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Dynamic Pricing with Periodic Review and a Finite set of Prices with Cancellation

Mohammad Modarres[1*], Ehsan Bolandifar[2]

[1] Industrial Eng. Dept., Sharif University of Technology, Tehran, Iran
Modarres@sharif.edu

ABSTRACT

In this paper, three dynamic pricing models are developed and analyzed. We assume a limited number of a particular asset is offered for sale over a period of time. This asset is perishable and can be an inventory or a manufacturing capacity. During each period, the seller sets a price for this asset. This price is selected from a predetermined discrete set. The maximum amount which a customer is willing to pay is called "reservation price". Different customers have different reservation prices. The distribution function of the reservation prices for all potential customers is known. Demands arrive according to a nonhomogeneous Poisson process. To maximize the expected revenue, the price of this asset is controlled periodically, as sales evolve. Demand cancellation is also considered. Furthermore, we study the effect of cancellation as well as setting a sale limit for each period. The analysis of the models indicates that their properties are different from those of the basic models studied previously. By randomly generated examples, we show that the properties of "Inventory Monotonocity" and "Time Monotonocity" do not hold in our models, while these properties hold for continuous price review models.

***Key words*:** Dynamic pricing; Periodic review; Cancellation; Dynamic programming.

1. INTRODUCTION

Although the introduction of a generic model of dynamic pricing goes back to Kincaid and Darling (1963), this interesting and valuable topic was not seriously studied by researchers for more than three decades. However, during the same period, many industries such as retailers were applying and experiencing it. Thus, its extensive applications by many practitioners and firms motivated researchers to resume it thorough study. Gallego and Van Ryzin (1994) studied the structural properties of optimal policies of dynamic pricing. Their work was extended later by the others, such as Bitran and Mondeschein (1997), Zhao and Zheng (2000) and Chatwin (2000).

Dynamic pricing arises in a variety of industries, which produce either manufactured goods with short shelf life (e.g. seasonal goods) or service products (e.g. flight seats). Actually, many industries including retailers have the chance of enhancing their revenue by applying dynamic pricing techniques to the pricing of their products. The readers are referred to Bitran and Caldenety (2002) and Elmaghrabi and Keskinocak (2003) for a thorough discussion of the dynamic pricing models.

[*] Corresponding Author

In dynamic pricing, the price of a given asset can be changed within the pricing periods in order to maximize the total revenue. Demand for a given inventory (or a manufacturing capacity) which has to be sold in a finite time horizon is stochastic and depends on its price as well as on the pricing period. The objective is to determining the optimal trade off between the following two types of potential losses in order to achieve the optimal total expected revenue:

- Yield loss-selling at a low price and losing the chance of a better price later.

- Spoilage loss-waiting to sell at a higher price and losing the opportunity of an earlier low price offer.

The following structural properties of the optimal policy were derived by Gallego and Van Ryzin(1994), provided the demand is homogeneous:

Inventory Monotonocity: at any given time, the optimal price decreases with respect to the number of items left.

Time Monotonocity: with any given number of items, the optimal price decreases over time. Gallego and Van Ryzin(1994) also generalized their results for nonhomogeneous demand when the ratio of demand intensities under any two prices remains unchanged over the sale horizon. Zhao and Zheng (2000) also proved the existence of inventory monotonocity by assuming that the intensity of customer arrival and the reservation price distribution may change over time. Similar to the model of Zhao and Zheng(2000) and with an assumption that the customers arrival rate decreases in time and under a discrete set of prices, Chatwin (2000) showed inventory and the time monotonocity properties are held.

Feng and Gallego (1995) studied a two-price model that allows a single price change. They obtained an optimal threshold control policy. The models by Feng and Xiao(2000a,b) and Chatwin (2000) draw an optimal control policy with a threshold structure, when the price set is discrete and controlled continuously over the pricing periods, while the number of price changes is not more than the number of prices in the price set. Bitran and Mondeschein (1997) considered periodic pricing policies, where price can be modified at K pre-determined times during the pricing periods. According to the computational experiments, there is only a small gap (less than 2.2%) between continuous and periodic pricing policies.

