Journal of Industrial and Systems Engineering
Vol. 3, No. 3, pp 189-212
Fall 2009

Integrated Procurement, Production and Delivery Scheduling in a Generalized three Stage Supply Chain

Naser Nikandish[1*], Kourosh Eshghi[2], S. Ali Torabi[3]

[1] Johnson Graduate School of Management, Cornell University, Ithaca, NY
nn93@cornell.edu

[2] Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran
eshghi@sharif.edu

[3] Department of Industrial Engineering, Faculty of Engineering, University of Tehran, Tehran, Iran
satorabi@ut.ac.ir

ABSTRACT

In this research, we investigate a three-stage supply chain with one supplier, several manufacturers and multiple retailers where the supplier provides a common raw material to each manufacturer, who in turn uses a single stage production facility to convert it into final products that are delivered at fixed lot sizes to retailers. An integrated economic procurement, production, and delivery model is developed whose objective is to find the common production cycle length, production sequences of final products at manufacturers and delivery frequencies of final products to retailers minimizing the total costs of considered supply chain. We propose an analytical solution procedure and an efficient heuristic solution method. The proposed heuristic solution algorithm is able to find the optimal solutions for the small and medium problem instances and consequently it is very promising for solving the large-sized instances in a reasonable time.

Keywords: Logistics, Supply chain management, Inventory, Integrated decision making

1. INTRODUCTION

A typical supply chain involves different suppliers (at possibly one or more tiers), assemblers/manufacturers, distribution centers, retailers and end customers. The goal of supply chain management is to optimize the entire system through coordination of the various processes (Simchi-Levi et al., 2000). Suppose a typical supply chain shown in Figure 1 where a manufacturer produces the products for a retailer. The main question of the retailer is that “How much products should she/he order each time to minimize the total costs?” To answer this question, the retailer considers its own cost elements. Since the cost elements of the manufacturer are not considered here, the answer usually is not acceptable from the manufacturer perspective. There is a similar scenario from the manufacturer point of view; the main question of the manufacturer is that “How much products should she/he produce in each production run to have minimum total cost?” and the optimal solution is usually unacceptable from the other parties.

---

* Corresponding Author

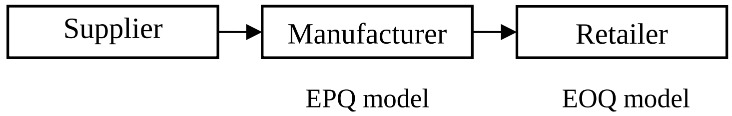

Figure 1 A typical three-stage supply chain.

The concept of integrated decision making in a supply chain was first introduced by Goyal (1977). After that time, several researches were conducted on the integrated decision-making in two-stage supply chains. Most of these researches dealt with a two stage supply chain consisting of a manufacturers' level and a retailers' level. For a complete review of integrated models in two-stage supply chain see Ben Daya et al. (2007). In contrary with two-stage supply chain, few researches have been carried out for the three stage supply chains. Muson and Rosenblatt (2001) have introduced the first three stage integrated supply chain. They considered a three-level chain (involving one supplier, one manufacturer, and one retailer) and explored the benefits of using quantity discounts on both ends of the supply chain to decrease the total costs through the chain. They showed that incorporating quantity discounts into both ends of the supply chain could significantly decrease the total costs of supply chain. Khouja (2003) considered a three-stage supply chain with multiple manufacturers and multiple retailers. In his paper a three mathematical model was presented considering three different coordination mechanisms. He showed that some of the coordination mechanisms could result in a significantly lower total cost than matching production and delivery along the chain. Another three-stage integrated supply chain was introduced by Lee (2005). In this supply chain structure, there is only one party at each level. The manufacturer orders raw materials from its supplier, converts them into the finished goods through its single-stage batch production process, and finally delivers the finished goods to the respective customer on the batch basis. Furthermore, an integrated inventory control model developed to find the joint economic lot sizes of manufacturer's raw material ordering, production batch, and buyer 's ordering minimizing the average total cost per unit time consisting of the raw materials ordering and holding, manufacturer's setup and finished goods holding as well as the buyer's ordering and inventory holding costs. Through some numerical examples, the author showed that considering all of the inventory costs in an integrated supply chain results in less mean total cost than considering all inventory costs separately in different parts of the chain. To the best of the authors' knowledge, the last three-stage integrated supply chain mathematical model was presented by Kim et al. (2006). They developed an analytical model to integrate and synchronize the procurement, production and delivery activities in a supply chain consisting of a single raw material supplier, a single manufacturer and multiple retailers. The objective is to find the production sequence of multiple items, the common production cycle length, and the delivery frequencies and quantities that minimizes the average total costs. In addition, an efficient heuristic algorithm is presented to solve the proposed problem. Through some numerical tests, they show that the proposed heuristic gives quite satisfactory solutions.

This study extends the previous research works presented by Khouja (2003), Lee (2005), and Kim et al. (2006). In this research, we investigate a three-stage supply chain with one supplier, several manufacturers, and retailers where the supplier prepares raw material from outside of the chain and converts it into one processed raw material. Each manufacturer orders the processed raw material from supplier and converts it to some final products through its single stage capacitated production facility. The final products are produced in batches at a finite rate. Each manufacturer produces several kinds of products and periodically delivers them at a fixed lot sizes to the retailers. The demand rates are constant. Each retailer is connected to just one manufacturer, receives just one kind of final products from corresponding manufacturer and continuously delivers it to the outside customers. In this paper, a joint economic procurement, production and delivery model is developed considering all of cost elements incurred in the different levels of the supply chain. In addition, an efficient solution procedure based on hybrid algorithm proposed by Clausen and Ju (2006) is proposed to solve the problem.

It is noteworthy that the considered problem is actually a variant of well-known Economic Lot and Delivery Scheduling Problem (ELDSP) which has been considerably studied in the literature (e.g., see Clausen and Ju (2006), Hahm and Yano (1992, 1995a, 1995b), Jensen and Khouja (2004)).

The remainder of this paper is organized as follows. The problem definition and notations used for model formulation are provided in the next Section. The proposed mathematical model is discussed in Section 3. Sections 4 is devoted to analytical solution and Section 5 discusses the proposed heuristic solution procedure. The numerical results are presented in Section 6. Finally, Section 7 is devoted to conclusion remarks.

2. PROBLEM DEFINITION

We consider a three-stage supply chain involving a single-supplier, multiple manufacturers, and mltiple retailers. Each retailer faces with a deterministic and continuous fixed-rate demand for just one final product from outside of the chain (similar to EOQ model). In order to fulfill the customer

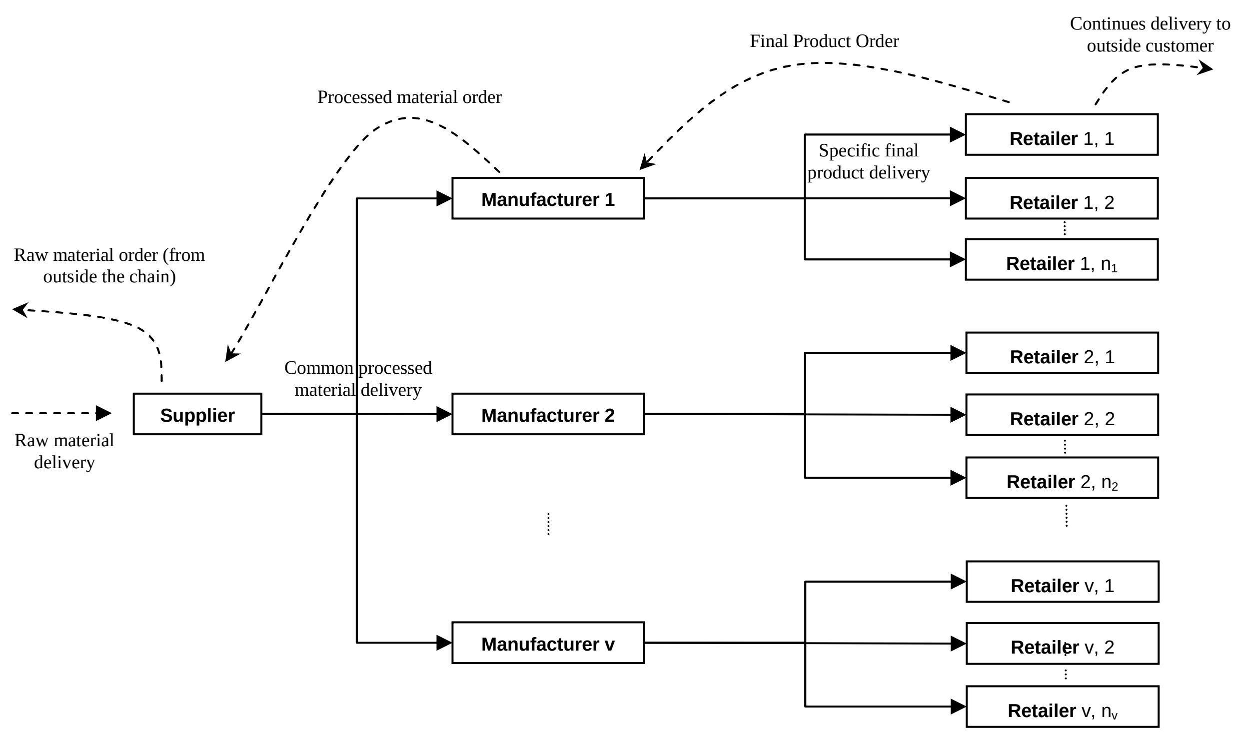

Figure 2 The considered supply chain structure.

demands, each retailer orders the respective final product to the specified manufacturer, receives the product in lots and continuously delivers it to the outside customers. Each manufacturer purchases the processed materials from a supplier, which in turn, through its single stage capacitated production facility, converts the processed materials into the several final products, and delivers them to respective retailers periodically (at the beginning of each production cycle). At each manufacturer, the consumption rate of processed material for each product is the proportion of the production rate to conversion factor of corresponding item. In this supply chain, a single supplier prepares required processed material for the manufacturers. In fact, the supplier orders input raw material from outside of the chain and converts it to the common processed material that is used by the manufacturers through its single stage production facility. The input raw material's consumption rate is the proportion of the production rate of output material to corresponding conversion factor. Figure 2 depicts the supply chain configuration.

