

Economic Production Quantity in Reworkable Production Systems with Inspection Errors, Scraps and Backlogging

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ABSTRACT

The classical economic production quantity (EPQ) model is a well-known and commonly used inventory control technique. Common assumptions in this model are that all units produced are of perfect quality and shortage is not allowed, since in practice shortage, non-conforming product or scrap items are possible, these assumptions will underestimate the actual required quantity. The objective of this paper is to provide a framework to integrate production of imperfect quality items, inspection errors, rework, scrap items and backlogging into a single EPQ model. To achieve this objective a suitable mathematical model is developed and the optimal production lot size which minimizes the total cost is derived. The sensitivity analysis results indicate the model is very sensitive to shortage cost per unit shortage and type-I error of inspection. While findings of this study show that the model is very sensitive to parameters type-I error of inspection, proportion of defectives in production process and shortage cost per unit short, it is much less sensitive to parameters like proportion of scraps in rework process and Type-II error of inspection. If the existence of such error and shortages are ignored, then the obtained results may differ considerably from the optimal outcome. This will impose additional costs to the system.

Keywords: EPQ, Rework, Inspection errors, backlogging, Scraps

1. INTRODUCTION

Profit acquisition is the essential aim of companies in the competitive world. To achieve this goal they have to be able to effectively utilize resources and their costs related to production and any other relevant factor. Economic Order Quantity (EOQ) model was the first attempt at using a mathematical model to assist corporations in minimizing overall inventory costs (Zipkin, 2000). A considerable amount of research has since been carried out to enhance the classical EOQ model by addressing its unrealistic assumptions (Nahmias, 2001). In the manufacturing sector, when items are produced internally instead of being obtained from an outside supplier, the Economic Production Quantity (EPQ) model is often used to determine the optimal production lot size in order to minimize the total production and inventory costs. The classical EPQ model has been used for a

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long time and is widely accepted and implemented (Bedworth, & Bailey, 1987). Nevertheless, the analysis for finding an economic lot size has been based on a number of unrealistic assumptions (Markland et al., 1998). The classical EPQ model shows that the optimal lot size will generate minimum manufacturing cost, thus leading to the minimum total setup cost and inventory cost. However, this is only true if all manufactured products are of perfect quality and no shortage is permitted. But in reality, on one hand, defective items are generated during a production run and on the other hand shortages are inevitable. Imperfect quality items can be reworked and repaired with additional repair and holding costs. For example, printed circuit board assembly (PCBA) in PCBA manufacturing, plastic goods in the plastic injection molding process, and production process in other industries, such as chemical, textile, metal components. Sometimes employ rework as an acceptable process in terms of level of quality. Rosenblatt and Lee (1986), proposed an EPQ model that deals with imperfect quality. They assumed that at some random point in time the process might shift from an in-control to an out-of-control state, and a fixed percentage of defective items are produced. Approximate solutions for obtaining an optimal lot size were developed in their paper. Zhang and Gerchak (1990), considered joint lot sizing and inspection policy in an EOQ model with random yield. They considered that the defective units cannot be used and must be replaced by non-defective ones. Salameh and Jaber (2000) hypothesized a production/inventory situation where items are not of perfect quality. The imperfect quality items could be used in another production/inventory situation. Their paper also considered that the imperfect items can be sold as a single batch at a lower price through a 100% inspection process. It shows that the economic lot size quantity tends to increase as the average percentage of imperfect items increases. However, they do not consider the impact of the rejection and the rework in their model. Furthermore, their paper only considered the EOQ model. Goyal and Cardenas-Barron (2002) presented a simple approach for determining the economic production quantity for an item with imperfect quality. It is suggested that this simple approach is comparable to the optimal method of Salameh and Jaber (2000). Jamal et al. (2004) developed a model of batch quantity in a single stage system considering the rework option of the defective items and adopted two policies as rework options. They assumed that all imperfect units will be reworked to become perfect items. Haji et al. (2010) developed a model to show the effect of inspection errors on the optimal batch size in reworkable single-stage production systems with scraps. Their research results indicate that the model is very sensitive to defective proportions and type-I error of inspection.

In this paper we develop a model, considering a production system in which the imperfect items are reworked to become either good items or scraps. Imperfect items are usually separated so that they are not passed to stock. While screening products at production process to separate imperfect quality items, errors may be committed. Imperfect quality items may be incorrectly accepted and good items incorrectly rejected. In addition to the above assumption, existence of the shortages is the other aspect of our model.

Assuming the rework process, scraps production, inspection errors (type-I and type-II errors) and shortage, a suitable mathematical model is defined, and then the optimal production lot size which minimizes the total cost of the system is obtained. A sensitivity analysis is also carried out for this model.

2. ASSUMPTION AND NOTATION

The following assumptions are made to develop the model:

- (a) Proportion of defectives in production process is constant in each cycle.

- (b) The production rate of non-defective items is constant and is greater than the demand rate.
- (c) Defective items produced at the production process are reworkable.
- (d) Reworked items are either good items or scraps.
- (e) Proportion of scraps and good items after rework process is constant in each cycle.
- (f) Production and rework are done using the same resources at the same speed.
- (g) Setup time for rework process is zero.
- (h) A 100% inspection is performed in order to identify the amount of good items, imperfect quality items and scraps in each lot and errors may be committed whose percentage is known.
- (i) All demands must be satisfied.
- (j) Backlogging permitted
- (k) The other assumption in classical EPQ model.

