$

$$w_1 = w_3 = \frac{1}{6}, w_2 = \frac{4}{6}$$

Where $w_1 + w_2 + w_3 = 1$, and w_1, w_2, w_3 , represent the weights of the most pessimistic, the most possible, and most optimistic value of the fuzzy demand respectively. The values of weights and β parameter are determined by decision maker. According to previous studies (Torabi & Hassini, 2008), the β parameter is set with 0.5 and the values of w_1, w_3 and w_2 are set with 1/6, 1/6 and 4/6 respectively. After applying the above mentioned method, the budget constraint can be represented as follows:

$$\begin{aligned}
 & \sum_{i=1}^{m_{11}} \sum_{k=1}^{k_i} p_{ik} (w_1 f_i^{p\beta} + w_2 f_i^{m\beta} + w_3 f_i^{o\beta}) x_{ik} + \sum_{i=m_{11}+1}^{m_{12}} \sum_{k=1}^{k_i} p_{ik} (w_1 f_i^{p\beta} + w_2 f_i^{m\beta} + w_3 f_i^{o\beta}) x_{ik} r_i \\
 & + \sum_{i=n_1+1}^{m_{21}} \sum_{k=1}^{k_i} [p_{ik} ((w_1 f_i^{p\beta} + w_2 f_i^{m\beta} + w_3 f_i^{o\beta}) x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] \\
 & + \sum_{i=m_{21}+1}^{m_{22}} \sum_{k=1}^{k_i} [p_{ik} ((w_1 f_i^{p\beta} + w_2 f_i^{m\beta} + w_3 f_i^{o\beta}) x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] r_i \leq B \tag{18} \\
 & w_1 + w_2 + w_3 = 1 \\
 & w_1 = w_3 = \frac{1}{6}, w_2 = \frac{4}{6}
 \end{aligned}$$

For constraints which have fuzzy parameters in the left hand side and right hand side, the fuzzy ranking concept which is proposed by (Lai & Hwang, 1992, 1994) is applied to convert these constraints into equivalent auxiliary crisp constraint. The capacity constraint can be represented as follows:

$$\tilde{p}t_i^j = (pt_i^{jp}, pt_i^{jm}, pt_i^{jo}) \tag{19}$$

$$\tilde{c}_i^j = (c_i^{jp}, c_i^{jm}, c_i^{jo}) \tag{20}$$

$$\sum_{k=1}^{k_i} \tilde{p}t_i^{p\beta} x_{ik} \leq \tilde{c}_i^{p\beta} \quad \forall i \in S \tag{21}$$

$$\sum_{k=1}^{k_i} \tilde{p}t_i^{m\beta} x_{ik} \leq \tilde{c}_i^{m\beta} \quad \forall i \in S \tag{22}$$

$$\sum_{k=1}^{k_i} \tilde{p}t_i^{o\beta} x_{ik} \leq \tilde{c}_i^{o\beta} \quad \forall i \in S \tag{23}$$

For treatment of fuzzy constraint with equality relation between two fuzzy numbers, Inuiguchi (2005) proposed an approach based on fuzzy goal G_i with linear membership function in flexible programming. The membership function which is considered for fuzzy goal G_i is as follows:

$$\mu_{G_i}(r) = \text{Max} \left(0, \text{Min} \left(1, \frac{b_i + k^+ - r}{k^+}, \frac{r - b_i + k^-}{k^-} \right) \right) \tag{24}$$

Where $k^+, k^- > 0$ show excess and shortage tolerances, given a satisfaction degree h_i , the constraint $a_i^T x \cong b_i$ with fuzzy goal G_i is treated by

$$\mu_{G_i}(a_i^T x) \geq h_i \Leftrightarrow a_i^T x \leq b_i + (1 - h_i)k^+ \quad \text{and} \quad a_i^T x \geq b_i - (1 - h_i)k^- \tag{25}$$

Therefore the mentioned constraint is treated as follows (Delgado et al, 1989):

$$\begin{aligned} \tilde{a}_i^T x &\cong_i \tilde{b}_i \\ \tilde{a}_i^T x &\leq \tilde{b}_i + (1 - h_i)\tilde{k}^+ \quad \text{and} \quad \tilde{a}_i^T x \geq \tilde{b}_i - (1 - h_i)\tilde{k}^- \end{aligned} \quad (26)$$

Where $h_i \in (0, 1]$ is a predetermined value and fuzzy numbers k^+ and k^- have membership function as follows:

$$\mu_{\tilde{k}^+}(r) = \mu_{\tilde{k}^-}(r) = 0 \quad \text{for all } r < 0 \quad (27)$$

By this way, the fuzzy equality relation between fuzzy numbers $a_i^T x$ and b_i is reduced to two inequality relations between fuzzy numbers.

$$\sum_{i \in S} \sum_{k=1}^{k_i} \tilde{f}_i x_{ik} \leq \tilde{d} + (1 - h)\tilde{k}^+ \quad (28)$$

$$\sum_{i \in S} \sum_{k=1}^{k_i} \tilde{f}_i x_{ik} \geq \tilde{d} - (1 - h)\tilde{k}^- \quad (29)$$

$$\tilde{d} = (\tilde{d}^p, \tilde{d}^m, \tilde{d}^o) \quad (30)$$

$$\tilde{k}^+ = (\tilde{k}^{+p}, \tilde{k}^{+m}, \tilde{k}^{+o}) \quad (31)$$

$$\tilde{k}^- = (\tilde{k}^{-p}, \tilde{k}^{-m}, \tilde{k}^{-o}) \quad (32)$$

$$\sum_{i \in S} \sum_{k=1}^{k_i} f_i^{p\beta} x_{ik} \leq d^{p\beta} + (1 - h)k^{+p\beta} \quad (33)$$

