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A Model for Sharing the Costs of Uncontrollable Risks among Contracting Parties

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ABSTRACT

The allocation of risks among the contracting parties in a contract is an important decision affecting the project success. Some risks in a project are uncontrollable; these are imposed to a project by external factors. Since contracting parties can neither control nor affect the occurrence of such risks, their allocation to a party would be inequitable. Therefore the cost overrun related to uncontrollable risks should be shared between contracting parties with a ratio which makes a win-win relationship between them in contract. This paper presents a mathematical model to achieve an equitable cost sharing ratio for uncontrollable risks between an owner and a contractor before contract conclusion using multi attribute utility theory.

Keywords: Cost sharing ratio, Uncontrollable risks, Contract, Contracting parties, Utility value, Win-win relationship

1. INTRODUCTION

Risks are an inseparable part of a project. They have been the part of attention because of time and cost overruns associated with projects (Kartam and Kartam, 2001). Risk management is an important tool to cope with risks in projects by: (a) assessing and ascertaining project viability; (b) analyzing and controlling the risks in order to minimize loss; (c) alleviating risks by proper planning; and (d) avoiding dissatisfactory projects and thus enhancing profit margins (Edwards and Bowen, 1998). In risk management all factors comprising a risk are to be identified, analyzed and evaluated to select the most suitable response. Risk response is a process of formulation of a management strategy leading to identifying action owners and the risk management plan (Lam et al, 2007). Contractual risk transfer is a strategy of risk management plan which has been employed for many years in the projects. It involves the allocation or distribution of the risks inherent to a project between or among contracting parties (Katz, 2001). If proper risk allocation is used, completion of the project will fulfill the owner's and contractor's expectations. On the other hand, incorrect perception of how risk should be assigned has often resulted in owners paying more than necessary for many projects (The American Council of Engineering Companies, 1998). The cost of improper risk allocation could be seen from the response from contractors such as adding a high contingency (premium) to the bid price or delivering low quality work. Also, during the project, the owner might

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spend more management resources for the increased work disputes. Upon completion of the works, litigation of contractual claims might come after. In the worst case, the owner pays for the risks twice including one in bidding contingencies and the other one in court (Fisk, 2000). Improper risk allocation may also cause costly delays in project completion (The American Council of Engineering Companies, 1998). If risk allocation is done effectively, risk transfer does not grossly or inequitably allocate all risk to one party. Ideally, the parties, in their contract, will assign the risks and liabilities to the party best equipped to manage and minimize them (Gandhi, 1979).

According to their source, project risks are divided into two groups: internal and external. Internal risks are those that are project related and usually fall under the control of the project management team. External risks are those risks that are beyond the control of the project management team (El-Sayegh, 2008), such as inflation, bad weather and acts of God. In cases that neither party has control on risk – external risk- it would be better that parties act together to mitigate and share the consequences of materialized risk, rather than incur a high premium for allocating all of these risks to just one party (Melbourne Victorian Department of Treasury and Finance, 2001). As a result, if the cost overrun related to the uncontrollable risks is shared between contracting parties, the probability of project success will increase. Now the question is how can the contracting parties arrive at a suitable cost sharing ratio before contract closure which makes a win-win relationship possible?

Various risk allocation principles had been suggested by a number of researchers such as, Chung (1991), Thompson and Perry (1992), Bolton and Harris (1997), American Council of Engineering Companies and Associated General Contractors of America (1998), Melbourne Victorian Department of Treasury and Finance (2001), Nabil and Saied A. Kartam (2001), Gerald I. Katz (2001), Diepenbrock et al. (2002), Wang and Chou (2003), Zaghloul and Hartman (2003), Cooper et al. (2005), Abednego and Ogunlana (2006), ANDI (2006), Burgert and Ruschendorf (2006), Lam et al (2007) , Francesca Medda (2007), Ng and Loosemore (2007), El-Sayegh (2008). These sources discuss about risk responsibility and in some cases risk sharing. Based on the authors' studies, there are a few research efforts dealing with the way contracting parties, owners and contractors, would arrive at a suitable cost sharing ratio prior to contract closure. Gandhi (1979) developed a strategy for reducing risk from the contractor's view point and, Kamal M Al-Subhi (1998) proposed a model in which each party based on the utility function determines an acceptable sharing ratio from his point of view, but they arrive at a suitable ratio through negotiation. Also, he considers actual cost of project as a discrete random variable.