The following notations are used to develop the model:

Production rate, units per planning period, (units/year).	P
Demand rate, units per planning period (units/year).	D
Setup cost	A
Processing cost in production process per item	C_p
Processing cost in rework process per item	C_r
Inspection cost per item	C_i
Net batch quantity which is needed per cycle to satisfy demand	Q
Input batch quantity required to be processed per cycle	Q_i
The number of backorders	B
inventory holding cost per unit per unit time	H
Shortage cost per unit short	π
Shortage cost per unit shortage per unit time	$\hat{\pi}$
proportion of defectives in production process in each cycle	β
proportion of scraps in rework process in each cycle	α
proportion of good items that incorrectly rejected in each cycle	E_1
proportion of bad items that incorrectly accepted in each cycle	E_2
Production process time in each cycle	t_p
Rework process time in each cycle	t_r
The cost per imperfect item which is incorrectly accepted in Production process and Rework process	v_1
The cost per perfect item which is incorrectly rejected as scrap	v_2
Cycle time	T

3. MATHEMATICAL MODEL

In any production system, production consists of mainly two operations: (i) setup before the production, and (ii) processing of jobs on the machine. Consequently, these two operations involve two types of costs, setup cost and processing cost. Since the production rate (P) is greater than the demand rate (D), during the production period, inventory is accumulated. This situation of inventory build-up sometimes helps to satisfy the demand during the period in which the production is stopped, due to various reasons. We further assume that the net production rate, after excluding the defective items or scraps, is constant and is greater than the demand rate, i.e.

$$P[(1-\beta)(1-E_1)+\beta E_2] > D \tag{1}$$

$$P[(1-\alpha)(1-E_1)+\alpha E_2] > D \tag{2}$$

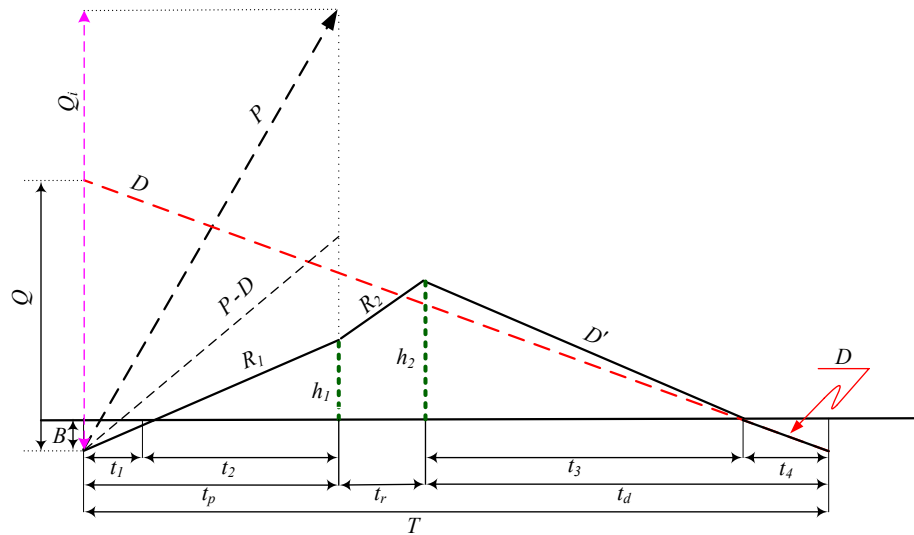


Figure 1 comparison of net inventory with defective and non-defective products

Figure 1 shows that the net inventory increases in the first phase of production (which includes t_1 and t_2), i.e., during the normal production time t_p , at rate R_1 which is equal to

$$R_1 = P[(1-\beta)(1-E_1)+\beta E_2]-D$$

At the end of this phase, the rework on the defective items starts. During the rework phase the net inventory increases at rate R_2 which is equal to

$$R_2 = P[(1-\alpha)(1-E_1)+\alpha E_2]-D$$

Also suppose that the inventory level comes down to zero at the end of t_3 when shortages start to build up until the end of cycle time. We suppose that customers are still waiting for the items to be backlogged. The current as well as the waiting customers' demands are totally satisfied during time t_1 at the start of the next cycle.

For simplification, let $L=(1-\beta)(1-E_1)+\beta E_2$ and $M=(1-\alpha)(1-E_1)+\alpha E_2$. Thus we can write R_1 and R_2 as follows:

$$R_1 = PL - D \tag{3}$$

$$R_2 = PM - D \tag{4}$$

Since Q_i stands for the input quantity in each cycle, then the required processing time for this quantity (the production time), t_p , is equal to Q_i/P , Includes t_1 and t_2 respectively. From Figure 1 we can write t_1 and t_2 as follow

$$t_1 = \frac{B}{R_1} \tag{5}$$

$$t_2 = t_p - t_1 \tag{6}$$

It is assumed that the quality of the output of the production process is not perfect. In each cycle at production process a fixed fraction of non-conforming items, β , may be produced. Therefore, the number of non-conforming items that are produced during the production process, N_1 , is equal to βQ_i and the number of perfect quality items at the end of the production process is $Q_i - N_1$. The processed items during the production process are inspected and put in the inventory to be used when necessary and non-conforming items are screened for rework process. It is assumed that raw materials and input products are of perfect quality. Since the inspection process is error prone, type-I and type-II errors may be committed. That is, non-conforming items may be incorrectly accepted and conforming items may be incorrectly rejected. From type-I error, the amount of correctly accepted items passed to inventory is $(Q_i - N_1)(1-E_1)$. From type-II error, the quantity of incorrectly accepted items is $N_1 E_2$. Hence the number of items recognized as good items is

$$Q_{op} = N_1 E_2 + (Q_i - N_1)(1 - E_1) \tag{7}$$

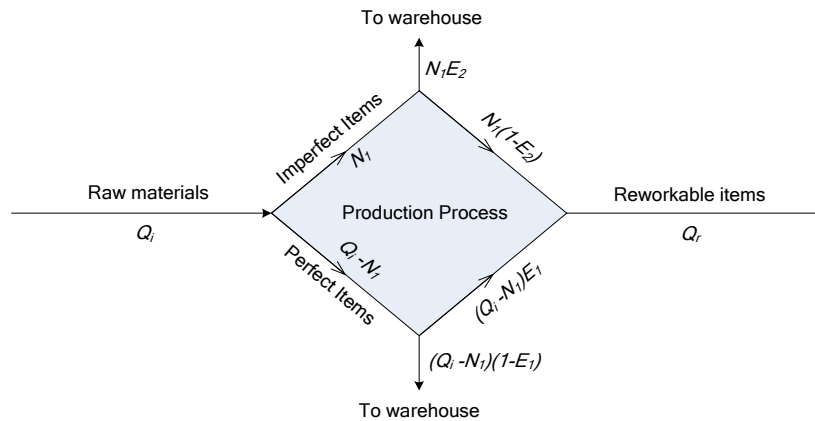


Figure 2 Product flow diagram in the regular production process

The number of reworkable items which is denoted by Q_r , consists of the number of incorrectly rejected good items and correctly accepted defective items for rework, which is computed as $Q_i -$

Q_{op} . It is shown in figure 2. The reworking on Q_r imperfect quality items starts immediately, when the regular production time t_p ends. The time t_r needed to do rework on imperfect quality items, is computed as Q_r/P .