It is noticeable that the topology of this chain is fixed over time and each retailer has been assigned to a specified manufacturer, in advance.

As illustrated in Figure 2, ten cost elements incurred in this supply chain. From upstream to downstream, there are supplier's ordering, setup, input raw material and processed material holding costs, manufacturers' ordering, setup, processed material and final product holding costs and finally retailers' final product ordering and setup costs. Our objective is to determine the economic production lot sizes at the supplier and manufacturers as well as the final products' delivery schedule minimizing the total costs of the supply chain subject to some constraints inspired from the problem nature.

The following assumptions are made to formulate the problem mathematically:

- External demands for final products at retailers are continuous with a given constant rate
- Each retailer fulfills demand for a single final product
- All inventories are imperishable
- All of the parameters are independent of production and delivery lot sizes
- The production facility at the supplier and each manufacturer is a capacitated single stage system
- All of the final products have the same production cycle length (i.e., one lot of each final product is produced in each rotation cycle considering a fixed products' sequence vector)
- A common cycle time is considered at the supplier and manufacturers
- Production batch size of final products are a multiplier of corresponding delivery lot sizes
- Synchronized activities is allowed (i.e., production and delivery of a final product can be carried out simultaneously)
- All of the lead times are constant
- Backorders are not allowed through the chain
- A similar processed material is used for producing different products at the manufacturers
- Delivery quantities of final products are equal-sized
- The required processed material for producing a single batch of each product is delivered to each manufacturer at the beginning of cycle time
- The required input material for one cycle is delivered to supplier at the beginning of the cycle time
- Planning horizon is infinite

The following notations are used for the model formulation:

Supplier level parameters:

S :	Production set up cost at the supplier
A :	Input raw material ordering cost of supplier
$h_{\circ}$:	Input raw material holding cost per unit per unit time at the supplier
$h'_{\circ}$:	Output (processed) material holding cost per unit per unit time at the supplier
f_s :	Supplier conversion factor of input raw material to output material
S :	Production set up cost of the supplier
v :	Number of manufacturers

Manufacturers level parameters:

M_i :	Manufacturer's index (i th manufacturer) $\forall\ i=1,\cdots,v$
n_i :	Number of M_i 's retailers (Number of M_i 's products)
ij :	Product's index (product j of manufacturer i) $\forall\ i=1,\cdots,v$ & $\forall\ j=1,\cdots,n_i$
A_i :	Input material ordering cost of M_i
H'_i :	Input material holding cost of M_i per unit per unit time
P_{ij} :	Production rate of product ij
$h_{ij}^{(2)}$:	M_i 's holding cost per unit per unit time for product ij
s_{ij} :	M_i 's setup time for producing ij
S_{ij} :	M_i 's setup cost for product ij
f_{ij} :	M_i 's conversion factor of input raw materials to final product ij
TCM :	Total costs of manufacturers per unit time

Retailers level parameters:

R_{ij} :	Retailer ij
$h_{ij}^{(3)}$:	R_{ij} 's holding cost per unit per unit time for product ij
D_{ij} :	Demand rate of product ij faced by the Retailer ij
A_{ij} :	Unit ordering cost of product ij
TCR_{ij} :	Total cost of Retailer R_{ij}
TCR :	Total cost of Retailers per unit time

Note that due to the value-added activities down through the supply chain, the following inequalities hold regarding the unit inventory holding costs:

$$h_{ij}^{(3)} \geq h_{ij}^{(2)} \geq \frac{H_i'}{f_{ij}} > \frac{h_\circ'}{f_{ij}} \geq \frac{h_\circ}{f_{ij}\, f_s} \quad \forall\ i = 1, \ldots, v \quad \forall\ j = 1, \ldots, n_i \tag{1}$$

The objective of the problem is to find optimum value of common cycle time, production sequences of final products at each manufacturer and delivery lot sizes to the retailers with respect to some relevant cost elements in different levels. Therefore, decision variables for the problem are as follows:

T :	Common cycle length
$\bar{Z}_i$:	Production sequence vector of final products on M_i
Q_{ij} :	Production quantity of product ij
m_{ij} :	Delivery frequency of product ij per production cycle
q_{ij} :	Delivery quantity of product ij
$[k]_i$:	Index for item at k th position in the production sequence vector $\bar{Z}_i$

It is noted that because of applying integer-ratio policy, the production batch sizes, delivery lot sizes and delivery frequencies have the relation of $Q_{ij} = m_{ij}.q_{ij}$. Moreover, for convenience, it has supposed that: $\bar{m}_i = \left(m_{i1}, m_{i2}, \ldots, m_{in_i}\right)\ \forall\, i = 1, \ldots, v$, $Z = \left\{\bar{Z}_1, \ldots, \bar{Z}_v\right\}$, and $m = \left\{\bar{m}_1, \ldots, \bar{m}_v\right\}$.

3. PROBLEM FORMULATION

In this section, a new mathematical model is presented for the problem. A total of ten relevant costs are incorporated in our model which are classified into the three categories: 1) *Retailer's level costs*, 2) *manufacturer's level costs* and 3) *supplier's level costs*. Our objective is to develop a model to minimize the sum of these cost elements.

3.1. Total cost function at retailers' level

Figure 3 shows the inventory evolution curve for the finished item ij at the retailer R_{ij}. Therefore, the total cost function at the retailers' level can be written as:

$$TCR = T\sum_{i=1}^{v}\sum_{j=1}^{n_i}\frac{h_{ij}^{(3)} D_{ij}}{2m_{ij}} + \frac{1}{T}\sum_{i=1}^{v}\sum_{j=1}^{n_i} m_{ij} A_{ij} \tag{2}$$

where the first and second statements are related to the holding costs of final products and ordering cost, respectively.

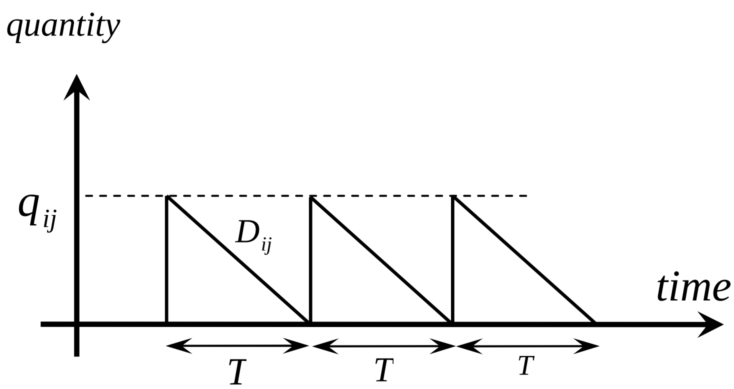

Figure 3 Inventory level of product ij at retailer R_{ij}

3.2. Total cost function at manufacturers' level

There are three major cost elements at the manufacturers' level as follows:

3.2.1. Manufacturers' ordering and setup costs

The average raw material ordering and setup costs per year for all of the manufacturers can be easily written by: $\sum_{i=1}^{v}\frac{1}{T}A_i+\frac{1}{T}\sum_{i=1}^{v}\sum_{j=1}^{n_i}S_{ij}=\frac{1}{T}\sum_{i=1}^{v}\left(A_i+\sum_{j=1}^{n_i}S_{ij}\right)$

