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Budgetary Constraints and Idle Time Allocation in Common-Cycle Production with non-zero Setup Time

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ABSTRACT

Economic lot size scheduling problem (ELSP) for a multi-product single machine system is a classical problem. This paper considers ELSP with budgetary constraint as an important aspect of such systems. In the real world situations the available funds for investment in inventory is limited. By adopting the common cycle time approach to ELSP, we obtain the optimal common cycle which minimizes the total inventory ordering and holding costs for the case of nonzero setup times. One aspect of the scheduling is to decide what should be the sequence of production runs and how the idle times shall be distributed in the common cycle time. For such a sequencing problem, we consider two cases: a) the common cycle time is given, and b) the common cycle time is a decision variable. In the literature, scheduling rules are introduced for both cases, which assume that the total idle time is located at the end of each cycle. This paper relaxes this assumption and provides: i) a rule to optimize the production sequence and the length of idle times before (or after) producing each item, for both cases (a) and (b), and ii) the optimal common cycle for case (b). The presented rule is interestingly general, simple and easy-to-apply.

***Keywords*:** ELSP, Sequencing, Inventory Control.

1. INTRODUCTION

Realizing the importance of the effects of a decision made by one organizational unit on another unit, managers and enterprise system developers prefer decision support models that are capable of integrating a variety of inter- and intra-departmental relationships. This paper examines two such problems. These two problems involve financial and operational issues in common-cycle production of a group of products on a single machine. Specifically, allowing non-zero setup times, this paper considers a situation in which common-cycle includes idle time, and develops a method for determining the amount and the time of idle times between the production runs such that the total investment in inventory is minimized. Also, this paper considers the problem of determining the cycle-time that

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minimizes total inventory cost while keeping total investment in inventory under a specified budgetary level.

Economic lot scheduling problem is a challenging operational problem that managers frequently face. This problem has attracted the attention of many researchers. The objective is to economically schedule lots (i.e. production runs) of one or more products on a single machine, to satisfy demand for each product immediately while minimizing the average holding and set up cost per period. Elmaghraby (1978) presents a survey of approaches to this problem. This problem is NP-hard (Hsu, 1983), and there are no algorithms available to find the optimum solution. Boctor (1982), Carreno (1990), Cook et al. (1980), Dobson (1987), Fujita (1978), Goyal (1973 and 1984), Graves (1979), Gunter and Swanson (1986), Haessler (1979), Jones and Inmann (1989), Park and Yun (1984), and Zipkin (1988) have developed heuristics for solving this problem and some of its variants. Hanssmann (1962) reports on a common-cycle approach to this problem that results in the same production cycle for all of the products. One batch of one or more units of each product is produced only once in each cycle. This approach is computationally less cumbersome than other procedures and guarantees a feasible solution.

This paper deals with two financial and operational issues in common-cycle production. An important financial consideration is the maximum investment in inventory. Solutions that minimize total inventory cost could be infeasible when the necessary funds are unavailable. Therefore, minimizing the total investment in inventory or limiting its magnitude is an important managerial consideration. Parsons (1966) and Haji and Mansouri (1995) have included the total investment in inventory in common-cycle problem. Parsons (1966) makes the unrealistic assumption that the total investment in inventory is equal to the sum of the lot sizes of all of the products. Haji and Mansouri (1995) assume that a cycle starts with the setup and production of the first product followed by the setup and production of the other products with no idle time until the last product in the group is produced. Upon completion of the production of the last product the machine goes into an idle stage until the time when it must be set to produce the first product of the next cycle.

This study examines the common-cycle approach by considering the total investment in inventory, and allowing the occurrence of idle times between production of any two consecutive products within a cycle. These idle times may provide more frequent rest periods for operators and machinery which in turn result in higher levels of operational flexibility. They could also be necessary for machine maintenance within a common-cycle. This paper considers the following two cases in both of which the setup times are allowed to be non zero:

- Case 1. When a common cycle schedule is already determined. In this case, the paper presents a procedure for determining the amount and the timing of the idle times so that the maximum investment in inventory is minimized.
- Case 2. When there is a budgetary constraint. In this case, the paper develops a procedure for determining the duration of the common-cycle so that the average setup and holding cost per unit time is minimized.