In this paper, we assume the firm reviews its pricing policies periodically at given point of times and the price is also selected from a finite set. Most of studies in dynamic pricing assume the price can be selected from a continuous interval. However, this is not always practical. For strategic as well as practical reasons, many firms restrict the price choices within a small discrete set. For example there is a preference to set a price as $9.99 rather than $10 or any other price within the range of [$9.5, $10.5]. Furthermore, in practice firms prefer to revise their prices periodically during the sale horizon and not continuously. Another reason to review the price change periodically is its hidden costs, which are not explicitly considered in the continuous models. There are also some other reasons, such as seasonal occasions only.

In this paper, we propose three models. The first one, (called *Basic Model*) discusses periodic pricing model in which a price has to be set for the product in each period. The length of each period is given at the beginning of the pricing periods. This model is similar to that of Bitran and Mondschein (1997), except that the price set is discrete.

The second model considers the possibility of order cancellation by customers. In this model, it is assumed that the refund rate of cancellation is independent of the purchasing price. Finally, in the third model the refund rate is assumed to be dependent on the purchasing price.

Our findings show that in these models, none of the structural properties, *i.e*. inventory or time monotonocity and concavity of the expected revenue function hold, even when the reservation price distribution is constant over the sale horizon. It is observed that setting a limit on the number of the products sold in each period does not have a considerable effect on the expected revenue in both models.

The remainder of the paper is organized as follows. In Section 2, the models and the appropriate solution approach are presented. In Section 3, we study the optimal properties of these models. Numerical experiments of the models are discussed in Section 4. Finally, in Section 5 some directions of future research and conclusion are given.

2. THE MODELS

A stock of C units of an inventory (or manufacturing capacity) must be sold within a period of T units of times, called "sale horizon". Otherwise, the remaining stock units have no value afterwards. The total pricing period is divided into N discrete intervals, called "pricing periods". The periods are indexed by $t \in \{0,1,2,...,N\}$ and run backward in time. Following this notation, period 0 represents the deadline of the sale. The proposed price and the maximum number of capacity units that can be sold to customers in each period must be controlled by the model dynamically. The following assumptions hold.

a) In each pricing period, a price is selected from a predetermined set of $S_p = \{p_1, p_2, ..., p_K\}$. A customer who arrives in that period purchases one unit, provided that the price is less the reservation price of that customer. Otherwise, he refuses to buy.

b) The number of customers who arrive during i^{th} pricing period is a random variable that follows a nonhomogeneous Poisson process with rate of λ_t, $t \in [0,T]$. Each customer demands one unit.

c) Each customer has a "reservation price", defined as the maximum amount that customer is willing to pay for one unit of stock, which also depends on the pricing period number. Since customers have different reservation prices, then $f_i(.)$ is defined as the density function of the reservation price of the customers arriving during i^{th} pricing period. Thus, if in a pricing period the price is set as p, then only the customers whose reservation price in that period is at least p, are willing to purchase. In other words, $F_i(p)$ represents the probability that a random customer arriving during i^{th} pricing period declines purchasing one unit if the price is more than p. Therefore, the rate of purchasing is $\lambda_i[1 - F_t(p)]$, $t \in [0,T]$.

d) Let $m_i(p_k)$ represent the average purchasing rate during the i^{th} pricing period in the interval of $[T_i, T_{i-1}]$, provided that the price is set equal to p_k. Furthermore, let $X_i(p_k)$ be the number of the customers that purchase in this pricing period. Then,

$$m_i(p_k) = \int_{T_i}^{T_{i-1}} \lambda_t (1 - F_t(p_k))dt$$

and,

$$\Pr[X_i(p_k) = j] = \frac{e^{-m_i(p_k)} m_i(p_k)^j}{j!}$$

2.1. Basic model with sale limits

In this model, by setting a limit for the total sale of each pricing period the firm can keep some inventories for the future sale. This is justified if the strategy of the firm is to achieve an acceptable service level by avoiding stockout in future periods. This policy may cause the demand exceeds the assigned stocks during a pricing period, which depends on the proposed price of that period. On the other hand, since the price which is set at the beginning of a pricing period can not be revised again during the same period, it is necessary to determine an appropriate level of the stocks to be assigned for sale in that period.

We apply dynamic programming technique to obtain the optimal solution of this problem. The elements of dynamic programming model are defined as follows:

(c): the state of system representing the number of available stocks at the beginning of this pricing period.

$V_i(c)$: the maximal expected revenue generated from pricing period i till the end of the sale horizon, if the state of the system is (c).

$V_i(c, p_k, b_i)$: the maximum expected revenue generated from period i till the end of sale horizon, if the state of the system is (c), the price is set equal to p_k and b_i is the maximum number of stocks allowed to be sold in this pricing period, where $0 \le b_i \le c_i \le C$.