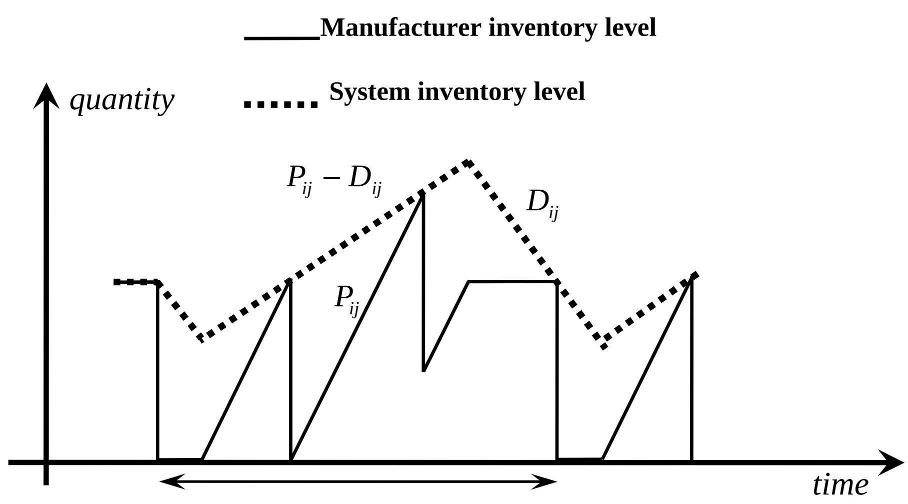

Figure 4 Inventory level of product *ij*

3.2.2. Manufacturers holding cost of the final products

Calculating the inventory holding costs at the manufacturers are somewhat complicated. Figure 4 represents the inventory evolution curve of final product ij at the manufacturer M_i along with respective system-wide inventory level (considering both Manufacturer and retailer). Using a similar method mentioned in (Ben Daya et al., 2007), doing some calculations and rearranging the equations results the following equation as the final products inventory holding cost:

$$T \cdot \sum_{i=1}^{v} \sum_{j=1}^{n_i} h_{ij}^{(2)} \left(\frac{D_{ij}^2}{m_{ij} P_{ij}} + \left(1 - \frac{D_{ij}}{P_{ij}}\right) \frac{D_{ij}}{2} - \frac{D_{ij}}{2m_{ij}} \right) \tag{3}$$

3.2.3. Manufacturers holding cost of the processed (input) material

As mentioned earlier, consumption rate and consequently average inventory level of processed material at each manufacturer depends on production sequence of the final products. Figure 5 shows the sample inventory level of processed material for a given sequence Z_i at manufacturer M_i. In this figure, $tp[k]_i$ and $s[k]_i$ denote production and setup times of the product at k th position of sequence Z_i, respectively. Hence, for a given production sequence Z_i for M_i, the average inventory holding cost of processed material at M_i can be given by

$$H_i' T \left\{ \sum_{j=1}^{n_i} \frac{D_{ij}^2}{2 f_{ij} \cdot P_{ij}} + \sum_{j=1}^{n_i - 1} \left(\frac{D[j]_i}{P[j]_i} \cdot \sum_{k=j+1}^{n_i} \frac{D[k]_i}{f[k]_i} \right) \right\} + H_i' \sum_{j=1}^{n_i} \left(s[j]_i \cdot \sum_{k=j}^{n_i} \frac{D[k]_i}{f[k]_i} \right) \tag{4}$$

In the above equation, the first and second terms correspond with the holding costs during the production periods and the setup periods, respectively.

Finally, the average total cost for the manufacturers can be written as follows:

$$\begin{aligned} TCM = T \cdot & \left\{ \sum_{i=1}^{v} \sum_{j=1}^{n_i} \frac{H_i'.D_{ij}^2}{2 f_{ij} \cdot P_{ij}} + \sum_{i=1}^{v} \sum_{j=1}^{n_i - 1} \left(H_i' \cdot \frac{D[j]_i}{P[j]_i} \cdot \sum_{k=j+1}^{n_i} \frac{D[k]_i}{f[k]_i} \right) \right\} + \left[\sum_{i=1}^{v} \sum_{j=1}^{n_i} \left(H_i' \cdot s[j]_i \cdot \sum_{k=j}^{n_i} \frac{D[k]_i}{f[k]_i} \right) \right] \\ & + T \cdot \sum_{i=1}^{v} \sum_{j=1}^{n_i} h_{ij}^{(2)} \left(\frac{D_{ij}^2}{m_{ij} \cdot P_{ij}} + \left(1 - \frac{D_{ij}}{P_{ij}}\right) \cdot \frac{D_{ij}}{2} - \frac{D_{ij}}{2m_{ij}} \right) + \frac{1}{T} \sum_{i=1}^{v} \left(A_i + \sum_{j=1}^{n_i} S_{ij} \right) \end{aligned} \tag{5}$$

Proof of the above relation is given in Appendix A.

3.3. Total cost function at supplier's level

At the supplier, the input raw material is procured and converted to the processed material (manufacturers common input material) with a conversion factor of f_s. Figure 6 shows the corresponding inventory levels of input raw materials and processed materials. Accordingly, the cost elements of the supplier (i.e., input raw material's ordering and holding costs, Setup costs and processed material holding costs) can be calculated by following expressions, respectively:

$$\frac{1}{T} A;\ \frac{h_\circ}{f_s} \sum_{i=1}^{v} \sum_{j=1}^{n_i} \frac{1}{2} \cdot \frac{D_{ij} \cdot T}{f_{ij}} = T.\sum_{i=1}^{v} \sum_{j=1}^{n_i} \frac{h_\circ}{2 f_s} \cdot \frac{D_{ij}}{f_{ij}};\ \ \frac{1}{T} S;\ h_\circ' \sum_{i=1}^{v} \sum_{j=1}^{n_i} \frac{1}{2} \cdot \frac{D_{ij} \cdot T}{f_{ij}} = T.\sum_{i=1}^{v} \sum_{j=1}^{n_i} \frac{h_\circ' \cdot D_{ij}}{2 f_{ij}}$$

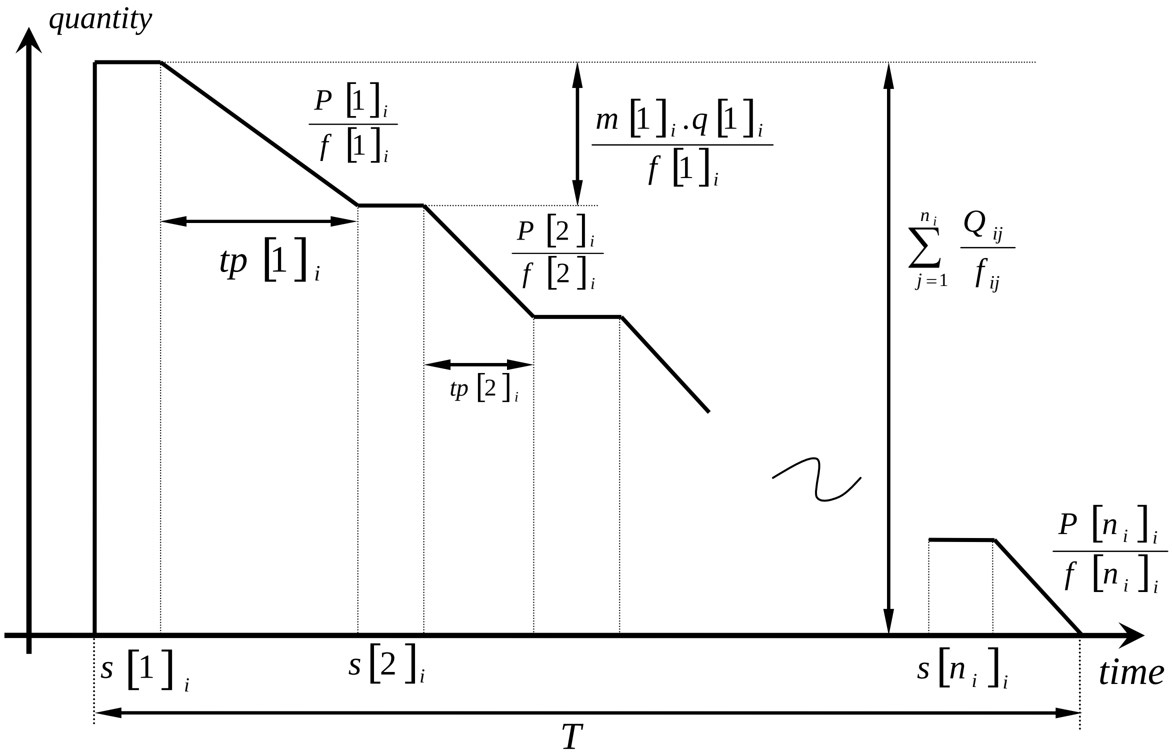

Figure 5 M_i's inventory level for input material

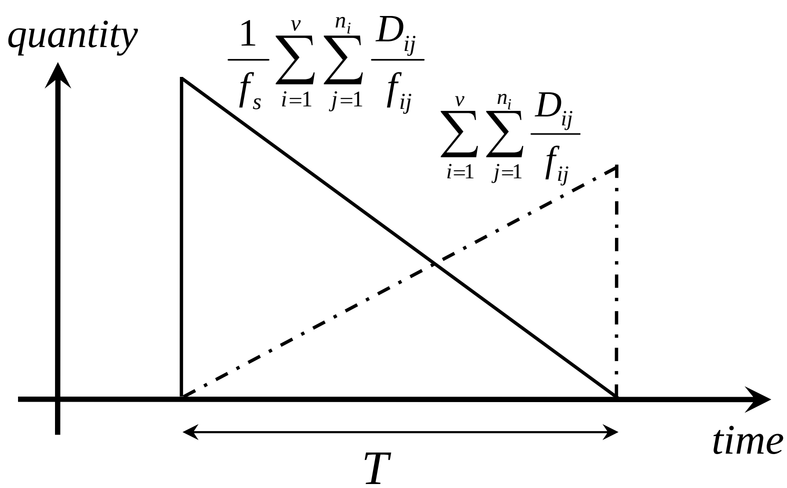

Figure 6 Inventory trajectory at the supplier level

Therefore, the total cost factor at the supplier level is

$$TCS = T \sum_{i=1}^{v} \sum_{j=1}^{n_i} \left(h'_{\circ} + \frac{h_{\circ}}{f_s} \right) \left(\frac{D_{ij}}{2 f_{ij}} \right) + \frac{1}{T} (A + S) \tag{6}$$

3.4. Model constraints

There is no constraint on the production sequences, but delivery factors must be positive and integer values, mathematically $\left(\text{i.e., } m_{ij} \in \mathbb{Z}^{+};\ \forall i = 1,\dots,v;\ \forall j = 1,\dots,n_i\right)$.