Haji and Haji (2002) considered only case 1 and assumed that all the setup times are zero. In this paper we relax this restriction and consider a more general and practical case in which the setup times are allowed to be non-zero.

Notation and Assumption

The following notations are used throughout the paper:

N	number of products
A_j	setup cost for production of product j, $j = 1, 2,..., N$
S_j	setup time for production of product j, $j = 1, 2,..., N$
B	maximum available budget
d_j	monetary value of demand rate per unit time for product j, $j = 1, 2, ..., N$
D	monetary value of total demand per unit time for all of the N products
h	inventory holding cost per monetary unit per unit time
I_j	monetary value of inventory of product j just before the start of production of fist product in a cycle
K	average setup and inventory costs per unit time
m_k	monetary value of total inventory just before the production of product k
M_k	monetary value of total inventory at the completion time of production of product k
P_j	monetary value of production rate per unit time for product j, $j = 1, 2,..., N$
t_j	production run-time of product j, $j = 1, 2, ..., N$
T	duration of common-cycle time
X_j	duration of idle time occurring just before the production of product j, $j = 1, 2, ..., N$

We make the following assumptions:

1. There is an infinite planning horizon.
2. Only one product can be produced at any point in time.
3. In each cycle, all of the N products are produced.
4. Each product is produced once in each cycle.
5. Setups take place prior to production of each product. Setup times are constant and independent of production sequence.
6. Demand rate for each product is constant and known.
7. The production rate of each product is constant and known.
8. In each cycle, the increase of monetary value of the aggregate inventory during production run of any product is at least equal to monetary value of the aggregate demand during the set up time of that product.
9. No shortages are allowed.

2. CASE 1 - DETERMINING THE DURATION AND TIMING OF IDLE TIMES

In this section we allow the setup times to be non-zero and analyze the allocation of the total idle time in a common-cycle time among the production runs of N products. The objective is to determine the durations and times of idle times such that the total investment in inventory is minimized.

To achieve this purpose first we select a cycle time T which begins just at the start of the production run of a particular product. We denote this particular product by k_1 and the product that will be produced next in the cycle by k_2, and so on. Then we present the following remarks and a theorem.

***Remark 1*:** Since in each cycle time T the total investment in inventory decreases during idle times and increases during the production run of any product, it is clear that z, the maximum inventory investment, occurs at the end of production run of one of the N products, i.e., at an M_{k_j}, j=1,…, N. Hence,

$$z = \underset{1 \le j \le N}{Max} \quad M_{k_j} \tag{1}$$

and

$$M_{k_{j+1}} = M_{k_j} - DX_{k_{j+1}} + (P_{k_{j+1}} - D)T_{k_{j+1}} \qquad j = 0,1,\dots,N \ , \tag{2}$$

where the index k_{N+1} is equivalent to k_1.

Remark 2: From (2) we can write

$$\text{(a)}\, M_{k_{j+1}} = M_{k_j} \text{ if } DX_{k_{j+1}} = (P_{k_{j+1}} - D)T_{k_{j+1}} \tag{3}$$

or

$$M_{k_{j+1}} = M_{k_j} \text{ if } X_{k_{j+1}} = \frac{P_{k_{j+1}} - D}{D} T_{k_{j+1}} \tag{4}$$

and

$$\text{(b) } M_{k_{j+1}} < M_{k_j} \text{ if } X_{k_{j+1}} > \frac{P_{k_{j+1}} - D}{D} T_{k_{j+1}} \tag{5}$$

Remark 3: For feasibility of the problem the following constraints must be satisfied:

$$X_{k_j} \ge S_{k_j} \ , \qquad j = 1,2,\dots N. \tag{6}$$

Theorem 1: For a feasible solution, suppose for some j, $j = 1,2,\dots N$, $M_{k_j} > M_{k_{j+1}}$ where index k_{N+j} is equivalent to k_j. If we decrease $X_{k_{j+1}}$ by an amount $w = \dfrac{M_{k_j} - M_{k_{j+1}}}{D}$ and increase $X_{k_{j+2}}$ by the same amount and fix all other X_{k_j}, then the new values of idle times are still feasible.