The purpose of this paper is to present a mathematical model to achieve an equitable cost sharing ratio for uncontrollable -external- risks between contracting parties, to achieve a win-win relationship. This ratio would be beneficial to both owner and contractor. The proposed model aims to support the decision making before contract conclusion. As neither contracting parties can create or control the uncontrollable risks, it is very difficult to determine some criteria according to which to allocate these risks equitably. Since it is reasonable to consider the amount of satisfaction of each party from participating in project execution for different values of cost sharing ratio, then this is a proper way to allocate these risks. Based on this reasoning multi attribute utility theory is utilized in building the proposed model.

2. EXTERNAL RISKS

Risk identification determines which risks might affect the project and documents their characteristics (An American National Standard, 2004). Figure 1 shows a sample of Risk Breakdown Structure (RBS) to organize the different categories of risks based on their source (El-

Sayegh, 2008). This figure indicates that internal risks originate from owners, contractors, subcontractors, designers, suppliers and so on. For optimal risk allocation, risks should be allocated to the party in the best position to control them. In most cases the originator of a risk is in the greatest position of controlling that risk, and could be known as the party responsible for such risk. However, external risks are consequence of political, social, economical and natural factors that are beyond the control of the contracting parties. In these cases, a contractual mechanism is needed under which identifiable risks beyond the control of the parties can be shared to achieve optimal risk allocation (Melbourne Victorian Department of Treasury and Finance, 2001).

The proposed model in this paper is applicable for sharing the costs of uncontrollable risks.