During the rework process a fixed fraction α of scraps may be produced and also errors may be committed. We denote the number of scraps which are produced during the rework process by N_2 that is equal to αQ_r which implies that $Q_r - N_2$ good items have been produced. Based on the errors involved in the inspection process, that the number of scraps and the number of perfect items are $N_2(1 - E_2) + (Q_r - N_2)E_1$ and $N_2E_2 + (Q_r - N_2)(1 - E_1)$ respectively. It is shown in figure 3. The quantity of the Perfect items obtained during the rework process, denoted by Q_{or} , is equal to

$$Q_{or} = N_2E_2 + (Q_r - N_2)(1 - E_1) \tag{8}$$

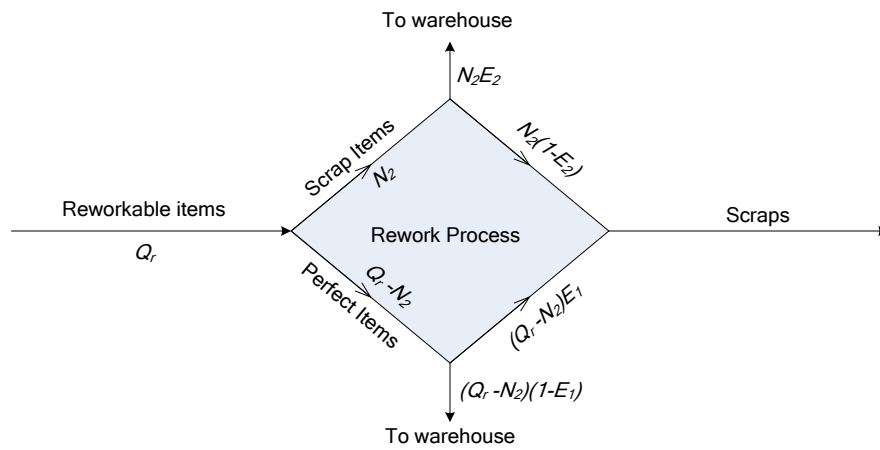


Figure 3 Product flow diagram in the rework process

Cycle time, T , includes regular production time, rework process time and the idle time, i.e.,

$$T = t_p + t_r + t_d$$

t_d includes t_3 and t_4 , where $t_3 = T - t_4 - t_r - t_p$, $t_4 = B/D$ and

$$T = \frac{Q}{D} \tag{9}$$

In which Q is the net batch quantity which is needed to satisfy demand per cycle and is calculated in the following relation (15). The maximum level of on-hand inventory at the end of production process, t_p , is denoted by h_1 , and at the end of the rework process t_r , is denoted by h_2 , (Figure 1).

$$h_1 = R_1 t_2 \tag{10}$$

$$h_2 = h_1 + R_2 t_r \tag{11}$$

Therefore, the quantity which is accepted as perfect items in each cycle and is sent to customers is:

$$Q_o = Q_{op} + Q_{or} \quad (12)$$

From the above equations we can write

$$Q_o = Q_i \lambda \quad (13)$$

$$Q_r = Q_i \omega \quad (14)$$

$$Q = Q_i \psi \quad (15)$$

Where:

$$\lambda = \beta(E_1 E_2 + E_1^2 - E_1) + \alpha(E_1 E_2 + E_1^2 - E_1) + \alpha\beta(2E_1 + 2E_2 - 2E_1 E_2 - E_1^2 - E_2^2 - 1) + 1 - E_1^2,$$

$$\omega = \beta(1 - E_1 - E_2) + E_1 \quad (16)$$

$$\psi = \lambda - E_2(\beta + \alpha\omega) \quad (17)$$

The total production cost of the system consists of the setup cost, processing-rework-inspection cost, inventory holding cost and the shortage cost (time dependent and time independent). These various cost items are now derived.

3.1. Setup cost

The annual setup cost is simply

$$TC_S = \frac{DA}{Q} \quad (18)$$

3.2. Processing, rework and inspection costs

In every cycle, the batch quantity Q_i is processed and the processing cost per item in a cycle is C_p . Hence, the total processing cost over the year is given by

$$TC_P = \frac{D}{Q} C_p Q_i \quad (19)$$

Defective items are reprocessed, and this leads to additional processing cost due to rework. For each batch quantity Q , the amount of defective items produced in each cycle is Q_r . The rework cost on the defective quantity Q_r over the year can be written as

$$TC_R = \frac{D}{Q} C_r Q_r \quad (20)$$

Since the processed and reworked items are inspected, the inspection cost over the year can be written as

$$TC_I = \frac{D}{Q} C_i (Q_i + Q_r) \quad (21)$$

Then the Processing, rework and inspection costs per year is simply

$$TC_{PRI} = \frac{D}{\psi} (C_p + \omega C_r + C_i (I + \omega)) \quad (22)$$

3.3. Inventory holding cost

From Figure 1 the average inventory, \bar{I} , can be computed as

$$\bar{I} = \frac{I}{2T} [h_1 t_2 + (h_1 + h_2) t_r + h_2 t_3] \quad (23)$$

Substituting Eqs. (3) – (6) and (9) – (17) into Eq. (23), we obtain

$$\bar{I} = \frac{B^2 PL}{2Q(PL - D)} + \frac{Q}{2\psi} \left[\frac{\omega D(L - M)}{P\psi} - \frac{D}{P} (I + \omega) + L + M\omega \right] - \frac{B}{2} \left(\frac{L + M\omega}{\psi} + I \right) \quad (24)$$

Hence, the inventory holding cost over the year is

$$TC_H = \frac{HB^2 PL}{2Q(PL - D)} + \frac{HQ}{2\psi} \left[\frac{\omega D(L - M)}{P\psi} - \frac{D}{P} (I + \omega) + L + M\omega \right] - \frac{HB}{2} \left(\frac{L + M\omega}{\psi} + I \right) \quad (25)$$