Another constraint of this model is related to the cycle time. That is, for each manufacturer the sum of production and setup times for all of the products per cycle time must be smaller than or equal to the cycle time, which can be written mathematically as follows:

$$\sum_{j=1}^{n_i} s_{ij} + \sum_{j=1}^{n_i} tp_{ij} \le T\,; \forall i = 1,\dots,v \Rightarrow T \ge \underset{i=1,\dots,v}{Max}\left\{ \frac{\sum_{j=1}^{n_i} s_{ij}}{\left(1 - \sum_{j=1}^{n_i} \frac{D_{ij}}{P_{ij}}\right)} \right\} \tag{7}$$

3.5. Model Structure

Considering all of the cost functions and the constraints gives the final mathematical model. After some calculations and rearrangements, the mathematical model of the problem can be written as follows:

$$Min\ TC\left(T, m, Z\right) = T\left(\sum_{i=1}^{v}\sum_{j=1}^{n_i}\left(\frac{\alpha_{ij}}{m_{ij}} + \beta_{ij}\right) + \sum_{i=1}^{v}\delta_i\right) + \frac{1}{T}\left(\lambda + \sum_{i=1}^{v}\sum_{j=1}^{n_i} m_{ij}A_{ij}\right) + \sum_{i=1}^{v}\gamma_i$$

Subject to:

$$T \ge \underset{i=1,\dots,v}{Max}\left\{ \frac{\sum_{j=1}^{n_i} s_{ij}}{\left(1 - \sum_{j=1}^{n_i} \frac{D_{ij}}{P_{ij}}\right)} \right\} \equiv T_{\min}\,;\ T \in R^{+};\ m_{ij} \in Z^{+}; \forall i = 1,\dots,v\,; \forall j = 1,\dots,n_i$$

where:

$$\alpha_{ij} = \left(h_{ij}^{(3)} - h_{ij}^{(2)}\right)\frac{D_{ij}}{2} + h_{ij}^{(2)}\frac{D_{ij}^{2}}{P_{ij}};\ \forall i, j$$

$$\beta_{ij} = \frac{1}{2}\left[\left(h_{ij}^{(2)} + \left(h'_{\circ} + \frac{h_{\circ}}{f_s}\right)\frac{1}{f_{ij}}\right)\cdot D_{ij} + \left(\frac{H'_i}{f_{ij}} - h_{ij}^{(2)}\right)\frac{D_{ij}^{2}}{P_{ij}}\right];\ \forall i, j$$

$$\lambda = A + S + \sum_{i=1}^{v} A_i + \sum_{i=1}^{v}\sum_{j=1}^{n_i} S_{ij}$$

$$\delta_i = H'_i \sum_{j=1}^{n_i - 1}\frac{D[j]_i}{P[j]_i}\sum_{k=j+1}^{n_i}\frac{D[k]_i}{f[k]_i};\ \forall i$$

$$\gamma_i = H_i' \sum_{j=1}^{n_i} s[j]_i \sum_{k=j}^{n_i} \frac{D[k]_i}{f[k]_i};\ \forall i$$

$$m = \{\bar{m}_1, \ldots, \bar{m}_v\};\ Z = \{\bar{Z}_1, \ldots, \bar{Z}_v\}.$$

It is noted that α_{ij}, β_{ij}, and λ are sequence independent and δ_i and γ_i are sequence-dependent parameters.

4. ANALYTICAL SOLUTION METHOD

To solve the problem, an analytical method has been proposed which is similar to that of Kim et al. (2006). The objective is to find the optimal value of cycle time (T), delivery frequencies (m) and optimal production sequences at each manufacturer (Z). As it is shown in this section, due to the recursive relation between the optimal production sequences and optimal cycle time, it is not applicable to use analytical solution's results for finding optimal solution. However, based on this section's results, an efficient heuristic algorithm will be developed in the next section.

The analytical solution procedure consists of two parts. In the first part, for a given sequences say Z the optimal value of cycle time and delivery frequencies is obtained, and in the next part the optimal production sequences is presented.
Since $TC(T \mid m, Z)$ is a convex function with respect to continuous cycle time variable, for a particular set of m and Z, the optimal value of T is obtained through the first derivative of $TC(T \mid m, Z)$ with respect to T and setting it equal to 0. Thus,

$$\frac{\partial TC(T \mid m, Z)}{\partial T} = 0 \Rightarrow T^*(m, Z) = \sqrt{\frac{\lambda + \sum_{i=1}^{v} \sum_{j=1}^{n_i} m_{ij} A_{ij}}{\sum_{i=1}^{v} \sum_{j=1}^{n_i} \left(\frac{\alpha_{ij}}{m_{ij}} + \beta_{ij} \right) + \sum_{i=1}^{v} \delta_i}} \tag{8}$$

Since $TC(T \mid m, Z)$ is a convex function and there is a constraint on minimum value of cycle time, so the optimal value of cycle time is obtained by $T_{opt} = Max\{T^*(m, Z), T_{\min}\}$.

For each of these possible values of T, the corresponding optimal value of delivery frequencies can be calculated as follows. Suppose $T_{opt} = T^*(m, Z)$, by substituting the value of $T^*(m, Z)$ from Eq. (8) to TC function we have:

$$TC(T^*(m, Z), m, Z) = 2\sqrt{\left\{\lambda + \sum_{i=1}^{v} \sum_{j=1}^{n_i} m_{ij} A_{ij}\right\}\left\{\sum_{i=1}^{v} \sum_{j=1}^{n_i} \left(\frac{\alpha_{ij}}{m_{ij}} + \beta_{ij} \right) + \sum_{i=1}^{v} \delta_i\right\}} + \sum_{i=1}^{v} \gamma_i \tag{9}$$

In Eq. (9), $\sum_{i=1}^{v}\gamma_i$ is independent of $m_{ij};\forall\ i,j$, so in determining the delivery frequencies we just consider the first statement of the Eq. (9) . For a given production sequences say Z we have:

$$f\left(m\right)=\left\{\lambda+\sum_{i=1}^{v}\sum_{j=1}^{n_i}m_{ij}A_{ij}\right\}\left\{\sum_{i=1}^{v}\sum_{j=1}^{n_i}\left(\frac{\alpha_{ij}}{m_{ij}}+\beta_{ij}\right)+\sum_{i=1}^{v}\delta_i\right\} \tag{10}$$

At First, we relax the $m_{ij}\in\mathbb{Z}^{+};\ \forall i,j$ and assume that $m_{ij}\in R^{+};\ \forall i,j$. Fortunately $f\left(m\right)$ is a strictly convex with respect to m_{ij} values. Solving $\frac{\partial f\left(m\right)}{\partial m_{ij}}=0$ gives the optimal values of delivery frequencies as follows:

$$\frac{\partial f\left(m\right)}{\partial m_{ij}}=0\ \Rightarrow\ A_{ij}\left(\sum_{i=1}^{v}\sum_{j=1}^{n_i}\left(\frac{\alpha_{ij}}{m_{ij}}+\beta_{ij}\right)+\sum_{i=1}^{v}\delta_i\right)=\frac{\alpha_{ij}}{m_{ij}^2}\left\{\lambda+\sum_{i=1}^{v}\sum_{j=1}^{n_i}m_{ij}A_{ij}\right\}\quad\forall i,j \tag{11}$$

For solving the above equation, suppose that $\theta=\dfrac{\sum_{i=1}^{v}\sum_{j=1}^{n_i}\left(\frac{\alpha_{ij}}{m_{ij}}+\beta_{ij}\right)+\sum_{i=1}^{v}\delta_i}{\lambda+\sum_{i=1}^{v}\sum_{j=1}^{n_i}m_{ij}A_{ij}}$ by substituting the value of θ in Eq. (11) the following result is obtained:

$$A_{ij}\theta=\frac{\alpha_{ij}}{m_{ij}^2}\quad\forall i,j\ \Rightarrow\ \theta=\frac{1}{\lambda}\left(\sum_{i=1}^{v}\sum_{j=1}^{n_i}\beta_{ij}+\sum_{i=1}^{v}\delta_i\right)\ \Rightarrow\ m_{ij}^{*}=\sqrt{\frac{\alpha_{ij}\cdot\lambda}{A_{ij}\left(\sum_{i=1}^{v}\sum_{j=1}^{n_i}\beta_{ij}+\sum_{i=1}^{v}\delta_i\right)}};\ \forall i,j \tag{12}$$