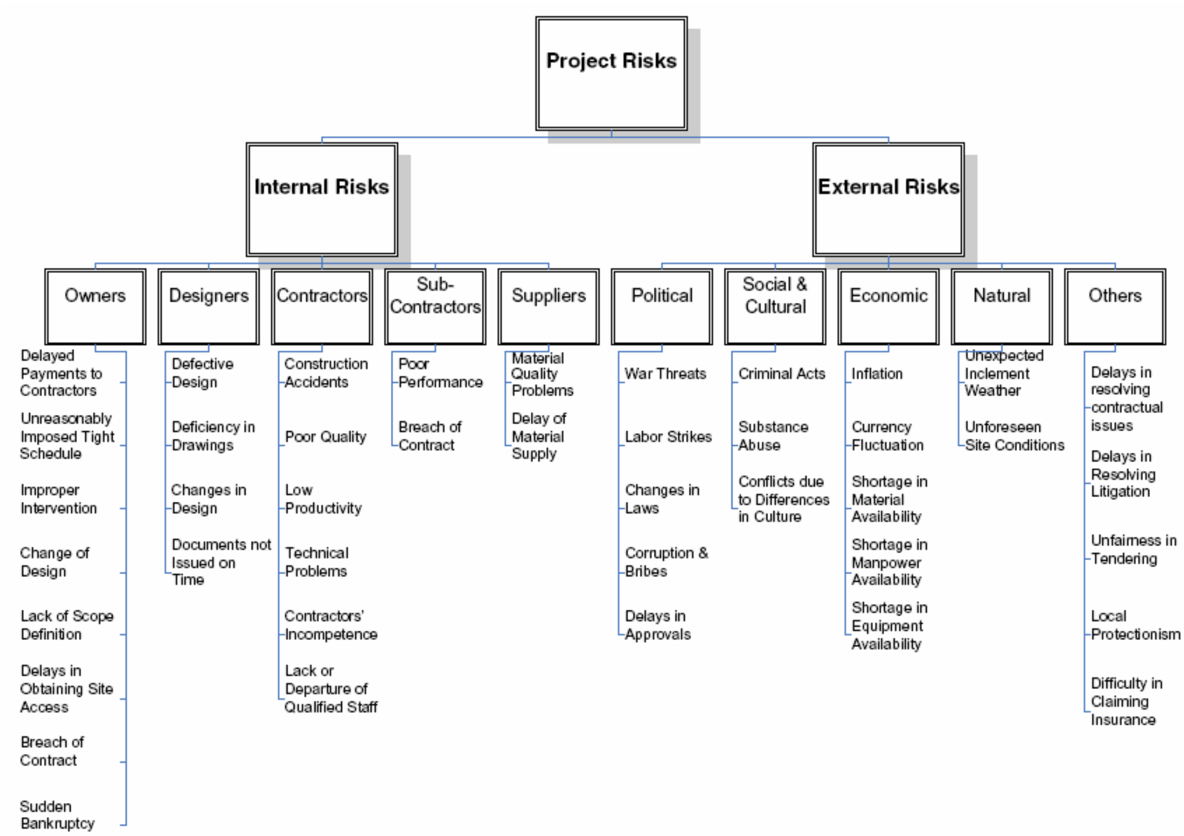

Figure 1 Risk breakdown structure based on the source of risk

3. MULTI-ATTRIBUTE UTILITY FUNCTION

In multi-criteria decision making the basic problem stated by analysts and decision makers concerns the way that the final decision should be made. The philosophy of preference dis-aggregation in multi-criteria analysis is to assess/infer preference models from given preferential structures and to address decision-aiding activities through operational models within the specified framework. So having a preference structure on a set of actions, the problem is to adjust additive value or utility functions based on multiple criteria, in such a way that the resulting structure would be as consistent as possible with the initial structure (Figueira et al., 2005).

There are many different methods in multi-criteria analysis which can be recommended depending upon the circumstances of decision making. Among these, the UTA* method initially proposed by Jacquet-Lagreze and Siskos (1978, 1982) has several interesting features: it makes possible the estimation of a nonlinear additive function, which is obtained by the use of a linear program which provides a convenient piecewise linear approximation of the function, and the only information required from the decision maker is global stated preferences between alternatives (Baourakis et al., 1996). This fact which states "the key condition for a utility function to be additive, is the marginality condition which states that the preferences should depend only on the marginal probabilities of the attribute values, and not on their joint probability distribution" (Figueira, 2005) may limit the use of UTA method. But, Michel Beuthe and Giuseppe Scannella (2001) show that the UTA approach of estimating utility function on the basis of the stated global preferences between projects provides in practice rather good results, even in cases where there is interdependence between attributes (Beuthe and Scannella, 2001). The UTASTAR method proposed by Figueira et al (2005) is an improved version of the original UTA model. When errors remain in the utility function's estimation, the UTASTAR model appears more reliable than other methods of UTA. However, the presence of errors is likely to reveal some irrational preference judgment by the decision maker (Beuthe and Scannella, 2001). A major weakness of multi attribute utility methods is their reliance on rationality, stability, consistency and coherence of the decision maker. The utilities are generated based on the assumption that the preferences of the decision maker would stay constant throughout the preference elicitation process. Even though human beings are rational, a decision maker's preferences may change dynamically during the process and making the assumption of constant utilities less valid (Zaghloul and Hartman, 2003). In these cases UTASTAR method estimates utility values of each decision maker more reliably than other UTA methods. Therefore we use the UTASTAR technique for calculating utility functions in this paper.

4. PROPOSED MODEL FOR SHARING THE COST OF UNCONTROLLABLE RISKS

This study initially focuses on the main contract and its uncontrollable risk allocation between owner and contractor. A mathematical model based on the utility theory is developed to support the risk allocation decision.

The model mainly consists of four main parts, namely: utility functions calculation, modeling the final cost of owner and the final profit of contractor, determination of actual cost density function, finding the proper cost sharing ratio that makes a win-win relationship possible.

In the proposed model we use utility function of owner's final cost and contractor's final profit to determine their preference structure considering different criteria and to measure the satisfaction of each party from the proposed conditions of project execution. The final cost of owner and the final profit of contractor can be written as a linear function of cost sharing ratio and project actual cost.

Since project actual cost is not determinable before project execution, we can not calculate owner and contractor's utilities in the contract conclusion stage. But in association with an expert team we can estimate probability of project completion for any given actual cost, and determine actual cost probability distribution – which usually follows a gamma distribution. Ascertaining the actual cost distribution we can use expected utility values instead of utility values.