Then, the recursive equation for pricing period i is as follows:

$$V_i(c_i, p_k, b_i) = \sum_{j=0}^{b_i} \Pr[X_i(p_k) = j].[jp_k + V_{i-1}(c_i - j)] + \Pr[X_i(p_k) > b_i].[b_i p_k + V_{i-1}(c_i - b_i)]$$

Boundary conditions:

$$V_i(0) = 0, \forall i, \qquad V_i(c) = 0, \forall c \tag{1}$$

Obviously, in each pricing period:

$$V_i(c_i) = Max\{V(c_i, \ p_k, \ b_i)\} \tag{2}$$

This dynamic formulation can be solved backward in pricing periods. For each stage and state, the optimal price and sale limit can be found easily.

2.2. Basic model with sale limits and cancellation (independent refund rate)

In some cases, customers are allowed to cancel their purchases. In that case, they are paid back with some penalties. The following assumptions are added to the previous ones of the basic model.

- Cancellations occur only at the beginning of each pricing period.
- The returned products can be sold again.
- The penalty (and consequently the returned money to the customers) is independent of the purchase price and is determined by the firm, at the beginning of each pricing period.

Following the notation of the basic model, let $Y = (Y_{i1}, Y_{i2}, ... - Y_{iK})$, where Y_{ik} indicates the number of customers who have purchased the product during ith pricing period. We assume that the distribution function of Y_i is known. Clearly, the number of cancellations cannot be more than the number of the sold units.

Following the notation introduced for the basic model,

$$V_i(c_i, p_k, b_i) = \sum_{m=0}^{C-c_1} \Pr[Y_i = m].[\sum_{j=0}^{b_i} \Pr[X_i(p_k) = j].(jp_k + V_{i-1}(c_i + m - j) - ms_i) + \Pr[X_i(p_k) > b_i].(b_i p_k + V_{i-1}(c_i + m - b_i) - ms_i)] \qquad \text{if} \quad b_i \le c_i$$

and

$$\begin{aligned} V_i(c_i, p_k, b_i) = & \sum_{m=0}^{b_i - c_i} \Pr\{Y_i = m\} \times [\sum_{j=0}^{c_i+m} \Pr\{X_i(p_k) = j\} \times (jp_k + V_{i-1}(c_i + m - j) - ms_i) + \\ & + \Pr\{X_i(p_k) > c_i + m) \times ((c_i + m) p_k + V_{i-1}(0) - ms_i)] + \\ & \sum_{m=b_i - c_i + 1}^{C} \Pr\{Y_i = m\} \times [\sum_{j=0}^{b_i} \Pr\{X_i(p_k) = j\} \times (jp_k + V_{i-1}(c_i + m - j) - ms_i) + \\ & \Pr\{X_i(p_k) > b_i\} \times (b_i p_k + V_{i-1}(c_i + m - b_i) - ms_i)] \qquad \textit{if} \quad c_i \le b_i \le C \end{aligned} \tag{3}$$

Like the previous model, the optimal pricing policy can be determined by solving this dynamic programming formulation backward in periods.

2.3. Basic Model with Sale limits and cancellation (dependant refund rate)

In the previous model, it was assumed the refund rate for cancellation is predetermined and independent of the purchase price. Now, by relaxing this assumption, the third model is developed.

Let $S'_p = \{p'_1, p'_2, ..., p'_K\}$ be the refund rate, where p'_k represents the refund rate if the customer has paid p_k to purchase one stock unit. Without loss of generality, we assume p'_k is independent of the pricing period and depends only on p_k. This set is determined by the firm at the beginning of the sale horizon. The elements of dynamic programming model are as follows.

$x_{i1}, x_{i2}, ..., x_{ik}$: the number stocks sold in previous periods with prices $p_1, p_2, ..., p_{1k}$, respectively.
$V_i(x_{i1}, x_{i2}, ..., x_{ik})$: the maximal expected revenue generated from period i till the end of sale horizon, provided the state of the system is $x_{i1}, x_{i2}, ..., x_{ik}$.

$V_i(x_{i1}, x_{i2}, ..., x_{ik}, b_i, p_k)$: the maximal expected revenue generated from period i till the end of sale horizon, provided the state of the system is $x_{i1}, x_{i2}, ..., x_{ik}$, p_k is the price and b_i is the sale limit for the i th period. Two cases may happen.