Finally by substituting Eq. (12) to Eq. (8), the value of $T^{*}\left(Z\right)$ is obtained by the equation:

$$T^{*}\left(Z\right)=\sqrt{\frac{A+S+\sum_{i=1}^{v}A_i+\sum_{i=1}^{v}\sum_{j=1}^{n_i}S_{ij}}{\sum_{i=1}^{v}\sum_{j=1}^{n_i}\beta_{ij}+\sum_{i=1}^{v}\delta_i}} \tag{13}$$

$$\Rightarrow\ m_{ij}=T^{*}\left(Z\right)\sqrt{\frac{\alpha_{ij}}{A_{ij}}};\ \forall i,j \tag{14}$$

On the other hand, if $T_{opt}=T_{\min}$ then assuming $m_{ij}\in R^{+};\ \forall i,j$, the optimal value of delivery frequencies, similar to previous case, can be obtained as follows:

$$\frac{\partial TC\left(T_{\min}, m, Z\right)}{\partial m_{ij}} = 0 \Rightarrow \; m_{ij} = T_{\min}\sqrt{\frac{\alpha_{ij}}{A_{ij}}};\ \forall i, j \tag{15}$$

As mentioned previously,

$$T_{opt} = Max\left\{T^{*}\left(Z\right), T_{\min}\right\} \tag{16}$$

Therefore, it can be concluded that for a given value of cycle time say T , the delivery frequencies can be obtained using the following equation:

$$m_{ij}\left(T\right) = T\sqrt{\frac{\alpha_{ij}}{A_{ij}}};\ \forall i, j \tag{17}$$

Now, we can turn to $m_{ij} \in \mathbb{Z}^{+};\ \forall i, j$ constraints. If all of the obtained values for delivery frequencies from Eq. (16) will be integer, the resulting values are optimal. Otherwise, we consider two possible candidate $\lfloor m_{ij} \rfloor$ and $\lceil m_{ij} \rceil$ for non-integer m_{ij} values. Considering at most $2^{\sum_{i=1}^{v} n_i}$ number of sets as the candidate sets for the optimal values of delivery frequencies, the optimal set can be distinguished by calculating and comparing total cost functions of these delivery frequencies. After obtaining the best values of delivery frequencies, $T^{*}\left(m, Z\right)$ can be calculated using Eq. (8) and substituting the final integer values of $m_{ij};\ \forall i, j$.

The last decision variables of the problem are production sequences. The only sequence-dependent part of the objective function is $T\sum_{i=1}^{v}\delta_i + \sum_{i=1}^{v}\gamma_i$. In order to minimize TC , we must minimize $T\sum_{i=1}^{v}\delta_i + \sum_{i=1}^{v}\gamma_i = \sum_{i=1}^{v}\left(T\delta_i + \gamma_i\right)$. It is clear that in this expression, the production sequences of manufacturers are independent of each other, and the production sequence that is optimal for each manufacturer (independent of other manufacturers) will be optimal for the whole problem. According to theorem 2.4 of Baker (1974), each manufacturer of the problem like M_i is equivalent to a single-machine weighted completion time problem with $t_{ij} = s_{ij} + T\,\frac{D_{ij}}{P_{ij}}$ and $w_{ij} = \frac{Q_{ij}}{f_{ij}} = \frac{D_{ij} \cdot T}{f_{ij}}$. Therefore, the WSPT (Weighted Shortest Processing Time) rule gives an optimal solution. Thus, for a given cycle time say T , an optimal production sequence of manufacturer M_i can be obtained by arranging the items in non-decreasing order of $\dfrac{s_{ij}+T\dfrac{D_{ij}}{P_{ij}}}{\dfrac{D_{ij}}{f_{ij}}\cdot T}$. It is obvious that the cycle time term can be eliminated from the nominator. Therefore, the optimal sequence at manufacturer M_i is as follows:

$$\frac{\dfrac{D[1]_i}{f[1]_i}}{s[1]_i+T\dfrac{D[1]_i}{P[1]_i}} \geq \frac{\dfrac{D[2]_i}{f[2]_i}}{s[2]_i+T\dfrac{D[2]_i}{P[2]_i}} \geq \cdots \geq \frac{\dfrac{D[n_i]_i}{f[n_i]_i}}{s[n_i]_i+T\dfrac{D[n_i]_i}{P[n_i]_i}}; \quad \forall i=1,\ldots,v \tag{18}$$

5. HEURISTIC PROCEDURE

As obtained in previous section and depicted in Figure 7, there is a recursive relation between optimal value of cycle time $\left(T^*(Z)\right)$ and optimal production sequences and it makes hard to find the optimal solution of the problem using analytical solution method. So, in this section, an efficient heuristic solution method is presented to find a good feasible solution (ideally optimal one).

As mentioned earlier, this problem is a generalized form of the so-called ELDSP problem. Therefore, a new heuristic solution method has been developed inspired from algorithms suggested for ELDSP problem. Hahm and Yano (1995) presented mathematical model of ELDSP problem and introduced a heuristic solution method for the problem called H&Y. Jensen and Khouja (2003) devised a polynomial time algorithm called J&K, which solves the ELDSP problem to optimality. Finally, Clausen & Ju (2006) combined two previously suggested algorithms and constructed a new hybrid algorithm. The hybrid algorithm uses H&Y algorithm twice as the preprocessor of the J&K algorithm to decrease the computational time of the J&K algorithm then uses the J&K algorithm to solve the problem up to optimality.

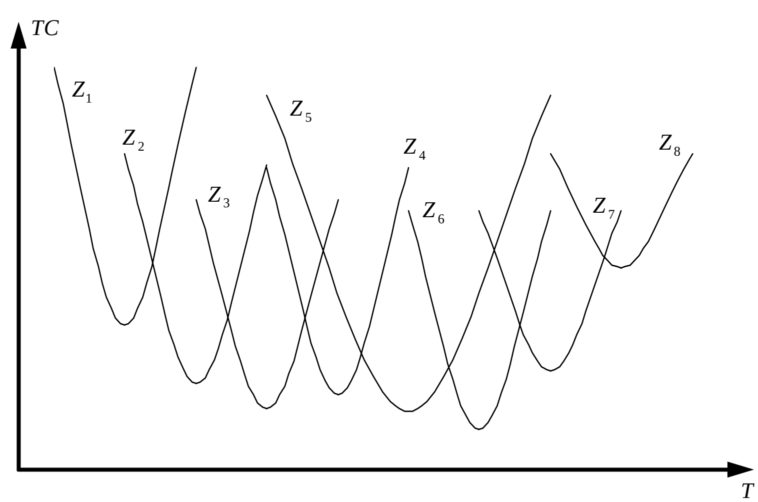

Figure 7 Recursive relationship between optimal value of cycle time and production sequences

We construct a new heuristic using the hybrid algorithm as its core for finding the value of cycle time and production sequences. In addition, we use a method similar to that of Kim et al. (2006) for finding the value of delivery frequencies.

The proposed algorithm consists of two main phases. In *phase one,* using hybrid algorithm the optimal production sequences are determined and then, in *phase two* the final value of the cycle time and delivery frequencies are calculated. At the rest of this section, a complete description of the algorithm along with corresponding pseudo code are given.

5.1. Phase one of the algorithm

Step 1: calculates the lower and upper bounds of the optimal cycle time value. The $T_{\min}$ could be obtained using Eq. (7). Moreover, $T_{\max}$ can be found by considering $\sum_{i=1}^{v}\delta_i = 0$ in Eq. (13); therefore:

$$T_{\min} = \underset{i=1,\ldots,v}{Max}\left\{\sum_{j=1}^{n_i} s_{ij} \Bigg/ \left(1-\sum_{j=1}^{n_i}\frac{D_{ij}}{P_{ij}}\right)\right\} \; and \; T_{\max} = \sqrt{\left(A+S+\sum_{i=1}^{v}A_i+\sum_{i=1}^{v}\sum_{j=1}^{n_i}S_{ij}\right)\Bigg/\sum_{i=1}^{v}\sum_{j=1}^{n_i}\beta_{ij}} \tag{19}$$

If $T_{\min} \geq T_{\max}$ *Step 2* sets $T_{temp} = T_{\min}$ and goes to *Step 6*. If $T_{\min} < T_{\max}$, the algorithm enters *Step 3* and iterates the H&Y algorithm twice using $T_{\max}$ and $T_{\min}$ and finally returns $\left(T_l, Z_{(l)}, m_{(l)}, TC_l\right)$ and $\left(T_u, Z_{(u)}, m_{(u)}, TC_u\right)$ as two local optimum solutions in *Step 4*. *Step 5* explores the interval $\left[T_l, T_u\right]$ in order to find the best value of cycle time. If $T_l = T_u$ algorithm goes to *Step 6*. Otherwise, else the interval $\left[T_l, T_u\right]$ is divided into a number of sub-intervals where the optimal sequences of all of the manufacturers within each are unchanged. An example of this case is shown in Figure 8. In this figure, there are two manufacturers M_1 and M_2. The top part of the figure represents the $\left[T_l, T_u\right]$ break points for M_1 where the production sequence of M_1 is unchanged in each sub-intervals. Similarly, the middle part is for M_2. Combination of these subintervals for two manufacturers is shown in bottom part of Figure 8 in which the production sequences of both of the manufacturers are unchanged in each of the determined sub-intervals.