Should the owner's final cost or contractor's final profit be a continuous random variable– they have the same distribution as actual cost- the expected utility value can be calculated as:

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* UTilites Additives

$$EUV = E[u(X)] = \int_{-\infty}^{+\infty} u(x) f(x) dx \tag{1}$$

Where

X = owner's final cost or contractor's final profit

$u(X)$ = utility of X (can be exponential, quasilinear, homothetic or other types of utility function)

$f(x)$ = Probability density function of X

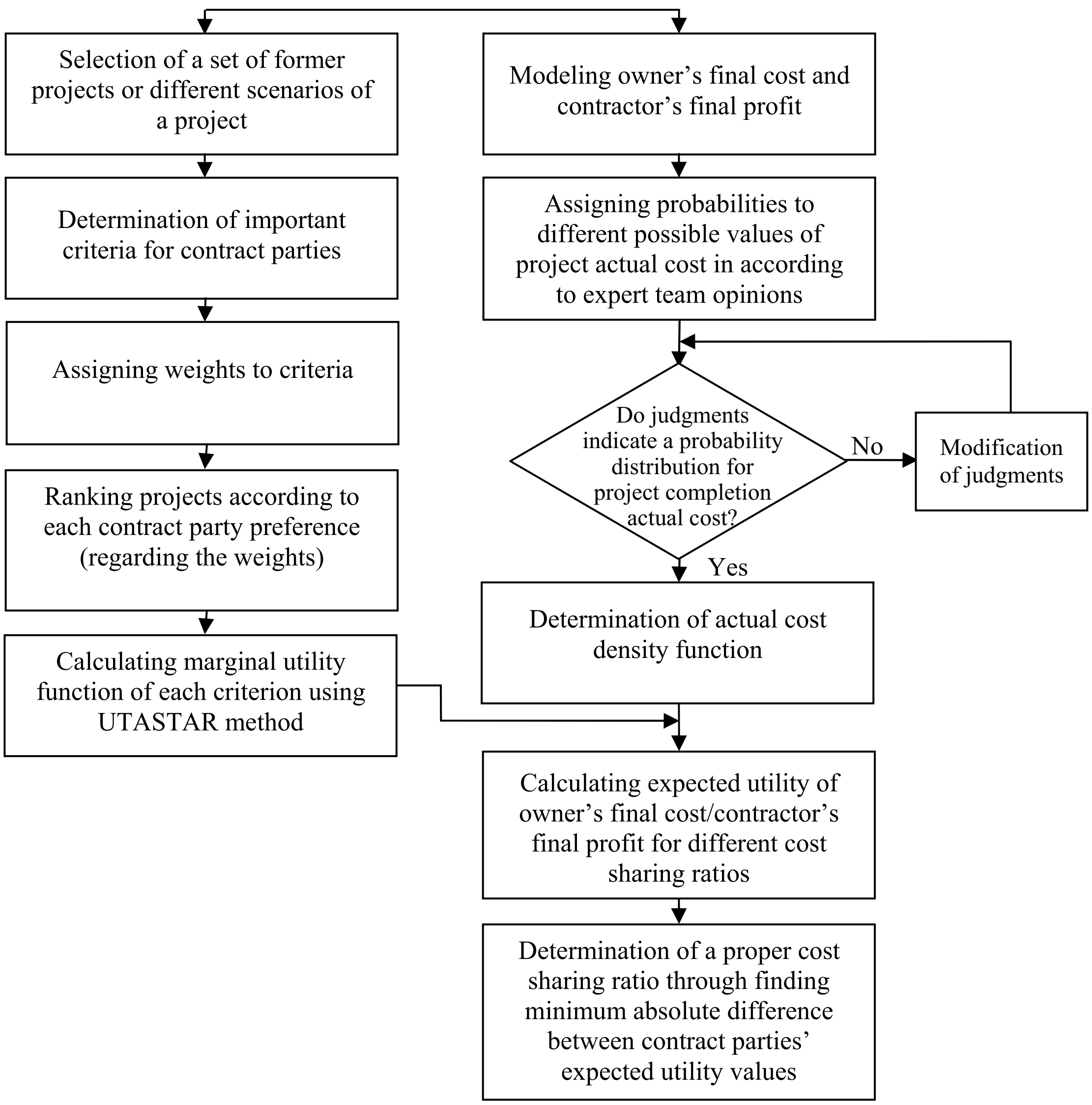

Figure 2 Proposed model for allocation of uncontrollable risks in contract

Hence, through determining density function of actual cost we can obtain expected utility of owner and contractor only in terms of cost sharing ratio. In the next stage with assigning different values to this ratio and comparison of expected utility values of owner's final cost and contractor's final profit, the proper cost sharing ratio would be determinable. The model is illustrated by Figure 2.

In order to further explain our model lets consider a practical example with actual data, where a contract is to be singed between a contractor and owner, with target cost of 2000 M$; target profit of 300 M$; quantity of production of 2300 tons per day and target time of 82 months; our aim is to find the proper cost sharing ratio for uncontrollable risks before contract conclusion, to achieve a win-win relationship between contracting parties. We would like to reemphasize that time, cost and production amounts are the preferred criteria for the owner, whereas time and profit are the most important criteria for the contractor. These criteria would be changeable depending on readers preference and projects conditions.

4.1. Calculation of Contracting Parties' Utility Functions

In order to use multi-attribute utility function, Jacquet-Lagreze and Siskos note that the clarification of the DM's global preference necessitates the use of a set of reference actions. Usually this set could be: a set of past decision alternatives, a subset of decision actions or a set of fictitious actions. In each of the mentioned cases, the DM is asked to externalize and/or confirm his/her global preferences on the set of reference actions taking into account their performances on all criteria (Figueira et al., 2005).