$$\sum_{i \in S} \sum_{k=1}^{k_i} f_i^{m\beta} x_{ik} \leq d^{m\beta} + (1 - h)k^{+m\beta} \quad (34)$$

$$\sum_{i \in S} \sum_{k=1}^{k_i} f_i^{o\beta} x_{ik} \leq d^{o\beta} + (1 - h)k^{+o\beta} \quad (35)$$

$$\sum_{i \in S} \sum_{k=1}^{k_i} f_i^{p\beta} x_{ik} \geq d^{p\beta} - (1 - h)k^{-p\beta} \quad (36)$$

$$\sum_{i \in S} \sum_{k=1}^{k_i} f_i^{m\beta} x_{ik} \geq d^{m\beta} - (1 - h)k^{-m\beta} \quad (37)$$

$$\sum_{i \in S} \sum_{k=1}^{k_i} f_i^{o\beta} x_{ik} \geq d^{o\beta} - (1 - h)k^{-o\beta} \quad (38)$$

Treating the fuzzy objective functions:

Due to treating the imprecise objective functions, we applied Lai and Hwang method (Lai & Hwang, 1992, 1994). By considering triangular possibility distribution for parameters in objective functions, the objective functions would have a triangular possibility distribution as well. Therefore,

two objective functions can be defined three important points. By instance, profit objective function (profit^p,0), (profit^m,1) and (profit^o,0). Hence, due to maximizing the imprecise profit objective function or (minimizing the imprecise cost objective function) needs maximizing profit^p,profit^m and profit^o simultaneously. Due to Lia and Hwang's approach (Lai & Hwang, 1992, 1994), we should minimize (profit^m-profit^p), maximize (profit^m) and maximize (profit^o-profit^m) instead of maximizing profit^p, profit^m and profit^o simultaneously. The two objective functions are converted into six objective functions by applying Lai and Hwang's method which are indicated by formulas 39, 40, 41, 42, 43, 44.

$$z_1 = profit^p = Min \left(\begin{aligned} & - \sum_{i=1}^{n_1} \sum_{k=1}^{k_i} p_{ik} (f_i^m - f_i^p) x_{ik} + \sum_{i=m_{11}+1}^{m_{12}} \sum_{k=1}^{k_i} p_{ik} (f_i^m - f_i^p) x_{ik} (1 - r_i) (1 + ir)^{l_i} \\ & + \sum_{i=m_{12}+1}^{m_{13}} \sum_{k=1}^{k_i} p_{ik} (f_i^m - f_i^p) x_{ik} r_i (1 + ir)^{l_i} \\ & + \sum_{i=m_{12}+1}^{m_{13}} \sum_{k=1}^{k_i} p_{ik} (f_i^m - f_i^p) x_{ik} (1 - r_i) (1 + ir)^{\theta_i + l_i} \\ & - \sum_{i=n_1+1}^S \sum_{k=1}^{k_i} [p_{ik} (f_i^m - f_i^p) x_{ik} - y_{ik} u_{i,k-1}] + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1}) \\ & + \sum_{i=m_{21}+1}^{m_{22}} \sum_{k=1}^{k_i} [p_{ik} ((f_i^m - f_i^p) x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] (1 - r_i) (1 + ir)^{l_i} \\ & + \sum_{i=m_{22}+1}^{m_{23}} \sum_{k=1}^{k_i} [p_{ik} ((f_i^m - f_i^p) x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] r_i (1 + ir)^{l_i} \\ & + \sum_{i=m_{22}+1}^{m_{23}} \sum_{k=1}^{k_i} [p_{ik} ((f_i^m - f_i^p) x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] (1 - r_i) (1 + ir)^{\theta_i + l_i} \end{aligned} \right) \tag{39}$$

$$z_2 = profit^m = Max \left(\begin{aligned} & - \sum_{i=1}^{n_1} \sum_{k=1}^{k_i} p_{ik} (f_i^m) x_{ik} + \sum_{i=m_{11}+1}^{m_{12}} \sum_{k=1}^{k_i} p_{ik} (f_i^m) x_{ik} (1 - r_i) (1 + ir)^{l_i} \\ & + \sum_{i=m_{12}+1}^{m_{13}} \sum_{k=1}^{k_i} p_{ik} (f_i^m) x_{ik} r_i (1 + ir)^{l_i} \\ & + \sum_{i=m_{12}+1}^{m_{13}} \sum_{k=1}^{k_i} p_{ik} (f_i^m) x_{ik} (1 - r_i) (1 + ir)^{\theta_i + l_i} \\ & - \sum_{i=n_1+1}^S \sum_{k=1}^{k_i} [p_{ik} (f_i^m) x_{ik} - y_{ik} u_{i,k-1}] + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1}) \\ & + \sum_{i=m_{21}+1}^{m_{22}} \sum_{k=1}^{k_i} [p_{ik} ((f_i^m) x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] (1 - r_i) (1 + ir)^{l_i} \\ & + \sum_{i=m_{22}+1}^{m_{23}} \sum_{k=1}^{k_i} [p_{ik} ((f_i^m) x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] r_i (1 + ir)^{l_i} \\ & + \sum_{i=m_{22}+1}^{m_{23}} \sum_{k=1}^{k_i} [p_{ik} ((f_i^m) x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] (1 - r_i) (1 + ir)^{\theta_i + l_i} \end{aligned} \right) \tag{40}$$