Proof: Denote the new values of idle times by X'_{k_l}, $l = 1,2,\ldots,N$. Clearly $X'_{k_l} = X_{k_l}$ for all l, $l \neq j+1$ and $j+2$. From the statement of the theorem $w > 0$, and from feasibility of $X_{k_{j+2}}$,i.e., $X_{k_{j+2}} \geq S_{k_{j+2}}$, we can write

$$X'_{k_{j+2}} = (X_{k_{j+2}} + w) > S_{k_{j+2}}$$

which shows that $X'_{k_{j+2}}$ is feasible. It remains to prove that $X'_{k_{j+1}}$ is also feasible. That is

$$X_{k_{j+1}} - w = X'_{k_{j+1}} \geq S_{k_{j+1}}$$

To do this we first note that from (2) and the assumption of the theorem

$$w = \frac{M_{k_j} - M_{k_{j+1}}}{D} = X_{k_{j+1}} - \frac{P_{k_{j+1}} - D}{D} T_{k_{j+1}} \tag{7}$$

We also note that the assumption number 8 implies that

$$(P_{k_{j+1}} - D) T_{k_{j+1}} \geq D S_{k_{j+1}}$$

or

$$\frac{P_{k_{j+1}} - D}{D} T_{k_{j+1}} \geq S_{k_{j+1}} \tag{8}$$

Thus from (8) and part (b) of remark 2 we can write

$$X_{k_{j+1}} = \beta + S_{k_{j+1}} \tag{9}$$

Where β is a positive number. Therefore if we show that $w \leq \beta$, then from (9) we can write

$$X_{k_{j+1}} - w = X'_{k_{j+1}} \geq S_{k_{j+1}} \tag{10}$$

Which means $X'_{k_{j+1}}$ is feasible.

To show that $w \leq \beta$, substitute (9) in (7) to get

$$w = \beta + S_{k_{j+!}} - \frac{P_{k_{j+1}} - D}{D} T_{k_{j+1}} \tag{11}$$

It is clear from (8) and (11) that $w \leq \beta$ which proves (10). This completes the proof of the theorem.

2.1. Optimal Inventory Investment for a Given Cycle

In this section we obtain the optimal sequence of production runs and optimal allocation of idle times which minimizes the total investment in inventory for a given cycle time T

Inventory value at the end of a product

Because no shortages are allowed, the inventory level for product k_1 at the start of its production is zero, i.e. $I_{K_1}=0$. Furthermore, the inventory level for product k_j $(j\geq 2)$, at the start of production run of product k_1 is equal to its demand during F_{k_j}, the time interval from the start of production of k_1 to the start of production of product k_j, Figure(1). Thus, for any feasible idle times $X_{k_j}, j=1,2,...,N$ we can write