3.4. Shortage costs

Shortage costs, include the time dependent and time independent costs. From Figure 1, the average backorder, \bar{B} , can be computed as

$$\bar{B} = \frac{B(t_1 + t_4)}{2T} \quad (26)$$

Substituting Eqs. (3) – (5) and (9) into Eq. (26), we obtain

$$\bar{B} = \frac{B^2 PL}{2Q(PL - D)} \quad (27)$$

Hence the time dependent shortage cost over the year is computed as

$$TC_{\bar{B}} = \hat{\pi} \frac{B^2 PL}{2Q(PL - D)} \quad (28)$$

and the annual time independent shortage cost is

$$TC_{\hat{B}} = \frac{D}{Q} \pi B \quad (29)$$

Then the total shortage costs will be

$$TC_B = \hat{\pi} \frac{B^2 PL}{2Q(PL-D)} + \frac{D}{Q} \pi B \quad (30)$$

3.5. Inspection error costs

The inspection error costs during each cycle include the following costs:

- 1- The costs of non-conforming items which is incorrectly accepted in the production process and rework process where is equal to $v_1(Q_i \beta + \alpha Q_r) e_2$.
- 2- The cost of perfect items which is incorrectly rejected at rework process that is $v_2(1 - \alpha) e_1 Q_r$.

By using equation (7) and (13) and the above costs, we can compute the Inspection error costs per year as $TC_{QR} = \frac{D\gamma}{\psi}$.

Where $\gamma = v_1(\beta + \omega\alpha)e_2 + v_2\omega(1 - \alpha)e_1$.

3.6. Total cost

The total system cost $TC(Q, B)$ can be obtained by adding the individual costs expressed in Eqs. (18), (22), (25) and (30).

$$TC(Q, B) = \frac{DA}{Q} + \frac{HB^2 PL}{2Q(PL-D)} + \frac{HQ}{2\psi} \left[\frac{\omega D(L-M)}{P\psi} - \frac{D}{P}(1+\omega) + L + M\omega \right] - \frac{HB}{2} \left(\frac{L + M\omega}{\psi} + I \right) + \hat{\pi} \frac{B^2 PL}{2Q(PL-D)} + \frac{D}{Q} \pi B + \left(\frac{D}{\psi} \right) (C_p + C_r \omega + C_i(1+\omega) + \gamma) \quad (31)$$

By defining G , F , K and θ as

$$G = \frac{PL(H + \hat{\pi})}{2(PL-D)}$$

$$F = \frac{1}{2\psi} \left[\frac{\omega D(L-M)}{P\psi} - \frac{D}{P}(1+\omega) + L + M\omega \right]$$

$$K = \frac{1}{2} \left(\frac{L + M\omega}{\psi} + I \right)$$

$$\theta = \frac{C_p + C_r\omega + C_i(I + \omega) + \gamma}{\psi}$$

We can write (31) as

$$TC(Q, B) = \frac{D}{Q}(A + \pi B) + \frac{B^2 G}{Q} + HQF - HBK + D\theta \quad (32)$$

3.7. Optimal Solution

It can be shown that $TC(Q, B)$ is a convex function of Q and B . to establish the convexity of $TC(Q, B)$, one can utilize the Hessian matrix equation and obtain the following (Rardin, 1997):

$$[Q \ B] \begin{bmatrix} \frac{\partial^2 TC(Q, B)}{\partial Q^2} & \frac{\partial^2 TC(Q, B)}{\partial Q \partial B} \\ \frac{\partial^2 TC(Q, B)}{\partial B \partial Q} & \frac{\partial^2 TC(Q, B)}{\partial B^2} \end{bmatrix} \begin{bmatrix} Q \\ B \end{bmatrix} = \frac{2AD}{Q} > 0 \quad (33)$$

Equation (33) is strictly positive, because all parameters (A , D , and Q) are positive. Hence, the total cost function $TC(Q, B)$ is a strictly convex function for all Q and B different from zero. It follows that for the optimal production lot size Q and the maximal level of backorder B , one can differentiate $TC(Q, B)$ with respect to Q and with respect to B , and solve the linear system of Eqs. (34) and (35), by setting these partial derivatives equal to zero.

$$\frac{\partial TC(Q, B)}{\partial Q} = -\frac{D(A + \pi B)}{Q^2} - \frac{B^2 G}{Q^2} + HF \quad (34)$$

$$\frac{\partial TC(Q, B)}{\partial B} = \frac{2BG + \pi D}{Q} - HK \quad (35)$$

By isolating B from (35) and substituting it in (34), we will get

$$Q^* = \sqrt{\frac{D(4AG - \pi^2 D)}{H(4FG - HK^2)}} \quad (36)$$

And

$$B^* = \frac{HKQ^* - \pi D}{2G} \quad (37)$$

3.8. Sensitivity analysis

A sensitivity analysis is carried out for this model to study how the batch quantity, amount of backlog and total cost of the system are affected due to the changes of parameters: β , α , E_1 , E_2 , H , π and $\hat{\pi}$. The rate and direction of changes of Q and B with respect to β , α , E_1 , E_2 , H , π and $\hat{\pi}$ by

mathematical expression are $\left\{ \frac{\partial Q^*}{\partial \beta}, \frac{\partial Q^*}{\partial \alpha}, \frac{\partial Q^*}{\partial E_1}, \frac{\partial Q^*}{\partial E_2}, \frac{\partial Q^*}{\partial H}, \frac{\partial Q^*}{\partial \pi}, \frac{\partial Q^*}{\partial \hat{\pi}} \right\}$ and $\left\{ \frac{\partial B^*}{\partial \beta}, \frac{\partial B^*}{\partial \alpha}, \frac{\partial B^*}{\partial E_1}, \frac{\partial B^*}{\partial E_2}, \frac{\partial B^*}{\partial H}, \frac{\partial B^*}{\partial \pi}, \frac{\partial B^*}{\partial \hat{\pi}} \right\}$ respectively, this is depend on the parametric values of variables. For a problem with below data, we study the rate and direction of changes of production quantity, shortage and total cost of the system.