$$z_3 = profit^O = Max \left(\begin{aligned} & - \sum_{i=1}^{n_1} \sum_{k=1}^{k_i} p_{ik} (f_i^o - f_i^m) x_{ik} + \sum_{i=m_{11}+1}^{m_{12}} \sum_{k=1}^{k_i} p_{ik} (f_i^o - f_i^m) x_{ik} (1-r_i)(1+ir)^{le_i} \\ & + \sum_{i=m_{12}+1}^{m_{13}} \sum_{k=1}^{k_i} p_{ik} (f_i^o - f_i^m) x_{ik} r_i (1+ir)^{le_i} \\ & + \sum_{i=m_{12}+1}^{m_{13}} \sum_{k=1}^{k_i} p_{ik} (f_i^o - f_i^m) x_{ik} (1-r_i)(1+ir)^{\theta_i+le_i} \\ & - \sum_{i=n_1+1}^S \sum_{k=1}^{k_i} [p_{ik} (f_i^o - f_i^m) x_{ik} - y_{ik} u_{i,k-1}] + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1}) \\ & + \sum_{i=m_{21}+1}^{m_{22}} \sum_{k=1}^{k_i} [p_{ik} ((f_i^o - f_i^m) x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] (1-r_i)(1+ir)^{le_i} \\ & + \sum_{i=m_{22}+1}^{m_{23}} \sum_{k=1}^{k_i} [p_{ik} ((f_i^o - f_i^m) x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] r_i (1+ir)^{le_i} \\ & + \sum_{i=m_{22}+1}^{m_{23}} \sum_{k=1}^{k_i} [p_{ik} ((f_i^o - f_i^m) x_{ik} - y_{ik} u_{i,k-1}) + y_{ik} \sum_{h=1}^{k-1} p_{ih} (u_{ih} - u_{ih-1})] (1-r_i)(1+ir)^{\theta_i+le_i} \end{aligned} \right) \quad (41)$$

$$z_4 = TVP^P = Min \sum_{i \in S} sc_i \sum_{k=1}^{k_i} (f_i^m - f_i^P) x_{ik} \quad (42)$$

$$z_5 = TVP^m = Max \sum_{i \in S} sc_i \sum_{k=1}^{k_i} (f_i^m) x_{ik} \quad (43)$$

$$z_6 = TVP^o = Max \sum_{i \in S} sc_i \sum_{k=1}^{k_i} (f_i^P - f_i^m) x_{ik} \quad (44)$$

At the second stage, an interactive fuzzy programming approach, the augmented ε -constraint method and reservation level driven Tchebycheff procedure are applied to solve the multiple objective mixed integer programming model.

4.1. An interactive fuzzy programming approach (TH method)

Torabi and Hassini's method (TH method) is applied to solve the proposed multiple objective mixed integer programming model. This method is a hybridization of Lai and Hwang's method (LH method) and modified Werner's approach (MW method). Their procedure to solve multiple objective possibilistic mixed integer linear programming model is as follows (Torabi & Hassini, 2008):

Step1: An appropriate triangular possibility distribution is determined for the imprecise parameters in the model.

Step2: convert two fuzzy objective functions (profit) and (TVP) into six equivalent crisp objectives.

Step3: convert fuzzy constraints into corresponding crisp ones, determine the minimum acceptable possibility level for an imprecise parameter, β . In this step, the equivalent crisp model is built.

Step4: determine the positive ideal solution (PIS) and negative ideal solution (NIS) for each objective function by solving the corresponding mixed integer linear programming as follows:

$$\begin{aligned}
 z_1^{PIS} &= \min(profit^m - profit^p) & z_1^{NIS} &= \max(profit^m - profit^p) \\
 z_2^{PIS} &= \max(profit^m) & z_2^{NIS} &= \min(profit^m) \\
 z_3^{PIS} &= \max(profit^o - profit^m) & z_3^{NIS} &= \min(profit^o - profit^m) \\
 z_4^{PIS} &= \min(TVP^m - TVP^p) & z_4^{NIS} &= \max(TVP^m - TVP^p) \\
 z_5^{PIS} &= \max(TVP^m) & z_5^{NIS} &= \min(TVP^m) \\
 z_6^{PIS} &= \max(TVP^o - TVP^m) & z_6^{NIS} &= \min(TVP^o - TVP^m)
 \end{aligned} \tag{45}$$

subject to

$$x \in f(x)$$

Due to reduce the difficulty of solving twelve mixed integer linear programming models to determine ideal solutions, Hassini and Torabi (2008), proposed the following heuristic rules:

1. The positive ideal solution is obtained by solving the corresponding mixed integer linear programming heuristically until a satisfactory feasible integer solution is obtained.
2. The negative ideal solution is estimated by using the positive ideal solutions. Let x_m^* and $z_m(x_m^*)$ denote the decision vector associated with PIS of m^{th} objective function and the corresponding value of m^{th} objective function, respectively. So the related NIS could be estimated as follows:

$$z_m^{NIS} = \min_{k=1,2,3,4,5,6} \{z_m(x_k^*)\} \text{ for } m = 1,2,3,4,5,6 \tag{46}$$

Step5: define a linear membership function for each objective function such as indicates in formula (47):

$$\mu_{z_i}(x) = \begin{cases} 1 & z_i > z_i^{PIS} \\ \frac{z_i - z_i^{NIS}}{z_i^{PIS} - z_i^{NIS}} & z_i^{NIS} \leq z_i \leq z_i^{PIS} \\ 0 & z_i < z_i^{NIS} \end{cases} \tag{47}$$

for $i = 1,2,3,4,5,6$

Where $\mu_{z_i}(x)$ denote the satisfaction degree of objective function z_i for given solution vector x . The graphs of this membership function are represented in Figure 1.