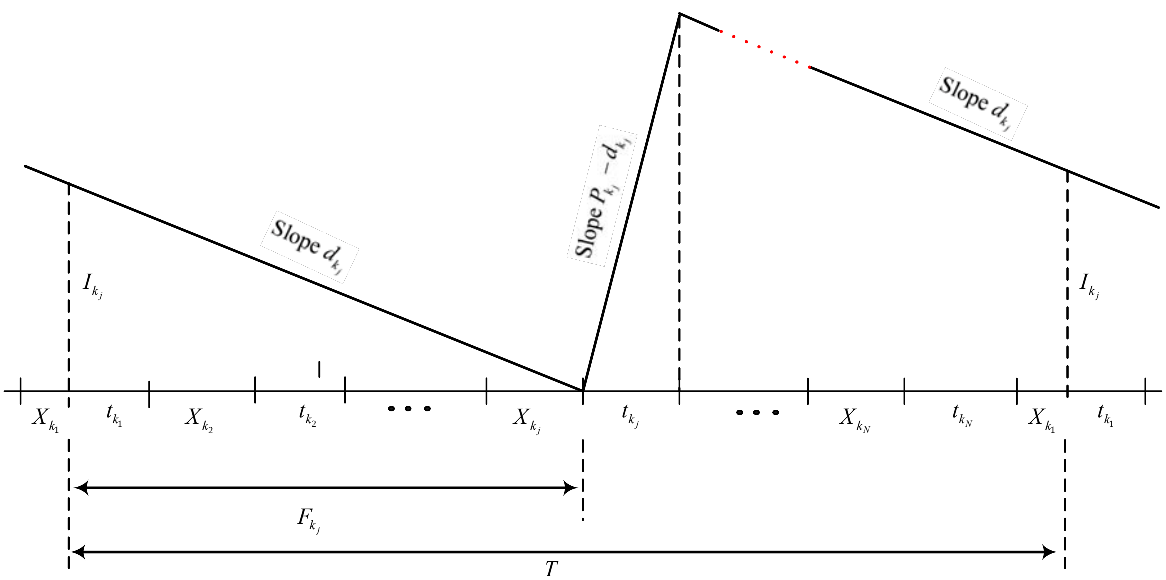

Figure 1 Monetary value of inventory of product *j* over time

$$I_{k_j}=d_{k_j}\left(\sum_{y=1}^{j-1}t_{k_y}+\sum_{y=2}^{j}X_{k_y}\right), \qquad 2\leq j\leq N \tag{12}$$

The total inventory level at the start of production run of product k_1 is:

$$m_{k_j}=\sum_{j=1}^{N}I_{k_j} \tag{13}$$

Hence, from (12), (13) and the fact that $I_{K_1}=0$ we can write

$$m_{k_1}=\sum_{j=2}^{N}d_{k_j}\left(\sum_{y=1}^{j-1}t_{k_y}+\sum_{y=2}^{j}X_{k_y}\right) \tag{14}$$

Furthermore, the total inventory at the end of production runs of product k_j is (Figure2):

$$M_{k_1} = m_{k_1} + (P_{k_1} - D)\, t_{k_1}, \qquad j=1 \tag{15}$$

and

$$M_{k_j} = M_{k_1} - D\sum_{l=2}^{j} X_{k_l} + \sum_{l=2}^{j}(P_{k_l} - D)t_{k_l}, \qquad 2 \le j \le N \tag{16}$$

The level of $M_{k_j}\ \ (j = 1,2,\ldots,N)$ depends not only on the $(N-1)!$ different production sequences but it also depends on the duration of the idle times X_{k_j}, $j=1,\ldots N$. The objective in this case is to determine the production sequence and the duration of the idle times to minimize the maximum inventory investment. To achieve this purpose first we state the following theorem for the case in which the setup times are allowed to be non-zero and the cycle time T is known and is feasible. That is

$$\sum_{J=1}^{N} S_j < T \tag{17}$$

Theorem 2: For any setup time values, satisfying assumption 8 and a given sequence, the optimal solution, z^*, for any feasible production cycle T has the following property:

$$z^* = M_1 = M_2 = \ldots = M_N \tag{18}$$

Proof: We prove the theorem by contradiction. Suppose for the given production sequence there exists an optimal solution z_0 for which, all M_{k_j}, are not equal. This implies that, in the optimal solution, there exist two consecutive products, denoted by i_1 and i_2, for which $M_{i_1} > M_{i_2}$, and $z_0 = M_{i_1}$ Thus we can write

$$z_0 = \underset{1\le j\le N}{Max}\ M_{i_j} = \underset{1\le j\le N \text{ and } j\ne 2}{Max}\ M_{i_j} \tag{19}$$

where i_j means the j^{th} production run, in a cycle which starts at the beginning of production run of product i_1.