If $b_i \le C - x_{i1} - x_{i2} ... - x_{iK}$, then, the recursive equation is as follows.

$$V_i(x_{i1}, x_{i2}, ..., x_{ik}, b_i, p_k) = \sum_{y_1+...+y_K \le x_{i1}+...+x_{iK}} \Pr\{Y_{i1} = y_1, ..., Y_{iK} = y_K\} \times [\sum_{j=0}^{b_i} \Pr\{X_i(p_k) = j\} \times$$

$$(jp_k - y_1 p'_1 - ... - y_K p'_K + V_{i-1}(x_{i1} - y_1, ..., x_{ik} - y_k + j, ..., x_{iK} - y_K)) + \Pr\{X_i(p_k) > b_i)$$

$$\times (b_i p_k - y_1 p'_1 - ... - y_K p'_K + V_{i-1}(x_{i1} - y_1, ..., x_{ik} - y_k + b_i, ..., x_K - y_K)]$$

However, if $C - x_{i1} - x_{i2} - ... - x_{iK} \le b_i \le C$, then,

$$V_i(x_{i1}, x_{i2}, ..., x_{iK}, b_i, p_k) = \sum_{y_1+...+y_K \le b_i - (C - \sum_{l=1}^{K} x_{il})} \Pr\{Y_{i1} = y_1, ..., Y_{iK} = y_K\} \times [\sum_{j=0}^{C - \sum_{l=1}^{K} x_{il} + \sum_{l=1}^{K} y_l} \Pr\{X_i(p_k) = j\} \times$$

$$(jp_k - y_1 p'_1 - ... - y_K p'_K + V_{i-1}(x_{i1} - y_1, ..., x_{ik} - y_k + j, ..., x_{iK} - y_K)) + \Pr\{X_i(p_k) > C - \sum_{l=1}^{K} x_{il}$$

$$+ \sum_{l=1}^{K} y_l) \times ((C - \sum_{l=0}^{K} x_{il} + \sum_{l=0}^{K} y_l) p_k - y_1 p'_1 - ... - y_K p'_K + V_{i-1}(x_{i1} - y_1, ..., x_{ik} - y_k +$$

$$(C - \sum_{l=0}^{K} x_{il} + \sum_{l=0}^{K} y_l), ..., x_{iK} - y_K) +$$

$$\sum_{b_i - (C - \sum_{l=0}^{K} x_{il}) \le \sum_{l=0}^{K} y_l} \Pr\{Y_{i1} = y_1, ..., Y_{iK} = y_K\} \times [\sum_{j=0}^{b_i} pr\{X_i(p_k) = j\} \times (jp_k$$

$$- y_1 p'_1 - ... - y_K p'_K + V_{i-1}(x_{i1} - y_1, ..., x_{ik} - y_k + j, ..., x_{iK} - y_K)) + \Pr\{X_i(p_k) > b_i\} \times$$

$$(b_i p_k - y_1 p'_1 - ... - y_K p'_K + V_{i-1}(x_{i1} - y_1, ..., x_{ik} - y_k + b_i, ..., x_{iK} - y_K))],$$

Where $0 \le x_{i1} + x_{i2} + ... + x_{iK} \le C$. In each pricing period,

$$V_i(x_{i1}, ..., x_{iK}) = \underset{b_i, p_k}{Max}\{V_i(x_{i1}, ..., x_{iK}, b_i, p_k)\}$$

Like the previous models, the optimal pricing policy can be found by solving the dynamic programming model backward in period. In the next section, we study the structural properties of the first and second models.

3. STRUCTURAL PROPERTIES

In this section, we investigate the structural properties of the models introduced in previous section. To do that, 1000 examples were examined. The parameters of the models were generated randomly from uniform distribution within the following ranges.

The number of sale periods, [2, 10];

the starting inventory, [10, 50];

maximum price, [10, 50];

prices, [1, the maximum price];

reservation price , [0, the maximum price+1];

average rate of customers, [5, 10].

In our numerical examples we assume that s_i is equal to $\alpha_i p_k$.

In each pricing period, the optimal price as well as the optimal sale limit is obtained through equation (1) and the boundary conditions are represented by (2).

3.1. Basic model with sale limit

The structural properties of the first model are derived from the randomly generated examples and are described in more detail in the following subsections.