In this paper we consider 15 past project data as a set of reference action. These project's data are listed in Table 1.

Table 1 The set of reference actions

Project	Time	Cost∗ (M$)	Production per day (ton)	Profit	Ranking of the owner	Ranking of the contractor
1	60	1771	2000	265.65	4	5
2	90	3000	3900	900	13	2
3	85	1870	2500	374	12	6
4	80	1700	1500	340	15	7
5	72	1500	2100	300	6	9
6	90	2096	3400	628.8	8	3
7	46	800	1000	80	1	15
8	66	1600	1500	240	10	10
9	38	1000	700	100	2	11
10	96	3100	4000	930	14	1
11	88	2100	3000	430	11	4
12	76	1600	2200	320	7	8
13	54	1100	1200	165	3	13
14	72	1400	2000	210	5	14
15	64	1500	1400	225	9	12

∗ Assume the difference between projects costs is not consequent of inflation.

Time, cost and quality have been assumed as owner's criteria and profit and time for contractor. The UTASTAR method is applied in order to estimate the marginal utilities assigned implicitly to each project by each party. The method needs an initial ranking of projects based on decision maker preference, for this reason we use simple additive weighting method, in this method an evaluation score is calculated for each project by multiplying the dimensionless value of each criterion by the weights of relative importance directly assigned by decision maker followed by summing of the products for all criteria.

Therefore, the importance of each criterion would be considered in calculating utility functions. The two last columns of Table 1 show the owner and the contractor's ranking with respect to the 15 past projects. Also estimated marginal utilities through utilizing UTASTAR method are shown in Table 2.

Table 2 Estimated utilities

Project	Owner's marginal utilities			Contractor's marginal utilities	
	Time	Cost	Production	Time	Profit
1	0.2431	0.1592	0.1306	0.1360	0.4740
2	0	0.0250	0.3561	0.0082	0.7903
3	0.0185	0.3219	0.2600	0	0.5711
4	0.0648	0.1655	0.0431	0.0193	0.5407
5	0.1187	0.2274	0.1762	0.0307	0.5043
6	0	0.1592	0.3513	0.0082	0.6412
7	0.3031	0.3344	0	0.2030	0
8	0.1589	0.1965	0.0431	0.0528	0.4327
9	0.3031	0.3594	0	0.4052	0.0552
10	0	0	0.3561	0.0012	0.8223
11	0	0.1592	0.3124	0.0106	0.6138
12	0.0979	0.1965	0.2034	0.0241	0.5225
13	0.2906	0.3219	0	0.1777	0.2302
14	0.1187	0.2586	0.13063	0.0307	0.3521
15	0.1869	0.2275	0.0256	0.0805	0.3926