In order to find the end points of these sub-interval, the following equation is solved for each manufacturers:

$$\frac{\dfrac{D[j]_i}{f[j]_i}}{s[j]_i + T\dfrac{D[j]_i}{P[j]_i}} = \frac{\dfrac{D[k]_i}{f[k]_i}}{s[k]_i + T\dfrac{D[k]_i}{P[k]_i}} \qquad \forall j,k = 1,\ldots,n_i \;\Big|\; j \neq k \tag{20}$$

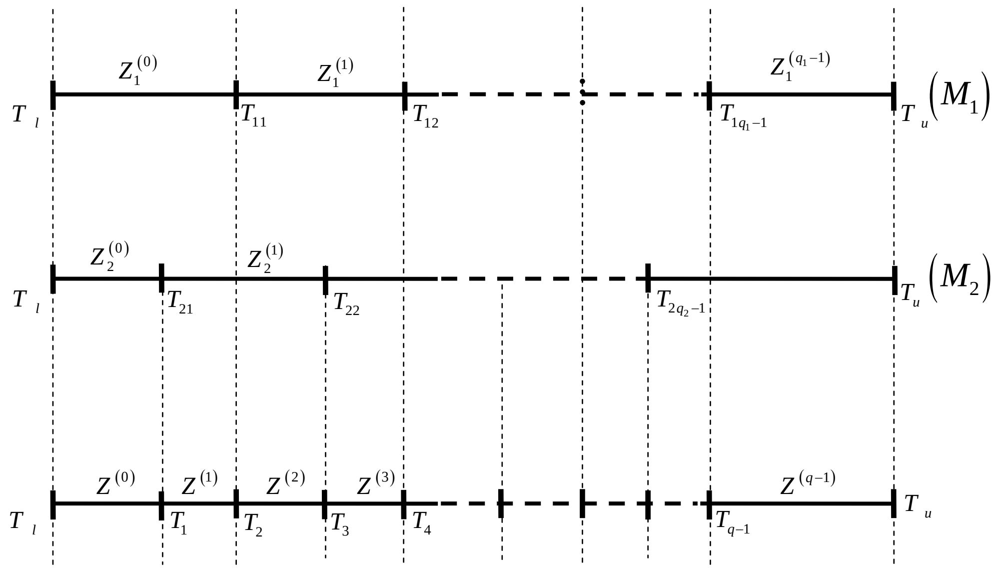

Figure 8 Combination of different sub-intervals.

Solving the above equations for M_i gives the values $T_{i1}, T_{i2}, \ldots, T_{iq_{i-1}}$. The values within $[T_l, T_u]$ are the potential sub-intervals' end points. In each manufacturer, there are at most $n_i(n_i - 1)/2$ different values for *T*. Considering all of the intervals, at most $1 + \sum_{i=1}^{v} n_i(n_i - 1)/2$ different subintervals can exist. In *Step 5*, algorithm finds the best value of cycle time in each sub-interval. The best-found value of cycle time and corresponding delivery frequencies among all of the sub-intervals is used in *Section 2* in order to find the final values of these decision variables.

5.2. Phase two of the algorithm

If all the values of delivery frequencies are integer, the algorithm terminates. Otherwise, it fixes all of the integer $(m_{ij})_{temp}$ and determines two nearest integer values for non-integer delivery frequencies. These values are the alternative values for non-integer delivery frequencies. In Step 7, all permutations of delivery frequencies using $(m_{ij})_{temp}$ for integer-value delivery frequencies as well as $\lfloor m_{ij} \rfloor$ and $\lceil m_{ij} \rceil$ for each non-integer value $(m_{ij})_{temp}$ are considered and by using Eq. (8) the relevant value of cycle time is calculated. Finally, the delivery frequencies and cycle time relevant to the least objective function are chosen as the final values for these decision variables. The complete pseudo code of the algorithm has been provided below for more clarification. In the pseudo code, T_{final}, Z_{final} and m_{final} are the final values of the decision variables and the T_{temp}, Z_{temp}, m_{temp} and TC_{temp} represent the temporary values of these decision variables.

Phase one

Step 1: Calculate $T_{\min}$ and $T_{\max}$ (Eq. 18)

Step 2: If $T_{\min} \geq T_{\max}$ then set $T_{temp} = T_{\min}$ and go to ***Step 6***

Step 3: $T_l = T_{\min}$, $T_u = T_{\max}$
$stop = 0$
$While \left(stop = 0 \right) \; do$ {
For a given T_u, find $Z = Z\left(T_u \right)$ (using Eq. 18) and $T^*\left(Z\right)$ (using Eq. 13)
If $T^*\left(Z\right) \neq T_u \wedge T^*\left(Z\right) \geq T_{\min}$ then $T_u = T^*\left(Z\right)$
Else if $T^*\left(Z\right) \neq T_u \wedge T^*\left(Z\right) < T_{\min}$ then $T_{temp} = T_{\min}$ and go to ***Step 6***
Else $stop = 1$
}
$stop = 0$
$While \left(stop = 0 \right) \; do$ {
For a given T_l, find $Z = Z\left(T_l \right)$ (using Eq. 18) and $T^*\left(Z\right)$ (using Eq. 13)
If $T^*\left(Z\right) \neq T_l \wedge T^*\left(Z\right) \geq T_{\min}$ then $T_l = T^*\left(Z\right)$
Else if $T^*\left(Z\right) \neq T_l \wedge T^*\left(Z\right) < T_{\min}$ then $T_l = T_{\min}$, $stop = 1$
Else $stop = 1$
}

Step 4: Return $\left(T_l, Z\left(T_l\right), m\left(T_l\right), TC_l\right)$ and $\left(T_u, Z\left(T_u\right), m\left(T_u\right), TC_u\right)$ as two local optimum solutions

Step 5: if $\left(T_l = T_u\right)$ then $T_{temp} = T_l$ and go to ***Step 6***
Else
$TC_{temp} = \min\left(TC_l, TC_u\right)$ Return the corresponding T, m, Z as $T_{temp}, Z_{temp}, m_{temp}$
In each manufacturer such as M_i, for each pair of products j and k, $j \neq k$ solve

$$\frac{\dfrac{D[j]_i}{f[j]_i}}{s[j]_i + T\dfrac{D[j]_i}{P[j]_i}} = \frac{\dfrac{D[k]_i}{f[k]_i}}{s[k]_i + T\dfrac{D[k]_i}{P[k]_i}} \qquad \forall j,k = 1,\ldots,n_i \mid j \neq k$$

and find the resulting values for T Store the values within $\left[T_l, T_u\right]$ into $W = \left[T_l, T_1, T_2, \ldots, T_i, \ldots, T_u\right]$.
Sort W in increasing order. $w = size\left(W\right)$
For $\left(i = 1; i < w; i = i + 1\right)$ {
For $T_w = \frac{1}{2}\left(W_i + W_{i+1}\right)$, find
$Z = Z\left(T_w\right)$ (using Eq. 18) and $T^*\left(Z\right)$ (using Eq. 13)

If $\left(T^*(Z)\in[W_i\ ,W_{i+1}]\wedge TC\left(T^*(Z),Z\left(T^*\right),m\left(T^*\right)\right)<TC_{temp}\right)$ then

$T_{temp}=T^*(Z)$, $Z_{temp}=Z\left(T_{temp}\right)$, $m_{temp}=m\left(T_{temp}\right)$, $TC_{temp}=TC\left(T_{temp},Z_{temp},m_{temp}\right)$

Else if $T^*(Z)\notin[W_i\ ,W_{i+1}]$ then

Select $Min\left\{TC\left(W_i,Z(W_i),m(W_i)\right),TC\left(W_{i+1},Z(W_{i+1}),m(W_{i+1})\right),TC_{temp}\right\}$ as TC_{temp} and the corresponding (T,Z,m) as $\left(T_{temp},Z_{temp},m_{temp}\right)$

}

Step 6: Return $Z_{final}=Z\left(T_{temp}\right)$ (using Eq. 18) as the final production sequences. Put T_{temp} and $m_{temp}=m\left(T_{temp}\right)$ (using Eq. 17) as the best found values of cycle time and delivery frequencies in ***Section One***.

Phase two

Step 7: if all of the $\left(m_{ij}\right)_{temp}$ values are integer,

Go to ***Step 8***

Else

$TC_{temp}=\infty$

For all integer values of $\left(m_{ij}\right)_{temp}$, set $m_{ij}^*=\left(m_{ij}\right)_{temp}$

Set $k=2^{\text{number of non-integer }\left(m_{ij}\right)_{temp}}$ and $t=0$

For $\left(t=0;t\le k;t=t+1\right)$ {

Generate a new permutation of delivery frequencies $\left(m^*\right)$ using $\left(m_{ij}\right)_{final}$ for integer-value $\left(m_{ij}\right)_{temp}$, and $\left\lfloor\left(m_{ij}\right)_{temp}\right\rfloor$ & $\left\lceil\left(m_{ij}\right)_{temp}\right\rceil$ for each non-integer value $\left(m_{ij}\right)_{temp}$

Calculate $T\left(m^*,Z_{final}\right)$ using Eq. 8

If $TC\left(T,Z_{final},m^*\right)<TC_{temp}$ then

$T_{temp}=T$, $m_{temp}=m^*$, and $TC_{temp}=TC\left(T_{temp},Z_{final},m_{temp}\right)$

}

Step 8: $T_{final}=T_{temp}$, $m_{final}=m_{temp}$. Return $\left(T_{final},Z_{final}\ ,m_{final}\right)$ and $TC_{final}=TC^*\left(T_{final},Z_{final}\ ,m_{final}\right)$ as the final solution of the problem.