We decrease X_{i_2} by an amount $w = \dfrac{M_{i_1} - M_{i_2}}{D} > 0$, increase X_{i_3} by the same amount, and fix all other $X_{i_j}\ \ (j \ne 2,3)$ and denote the new values of these idle times by X'_{i_2} and X'_{i_3}, that is

$$X'_{i_2} = X_{i_2} - w \tag{20}$$

$$X'_{i_3} = X_{i_3} + w \tag{21}$$

First we need to show that the new values of idle times $X'_{i_j},\ j = 1,\ldots,N$ are feasible. From theorem 1 one can easily show that these idle times are feasible.

Now by denoting the new values of total inventory at the end of production run of product i_j (j=1,…,N) by M'_{i_j}, we show that:

a) $M'_{i_j} = M_{i_j} - d_{i_2} w < M_{i_j}$, $\qquad j = 1,3,\ldots,N$, $(j \neq 2)$

and

b) $M'_{i_2} < M'_{i_1}$, $\qquad j = 2$

which implies that the new value of maximum aggregate inventory, denoted by z'_0, is less than its pervious value, z_0 in equation (19), contradicting the assumption that z_0 was optimal.

$$z'_0 = \underset{1\le j\le N}{Max} M'_{i_j} = \underset{1\le j\le N \ (j\neq 2)}{Max} M'_{i_j} ,$$

To show that (a) is true, note that from (12), replacing k by i, for $j \neq 2$, decreasing X_{i_2} by an amount w will decrease the inventory of product i_2 at the start of the production run of product i_1 by an amount $d_{i_2} w \ (w > 0)$. But, since $X'_{i_2} + X'_{i_3} = X_{i_2} + X_{i_3}$, from (12) we see that, replacing k by i, all other I_{i_j} 's, $j \geq 3$, remain unchanged. Thus, the new value of total inventory at the start of production run of item i_1, denoted by m'_{i_1}, differs from its pervious value m_{i_1} by an amount $d_{i_2} w$, that is,

$$m'_{i_1} = m_{i_1} - d_{i_2} w \tag{22}$$

Now, denoting the new value of total inventory at the end of production run of item i_j by M'_{i_j}, then as we derived (15), we can write

$$M'_{i_1} = m'_{i_1} + (P_{i_1} - D) t_{i_1} , \qquad j = 1 \tag{23}$$

Replacing (22) in (23) we have

$$M'_{i_1} = m_{i_1} + (P_{i_1} - D) t_{i_1} - d_{i_2} w$$

or from (15), replacing k by i,

$$M'_{i_1} = M_{i_1} - d_{i_2} w , \qquad j = 1 \tag{24}$$

Now for $j \geq 3$, noting that $X'_{i_2} + X'_{i_3} = X_{i_2} + X_{i_3}$, according to(16) we can write

$$M'_{i_j} = M'_{i_1} - D \sum_{l=2}^{j} X_{i_l} + \sum_{l=2}^{j} (P_{i_l} - D) t_{i_l} , \qquad 3 \le j \le N \tag{25}$$

or from (24)

$$M'_{i_j} = M_{i_1} - D\sum_{l=2}^{j} X_{i_l} + \sum_{l=2}^{j}(P_{i_l} - D)t_{i_l} - d_{i_2}w, \qquad j \geq 3$$

Hence, from (16) (replacing *k* by *i*)

$$M'_{i_j} = M_{i_j} - d_{i_2}w, \qquad 3 \leq j \leq N \tag{26}$$

Thus, from (24) and (26), we have

$$M'_{i_j} < M_{i_j}, \qquad j = 1,3,\ldots,N \ , \ (j \neq 2)$$

which shows that (a) is true.