Based on estimated marginal utilities of owner's cost and contractor's profit in Table 2 and using mathematical software the following marginal utility functions are obtained:

$$\begin{cases} u_1(c) = 1\times10^{-7}\,c^2 + 1.84\times10^{-4}c + 3.109\times10^{-1} & c \le 1700 \\ u_2(c) = 1\times10^{-7}\,c^2 + 4.69\times10^{-4}c + 2.932\times10^{-1} & c > 1700 \end{cases} \quad (2)$$

$$\begin{cases} u_1(p) = -5\times10^{-6}\,p^2 + 4.339\times10^{-3}p - 3.230\times10^{-1} & p \le 400 \\ u_2(p) = 1\times10^{-6}\,p^2 - 8.52\times10^{-3}p + 8.090\times10^{-1} & p > 400 \end{cases} \quad (3)$$

Where $u(c)$ is the utility of final cost of owner and $u(p)$ is the utility of final profit of contractor. Also utility functions of other criteria can be calculated. But since time and quality of a project are not directly a functions of cost sharing ratio, and changes in this ratio do not influence their utility values, therefore their utility functions are not necessary, and for brevity we relinquish from calculating them. Also, it is important to mention that, quadratic function is the best for fitting our data; even though our model is applicable for all types of utility functions.

4.2. Modeling Final Cost of Owner and Final Profit of Contractor

With mutual agreement between owner and contractor to share the cost of uncontrollable risk associated with the project execution, the two parties negotiate and agree on a target cost and a target profit. Any deviation between the actual and target resulting from uncontrollable risks will be shared by both of them on an agreed upon ratio. On completion, the contractor receives the target fee plus or minus his share in cost deviation. This could be modeled mathematically as:

$$c = x + b + \rho_c (x_o - x) \tag{4}$$

$$p = b + \rho_c (x_o - x) \tag{5}$$

Where:

c = the owner’s final cost (owner's new fee);

p = the contractor’s final profit (contractor's new fee);

x_o = the target cost;

x = the actual cost;

b = the target profit;

ρ_c = the contractor's sharing fraction, $0 < \rho_c < 1$, also equal to $\rho_c = 1 - \rho_o$, where ρ_o is owner's sharing fraction.

4.3. Determination of Actual Cost Density Function

Actual cost for project completion may have overrun or under run the estimated cost. We can ascertain probability of different possible values of actual cost in association with an expert team, before the project execution. Therefore actual cost can be assumed as a continuous random variable with a distinct distribution. Figure 3 shows different values of actual project cost vs. their probability of occurrence for our case.

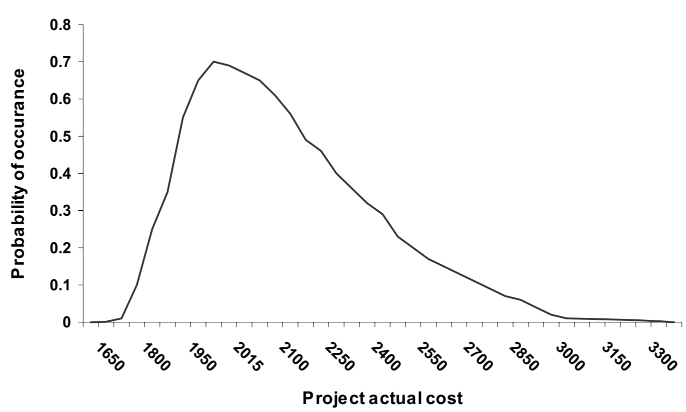

Figure 3 Probability density of actual project completion cost

Using a mathematical software, the gamma distribution with k =24 and θ =103 is obtained for actual cost distribution in our example, where k and θ are respectively the shape parameter and the scale parameter of gamma distribution.