3.1.1. Concavity of the revenue function in inventory

It has been proved that the revenue function of continuous dynamic pricing models where the price is revised continuously in time is concave with respect to inventory, for example Zhao and Zheng (2000) and Chatwin (2000). In the model by Zheng (2000) and Chatwin (2000) the price set is a continuous interval while in the model by Chatwin (2000) the set of prices is discrete.

In our first model, our numerical experiments indicate that this property does not hold. As one can observe from Table 1, there exist some contradictions in concavity of the revenue function, although these contradictions are not significant. In the first set of numerical experiments we have studied, out of 174881 random generated examples, 14 contradictions in the concavity of the revenue function were observed.

3.1.2. Time monotonocity

In the first set of numerical examples to check the property of "time monotonocity", no contradiction was observed. However, in the second set 163 contradictions for this property were observed. The only difference between these two sets is their average arrival rate of customers. As one can see, only in *0.1%* of examples this property does not hold.

3.1.3. Inventory monotonocity

No contradiction in "inventory monotonocity" and "time monotonocity" properties was observed in the first and second set of numerical examples. However, in the third set, 21 contradictions in inventory monotonocity were observed. In this set of numerical examples, the average arrival rate of customers in each period is between 0 and 50. Although, in reality it is impossible to have such disperse rates of arrival in consecutive periods, there are some contradiction for this property.

Table 1 Results of numerical examples

Optimal properties	Concavity	Inventory Mo.	Time Mo.	total per-inv
example set 1	14	0	0	174881
example set 2	72	0	163	174881
example set 3	5522	21	338	174881
example set 4	36957	10529	8327	145741

3.2. Basic model with Sale limits and cancellation

In order to study the optimal properties of the second model, we generated the forth set of numerical examples. In this set, the probability of cancellation as well as the refund rate was generated randomly. The other characteristics of this set are the same as the first set of numerical examples. In the second model, the probability distribution of the number of cancelled purchases in each period is assumed to be binomial. Thus, the probability depends on the number of sold items. Accordingly, we have studied the optimal properties for a fixed initial inventory. In other words, the optimal solution depends not only on the on-hand inventory but on the number of sold items.

According to Table 1, in addition to the on-hand inventory, none of the optimal properties holds in the model. In the forth set of numerical examples, we find that unlike the first one, lots of contradictions to the optimal properties of the continuous models are observed. Numerical examples show that in 25% of period-inventories the revenue function is not concave. In the first model, with no cancellation, this percentage was about 0.008%. On the other hand, 7.2% and 5.7% of studied period-inventories the inventory and time monotonocity properties do not held, respectively.

4. NUMERICAL EXPERIMENTS

In this section, we present a set of computational experiments that shows the optimal price path in our models. As a benchmark for our first model we use the continuous time dynamic pricing model with discrete set of prices, presented by Chatwin (2000).

4.1. Example 1

Consider a firm selling its inventory within a period of 35 days. The arrival rate of customers has a nonhomoheneous Poisson distribution with the rate of $\lambda_t = t/18$.

The price set is $S_p = \{10,11,...,24,25\}$. The firm plans to review its pricing policy only at the beginning of each week of the sale horizon. The reservation price of a customer has a uniform distribution within the interval of $[0,30]$ and it is independent of the time of sale.

Table 2 shows the expected revenue earned from the first model and compared with the expected revenue of the continuous dynamic pricing model, based on the initial inventory at the beginning of the sale horizon. The greatest difference between the expected revenue of these models is 0.99%. As the initial inventory increases, the difference between two models increases first and then decreases.

Table 2 Difference between the discrete and continuous time models

Inventory	**5**	**10**	**15**	**20**	**25**	**30**
Expected revenue in continuous time model	115.55	191.74	233.57	250.52	254.68	255.21
Expected Revenue in the first model	114.83	189.84	231.96	249.86	254.55	255.17
Difference	0.72	1.91	1.61	0.66	0.14	0.03
Percentage	0.63%	0.99%	0.69%	0.26%	0.05%	0.01%

4.2. Example 2

We assume that the probability distribution of the number of cancelled purchases in each period has a binomial distribution with $C - c_i$ and 0.05, where $C - c_i$ is the number of sold inventories at the beginning of the i th period, 0.05 is the cancellation probability of any sold item in each period and 0.9 is the refund rate. The other characteristics are the same as the first example.