Next, to see that (b) is also true, note that for $j = 2$ as we derived (2) we can write

$$M'_{i_2} = M'_{i_1} - DX'_{i_2} + (P_{i_2} - D)t_{i_2}$$

Since $X'_{i_2} = (X_{i_2} - w)$, we can write

$$M'_{i_2} = M'_{i_1} + Dw - DX_{i_2} + (P_{i_2} - D)t_{i_2} \tag{27}$$

Now from the fact that $\frac{M_{i1} - M_{i_2}}{D} > w$, replacing Dw in (27) by $(M_{i_1} - M_{i_2})$, we can write

$$M'_{i_2} < M'_{i_1} + M_{i_1} - M_{i_2} - DX_{i_2} + (P_{i_2} - D)t_{i_2} \tag{28}$$

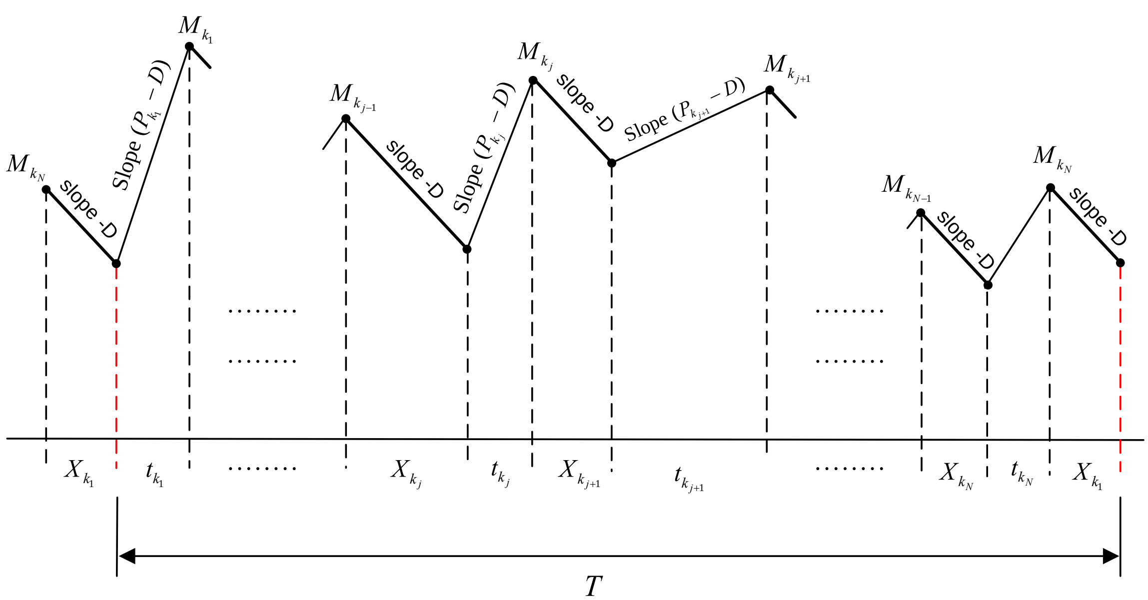

Figure 2 Monetary value of total inventory in a cycle.

Noting that, from (2), replacing *k* by *i*, we have

$$M_{i_2} = M_{i_1} - DX_{i_2} + (P_{i_2} - D)t_{i_2}$$

which implies the sum of the last four terms on the right hand side of (28) is zero and we can write (28) as

$$M'_{i_2} < M'_{i_1}$$

which shows that (b) is also true and this completes the proof of theorem 2.

Finally, for zero as well as non-zero setup times, we prove the following theorem

Theorem 3: For any setup time values, satisfying the assumption 8, the optimal solution is sequence independent.

