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Optimal Batch Production for a Single Machine System with Accumulated Defectives and Random Rate of Rework

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ABSTRACT

In this paper we consider an imperfect production system which produces good and defective items and assume that defective items can be reworked. Due to the nature of rework process we do not restrict the rework rate to be equal to normal production rate or constant and assume that it is a random variable with an arbitrary distribution function. We also assume as it is true in most real world situation that the unit rework processing cost need not to be equal to unit normal processing cost. We consider the case in which during a specified period of time consisting of several cycles the rework process takes place only in its last cycle. For practical reasons such as the ease of production scheduling and resource planning we require that the length of the last cycle must have the same length as the other cycles of this period. For this purpose we assume in the last cycle of this period first the normal production starts after which the rework of all defective units produced in the period begins. After the end of the last cycle of the period the whole process will start all over again. Further we assume that, for reduction of set up cost for rework per unit time, the total number of setups for rework must be as small as possible. For this system we derive the expected total cost function consisting of inventory holding, waiting time of defectives for rework, setup, and processing costs per unit time. Then we obtain the economic batch quantity which minimizes this expected cost.

***Keywords*:** Economic batch quantity, Accumulated rework, Random rework rate

1. INTRODUCTION

To obtain the economic batch quantity (*EBQ)* in a single stage production system, the classic inventory control assumes that the manufacturing process will never produce defective items in a production cycle Johnson (1974), Love (1979), Nahmias (2005). But, defective items will be produced in each cycle of production in most practical situations. It is clear that there are many instances in which the produced imperfect quality items should be reworked or repaired with additional costs. Several Scholars have investigated the effect of imperfect quality production on economic production models.

Kalro and Gohil (1982) considered the problem of determining the economic lot size for the case in which due to several reasons such as rejections during inspection process or damage in transit the quantity of the lot received is uncertain. Rosenblatt and Lee (1986), Lee (1992), and Hayek and Salameh (2001) are among those who have investigated the issue of imperfect production and

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quality control. Lee and Rosenblatt (1987) studied an *EBQ* model with joint determination of production cycle time and inspection schedules, and they derived a relationship that can be used to determine the effectiveness of maintenance by inspection. Porteus (1986) used Markov chains to model the shifting behavior of a production system from in-control status to out-of-control status and derived the optimal lot size for the system. Schwaller (1988) addressed the *EBQ* problem when in detecting a given portion of defective items both fixed and variable costs are considered.

Zhang and Gerchak (1990) considered joint lot sizing and inspection policy in an *EBQ* model with random yield. Cheng (1991) considered an imperfect production process with demand-dependent unit production cost and derived the optimal solution for the *EBQ* model. Salameh and Jaber (2000), Lee et al. (1997), Chakrabarty and Rao (1988), Gupta and Chakrabarty (1984) and Sarker et al. (2008) are among the notable efforts that have considered different variants of imperfect production processes. Buscher and Lindner (2007) like Sarker et al. (2008) investigated a multistage imperfect production system. In this work they simultaneously addressed the lot sizing analysis and the scheduling problem. Haji R. et al. (2008) studied an imperfect production process with rework in which several products are produced on a single machine. For more references one can see Cárdenas-Barrón L.E. (2009).

Jamal et al. (2004) developed two models for obtaining the economic batch quantity for the single product. In the first model they assumed that in each cycle the rework process starts immediately after normal production process. In the second model they considered the case in which defective items from each cycle are accumulated until N equal cycles are completed after which all defectives are reworked in a new cycle, rework cycle. In their paper, the length of the rework cycle is not the same as the regular production cycles. Also in their derivation of the total cost they Restricted N to be equal to the ratio of the demand rate to the order size. Further they ignored the waiting time of defectives for rework during the normal production of the first cycle of their both models. Haji B. et al. (2008) relaxed these three restrictions and assumed that the rework cycle has the same length as the normal cycles. They also as Jamal et al (2004) (a) assumed that the rework process rate is deterministic, constant, and equal to the normal production rate and (b) they also assumed that the unit rework processing cost is equal to unit normal processing cost which may not be true in most practical situations.