6. NUMERICAL EXPERIMENTS

To verify the efficiency of the proposed algorithms in terms of the solution quality and the required computational time, some numerical experiments have been generated. It is required to find the optimal solution of each problem instance in order to evaluate the ability of the heuristic algorithm in approaching the optimal solution. Consequently, an explicit enumeration algorithm has also been developed to find the optimal solution of each problem instance. This algorithm generates all of the possible production sequences among the manufacturers in order to find the optimal solution. The pseudo code of the explicit enumeration algorithm is given in Appendix B. The proposed explicit enumeration method as well as the analytical and heuristic algorithms were coded in Borland Delphi 7.0 language and run on a personal computer AMD 2200 M.Hz with 256 MB of RAM.

We have tested the heuristic algorithm on different supply chain configurations using different set of parameter combinations. Required parameters are randomly generated using uniform distributions. As mentioned previously, the core of our algorithm is ELDSP and it is predictable that in most of the parameter combinations $T_{\min} \geq T_{\max}$. In these cases, the optimal sequences are uniquely determined by $T_{\min}$ and algorithm stops at the very beginning before iterating. In order to gain insight to the efficiency of the algorithm, the generated problem instances should satisfy $T_{\min} < T_{\max}$ to allow the algorithm enters into the iterative parts. It should be noted that similar to ELDSP, as the number of the retailers increase, $T_{\min}$ and $T_{\max}$ values increase. However, the increase rate of $T_{\min}$ is expected to be much greater than the rate of $T_{\max}$ if the ranges from which the parameters are drawn are kept constant. Consequently, this increases the occurrence of the $T_{\min} \geq T_{\max}$ situation. To resolve this problem, the parameters' range are selected in such a way that s_{ij} (and consequently $T_{\min}$) decreases as the number of the retailers increases. Similarly, A, S, A_i, and S_{ij} (and consequently $T_{\max}$) increase as the number of the components increase. This ceases the frequent occurrence of the situations in which $T_{\min} \geq T_{\max}$. Further, parameters' ranges are selected in such a way that $\sum_{j=1}^{n_i} \frac{D_{ij}}{P_{ij}} < 1;\ \forall i$, for the relevance of the proposed model. In addition, as mentioned earlier, because of the value added activities down through the supply chain, the relations $H_i' > h_\circ' \geq \frac{h_\circ}{f_s};\ \forall\, i$ and $h_{ij}^{(3)} \geq h_{ij}^{(2)} \geq \frac{H_i'}{f_{ij}};\ \forall\, i, j$ must hold among the unit inventory holding costs. The following uniform distributions have been used for the instances generation:

$S \sim U(100J, 200J)$, $A \sim U(50J, 100J)$, $h_\circ \sim U(0.1, 0.15)$, $h_\circ' \sim U(0.16, 0.2)$, $f_s \sim U(0.95, 1)$, $A_i \sim U(300n_i, 400n_i)$, $H_i' \sim U(0.2, 0.25)$, $P_{ij} \sim U(100, 200)$, $h_{ij}^{(2)} \sim U(0.27, 0.3)$, $s_{ij} \sim U\left(0, 0.25/\sqrt{n_i}\right)$, $S_{ij} \sim U(500n_i, 1000n_i)$, $f_{ij} \sim U(0.95, 1)$, $h_{ij}^{(3)} \sim U(0.3, 0.35)$, $D_{ij} \sim U(10, 20)$, $A_{ij} \sim U(50, 100)$

Note that $J = \sum_{i=1}^{v} n_i$ denotes the total number of retailers in the chain. In addition, $U(a, b)$ denotes the uniform random variable between *a* and *b*.

By using above parameter sets, different kinds of chain configurations in terms of the number of the manufacturers and number of the retailers are investigated. The simplest chain structure encompasses a single manufacturer and a single retailer. Table 1 represents the result of investigated experiments. In all of the investigated problem instances, the heuristic algorithm end up at the optimal solution. This result is somehow predictable, because the proposed algorithm is based on the hybrid algorithm proposed in Clausen and Ju (2006) which explores all of the feasible cycle time values between $[T_l, T_u]$ and consequently finds the optimal cycle time value. Sequencing and delivery frequency decision variables depend on the cycle time. As the algorithm finds the optimal cycle time value, the sequence vector as well as delivery frequencies are expected to be optimal.

It is noticeable that, as the number of retailers in each configuration increases, the required computation time for finding the optimal solution significantly increases. As it is shown in table one, in the case of two manufacturers, by increasing one retailer to each manufacturer, the computational time of the explicit enumeration method increased 22 times (from 5.688 to 137.04 seconds). Further, the computational time of the proposed algorithm increased nearly 18 times. In addition, in the case of four manufacturers, by increasing only one retailer to the retailers of just one manufacturer, the computational time of the explicit enumeration method increased four times. Also, the computational time of the proposed algorithm increased about three times. This observation is due to the tremendous increase in the number of possible sequence vectors and different delivery frequencies combinations.

Table 1 Computational results

No. of manufacturers	**Manufacturer index**	**No. of retailers**	**No. of instances**	**Cumulative computational time (seconds)**	
				Heuristic algorithm	**Explicit Enumeration algorithm**
1	1	1	1000	negligible	0.016
1	1	6	1000	3.224	6.327
2	1 2	3 4	1000	2.861	5.688
2	1 2	4 5	1000	53.606	137.04
3	1 2 3	3 2 3	1000	4.673	10.907
3	1 2 3	3 3 3	1000	13.052	51.642
3	1 2 3	4 3 3	1000	60.3	256.045
4	1 2 3 4	2 3 3 2	1000	36.805	128.062
4	1 2 3 4	2 3 3 3	1000	118.2	512.531
5	1 2 3 4 5	1 2 2 3 3	1000	45.972	334.908
5	1 2 3 4 5	2 2 2 3 3	1000	263.920	1162.141

Totally, the proposed solution algorithm seems to be very promising in finding the optimal solution for at least the small and moderate problem instances.

7. CONCLUSION REMARKS

This paper analyzes a supply chain with multiple manufacturers and multiple retailers, to determine a joint procurement–production–delivery policy. Each manufacturer procures common raw material, produces multiple items on a single production facility based on the common rotation cycle policy, and delivers them to the corresponding retailers. The goal is to derive production sequences along with the common cycle length for the manufacturers, and delivery lot sizes for the multiple retailers minimizing the average total cost. The proposed model can readily be applied to many practical manufacturing systems such as chemical and petrochemical industries. Numerical experiments show that in small to moderate instances of problems, the proposed algorithm find the optimal solution.

Further research is needed to analyze more generalized case of multiple items and multiple retailers, where any retailer could order any number of the items. Also, negotiation and/or coordination mechanisms may be worthy of future study through investigating the negotiation mechanism in which anticipated losses caused by accepting the joint procurement–production–delivery policy are compensated for the parties involved in the supply chain.

REFERENCES

[1] Baker K. (1974), Introduction to sequencing and scheduling; John Wiley & Sons Inc., New York; 20-22.

[2] Ben-Daya M., Darwish M., Ertogral K. (2007), The joint economic lot sizing problem: Review and extensions; *European Journal of Operational Research*; In press.

[3] Clausen J., Ju S. (2006), A hybrid algorithm for solving the economic lot and delivery scheduling problem in a common cycle case; *European Journal of Operational Research* 175; 1141-1150.

[4] Goyal S.K. (1977), An integrated inventory model for a single supplier–single customer problem; *International Journal of Production Research* 15; 107–111.

[5] Hahm J., Yano C.A. (1992), The economic lot and delivery scheduling problem: The single item case; *International Journal of Production Economics* 28; 235-252.

[6] Hahm J., Yano C.A. (1995a), The economic lot and delivery scheduling problem: Models for nested schedule; *IIE Transactions* 27; 126-139.

[7] Hahm J., Yano C.A. (1995b), The economic lot and delivery scheduling problem: power of two policies; *Transportation Science* 29(3); 222-241.

[8] Jensen M.T., Khouja M. (2004), An optimal polynomial time algorithm for the common cycle economic lot and delivery scheduling problem; *European Journal of Operational Research* 156(3); 305-311.

[9] Khouja M. (2003), Optimizing inventory decisions in a multi-stage multi-customer supply chain; *Transportation Research Part E* 39; 193–208.

[10] Kim T., Hong Y., Chang S. (2006), Joint economic procurement-production-delivery policy for multiple items in a single-manufacturer, multiple-retailer system; *International Journal of Production Economics* 103; 199-208.

[11] Lee W. (2005), A joint economic lot size model for raw material ordering, manufacturing setup, and finished goods delivering; *Omega* 33; 163-174.