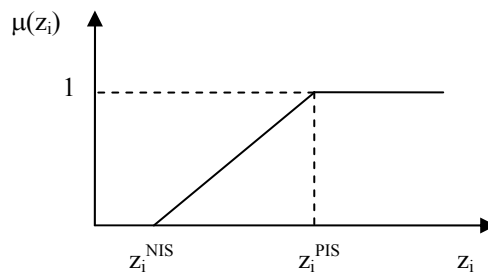


Figure 1 Linear membership function for z_i

Step6: the proposed multiple objective mixed linear programming model is converted into single objective mixed integer linear programming model as follows:

$$\begin{aligned}
 \max \quad & \lambda(x) = \gamma\lambda_0 + (1 - \gamma)\sum_i \theta_i \mu_{z_i}(x) \\
 \text{s.t.} \quad & \lambda_0 \leq \mu_{z_i}(x) \quad i = 1, \dots, 6 \\
 & x \in F(x), \quad \lambda_0 \quad \text{and} \quad \gamma \in [0,1] \\
 & \lambda_0 = \min_i \{\mu_{z_i}(x)\}
 \end{aligned} \tag{48}$$

Where $\mu_{z_i}(x)$ and λ_0 denote the satisfaction degree of objective function z_i and the minimum satisfaction degree of objectives, respectively (Torabi & Hassini, 2008). Furthermore, θ_i and γ show the relative importance of the i^{th} objective function and the coefficient of compensation, respectively. θ_i parameters are determined by the decision maker based on his/her preferences such that $\sum_i \theta_i = 1, \theta_i > 0$.

Also, γ controls the minimum satisfaction level of objectives as well as the compromise degree among the objectives implicitly. By adjusting the value of parameter, γ , the proposed model is capable to yield both unbalanced and balanced compromised solutions for a problem.

Step7: after adjusting the value of γ parameter and θ vector, the proposed auxiliary crisp model is solved by MIP solver. If the decision maker is satisfied with the current efficient compromise solution, stop. Otherwise, obtain another efficient solution after changing the value of some controllable parameters say β and γ and then go back to step 3.

4.2. Augmented ϵ -constraint method

In the ϵ -constraint method which is proposed by Haimes et al. (1971) one of the objective function is selected as the main objective function to be optimized and all other objective functions are converted into constraint by considering an upper bound for each of them. The augmented ϵ -constraint method consist of four following stages.

Step1: set the first objective function as the main objective function to be optimized and convert other objective function into constraint. Determine the appropriate interval for ϵ_k by using the payoff Table which is shown in Table 1. The payoff Table is constructed by solving p (the number of objective function) different model. In each model, one objective function is selected to optimize and other objective function are omitted. The ideal value and the nadir value for each objective function are obtained by solving these (p) models separately.

Table 1 The payoff Table for augmented ϵ -constraint method

The optimal solution for single objective model	$f_2(x)$	$f_3(x)$...	$f_p(x)$
x_2^*	$f_2(x_2^*)$	$f_3(x_2^*)$		$f_p(x_2^*)$
x_3^*	$f_2(x_3^*)$	$f_3(x_3^*)$		$f_p(x_3^*)$
...				
x_p^*	$f_2(x_p^*)$	$f_3(x_p^*)$		$f_p(x_p^*)$
Ideal value	$f_2(x_2^*)$	$f_3(x_3^*)$		$f_p(x_p^*)$
Nadir value	$\text{Min}_{k \neq 2}(f_2(x_k^*))$	$\text{Min}_{k \neq 3}(f_3(x_k^*))$		$\text{Min}_{k \neq p}(f_p(x_k^*))$

Step2: determine grid points (ε_k)

When the nadir value and ideal value of objective functions which are converted into constraint, are obtained, the interval for ε_k ($k=2, \dots, p$) is created.

$$\varepsilon_k \in [\text{nadir value for objective function } k, \text{ ideal value for objective function } k]$$

The above mentioned interval can be divided into some equal part (e.g q_k equal parts) therefore these divided part create q_k+1 points on interval. The values of ε_k are obtained by applying this method.

Step3: the augmented ε -constraint model is solved for each vector of ε which is obtained in previous step.

$$\begin{aligned}
 &\max f_j(x) + \zeta \sum_{k \neq j} s_k \\
 &s.t. \\
 &x \in X \\
 &f_k(x) - s_k = \varepsilon_k \quad k \neq j \\
 &s_k \in R^+
 \end{aligned} \tag{49}$$

where ζ is a scalar between 0.001, 0.000001.

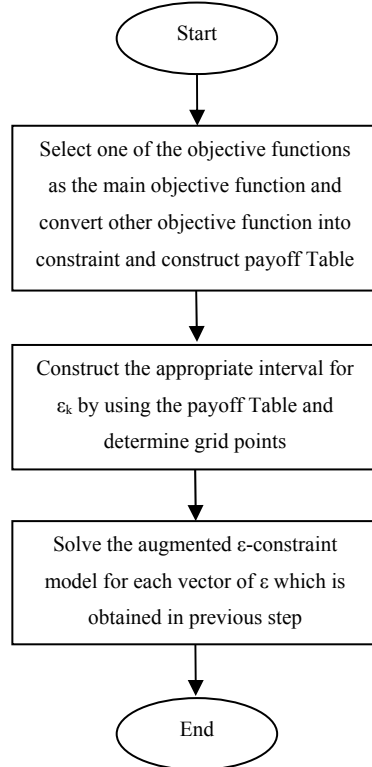


Figure 2 Flowchart of augmented ε -constraint procedure

4.3. Reservation level driven Tchebycheff procedure (RLTP) method

This method is one of the Tchebycheff metric -based approach in which the reservation level is used to reduce the space of objective function. This procedure consists of five stages.

Step1: Initialization

Define the number of solution which can be presented to the DM at each iteration ($N \geq P$, where P is the number of objective functions).