In this paper we extend the work of Haji B. et al. (2008) by relaxing the unrealistic assumption that the rework process rate need to be restricted to have the same rate as the normal production rate. Further we consider the case which is more practical in most real world situations. That is we assume that the rework rate is a random variable with an arbitrary distribution function. We also assume as it is true in most real world situation that the unit rework processing cost may not be equal to unit normal processing cost

This paper deals with the issue of economic batch quantity (*EBQ*) in a single machine system in which defective items are produced in each cycle of production. We assume that the accumulated defective items produced in a period, consisting of $N + 1$ cycle, are all reworked in the last cycle known as the rework cycle. At the end of each period the whole process will start all over again. Due to the nature of rework process we do not restrict the rework rate to be equal to normal production rate or constant and assume that it is a random variable with an arbitrary distribution function. Further as in Haji B. et al (2008) we assume that the number of rework cycles be as small as possible, which is mainly due to changeover costs needed for going from normal production to rework as well as special attention required for rework to satisfy the zero defect criterion. Finally, for the ease of production and resource planning, the rework cycle has to have the same length as the other cycles. In the last cycle first normal production starts then at the end of this normal production the rework process on all defectives of the period begins.

For this model, assuming no shortages are permitted, we formulate the total system cost function consisting of setup, regular production and rework process, waiting time of defectives for rework, and inventory holding costs per unit time. Then, we obtain the optimal order quantity which minimizes this total cost.

2. ASSUMPTIONS

In this paper, all the standard assumptions of the general *EBQ* hold true Love (1979) . The most relevant assumptions used in this paper are as follows:

- The rework rate of defective items is a random variable with a known distribution function.
- The unit rework processing cost need not to be equal to unit normal processing cost.
- Demand rate for the product is constant, known, and finite.
- Normal production rate for the product is constant, known, and finite.
- No scrap is produced during Normal and rework processing.
- No shortages are permitted.
- Proportion of defectives is constant in each cycle,
- The production rate of non-defectives is constant and greater than the demand rate

3. NOTATIONS

The following notations are used in this paper

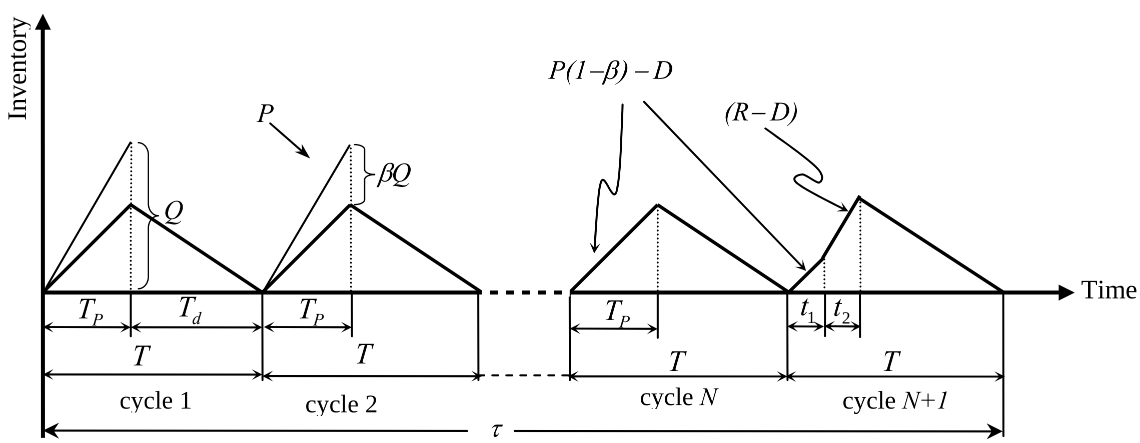

Figure 1 The inventory during τ

P	Normal production rate, units/unit time
R	Rework production rate a random variable, units/unit time
$E[1/R]$	Expected value of random variable $1/R$