[12] Muson C.L., Rosenblatt M.J. (2001), Coordinating a three-level supply chain with quantity discounts; *IIE Transactions* 33; 371-384.

[13] Simchi-Levi D., Kaminsky P., Simchi-Levi E. (2000), Designing and Managing the Supply Chain; Irwin McGraw-Hill; New York.

[14] Torabi S.A., Fatemi Ghomi S.M.T., Karimi B. (2006), A hybrid genetic algorithm for the finite horizon economic lot and delivery scheduling in supply chains; *European Journal of Operational Research* 173; 173-189.

Appendix A

Manufacturer's holding cost of input material Suppose a given manufacturer M_i. The following additional notations are required to facilitate the calculation of input material's holding cost:

$\bar{I}(Z_i)_{np}$: Input material's average inventory level of M_i during the set up times for a given sequence Z_i

$\bar{I}(Z_i)_{p}$: Input material's average inventory level of M_i during the production times for a given sequence Z_i

$\bar{I}(Z_i)$: M_i 's average inventory level for input material for a given sequence Z_i

According to Figure 5, average Inventory level of input material during the production and setup periods can be calculated as follows:

$$
\begin{aligned}
\bar{I}(Z_i)_p = \frac{1}{T}\Bigg\{ & \frac{1}{2}tp[1]_i \cdot \left(\sum_{k=1}^{n_i} \frac{m[k]_i \cdot q[k]_i}{f[k]_i} + \sum_{j=2}^{n_i} \frac{m[j]_i \cdot q[j]_i}{f[j]_i} \right) + \\
& + \frac{1}{2}tp[2]_i \cdot \left(\sum_{k=2}^{n_i} \frac{m[k]_i \cdot q[k]_i}{f[k]_i} + \sum_{j=3}^{n_i} \frac{m[j]_i \cdot q[j]_i}{f[j]_i} \right) + \cdots + \\
& + \frac{1}{2}tp[n_i-1]_i \cdot \left(\frac{m[n_i-1]_i \cdot q[n_i-1]_i}{f[n_i-1]_i} + \frac{m[n_i]_i \cdot q[n_i]_i}{f[n_i]_i} + \frac{m[n_i]_i \cdot q[n_i]_i}{f[n_i]_i} \right) + \\
& + \frac{1}{2}tp[n_i]_i \cdot \frac{m[n_i]_i \cdot q[n_i]_i}{f[n_i]_i} \Bigg\} =
\end{aligned}
$$

$$=\frac{1}{T}\left\{\frac{D[1]_i\cdot T}{2P[1]_i}\cdot\left(\sum_{k=1}^{n_i}\left(\frac{D[k]_i\cdot T}{f[k]_i}\right)+\sum_{j=2}^{n_i}\left(\frac{D[j]_i\cdot T}{f[j]_i}\right)\right)+\right.$$

$$+\frac{D[2]_i\cdot T}{2P[2]_i}\cdot\left(\sum_{k=2}^{n_i}\left(\frac{D[k]_i\cdot T}{f[k]_i}\right)+\sum_{j=3}^{n_i}\left(\frac{D[j]_i\cdot T}{f[j]_i}\right)\right)+\cdots+$$

$$\left.+\frac{D[n_i-1]_i\cdot T}{2P[n_i-1]_i}\left(\left(\frac{D[n_i-1]_i\cdot T}{f[n_i-1]_i}+\frac{D[n_i]_i\cdot T}{f[n_i]_i}+\frac{D[n_i]_i\cdot T}{f[n_i]_i}\right)\right)+\frac{D[n_i]_i\cdot T}{2P[n_i]_i}\cdot\frac{D[n_i]_i\cdot T}{f[n_i]_i}\right\}=$$

$$=T\left\{\sum_{j=1}^{n_i}\frac{D_{ij}^2}{2f_{ij}\cdot P_{ij}}+\sum_{j=1}^{n_i-1}\left(\frac{D[j]_i}{P[j]_i}\cdot\sum_{k=j+1}^{n_i}\frac{D[k]_i}{f[k]_i}\right)\right\}$$

$$\Rightarrow\qquad \bar{I}(Z_i)_p=T\left\{\sum_{j=1}^{n_i}\frac{D_{ij}^2}{2f_{ij}P_{ij}}+\sum_{j=1}^{n_i-1}\left(\frac{D[j]_i}{P[j]_i}\sum_{k=j+1}^{n_i}\frac{D[k]_i}{f[k]_i}\right)\right\}$$

$$\bar{I}(Z_i)_{np}=\frac{1}{T}\cdot\left(s[1]_i\cdot\sum_{k=1}^{n_i}\frac{m[k]_i\cdot q[k]_i}{f[k]_i}+s[2]_i\cdot\sum_{k=2}^{n_i}\frac{m[k]_i\cdot q[k]_i}{f[k]_i}+\cdots+s[n_i]_i\cdot\frac{m[n_i]_i\cdot q[n_i]_i}{f[n_i]_i}\right)=$$

$$=\frac{1}{T}\left(\sum_{j=1}^{n_i}\left(s[j]_i\cdot\sum_{k=j}^{n_i}\frac{m[k]_i\cdot q[k]_i}{f[k]_i}\right)\right)=\sum_{j=1}^{n_i}\left(s[j]_i\cdot\sum_{k=j}^{n_i}\frac{\frac{m[k]_i\cdot q[k]_i}{f[k]_i}}{\frac{m[k]_i\cdot q[k]_i}{D[k]_i}}\right)=\sum_{j=1}^{n_i}\left(s[j]_i\cdot\sum_{k=j}^{n_i}\frac{D[k]_i}{f[k]_i}\right)$$

$$\Rightarrow\qquad \bar{I}(Z_i)_{np}=\sum_{j=1}^{n_i}\left(s[j]_i\cdot\sum_{k=j}^{n_i}\frac{D[k]_i}{f[k]_i}\right)$$

$$\bar{I}(Z_i)=\bar{I}(Z_i)_p+\bar{I}(Z_i)_{np}=T\cdot\left\{\sum_{j=1}^{n_i}\frac{D_{ij}^2}{2f_{ij}\cdot P_{ij}}+\sum_{j=1}^{n_i-1}\left(\frac{D[j]_i}{P[j]_i}\cdot\sum_{k=j+1}^{n_i}\frac{D[k]_i}{f[k]_i}\right)\right\}+\sum_{j=1}^{n_i}\left(s[j]_i\cdot\sum_{k=j}^{n_i}\frac{D[k]_i}{f[k]_i}\right)$$

Considering all of the manufacturers results the following equation as the total holding cost of input material:

$$\sum_{i=1}^{v}H_i'.\bar{I}(\bar{Z}_i)=\sum_{i=1}^{v}H_i'.\left\{T\cdot\left[\sum_{j=1}^{n_i}\frac{D_{ij}^2}{2f_{ij}\cdot P_{ij}}+\sum_{j=1}^{n_i-1}\left(\frac{D[j]_i}{P[j]_i}\cdot\sum_{k=j+1}^{n_i}\frac{D[k]_i}{f[k]_i}\right)\right]+\sum_{j=1}^{n_i}\left(s[j]_i\cdot\sum_{k=j}^{n_i}\frac{D[k]_i}{f[k]_i}\right)\right\}=$$

$$=T\cdot\left\{\sum_{i=1}^{v}\sum_{j=1}^{n_i}\frac{H_i'.D_{ij}^2}{2f_{ij}\cdot P_{ij}}+\sum_{i=1}^{v}\sum_{j=1}^{n_i-1}\left(H_i'\cdot\frac{D[j]_i}{P[j]_i}\cdot\sum_{k=j+1}^{n_i}\frac{D[k]_i}{f[k]_i}\right)\right\}+\left[\sum_{i=1}^{v}\sum_{j=1}^{n_i}\left(H_i'\cdot s[j]_i\cdot\sum_{k=j}^{n_i}\frac{D[k]_i}{f[k]_i}\right)\right]$$

Appendix B

The pseudo code of the explicit algorithm is as follows:

For ($i=0$; $TC_{temp}=+\infty; i<\prod_{i=1}^{v}\left(n_i!\right); i=i+1$) {

Generate a new set of sequences (Z)

Calculate T using Eq. 16 , m^* using Eq. 17, and $TC=TC\left(T,m^*,Z\right)$

If $TC<TC_{temp}$ then

If all of the m_{ij}^* values are integer then $TC_{temp}=TC$, $T_{temp}=T$

Else

Fix all of the integer-value m_{ij}^*

Set $k=2^{\text{number of non-integer } m_{ij}^*}$

For ($t=0$; $t<k; t=t+1$) {

Generate a new permutation of delivery frequency factors $\left(m^*\right)$ using m_{ij}^* for integer-value m_{ij} , and $\lfloor m_{ij}^* \rfloor$ & $\lceil m_{ij}^* \rceil$ for each non-integer value m_{ij}^*

Calculate $T=T\left(m^*,Z\right)$ using Eq. 8 and $TC=TC\left(T,m^*,Z\right)$

If $TC<TC_{temp}$ then $T_{temp}=T$ and $TC_{temp}=TC$

}

}

Return $\left(T_{temp},m^*,Z\right)$, and TC_{temp} as the optimal solution of the problem.