Compute a reference objective vector which is presented by y^u .

$$y^u : y_k^u = \max \{ f_k(x); x \in X \} - \varepsilon_k \quad \varepsilon_k \geq 0 \quad (50)$$

Specify a first value for each reservation level ($RL_i, i=1, \dots, k$). In previous study this parameter is set with negative infinitive for maximization model but this parameter can be set by a value which is obtained by calculating the payoff Table. The value of objective function in payoff Table is obtained by solving p single objective model where each objective function should be optimized separately without considering other objectives.

Step2: Sampling

A group of $2N$ maximally dispersed weight vectors is generated by considering formula (51).

$$\Lambda = \{ \lambda \in R^p : \lambda_k \in (0,1) \quad , \sum_k \lambda_k = 1 \} \quad (51)$$

Step3: Solution

Solve the associated augmented weighted Tchebycheff procedure (AWTP) which is indicated by formula (52).

$$\begin{aligned} AWTP : \min \{ & \alpha - \rho \sum_k \lambda_k f_k(x) \\ s.t. & \\ \alpha \geq & \lambda_k (f_k(x) - y_k^u); \forall k \\ x \in & X \\ f_k(x) \geq & RL_k \end{aligned} \quad (52)$$

where ρ is a small positive scalar.

Step4: Filtering

Select (N) maximally dispersed solution between $2N$ solution which is obtained by solving $2N$ AWTP model. The filtering method which is proposed by Steuer and Harris (1980) is applied to select (N) maximally dispersed solution. By representing the (N) selected solution to DM. The most preferred solution can be selected with the DM, if the DM satisfied completely the procedure is

stopped otherwise goto step 5. In order to describe filtering method, let us consider an MOLP problem with k objective function such as follows:

$$\begin{aligned} & \text{Max } \{ c^1 x = z_1 \} \\ & \text{Max } \{ c^2 x = z_2 \} \\ & \vdots \\ & \text{Max } \{ c^k x = z_k \} \\ & \text{s.t.} \\ & x \in S \end{aligned}$$

Suppose we confronted with 18 distinct vectors. We have received instructions from the DM to present him only the 8 most representative candidates in the group for his inspection. Therefore there are 10 most redundant criterion vectors will have to be discarded. A relationship which can be used to determine the most redundant criterion vectors to discard in filtering method is as follows:

$$\left\{ \sum_{i=1}^k (\pi_i | z_i^t - z_i^h |)^p \right\}^{1/p} < d \quad (53)$$

where:

k is the number of objective functions,

π_i is the gradation weight to be associated with the i^{th} component in each criterion vector,

z_i is the i^{th} criterion vector component,

t is the identification superscript of a criterion vector undergoing the dissimilarity test,

h is the identification superscript of a criterion vector currently retained (i.e. held) by the filter,

p is the metric parameter that determines which of the family of L_p -metrics is to be applied, $p \in \{1, 2, \dots\} \cup \{+\infty\}$, and

d is the test-distance parameter that regulates the filtering process.

At the first stage of filtering, we choose a value for parameter d by experimentation. A large value of d may be increase the number of discarded vectors. Therefore to find an appropriate value for d , multiple filtering runs as different trial values of d , must be tested. At the next stage, the 18 criterion vectors are read into the filtering routine. As a convention, the first point read in will be called the forward seed point. The forward seed point "primes" the filter and is always retained. Then, in order to determine the 2nd criterion vector, the distance of the remaining 17 vectors from the forward seed point must be calculated by formula (53). The vectors which their distances from the forward seed point are less than d , must be discarded. Suppose 5 vectors are discarded. Of the 12 remaining, the one with the smallest weighted criterion distance away from the seed point is retained as the 2nd vector held by the filter. This procedure is continued until the 8 most dissimilar of the 18 criterion vector retained (Steuer and Harris, 1980).

Step5: Adjustment:

If the DM is not satisfied with presented solution, the value of (RL) should be adjusted. In this circumstance, the (N) selected solution is partitioned into two subsets which consist of the most

preferred solution and the less preferred solution. RLs can be adjusted based on the objective values of these subsets.

The RLs value can be adjusted automatically by the RLTP. This method which is used for space reduction is proposed by Reeves and Maclead (1991).

Let $CSWV_k$ and $MPWV_k$ be the worst values for the i th objective over the set of all current solution and the subset of most preferred current solution, respectively.

$$RL_k = MPWV_k - r (MPWV_k - CSWV_k)$$

Where, r is a reduction factor between 0 and 1. Smaller values for r would correspond range for the ρ parameter is from 0.001 to 0.01 given in steure (1986).