D	Demand rate, units/unit time
C	Processing cost for each unit of product, \$/unit
C_R	Rework processing cost for each unit of product, \$/unit
β	Proportion of defectives produced during each normal production
A	Setup cost for product, \$/batch
H	Inventory carrying cost, \$/unit/unit time
K	Waiting time cost of a defective unit for rework, \$/unit/unit time
T	Length of a single cycle
τ	A period equal to the total length of $(N+1)$ equal cycles
N	Number of production cycles of τ in which no rework is done
t_1	Length of normal production time in the $(N+1)^{th}$ cycle of τ, during which no rework is done.
t_2	Length of the rework processing time of all accumulative defectives produced in period τ
Q	Batch quantity, total of good and defective items produced in each cycle T of the first N cycles of τ
Q'	Total of good and defective items produced during t_1.
Q''	Total number of defective items produced during τ
T_1	$T_1 = \frac{Q''}{D}$
h_1	The maximum inventory at the end of normal production time t_1, in cycle $(N+1)$ of the period τ
h_2	$h_2 = \frac{Q''}{R}$
$\bar{I}$	The average inventory of the product
TC_S	The setup cost per unit time
TC_P	The total processing cost per unit time
TC_H	The expected total holding cost per time unit
W	The average waiting time for defective units for a given value of random variable R
$E(W)$	The expected value of W
TC_K	the expected total waiting time cost per unit time
$TC(Q)$	The expected total cost of the system per unit time

4. THE MODEL

This paper deals with the issue of economic batch quantity (*EBQ*) in a single machine system in which defective items are produced in each cycle of production. We assume that the accumulated defective items produced in a period τ, consisting of $N+1$ cycle each with length T, are all reworked in the $(N+1)^{th}$ cycle, last cycle of τ, known as the rework cycle. At the end of each period τ the whole process will start all over again. In cycle $N+1$, the items are produced for t_1 units of time during which time both good and defectives items are produced. At the end of t_1 the rework of defective items starts (Figure 1). We suppose in this cycle during t_2 units of time (Figure 2), the rework of all accumulated defectives of the period starts with a random rate R and are completed before the end of the end of the cycle. t_2 .

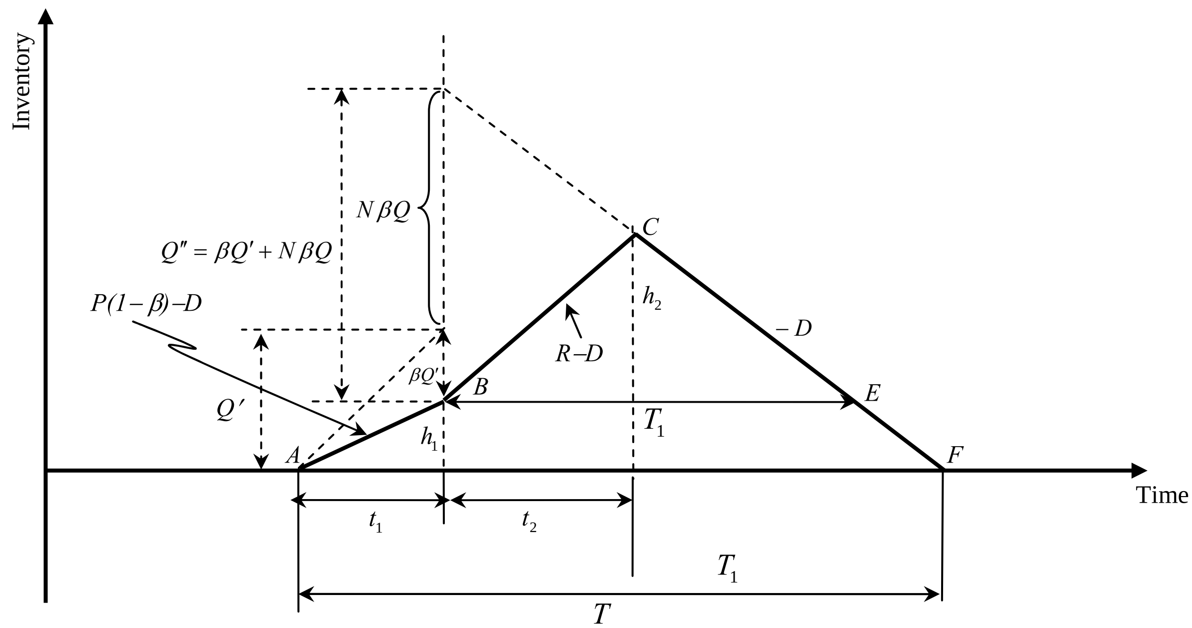

Figure 2 The inventory in the last cycle of τ

The rate of production of non-defective items during production in the first N cycles and during time t_1 in the last cycle is P, Figure 1. During t_2, the rate of production of non-defective items is R (Figure 2). Let $Q' =$ the total production in t_1, then from Figure 2 it is clear that