X ~Gamma(k,θ)~Gamma(24,103)

3.4. Calculation of Expected Utility Values

Regarding relation (1), the expected utility value (EUV) of the owner's final cost can be calculated as:

$$EUV_c = E[u(c)] = \int_{-\infty}^{+\infty} u(c)f(c)dc \tag{6}$$

Where c and $u(c)$ are defined based on relations (4) and (2) respectively, and $f(c)$ denotes the probability density function of owner's final cost. Thus:

$$\begin{aligned} EUV_c &= \overbrace{\int_{-\infty}^{0} u_1(c)f(c)dc}^{0} + \int_{0}^{1700} u_1(c)f(c)dc + \int_{1700}^{+\infty} u_2(c)f(c)dc = \\ &= \int_{0}^{1700} u_1(c)f(c)dc + (1 - \int_{0}^{1700} u_2(c))f(c)dc \\ \Rightarrow EUV_c &= \int_{0}^{1700} 1\times10^{-7}c^2 f(c)dc + \int_{0}^{1700} 1.84\times10^{-4} cf(c)dc + \\ & \int_{0}^{1700} 3.109\times10^{-1} f(c)dc + (1 - (\int_{0}^{1700} 1\times10^{-7}c^2 f(c)dc \\ & + \int_{0}^{1700} 4.69\times10^{-4} cf(c)dc + \int_{0}^{1700} 2.932\times10^{-1} f(c)dc)) \end{aligned} \tag{7}$$

As we know, X -project actual cost- has a gamma distribution with parameters k and θ. Regarding relation (4), c - owner's final cost - is a linear function of X; therefore it could be simply proven that c has a gamma distribution as well, with parameters k_1 and θ_1. We can ascertain these new parameters from relationship between the means and variances of X distribution and c distribution. The mean and variance of X may be written as:

$$\mu = E(x) = k\theta = 24*103 = 2472 \tag{8}$$

$$\sigma^2 = Var(x) = k\theta^2 = 24*103^2 = 254616 \tag{9}$$

The relation between the means and variances of X and c regarding relation (4), is as follows:

$$E[c] = E[x] + b + p_c x_0 - p_c E[x] = b + p_c x_0 + (1-p_c) E[x] = k_1\theta_1 \tag{10}$$

and

$$Var[c] = (1\text{-}p_c)^2 Var(x) = k_1\theta_1^2 \tag{11}$$

Thus

$$E[c] = k_1\theta_1 = 2772 - 472\,\rho_c \tag{12}$$

$$Var[c] = k_1\theta_1^2 = 254616(1-\rho_c)^2 \tag{13}$$

Table 3 shows k_1 and θ_1 values for different values of ρ_c.

Table 3 k_1 and θ_1 for different values of ρ_c.

ρ_c	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
K_1	30.179	36.000	43.997	55.458	72.799	101.035	152.046	260.148	562.921	2163.787	-
θ_1	91.853	75.690	60.858	47.431	35.484	25.100	16.369	9.385	4.254	1.085	0

For calculating $\int cf(c)dc$ and $\int c^2 f(c)dc$, we can write:

$$c \sim Gamma(k_1,\theta_1) \Rightarrow f(c) = \begin{cases} \dfrac{c^{k_1-1} e^{-c/\theta_1}}{\Gamma(k_1)\theta_1^{k_1}} & c \ge 0 \\ 0 & c < 0 \end{cases} \tag{14}$$

Thus

$$cf(c) = \frac{c^{k_1} e^{-c/\theta_1}}{\Gamma(k_1)\theta_1^{k_1}} \tag{15}$$

According to property of gamma distribution, the following relation holds:

$$\Gamma(k+1) = k\,\Gamma(k) \Rightarrow \Gamma(k) = \frac{\Gamma(k+1)}{k} \tag{16}$$

By substituting the relation (16) into relation (15) we arrive

$$cf(c) = k_1\theta_1 \frac{c^{k_1} e^{-c/\theta_1}}{\Gamma(k_1+1)\theta_1^{k_1+1}} \tag{17}$$

Thus

$$cf(c) \sim k_1\theta_1\, Gamma(k_1+1,\theta_1) \tag{18}$$

With the same reasoning, relation (19) could be justified.