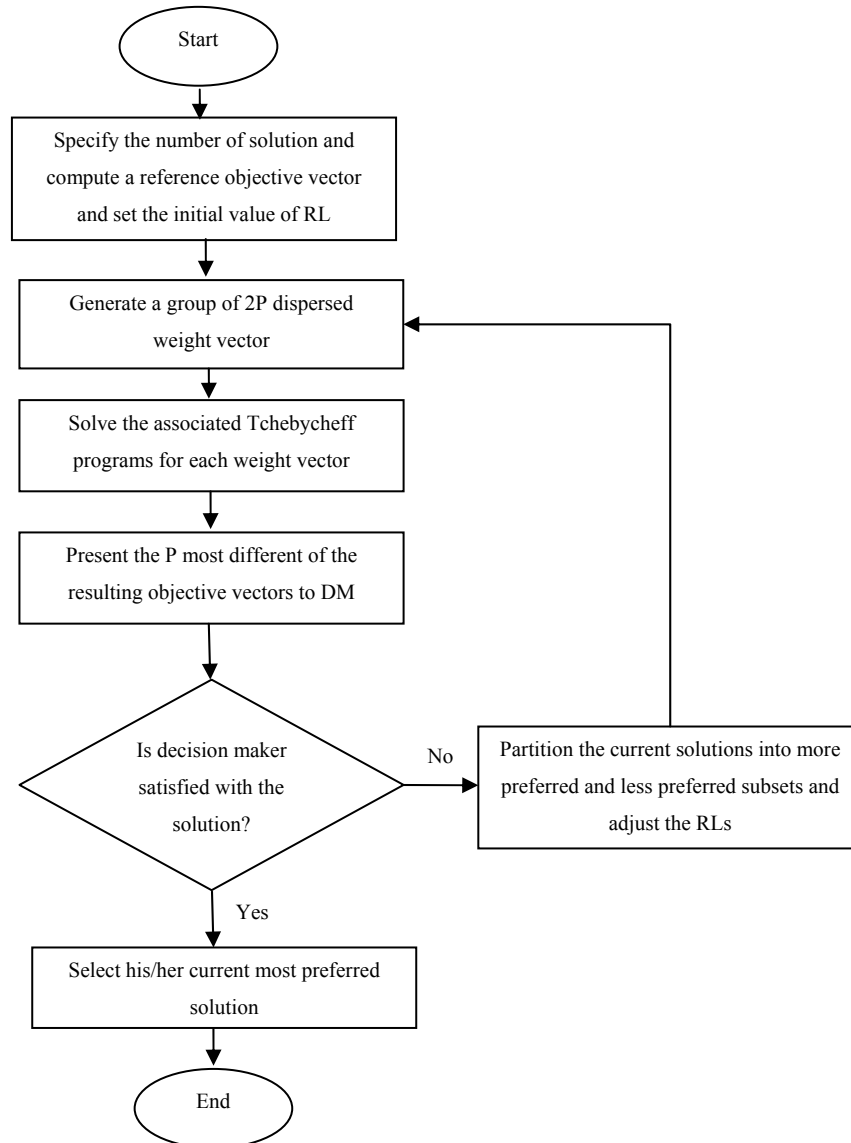


Figure 3 Flowchart of RLTP procedure

5. A NUMERICAL EXAMPLE

In this example, it is assumed that six different suppliers which can supply one part. Supplier A1, offer all-unit discount schedules and applied the first method of payment for sales. Supplier A2, offered all-unit discount schedules and proposed the second method of payment for sales. The prices offered by supplier A2 in different levels for part A are shown in Table 2. Supplier A3, proposed all-unit discount schedules and selected the third method of payment for sales. Suppliers A4, A5 and A6 offered incremental discount schedules and selected the first method, the second method and the third method of payment respectively. The offered prices in different levels are shown in Table 2. Two objective functions which consist of cost and total value of purchasing, are considered in proposed model. Our proposed criterion total value which include quality, delivery performance and after sale services for each supplier is taken into account in this model. The coefficient of total value of purchasing is considered to show the overall score of each supplier. In this circumstance the best supplier/s must be selected and purchased quantities must be determined by considering the limited budget supply uncertainty and capacity constraint which is related to each supplier. All other data which is used to solve this example, is shown in Table 3. Three methods of solutions are applied to solve this example.

Table 2 Discount intervals offered by suppliers

Supplier A1			Supplier A2			Supplier A3			Supplier A4			Supplier A5			Supplier A6		
<i>r</i>	l_{ik}^t	p_{ik}^t	<i>r</i>	l_{ik}^t	p_{ik}^t	<i>r</i>	l_{ik}^t	p_{ik}^t	<i>r</i>	l_{ik}^t	p_{ik}^t	<i>r</i>	l_{ik}^t	p_{ik}^t	<i>R</i>	l_{ik}^t	p_{ik}^t
1	50	4.10	1	50	4.22	1	50	4.26	1	50	4.10	1	50	4.22	1	40	4.26
2	125	3.72	2	125	3.79	2	125	3.76	2	125	3.72	2	150	3.79	2	200	3.76
3	250	3.55	3	250	3.48	3	250	3.51	3	250	3.55	3	300	3.48	3	300	3.51
4	500	3.39	4	500	3.36	4	500	3.32	4	500	3.39	4	500	3.36	4	450	3.32
5	∞	3.16	5	∞	3.13	5	∞	3.13	5	∞	3.16	5	∞	3.13	5	∞	3.13

Table 3 Data for random generation of parameters

Parameters	Distribution
Demand (d)	Uniform(16000,18000)
Capacity (c)	Uniform(500,5000)
Unit capacity requirement (pt)	Uniform(1,3)
Total score of suppliers	Uniform(0,1)
Supplier reliability	Uniform(0,1)

The symmetrical triangular possibility distribution is applied to generate fuzzy parameters. At the first, the most possible value of each imprecise parameter was generated with mentioned distribution, and then the corresponding most pessimistic and optimistic values were determined by multiplying the most possible value with 0.8 and 1.2, respectively. When the fuzzy constraints were converted into crisp auxiliary constraint and the two fuzzy objective functions are converted into six objective functions by applying Lai and Hwang method, the equivalent crisp model is obtained. Finally, three proposed methods which are included two interactive methods and augmented ϵ -constraint method, are applied to solve this multiple objective mixed integer linear programming model.

5.1. An interactive fuzzy programming approach

As it has been mentioned, TH method is applied to solve the proposed multiple objective possibilistic mixed integer linear programming model. In this procedure, at the first stage which mentioned in previous session, the equivalent crisp model is build. At the second stage, the

proposed crisp multiple objectives mixed integer linear programming model is converted into single objective mixed integer linear programming model which is indicated in formula (48). When the relative importance of objective functions is determined by decision maker, the weight vector (θ) will be adjusted. The value of parameter, γ , which is used in TH method, controls the minimum satisfaction level of objectives as well as the compromise degree among the objectives (Torabi & Hassini, 2008).