$D=8000, A=120, P=15000, H=80, \alpha=0.01, \beta=0.05, E_1=0.05, E_2=0.01, v_1=20, v_2=32, C_p=40, C_i=10, C_r=1.$

The results show that the model is very sensitive to parameters E_1, β and π whereas it is much less sensitive to parameters α and E_2 . In this case, changes in TC are directly related to changes in all parameters. Changes in Q are directly related to changes in α, E_1 and H whereas this change is inversely related to changes in π and E_2 . However changes in Q are directly related to changes in β from 0 to 0.22 and it is inversely related to changes in β from figures in excess of 0.23. Nowadays, given the progress made, inspection errors are very low and are often ignored. But findings of this study show that the changes in Q and TC are very sensitive to E_1 . If there are such errors in the system, and is not pay attention to, the obtained results will differ considerably from optimal outcome. This will impose additional costs to the system. Further information is given in Appendix.

4. CONCLUSION

In this paper we considered an imperfect production system in which a constant percent of defective items are produced. All defective items produced in each cycle are reworked in the same cycle immediately after the normal production ends. We assumed a 100% inspection takes place in both the normal production process and the rework process. We also assumed that type-I error (i.e., perfect items incorrectly rejected) and type-II error (i.e., perfect items incorrectly accepted) will be committed and backorders are permitted. For this system we obtained the optimal production quantity which minimizes the total cost of the system which is the sum of setup cost, normal and Rework processing costs, inspection cost, inventory holding cost and shortage costs. The sensitivity analysis results indicate the model is very sensitive to shortage cost per unit shortage and type-I error of inspection. Nowadays, given the progress made, inspection errors are rare which are often ignored. But findings of this study show that the changes in Q and total cost are very sensitive to type-I error of inspection. If the existence of such error is ignored, then the obtained results will differ considerably from the optimal outcome. This will impose additional costs to the system.

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Appendix

A sensitivity analysis is carried out for this model to study how the batch quantity, amount of backlog and total cost of the system are affected due to the changes of parameters: β , α , E_1 , E_2 , H , π and $\hat{\pi}$. The rate and direction of changes of Q and B with respect to β , α , E_1 , E_2 , H , π and $\hat{\pi}$ by mathematical expression are $\left\{ \frac{\partial Q^*}{\partial \beta}, \frac{\partial Q^*}{\partial \alpha}, \frac{\partial Q^*}{\partial E_1}, \frac{\partial Q^*}{\partial E_2}, \frac{\partial Q^*}{\partial H}, \frac{\partial Q^*}{\partial \pi}, \frac{\partial Q^*}{\partial \hat{\pi}} \right\}$ and $\left\{ \frac{\partial B^*}{\partial \beta}, \frac{\partial B^*}{\partial \alpha}, \frac{\partial B^*}{\partial E_1}, \frac{\partial B^*}{\partial E_2}, \frac{\partial B^*}{\partial H}, \frac{\partial B^*}{\partial \pi}, \frac{\partial B^*}{\partial \hat{\pi}} \right\}$ respectively, this is depend on the parametric values of variables. For a problem with below data:

$D=8000$	$\alpha=0.01$	$C_p=40$	$v_1=20$
$A=120$	$\beta=0.05$	$C_r=10$	$v_2=32$
$P=15000$	$E_1=0.05$	$C_i=1$	
$H=80$	$E_2=0.01$		

We study the rate and direction of changes of production quantity, shortage and total cost of the system. The results for each parameter are shown in the following figures and tables separately.

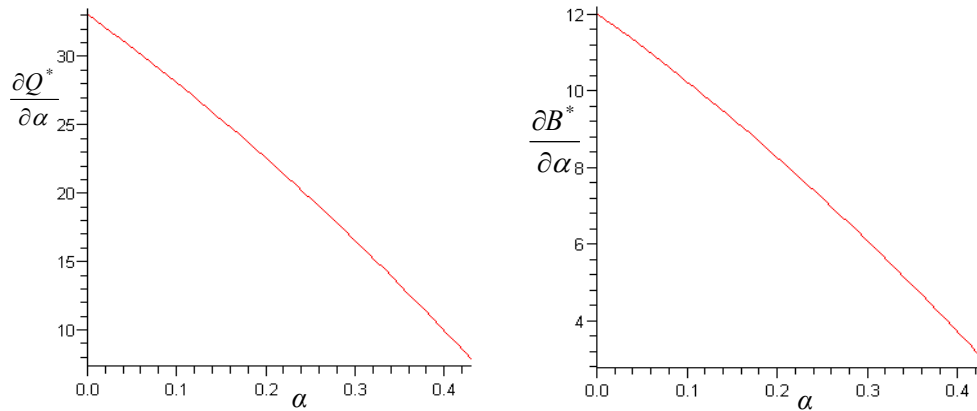
A1. Changing the α values

From which the solution is feasible if $\alpha \in [0, 0.44]$, then the effects of Q and B over the scrap proportion, α , are studied by changing the α values over the range from 0.00 to 0.44 (Figure 4).

It is observed that the total cost, $TC(Q^*, B^*)$, Q^* and B^* are directly related with defective proportion and its effect becomes less significant as the scrap rate increases.

Table 1 changing the α values

α	Q^*	B^*	$TC(Q^*, B^*)$
0.01	640.64	194.63	352639.73
0.05	641.90	195.09	353906.14
0.15	644.71	196.11	357115.46
0.25	646.97	196.94	360388.12
0.35	648.62	197.55	363726.02
0.40	649.20	197.76	365420.05
0.44	649.55	197.89	366787.56
0.444	---	---	---

Figure 4 Effect of scrap proportion, α , on batch and backlog size

A2. Changing the β values

Since the solution is feasible when $\beta \in [0, 0.44]$, then the effects of Q and B over the defective proportion are studied by changing the β values over the range from 0.00 to 0.44 (Figure 5). It is observed that the total cost, $TC(Q^*, B^*)$ and Q^* are directly related with defective proportion and B^* is inversely related with defective rate. Also its effect becomes more significant than scrap rate as the defective rate increases.