Price path through the periods based on inventory

Figure1 shows the price path with respect to the pricing periods for the first example. These price paths for the inventories 5, 10, 15 and 20 are depicted. As observed, the optimal price decreases with respect to period number and inventory. However, as was shown in the previous section this is not true for all cases and there are some contradictions as far as these properties are concerned. Although, according to our numerical experiments in most of practical cases these properties hold.

Figure 2 shows the optimal price paths with respect to sale periods, for different initial inventory of the second example. As it is depicted, the price paths in our two models are similar. The initial price in the second model is less than or equal to that of the first model, in most cases. However, as Figure 1 shows with inventory of 5 at the beginning of the pricing periods the optimal price in the second model is greater than that of the first model. Therefore, the optimal price is not necessarily decreasing. Although we have proposed a contradictory example, our numerical examples show that in most cases the firm must increase the price if cancellation is allowed.

Figure 3 shows the price path with respect to price periods, when the initial inventory is equal to 20. According to Figure 2, the optimal initial price at the beginning of the sale horizon is 17. At the last periods with fewer inventories, the optimal price decreases considerably. As a result, proposing lower prices can help us to reduce the remained products at the end of the sale period. On the other hand, since the refund depends on the current price, it can be appropriate to sell the remained inventories with lower prices in order to decrease the total refund.

According to Figure 3, the optimal properties of inventory and time monotonocity do not hold in the second example, when the possibility of cancellation exists.