The single objective model which is proposed in TH method is capable to obtain unbalanced and balanced solutions by adjusting the value of parameter, γ . For instance, a higher value for γ is applied to obtain a higher lower bound for the satisfaction degree of objectives (λ_0) and result in more balanced compromise solutions. Similarly, the lower value for γ is applied to obtain a solution with high satisfaction degree for some objectives with higher importance without considering the satisfaction degree of other objectives which is result in unbalanced compromise solution (Torabi & Hassini, 2008). Due to existence a correlation between γ and the range of θ_h value, a limited reasonable interval of γ can be considered for a given θ vector. Due to explicit performance of the decision maker for getting an unbalanced compromise solution, when the large values for θ is selected, a small value for γ parameter (e.g. smaller than 0.3) should be selected. Torabi & Hassini (2008) indicated that any value of γ between 0.3 and 0.8 could be appropriate for obtaining a compromise solution for their model. After adjusting the value of parameters, the proposed model is solved. If result satisfies decision maker the procedure is stopped else the new value for parameters is applied to solve the model for next time. The result is showed in Table 4 and Table 5.

Table 4 Solution obtained by applying TH method for different γ

No.	Z1	Z2	Z3	Z4	Z5	Z6	Additive utility
1($\gamma = 0.2$)	31038120	-31047000	-31038100	-434.734	2173.668	434.734	-4967169.013
2($\gamma = 0.3$)	31038120	-31047000	-31038100	-434.737	2173.685	434.737	-4967169.013
3($\gamma = 0.4$)	31038370	-31048200	-31038400	-431.551	2157.757	431.551	-4967169.013
4($\gamma = 0.5$)	31038370	-31048200	-31038400	-431.551	2157.757	431.551	-4967169.013
5($\gamma = 0.6$)	31038370	-31048200	-31038400	-431.551	2157.757	431.551	-4967169.013
6($\gamma = 0.7$)	31038370	-31048200	-31038400	-431.551	2157.757	431.551	-4967169.013
7($\gamma = 0.8$)	31038370	-31048200	-31038400	-431.551	2157.757	431.551	-4967169.013

Table 5 Result of purchased quantities by suppliers obtained using TH method

Suppliers:	1	2	3	4	5	6
Purchased quantity ($\gamma = 0.2$)	3246	2915	4623	1249	4304	4913
Purchased quantity ($\gamma = 0.3$)	3246	2915	4623	1249	4304	4913
Purchased quantity ($\gamma = 0.4$)	3246	2915	4623	1790	3906	4769
Purchased quantity ($\gamma = 0.5$)	3246	2915	4623	1790	3906	4769
Purchased quantity ($\gamma = 0.6$)	2992	2915	4623	2045	3906	4769
Purchased quantity ($\gamma = 0.7$)	2992	2915	4623	2045	3906	4769
Purchased quantity ($\gamma = 0.8$)	3246	2915	4225	1662	4304	4898

5.2. Reservation level driven Tchebycheff procedure (RLTP) method

This method has been applied to solve the example. At first, the numbers of weight vectors were generated and the solutions were related to these weight vectors obtained by solving the associated augmented weighted Tchebycheff procedure (AWTP) which is indicated by formula (52), then at the next step, ten maximally dispersed solutions were obtained by applying filtering method in RLTP procedure. The solutions which were obtained after applying filtering method can be

presented to DM. If DM to be satisfied with these result the procedure is stopped. Otherwise the *RL* value should be tightened by using the set of most preferred solution which is selected by DM. The solutions which are marked with (*) show the most preferred solutions which are selected by DM. The new value for *RLs* is computed automatically by using the formula which is indicated in RLTP method and the procedure is continued until DM to be satisfied. An additive utility function was used to show DM preference with weights of 0.5 and 0.5 assigned to cost objective function and the total value of purchasing objective function respectively. These weights can be changed on the base of DM preferences. The mathematical models in this study are solved by using the General Algebraic Modeling System (GAMS). The values of objective functions and additive utility are shown in Table 6. The result which was obtained at the first iteration was proposed to DM. suppose DM selects the values of objective functions with the number of 1, 3, and 5 as preferred solutions which are declared in Table 6. The new values for *RL* are calculated as follows:

$$RL_k = MPWV_k - r (MPWV_k - CSWV_k)$$

$$RL_1 = 31038140 - 0.3*(31038140-31038140) = 31038140$$

$$RL_2 = -31047000 - 0.3*(-31047000+31047900) = -31047270$$

$$RL_3 = -31038100 - 0.3*(-31038100+31038300) = -31038160$$

$$RL_4 = -434.692 - 0.3*(-434.692+434.692) = -434.692$$

$$RL_5 = 2173.692 - 0.3*(2173.692-2173.692) = 2173.692$$

$$RL_6 = 434.738 - 0.3*(434.738-434.738) = 434.738$$

At the second iteration, these values are entered in RLTP procedure and the new solution is generated to offer DM until to be satisfied. The new result is indicated in Table 7.