Proof: From part (a) of remark1 we note that if for an arbitrary sequence say $i_1, i_2, \ldots, i_N$, $X_{i_j} = \frac{P_{i_j} - D}{D} t_{i_j}$, $j = 1,2,\ldots,N$, then $M_{i_1} = M_{i_j}$, $j = 1,2,\ldots,N$. That is, from theorem 2, the optimal value of maximum inventory, z^*, for this sequence is:

$$z^* = \underset{1 \le j \le N}{Min} \; M_{i_j} = M_{i_1} = M_{i_2} = \cdots = M_{i_N} \tag{29}$$

We prove the theorem by showing that the optimum value of M_{i_j} i.e., z^*, or equivalently M_{i_1}, is constant and independent of the production sequence. To do this, we manipulate (14) as shown bellow

$$m_{i_1} = \sum_{j=1}^{N} d_{i_j} \sum_{y=1}^{j} (t_{i_y} + X_{i_y}) - \sum_{j=1}^{N} d_{i_j} t_{i_j} - X_{i_1} \sum_{j=1}^{N} d_{i_j}$$

Note that $\sum_{j=1}^{N} d_{ij} t_{ij}$ and $\sum_{j=1}^{N} d_{i_j}$ are constant and independent of production sequence. We denote them respectively by *C* and *D*. Thus, we can write

$$m_{i_1} = \sum_{j=1}^{N} d_{i_j} \sum_{y=1}^{j} \left(t_{i_y} + X_{i_y}\right) - C - X_{i_1} D \tag{30}$$

Replacing X_{i_y}, by $\frac{P_{i_y} - D}{D} t_{i_y}$, $y = 1,2,\ldots,N$, in equation (30), we have

$$m_{i_1} = \sum_{j=1}^{N} d_{i_j} \sum_{y=1}^{j} \frac{P_{i_y} t_{i_j}}{D} - C - (P_{i_1} - D)t_{i_1} \tag{31}$$

Thus from (15), replacing k by i , and then substituting m_{i_1} from (31) in (15), we can write

$$M_{i_1} = \sum_{j=1}^{N} d_{i_j} \sum_{y=1}^{j} (\frac{P_{i_y} t_{i_y}}{D}) - C \tag{32}$$

Since quantity demanded for each product during a cycle is produced during its production run in that cycle it follows that

$$P_{i_y} t_{i_y} = d_{i_y} T , \qquad y = 1,2,\ldots,N \tag{33}$$

Substituting (33) in (32) gives:

$$z^* = M_{i_1} = \frac{T}{D} \sum_{j=1}^{N} d_{i_j} \sum_{y=1}^{j} d_{i_y} - C \text{, or equivalently}$$

$$z^* = M_{i_1} = \frac{T}{2D} \left[\left(\sum_{j=1}^{N} d_{i_j} \right)^2 - \sum_{j=1}^{N} d_{i_j}^2 \right] - C$$

$$z^* = M_{i_1} = \frac{T}{2D} \left[D^2 - \sum_{j=1}^{N} d_j^2 \right] - C \tag{34}$$

From (33) and definition of C, $C = T \sum_{j=i}^{N} \frac{d_j^2}{p_j}$. Thus the right-hand side of the above equation is constant and independent of the production sequence. This completes the proof of theorem 3.

3. CASE 2- THE OPTIMUM COMMON CYCLE WITH NON-ZERO SETUP TIME AND BUDGET CONSTRAINT

In this section allowing non-zero production setup times, we derive a procedure for determining the duration of common-cycle such that the average setup and inventory holding cost per unit time is minimized and a given budgetary constraint is satisfied. The average setup and holding cost per unit time is (Johnson and Montgomery, 1974):

$$K = \frac{\sum_{j=1}^{N} A_j}{T} + \frac{T}{2} h \sum_{j=1}^{N} d_j \left(1 - \frac{d_j}{P_j} \right) \tag{35}$$