Table 2 changing the β values

β	Q^*	B^*	$TC(Q^*, B^*)$
0.05	640.64	194.63	352639.73
0.10	665.90	187.64	357783.92
0.15	688.72	176.52	362954.75
0.20	703.47	159.39	368165.58
0.30	681.62	103.27	378777.82
0.40	579.18	29.92	389753.84
0.443	522.11	0.17	394601.67
0.444	----	----	----

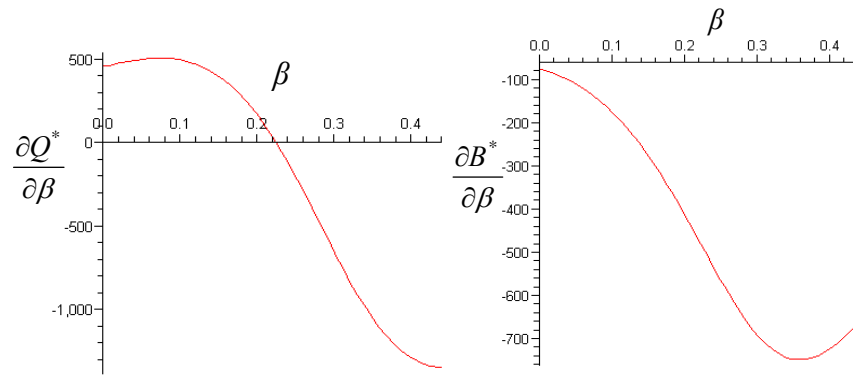


Figure 5 Effect of defective proportion, β , on batch and backlog size

A3. Changing the E_1 values

Since the solution is feasible if $E_1 \in [0, 0.44]$, then the effects of Q and B over the type-I error of inspection are studied by changing the E_1 values over the range from 0.00 to 0.45 (Figure 6). It is observed that the total cost, $TC(Q^*, B^*)$ and Q^* are directly related with type-I error of inspection and B^* is inversely related with that. Also its effect becomes more significant than scrap and defective rates as the type-I error of inspection increases.

Table 3 changing the E_1 values

E_1	Q^*	B^*	$TC(Q^*, B^*)$
0.05	640.64	194.63	352639.73
0.10	670.87	189.05	369372.28
0.20	757.59	172.74	412070.35
0.30	922.53	143.65	470615.14
0.40	1432.03	76.32	551925.21
0.4391	2377.82	0.08	592243.63
0.44	----	----	----

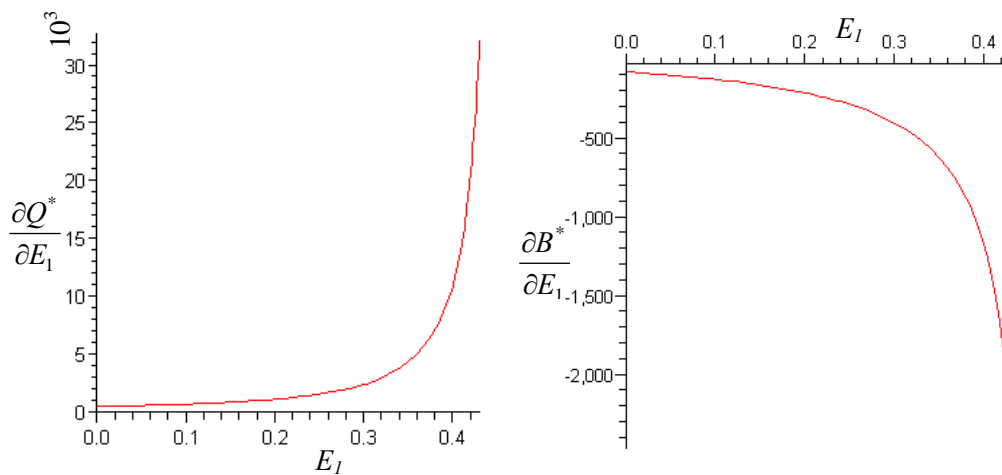


Figure 6 Effect of type-I error of inspection, E_1 , on batch and backlog size

A4. Changing the E_2 values

The effects of Q and B over the type-II error of inspection are studied by changing the E_2 values over the feasible range from 0.00 to 0.99 (Figure 7).

It is observed that the total cost, $TC(Q^*, B^*)$ and B^* are directly related with type-II error of inspection and Q^* is inversely related with that. Also its effect becomes less significant than type-I error and defective rates as the type-II error of inspection increases.

Table 4 changing the E_2 values

E_2	Q^*	B^*	$TC(Q^*, B^*)$
0.01	640.64	194.63	352639.73
0.20	632.55	195.73	357460.43
0.60	616.19	197.90	362618.10
0.90	604.60	199.46	367863.50
0.99	601.25	199.92	375901.54

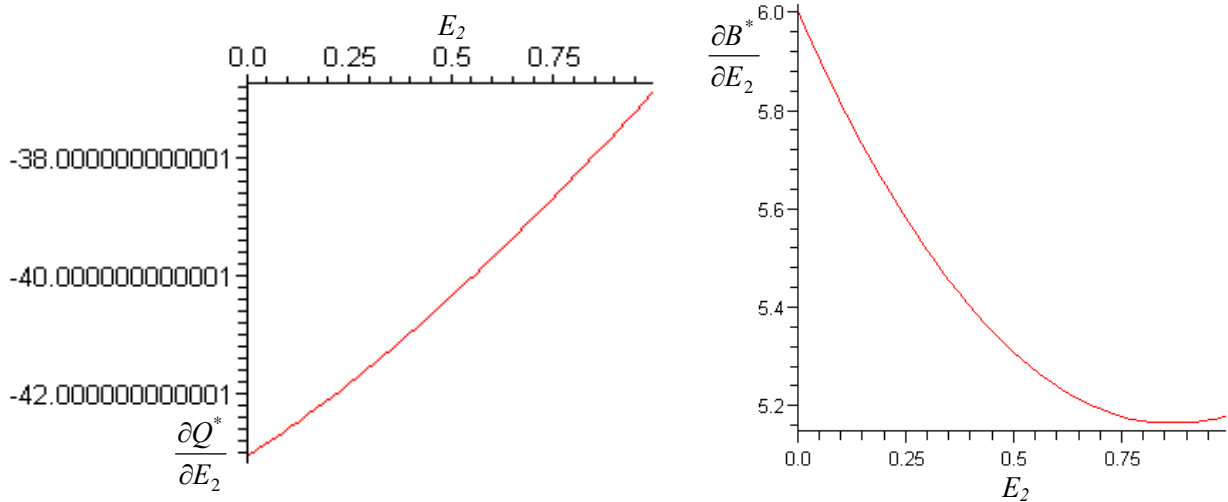


Figure 7 Effect of type-II error of inspection, E_2 , on batch and backlog size

A5. Changing the π values

Since the solution is feasible when $\pi \in [0, 2.2377]$, then the effects of Q and B over the shortage cost per unit shortage are studied by changing the π values over the range from 0.00 to 2.5 (Figure 8).