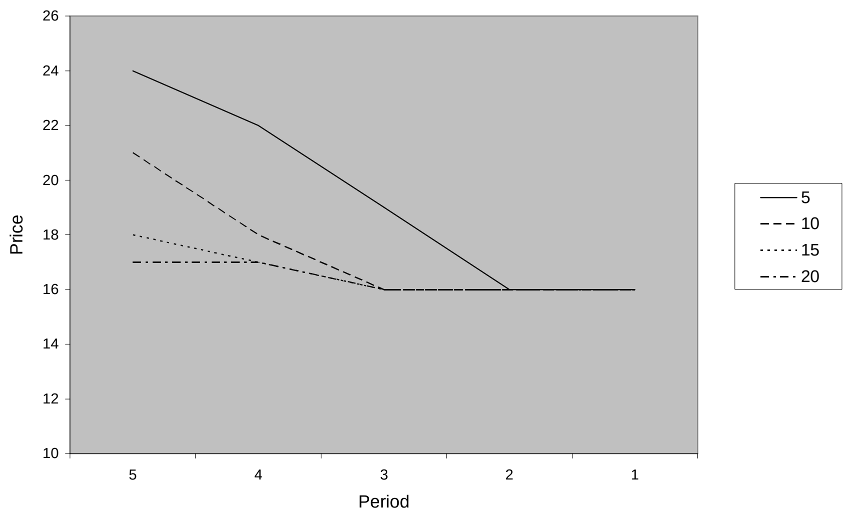

Figure 1 Initial price path through the sale periods

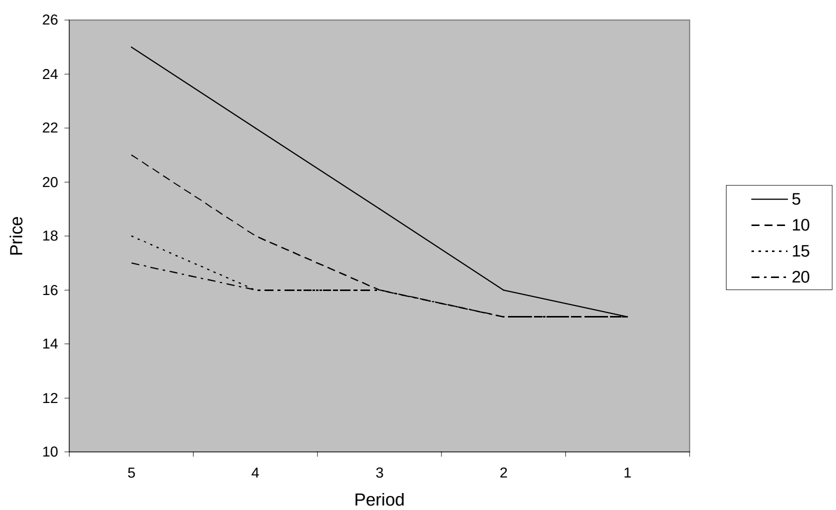

Figure 2 Initial price path through the periods

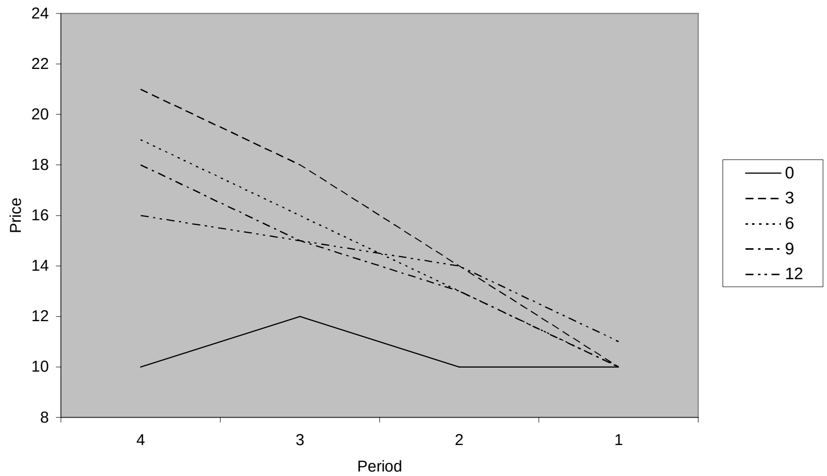

Figure 3 Price path through the price periods

The effect of cancellation probability and refund rate on the optimal initial price

In Figure 4, we have depicted the effect of cancellation probability in each sale period for different optimal initial prices. The initial inventory is equal to 10 and the refund rate is equal to 0.9. Our numerical experiments show that in each period, the more the probability of cancellation, the more the optimal price of the period. According to this figure, when we have enough time to sell our entire inventory or we have lots of inventory, compared to the potential customers in the left sale periods, the probability of cancellation does not have a significant effect on the optimal initial price.

Initial price path through the periods based on refund rate with 10 units of inventory and cancellation probability of 0.1 is depicted in Figure 5. We study the effect of increasing the refund rate in the next section. In the second example, we observe that as we increase the refund rate from 0 to 1, the optimal price increases accordingly. In low refund rates, it’s better to sell more because we can profit from every cancellation and therefore it seems that low prices can increase our expected revenue.

4.3. Results of random generated examples

From the randomly generated examples of the previous section, it can be concluded that setting a limit on the number of sales in the first model does not have a significant effect on the expected revenue. It was found out in the first model, setting a selling limit can only improve the expected revenue (between 0.02 and 7 percent for 1000 examples investigated). Our numerical experiments show that when the ratio of initial inventory to the number of periods is more than 4, we can expect that the difference between the expected revenue in the model with the sale limit and without it be at most 1%. As this ratio decreases, the above mentioned difference in the second model increases. On the other hand, we found out that the initial price in our first model is equal for both models with the sale limit and without it. However, it is possible in the second model to decrease the initial price of the model with the sale limit against the one and without it.

In the first and second models we investigated the effect of considering the possibility of cancellation on the optimal price of the period. Our observation shows that in most cases the optimal price in the second model is higher or equal to that of the first one. With the fourth set of numerical experiments, it was concluded that in 7.9% of cases the optimal price in the first model is higher than that of the second model.

According to our randomly generated examples, we studied the effect of increasing the refund rate or cancellation probability on the optimal price at the beginning of the pricing period of the second model when a sale limit is not considered. Figure 6 represents the results. In many examples, the optimal price at the beginning of the sale horizon is not decreased by increasing the probability of cancellation. As the refund rate decreases, we observe in most cases that the initial price is not increased.