$$Q' = Pt_1 \tag{1}$$

and

$$Q' + N\beta Q = DT \tag{2}$$

The length of the last cycle of τ is equal to

$$T_{N+1} = \frac{Q' + N\beta Q}{D}$$

The total non defective units that are needed to satisfy the demand during each cycle with length T is equal to DT. Clearly the total units of non defective units produced in each cycle T is equal to Q (1-β). Thus Q (1-β) = DT .Therefore the length of each cycle of the first N cycle of τ is

$$T = \frac{Q(1-\beta)}{D} \tag{3}$$

Note that for $N > \frac{1-\beta}{\beta}$ one can show that $T_{N+1} > T$. Since we require that as in Haji B. et al (2008) $T_{N+1} = T$, N must be the largest integer equal or less than $\frac{1-\beta}{\beta}$. When $\frac{1-\beta}{\beta}$ is not an integer we let $N = \left[(1-\beta)/\beta\right]$, where $\left[(1-\beta)/\beta\right]$ is the largest integer equal or less than $\frac{1-\beta}{\beta}$. In this case, the relation $T_{N+1} = T = \frac{Q(1-\beta)}{D}$ holds true only if we have some normal production, in cycle $N+1$, for a time t_1 during which defective and non-defective items are produced (Figure 2). Thus from (2) and (3) we can write $Q' = Q(1-\beta) - N\beta Q$ or

$$Q' = Q[1-\beta(N+1)] \tag{4}$$

Since the rate of production during t_1 is P, we can write $t_1 = Q'/P$ and from (1) and (4) we have

$$t_1 = \frac{Q'}{P} = \left(1-\beta-N\beta\right)\frac{Q}{P} \tag{5}$$

Now note that from Figure 2 we can write $DT_1 = Q''$ or

$$T_1 = \frac{Q''}{D} \tag{6}$$

In which

Q'' = the total number of defectives produced during τ and is equal to:

$$Q'' = \beta Q' + N\beta Q = \beta Q(1-\beta)(1+N) \tag{7}$$

Thus, from (6) and (7) we have

$$T_1 = \frac{\beta Q(1-\beta)(1+N)}{D} \tag{8}$$

Setup costs

The setup cost of the system for a period τ, denoted by $TC_S(\tau)$, can be written as $TC_S(\tau) = A(N+1)$ and the setup cost per unit time, denoted by TC_S, is

$$TC_S = \frac{A(N+1)}{\tau} = \frac{A(N+1)}{(N+1)T}$$

Or from (3)

$$TC_S = \frac{A}{Q(1-\beta)} \tag{9}$$

Processing Costs

The total processing cost per unit time during normal production is CD and during rework process is $\beta C_R D$. Thus the total processing cost per unit time, denoted by TC_P, is

$$TC_P = C\left(1+\beta C_R\right)D \tag{10}$$

Expected Inventory holding cost

To obtain the expected inventory cost let:

$\bar{I}_1 =$ the average inventory in the first N cycle of τ, and $\bar{I}_2 =$ the average inventory in the last cycle of τ. Then From Figure 1 the average inventory during τ can be written as:

$$\bar{I} = \frac{\bar{I}_1 NT + \bar{I}_2 T}{(N+1)T} = \frac{N\bar{I}_1 + \bar{I}_2}{N+1} \tag{11}$$

Also from Figure 1 we can write

$$\bar{I}_1 = \frac{Q}{2P}\left[P\left(1-\beta\right)-D\right] \tag{12}$$

Now, in Figure 2, let for a given value of the random variable R

$S_1 = \frac{1}{2}h_2T_1$ = the area of triangle BCE with height h_2 and base T_1,

$S_2 = \frac{1}{2}h_1(T+T_1)$ = the area of trapezoid ABEF with height h_1 and two parallel sides T_1 and T, and $S = S_1 + S_2$.