It is observed that the total cost, $TC(Q^*, B^*)$ is directly related with shortage cost per unit short. However Q^* and B^* are inversely related with that. Also its effect becomes more significant than scrap and defective rates as the shortage cost per unit shortage rate increases.

Table 5 changing the π values

π	Q^*	B^*	$TC(Q^*, B^*)$
0.00	694.85	250.13	349975.45
1.00	640.64	194.63	352639.73
1.50	565.61	149.63	353781.31
1.90	470.59	101.02	354555.37
2.00	439.56	86.26	354719.99
2.15	384.90	61.18	354934.58
2.20	363.87	51.81	354994.94
2.50	----	----	----

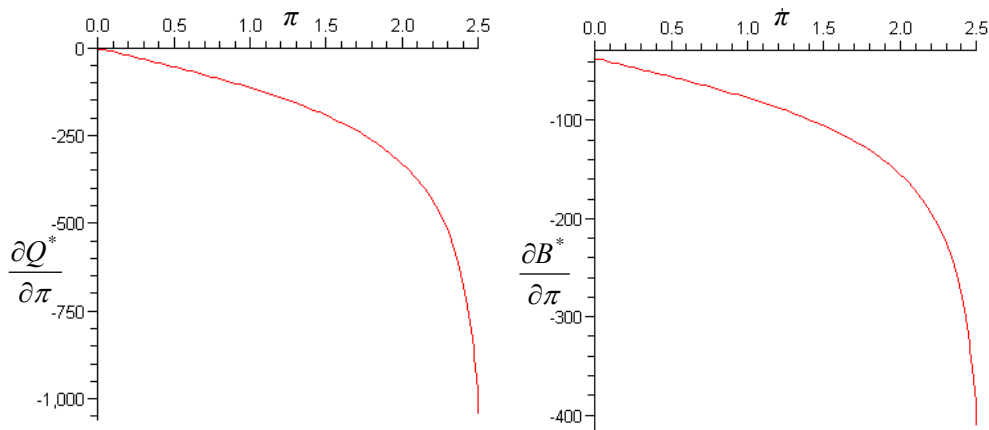


Figure 8 Effect of shortage cost per unit short, π , on batch size and backlog size

A6. Changing the $\hat{\pi}$ values

The effects of Q and B over the shortage cost per unit shortage per unit time are studied by changing the $\hat{\pi}$ values over the feasible range from 0.1 to 100 (Figure 9). It is observed that the total cost, $TC(Q^*, B^*)$ is directly related with shortage cost per unit shortage per unit time. However Q^* and B^* are inversely related with that. Also changes in Q^* and B^* are more sensitive to $\hat{\pi}$ from 0 to 10 and the sensitivity decreases as shown in the figures for values greater than 10.

Table 6 changing the $\hat{\pi}$ values

$\hat{\pi}$	Q^*	B^*	$TC(Q^*, B^*)$
0.1	5298.54	2126.06	350784.66
5	910.71	312.45	352065.20
10	667.43	206.55	352564.66
50	369.03	67.80	353884.82
100	312.08	38.60	354354.13
200	279.25	20.97	354688.58
1000	249.90	4.55	355040.84