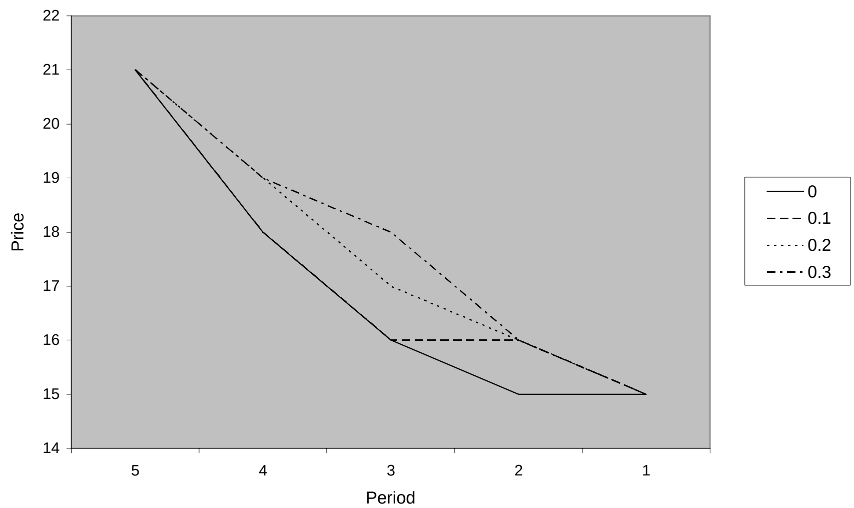

Figure 4 Price path through the periods based on probability of cancelation

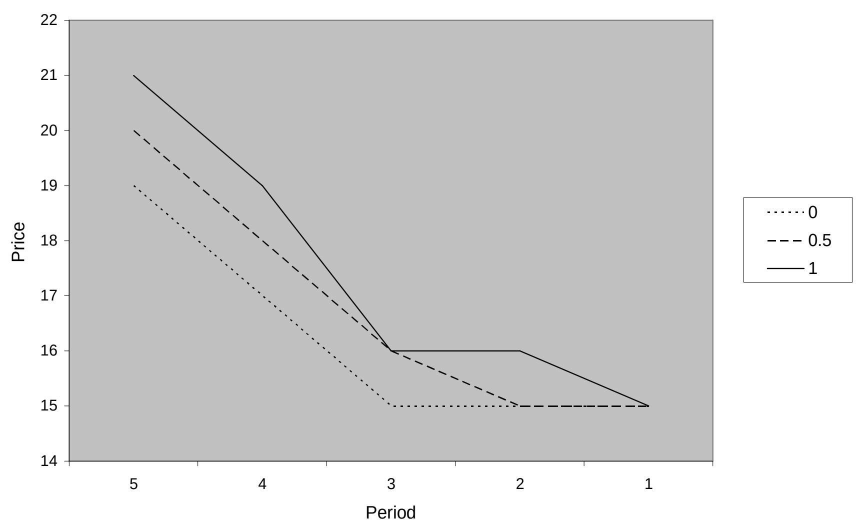

Figure 5 Price path through the periods based on refund rate.

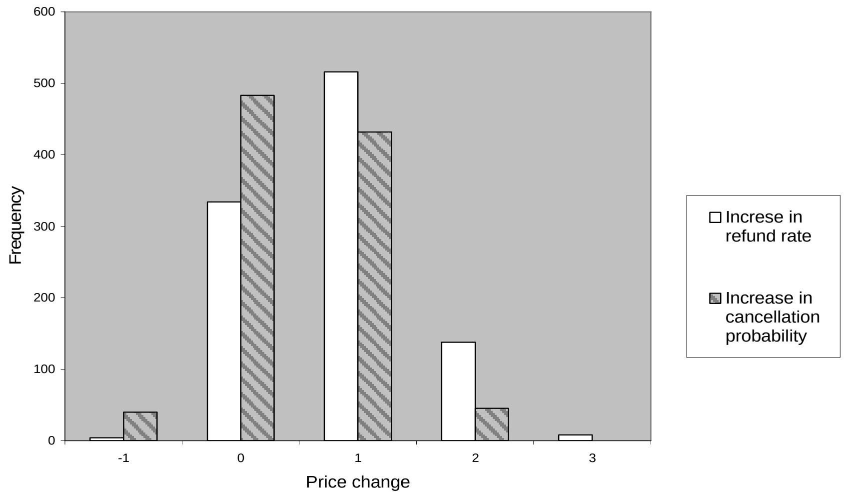

Figure 6 Dispersion of price changes in randomly generated examples

5. CONCLUSION AND FUTURE RESEARCH

In this paper, we introduced three dynamic pricing models that allow a firm to review its pricing policy periodically. It was concluded that in these models the Inventory and time Monotonocity do not hold, witch contradicts the results of the previous studies for the models with continuous review. We also studied two models that allow cancellation and investigate some aspects of considering the cancellation in dynamic pricing models.

Possible extension of this research include (i) incorporating the learning for both arrival rate and reservation price distribution as the sale evolves; (ii) incorporating costs of price changes and (iii) considering replenishment options during the sales horizon.

We did not consider competition in our models. Incorporating the competition would make it much more useful for practical applications.

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