Then the average inventory for a given value of the random variable R during the last cycle of τ can be written as:

$$\bar{I}_2 = \frac{E\left(S\right)}{T} = \frac{E\left(S_1\right)+E\left(S_2\right)}{T} \tag{13}$$

Where $E(x)$ is the expected value of x. To obtain $E(S)$ we first calculate $E(S_1)$ and $E(S_2)$. It is clear from Figure 2 that for a given value of the random variable R

$$h_2 = (R-D)t_2 \tag{14}$$

Now from Figure 2

$$t_2 = \frac{Q''}{R} \tag{15}$$

Thus for a given value of the random variable *R* we can rewrite (14) as

$$h_2 = (R - D)\frac{Q''}{R} \tag{16}$$

From (8) the area of triangle BCE, for a given value of the random variable *R*, can be written as $S_1 = \frac{1}{2}h_2T_1 = \frac{(R-D)}{2RD}Q''^2$, and from (7) one can write

$$S_1 = \frac{(R-D)\beta^2Q^2(1-\beta)^2(1+N)^2}{2RD} \tag{17}$$

To calculate the area of the trapezoid ABEF one can use (3) and (8) to write

$$S_2 = \frac{1}{2}h_1(T+T_1) = \frac{1}{2}h_1\left(\frac{Q(1-\beta)}{D} + \frac{\beta Q(1-\beta)(N+1)}{D}\right) \tag{18}$$

From (5) and Figure 2 we can write

$$h_1 = \left(P\left(1-\beta\right)-D\right).t_1 = \left(P\left(1-\beta\right)-D\right)\left(1-\beta-N\beta\right)\frac{Q}{P} \tag{19}$$

Substituting (19) in (18) yields S_2

$$S_2 = \frac{(P-\beta P-D)\left[1-\beta^2(N+1)^2\right]Q^2(1-\beta)}{2PD} \tag{20}$$

Clearly S_2 is independent from *R*. Using (3) for a given value of the random variable *R* we can write $S_1$and S_2 as

$$S_1 = \frac{1}{2}(1-\frac{D}{R})\beta^2Q(1-\beta)(1+N)^2\,T \tag{21}$$

$$S_2 = \frac{(P-\beta P-D)\left[1-\beta^2(N+1)^2\right]Q}{2P}T \tag{22}$$

Now the expected value of S_1 is $E(S_1) = \frac{1}{2D}\left[1-E(\frac{1}{R})\beta^2Q^2(1-\beta)^2(1+N)^2\right]$.

From (13) and the fact that $E(S) = E(S_1) + E(S_2)$ we have

$$\bar{I}_2 = \frac{E(S)}{T} = \frac{Q}{2P}\left\{\beta^2(N+1)^2D\left[1-E[\frac{1}{R}]P(1-\beta)\right]+P-\beta P-D\right\} \tag{23}$$

Now from (11), (12), and (23) the average inventory for a given value of the random variable R is $I=\frac{Q}{2P}\left\{\beta^2(N+1)D\left[1-E[\frac{1}{R}]P(1-\beta)\right]+P-\beta P-D\right\}$

Thus the expected total holding cost per time unit, denoted by TC_H, is

$$TC_H=HI=H\frac{Q}{2P}\left\{\beta^2(N+1)D\left[1-P(1-\beta)E[\frac{1}{R}]\right]+P-\beta P-D\right\} \tag{24}$$

Expected waiting time cost

To find the expected waiting time cost of all defective units per unit time we first obtain the average waiting time of a defective item (for rework) produced in cycle *i, i=1,…,N,* until the beginning of time period t_2 in the last cycle (Figure 2). Let

W_i =the average waiting time of a defective item produced in cycle *i, i=1,…,N,* until the end of the time period t_1 (beginning of the time period t_2) in the last cycle. Then we can write

$$W_i=\frac{1}{2}T_p+T_d+(N-i)T+t_1$$

From Figure 1 we have $T=T_p+T_d$ .Thus we can write W_i as

$$W_i=(N-i+1)T-\frac{1}{2}T_p+t_1 \tag{25}$$

From (3), (5), and the fact that $T_p=\frac{Q}{P}$ we can write W_i as

$$W_i=(N-i+1)T-\frac{DT}{2(1-\beta)P}+\frac{\left[1-\beta(N+1)\right]DT}{P(1-\beta)}$$