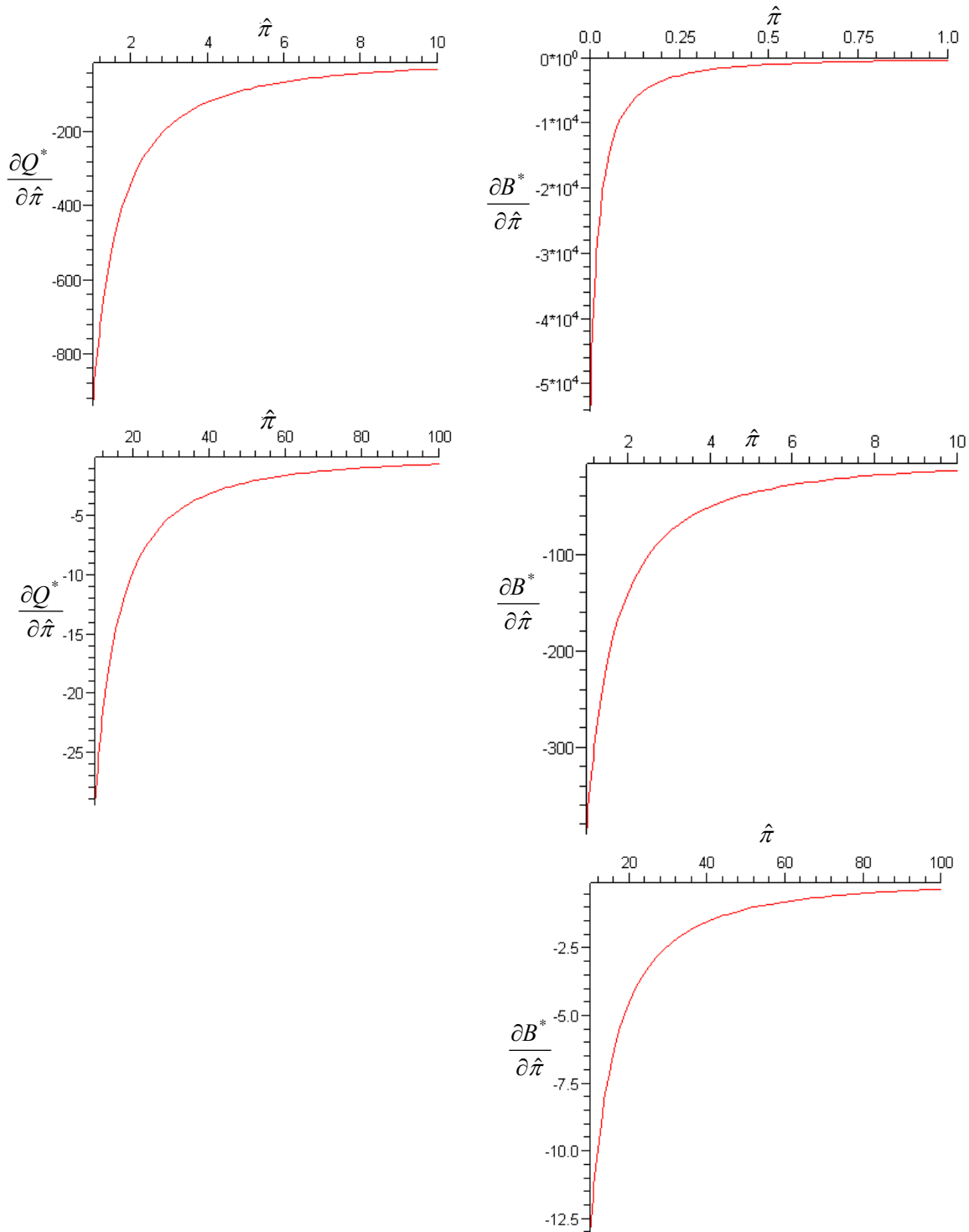


Figure 9 Effect of shortage cost per unit shortage per unit time, $\hat{\pi}$, on batch and backlog size

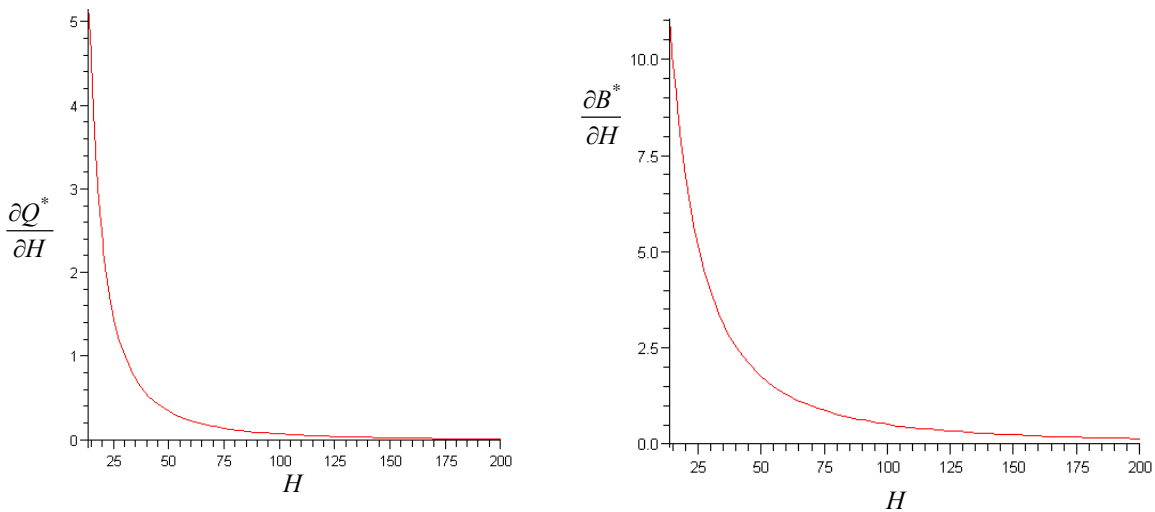
A7. Changing the H values

The effects of Q and B over the inventory holding cost per unit per unit time are studied by changing the H values over the feasible range from figures in excess of 13.7 (Figure 10).

It is observed that the total cost, $TC(Q^*, B^*)$ and B^* are directly related with inventory holding cost per unit per unit time and Q^* is directly related with H from 13.7 to 248.15 and it is inversely related with H from figures in excess of 248.15. However the changes in Q^* and B^* are more sensitive than total cost of the system.

Table 7 changing the H values

H	Q^*	B^*	$TC(Q^*, B^*)$
13.7	586.07	0.52	350495.39
20	607.68	54.89	351094.48
50	634.45	159.26	352246.67
100	642.54	207.53	352784.26
200	645.49	235.01	353099.47
300	645.52	244.45	353216.07
500	643.89	251.57	353319.86
1000	637.93	255.14	353421.59
2000	625.34	253.03	353519.70

Figure 10 Effect of inventory holding cost per unit per unit time, H , on batch and backlog size

A8. Changing the H and π values simultaneously

Since the production and backlog size are more sensitive to H and π , then, then behaviors of Q^* and B^* are studied due to π and H simultaneously. Figure 11 shows these behaviors.

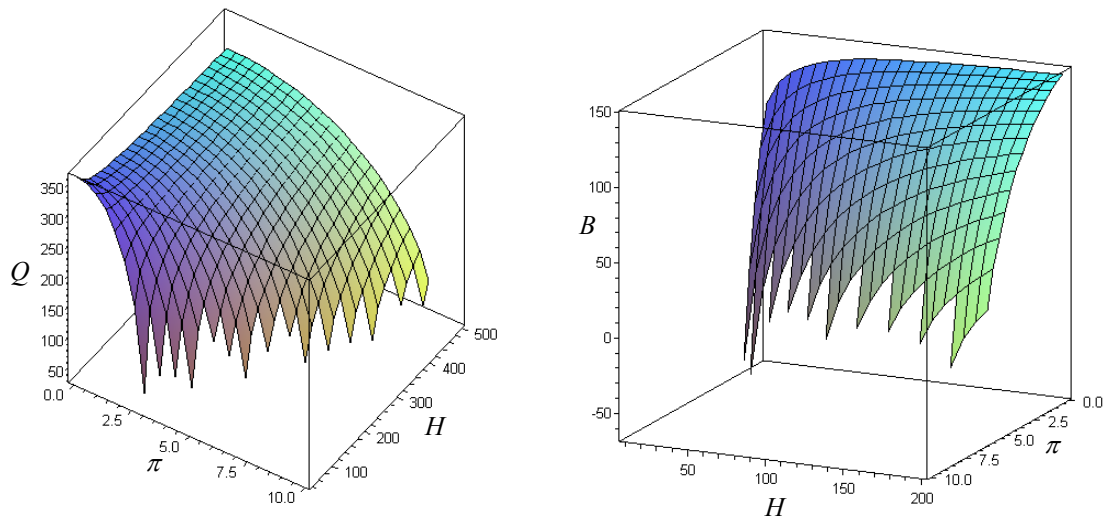


Figure 11 Effect of inventory holding cost per unit per unit time, H , and shortage cost per unit shortage, π on batch and backlog size simultaneously