Table 6 Initial solution obtained by RLTP method

No.	Z1	Z2	Z3	Z4	Z5	Z6	Additive utility
1*	31038140	-31047000	-31038100	-434.738	2173.692	434.738	-4967165.809
2	31038290	-31047800	-31038300	-434.738	2173.692	434.738	-4967301.809
3*	31038140	-31047000	-31038100	-434.738	2173.692	434.738	-4967165.809
4	31038180	-31047300	-31038200	-434.738	2173.692	434.738	-4967223.409
5*	31038140	-31047000	-31038100	-434.738	2173.692	434.738	-4967165.809
6	31038310	-31047900	-31038300	-434.738	2173.692	434.738	-4967314.609
7	31038190	-31047300	-31038200	-434.738	2173.692	434.738	-4967221.809
8	31038280	-31047700	-31038300	-434.738	2173.692	434.738	-4967287.409
9	31038300	-31047800	-31038300	-434.738	2173.692	434.738	-4967300.209

Table 7 Solution obtained by RLTP method after setting new values for RLs in second iteration

No.	Z1	Z2	Z3	Z4	Z5	Z6	Additive utility
1	31038160	-31047000	-31038200	-434.738	2173.692	434.738	-4967178.609
2	31038160	-31047000	-31038200	-434.738	2173.692	434.738	-4967178.609
3	31038160	-31047000	-31038200	-434.738	2173.692	434.738	-4967178.609
4	31038160	-31047000	-31038200	-434.738	2173.692	434.738	-4967178.609
5	31038160	-31047000	-31038200	-434.738	2173.692	434.738	-4967178.609
6	31038160	-31047000	-31038200	-434.738	2173.692	434.738	-4967178.609
7	31038160	-31047000	-31038200	-434.738	2173.692	434.738	-4967178.609
8	31038160	-31047000	-31038200	-434.738	2173.692	434.738	-4967178.609
9	31038160	-31047000	-31038200	-434.738	2173.692	434.738	-4967178.609

The purchased quantities for part from each supplier are shown in Table 8.

Table 8 The result of purchased quantities by suppliers obtained by RLTP method

Suppliers:	1	2	3	4	5	6
Purchased quantity	3246	2915	4623	1529	4304	4633

5.3. The augmented ε -constraint method

The result of this method is shown in Table 9 and Table 10. The values of objectives functions are shown in Table 9. In this method, six grid points were generated by using the obtained interval for ε_k . the purchased quantities of part, from each supplier are shown in Table 10.

Table 9 Solution obtained by augmented ε -constraint method

Grid point	Z1	Z2	Z3	Z4	Z5	Z6	Additive utility
1	31038616.6928	-31049400	-31038600	-434.738	2141.822	428.364	-4967559.657
2	31038567.1108	-31049200	-31038600	-434.101	2145.009	429.002	-4967534.877
3	31038567.1108	-31048900	-31038500	-433.464	2148.196	429.639	-4967470.163
4	31038614.8404	-31048700	-31038500	-432.826	2151.383	430.277	-4967429.812
5	31038616.6927	-31048400	-31038400	-432.189	2154.570	430.914	-4967364.802
6	31038616.6927	-31048200	-31038400	-431.551	2157.757	431.551	-4967332.088

Table 10 The result of purchased quantities by suppliers obtained by augmented ε -constraint method

Suppliers	1	2	3	4	5	6
Purchased quantity	3246	2915	4623	2045	4304	4117

The result of solving a numerical example by applying two methods (RLTP method and augmented ε -constraint method) shows that the delay of payment which can be offered by suppliers may be useful when there is a limited budget for purchasing. As shown in Table 5, 8 and 10, the maximum amount of order quantities is procured from three suppliers A3, A5, and A6 who offered delay in payments. As shown in the example, three scenarios for delay in payment are considered. It shows that the third scenario is the best one for selection and the buyer may acquire more profit than selecting other scenarios. Therefore, when the buyer faced with the limited budget to satisfy the demand in spot market, he prefers to purchase the whole amount of materials from suppliers which proposed a proper delay of payment.

6. CONCLUSIONS

In this paper, a fuzzy bi-objective model has been proposed for single part, single period supplier selection problem under budget limitations, supply uncertainty and capacity constraint. By considering the uncertainty of demand and capacity, the related constraints are modeled as fuzzy constraints. Also under condition of supply uncertainty, the supplier reliability is considered as a fuzzy parameter in objective functions. In this research, different kinds of discount policy such as all-unit discount and incremental discount can be offered by suppliers simultaneously. Furthermore, three kinds of method for delay in payment are tested in this supplier selection problem. Finally, a possibilistic multiple objective mixed integer programming model is proposed for this supplier selection problem. Three different method (an interactive fuzzy programming approach (TH method), augmented ε -constraint method and RLTP method) are proposed to solve the bi-objective model.

The result of solving a numerical example indicates that the delay of payment which can be offered by suppliers may be useful when there is a limited budget for purchasing. The maximum amount of

order quantities is procured from suppliers who offer delay in payments. Three scenarios for delay in payment are considered. The result of numerical example shows when buyer faces with limited budget, the alternative with more operational cost is the best one for selection. Three proposed methods are applied to solve this bi-objective model. The efficiency of RLTP method, TH method and augmented ε -constraint method is tested with additive utility function which is proposed by decision maker. The result indicates that the efficiency of the augmented ε -constraint method is better than RLTP method the TH method. But, TH method and the RLTP method can be applied as interactive methods. When we require considering decision maker's opinion in selection procedure constantly, two proposed interactive methods can be applied as proper tools for solving this problem. Decision makers can give their opinion through selection procedure and participate more actively in the interactive solution procedure and final solution can be affected by changes in the preferences of decision maker. The proposed model is very consistent with real condition in competitive market when the buyer is faced with unreliable suppliers. Furthermore, due to complexity of newsvendor model which is proposed for supplier selection under supply uncertainty condition, by considering the supply uncertainty as fuzzy parameter, the proposed possibilistic model can be applied as an appropriate and convenient tool for this problem. Therefore the proposed model can be applied as a well decision support system for practitioners when making the best decision is essential. It seems that if the number of suppliers will be increased the proposed model will be faced with problem. So it is recommended to create a meta-heuristic model to cope with practical problems. Furthermore the proposed model can be developed to a model by considering supplier selection problem under condition of multiple items and multiple period cases.

Acknowledgement

We would like to express our appreciation to the University of Tehran for the financial support for this study (Grant number 8108023/1/09). We are also grateful to the respected reviewers for their valuable comments in the preparation of the final revised manuscript.

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