The total number of defectives that are produced in each cycle of the first N cycle of τ is equal to βQ .Thus, the total waiting time for all defective units produced in the first N cycle of τ until the beginning of time period t_2 denoted by W_a is $W_a=\sum_{i=1}^{N}(\beta Q)W_i$ or

$$\begin{aligned} W_a &= \beta Q\left[\frac{N(N+1)}{2}T-\frac{NDT}{2P(1-\beta)}+\frac{N\left[1-\beta(N+1)\right]DT}{P(1-\beta)}\right] \\ &= \frac{N\beta Q}{2}\left[1-\frac{D\left[1-2\left[1-\beta(N+1)\right]\right]}{P(1-\beta)(N+1)}\right]T(N+1) \end{aligned} \tag{26}$$

It is clear that W_a is independent of the random variable R. Let:

$W^1_{N+1} =$ Total waiting time of defective units that are produced during t_1 in cycle N+1 until the start of rework process in the same cycle. Then, from Figure 2 we can write

$$W^1_{N+1} = \frac{1}{2}\beta Q' t_1 \tag{27}$$

From (3), (4), and (5), we can write (25) as $W^1_{N+1} = \frac{\beta Q^2}{2P}\left[1-\beta(N+1)\right]^2$

Or from (3)

$$W^1_{N+1} = \frac{N\beta Q}{2P}\left[\frac{\left[1-\beta(N+1)\right]^2}{N(1-\beta)}\right]DT \tag{28}$$

To obtain the total waiting time of all defective units (Q'' units) produced in the first N cycle of τ we need to find the waiting time of these defectives during time period t_2 in cycle $N+1$, (Figure 2). Let

$W^2_{N+1} =$ Total waiting time of all defective units during time period t_2 in cycle $N+1$
Now from Figure 2 we can write

$$W^2_{N+1} = \frac{1}{2}Q'' t_2$$

Or from (7) and (15) for a given value of the random variable R

$$W^2_{N+1} = \frac{\beta^2 Q^2 (1-\beta)^2 (N+1)^2}{2R}$$

Thus from (3) we can rewrite the above relation as

$$W^2_{N+1} = \frac{\beta^2 Q(1-\beta)(N+1)^2}{2R}DT \tag{29}$$

Thus, the total waiting time of all defective units, denoted by TW, for a given value of the random variable R is $TW = W_a + W^1_{N+1} + W^2_{N+1}$ That is,

$$TW = \frac{N\beta Q}{2}\left[1-\frac{D\left[1-2\left[1-\beta(N+1)\right]\right]}{P(1-\beta)(N+1)}\right]T(N+1) + \frac{\beta^2 Q(1-\beta)(N+1)^2}{2R}DT + \frac{N\beta Q}{2P}\left[\frac{\left[1-\beta(N+1)\right]^2}{N(1-\beta)}\right]DT$$

The average waiting time for defective units for a given value of the random variable R is $W = \frac{TW}{\tau}$ or from the above relation

$$W = \frac{TW}{\tau} = \frac{\beta Q}{2}\left[\frac{N}{D} + \frac{\beta(1-\beta)(N+1)}{R} + \frac{1+(N+1)(\beta^2-2\beta)}{P(1-\beta)}\right]D$$

The expected value of this random variable is

$$E(W) = \frac{TW}{\tau} = \frac{\beta Q}{2}\left[\frac{N}{D} + E(\frac{1}{R})\beta(1-\beta)(N+1) + \frac{1+(N+1)(\beta^2-2\beta)}{P(1-\beta)}\right]D$$

Now we can write the expected total waiting time cost per unit time, denoted by TC_K , as

$$TC_K = KE\left(W\right)$$

Replacing $E\left(W\right)$ in the above relation we have

$$TC_K = K\frac{\beta Q}{2}\left[\frac{N}{D} + E(\frac{1}{R})\beta(1-\beta)(N+1) + \frac{1+(N+1)(\beta^2-2\beta)}{P(1-\beta)}\right]D \tag{30}$$