K should be minimized subject to

$$z^* \leq B \tag{36}$$

and

$$\sum_{j=1}^{N}\left(t_j + S_j\right) \leq T \tag{37}$$

Constraint (36) limits the maximum of aggregate inventory to a given value *B*, and constraint (37) states that the total production and setup times in a cycle can not exceed the length of the cycle. Substituting z^* in (36) by the right hand side of (34) we have:

$$T \leq \frac{2D\ B}{\left[(D)^2 - \sum_{j=1}^{N} d_j^2\right] - 2D\ \sum_{j=i}^{N} \frac{d_j^2}{p_j}} \tag{38}$$

Also, Equation (33) implies $t_j = (d_j / P_j)T$, thus (37) can be written as:

$$T \geq \frac{\sum_{j=1}^{N} S_j}{1 - \sum_{j=1}^{N} \frac{d_j}{p_j}} \tag{39}$$

Equations (38) and (39) provide the limits for the duration of the common-cycle. Therefore, designating the right hand sides of equations (38) and (39) by T_M and T_m respectively, i.e.,

$$T_M = \frac{2D\ B}{\left[(D)^2 - \sum_{j=1}^{N} d_j^2\right] - 2D\ \sum_{j=i}^{N} \frac{d_j^2}{p_j}} \tag{40}$$

and

$$T_m = \frac{\sum_{j=1}^{N} S_j}{1 - \sum_{j=1}^{N} \frac{d_j}{p_j}} \tag{41}$$

Now we can write the problem as follows:

Min K

subject to:

$T_m \leq T \leq T_M$

K is a convex function (Johnson and Montgomery, 1974). Differentiating *K* with respect to *T* and solving for *T* gives:

$$T_o = \sqrt{\frac{2\sum A_j}{\sum h_j d_j (1 - \frac{d_j}{p_j})}} \tag{42}$$

Clearly, if $T_M < T_m$, then there is no feasible solution for T. But if $T_M \geq T_m$, first we obtain T_o from (42). Then due to convexity of *K*, we find the optimal cycle time, T^*, to be

$$T^* = \min[\max(T_m, T_o), T_M] \tag{43}$$

or equivalently

$$T^* = \begin{cases} T_m & \text{if} \quad T_o < T_m \\ T_o & \text{if} \quad T_m \leq T_o \leq T_M \\ T_M & \text{if} \quad T_o > T_M \end{cases} \tag{44}$$

The procedure can be summarized as follows:

1. Use Equation (40) to determine the maximum common-cycle, T_M.
2. Use Equation (41) to determine the minimum common-cycle, T_m.
3. If $T_M < T_m$ the problem has no solution, otherwise go to step 4.
4. Use Equation (42) to determine T_o.
5. Use Equation (44) to determine the optimum common-cycle.

4. CONCLUSION

Distributing idle times between production runs of products provides some flexibility for performing certain tasks such as preventive maintenance. Also, it may provide operators more frequent rest times which in turn results in a lower number of accidents and improve the quality of products. In this study, by adopting the common cycle time approach to lot size scheduling problem for a multi-product single machine system with budgetary constraint, we considered two common-cycle scheduling problems where non-zero setup times are allowed. One important aspect of these common cycle scheduling is to decide what should be the sequence of production runs and how the idle times shall be distributed in the cycle time.

For such a sequencing problem, we considered two cases: **a**) the common cycle time is given, and b) the common cycle time is a decision variable. In the literature, scheduling rules are introduced for both cases, which assume that the total idle time is located at the end of each cycle. This paper relaxed this assumption and presented a scheduling rule for both cases to optimize the production sequence and the length of idle times before (or after) producing each item. Furthermore, we provided a simple procedure which obtains the optimal common cycle for case (b) which minimizes the total inventory cost. We proved that for any setup time the optimal solutions in both cases (a) and (b) are sequence independent (assuming that in each cycle, the increase of monetary value of the inventory during production run of any product is at least equal to the monetary value of the aggregate demand during the set up time of that product). The presented scheduling rule is interestingly general, simple and easy-to-apply.

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