Expected total system costs

The expected total cost of the system per unit time is the sum of setup cost, inventory holding cost, waiting time cost of defective units for rework, and processing costs, that is

$$TC(Q) = TC_S + TC_P + TC_H + TC_K \tag{31}$$

Thus, from (9), (10) (24), (30), and (31) we can write

$$\begin{aligned} TC(Q) = {} & \frac{DA}{Q\left(1-\beta\right)} + C\left(1+\beta\right)D + \frac{HQ}{2P}\left\{\beta^2(N+1)D\left[1-E(\frac{1}{R})P(1-\beta)\right] + P - \beta P - D\right\} \\ & + K\frac{\beta Q}{2}\left[\frac{N}{D} + E(\frac{1}{R})\beta(1-\beta)(N+1) + \frac{1+(N+1)(\beta^2-2\beta)}{P(1-\beta)}\right]D \end{aligned}$$

5. THE OPTIMAL BATCH QUANTITY

The first derivative of $TC(Q)$ with respect to Q is

$$\begin{aligned} \frac{dTC(Q)}{dQ} = {} & -\frac{DA}{Q^2\left(1-\beta\right)} + \frac{H}{2P}\left\{\beta^2(N+1)D\left[1-E(\frac{1}{R})P(1-\beta)\right] + P - \beta P - D\right\} \\ & + K\frac{\beta}{2}\left[\frac{N}{D} + E(\frac{1}{R})\beta(1-\beta)(N+1) + \frac{1+(N+1)(\beta^2-2\beta)}{P(1-\beta)}\right]D \end{aligned}$$

One can easily show that the second derivative of $TC(Q)$ with respect to Q is positive.
Let Q^* be that value of Q which minimizes $TC(Q)$. To obtain Q^* we let the first derivative of $TC(Q)$ equal to zero. Therefore,

$$Q^* = \left(\frac{DA}{(1-\beta)}\right)^{\frac{1}{2}} \left\{ \frac{H}{2P} \left\{ \begin{array}{l} \beta^2 (N+1)D\left[1-E(\frac{1}{R})P(1-\beta)\right] \\ +P-\beta P - D \end{array} \right\} + K\frac{\beta}{2} \left[\begin{array}{l} \frac{N}{D} + E(\frac{1}{R})\beta(1-\beta)(N+1) \\ + \frac{1+(N+1)(\beta^2-2\beta)}{P(1-\beta)} \end{array} \right] D \right\}^{-\frac{1}{2}} \tag{32}$$

Remarks

a) It is interesting to note that (32) shows for obtaining the *EBQ* we do not need the distribution function of the random rate of rework, only knowledge of the expected value of the reciprocal of this random variable is sufficient.

b) When *R* is not a random variable and it is equal to *P* and we ignore the waiting time cost of the defective units (*K*= 0) which is an unrealistic assumption in most practical situation and let *N* = 0 (32) reduces to

$$Q^* = \left(\frac{2PDA}{(1-\beta)H\left(P-D-\beta P+\beta^3 D\right)}\right)^{\frac{1}{2}},$$

This can be easily shown to be the same result as obtained by Jamal et al (2004)[7] for their first policy, immediate rework process. For $\beta = 0$, (32) will reduce to the classical economic batch quantity formula

$$Q^* = \sqrt{\frac{2DA}{H(1-\frac{D}{P})}}$$

6. CONCLUSION

In this paper we considered an imperfect production system in which defective items are produced and the rework of these defectives is possible. We considered the case in which defective items produced in a period consisting of several equal cycles. We assumed that all defective items are accumulated and reworked in the last cycle. At the end of this period the process starts all over again. We studied the case in which for ease of production scheduling the rework cycle has to have the same length as the other cycles. We did not restrict the rework rate to have deterministic value and assumed that it is a random variable with an arbitrary distribution function. Further, due to the high change over costs of going from normal to rework process we assumed that the number of rework cycles per unit time to be as small as possible. For this system we derived the expected total cost function of the system consisting of inventory holding, waiting time of defectives for rework, setup, and processing costs per unit time. Then we obtained the economic batch quantity (*EBQ*) which minimizes this cost. The interesting result showed that to obtain the *EBQ* we do not need the distribution function of the random rate of rework, only knowledge of the reciprocal of this random variable is sufficient.

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