$$c^2 f(c) \sim (k_1+1)k_1\theta_1^{\,2}\, Gamma(k_1+2,\theta_1) \tag{19}$$

Now we can calculate the expected utility values of owner's final cost, as follows:

$$\begin{aligned} EUV_c &= 1\times10^{-7}\,\theta_1^{\,2}k_1(k_1+1)\overbrace{F(c=1700)}^{c\sim Gamma(k_1+2,\theta_1)} + 1.84\times10^{-4}\,\theta_1 k_1\overbrace{F(c=1700)}^{c\sim Gamma(k_1+1,\theta_1)} \\ &\quad + 3.109\times10^{-1}\overbrace{F(c=1700)}^{c\sim Gamma(k_1,\theta_1)} + (1-(1\times10^{-7}\,\theta_1^{\,2}k_1(k_1+1)\overbrace{F(c=1700)}^{c\sim Gamma(k_1+2,\theta_1)} \\ &\quad + 4.69\times10^{-4}\,\theta_1 k_1\overbrace{F(c=1700)}^{c\sim Gamma(k_1+1,\theta_1)} + 2.932\times10^{-1}\overbrace{F(c=1700)}^{c\sim Gamma(k_1,\theta_1)})) \end{aligned} \tag{20}$$

Where $F(c)$ is the cumulative distribution function of c.

Since k_1 and θ_1 are dependent on ρ_c, for different values of ρ_c we obtain different values of EUV_c. These values are shown in Table 4.

Table 4 EUV_c for different values of ρ_c

p_c	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
EUV_c	0.996638	0.997752	0.998680	0.999366	0.999780	0.999957	0.999997	1	1	1	-

In the same way, expected utility values of contractor's final profit, could be calculated as shown in Table 5.

Table 5 EUV_p for different values of ρ_c

ρ_c	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
EUV_p	-	1.7500	1.3554	1.0030	0.7291	0.5533	0.4915	-	-	-	-

4.4. Determination of the Proper Cost Sharing Ratio

Using expected utility values, contractor will be able to calculate cost sharing ratios which cause his loss -expected utility value of his final profit becomes negative- before contract conclusion.
This range of ratios is determined as follows:

$$\begin{aligned} &E(p) = b + \rho_c x_0 - \rho_c E[x] < 0 \\ &\Rightarrow 300 - 472\rho_c < 0 \\ &\Rightarrow \rho_c > 0.636 \end{aligned} \tag{21}$$

Since the main objective of this paper is seeking the proper cost sharing ratio which makes a win-win relationship between contract parties possible, the above range of ratios is not acceptable. Also in this range, scale parameter of gamma distribution had been negative before – in EUV_p calculation- and gamma distribution was undefinable.

Additionally, if $\rho_c = 0$, owner will be responsible for all of uncontrollable risks, therefore this cost sharing ratio is not acceptable either.

Table 6 shows expected utility values of the owner's final cost and the contractor's final profit for different acceptable values of ρ_c.

Table 6 Expected utility value of owner and contractor for different acceptable value of ρ_c

ρ_c	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
EUV_c	0.996638	0.997752	0.998680	0.999366	0.999780	0.999957	0.999997	1	1	1	-
EUV_p	-	1.7500	1.3554	1.0030	0.7291	0.5533	0.4915	-	-	-	-

Table 6 illustrates that any increment in ρ_c, diminishes expected utility of contractor and increases expected utility of owner. Since we are searching for a win-win ratio, the proper ratio would be a ratio that achieves the absolute minimum difference between contract parties' expected utilities.

Table 7 Absolute difference between contract parties' expected utilities

ρ_c	0.1	0.2	0.3	0.4	0.5	0.6
$\lvert EUV_c - EUV_p \rvert$	0.752288	0.356728	0.003662	0.270729	0.446619	0.508498

Regarding the contents of Table 7, $\rho_c = 0.3$ and $\rho_o = 0.7$ are the proper ratios for sharing cost of uncontrollable risks between contracting parties.

The ratio obtained here is acceptable when both the owner and the contractor wish to share the risk of overrunning the target cost and share the savings in the case of a better than expected contractor performance. Thus, for verification of ρ_c obtained here, the adverse effects of improper risk allocation on time, cost and quality of project can be explained to the owner and contractor in the first step. In fact, before that, each of the contracting parties endeavors to minimize his share of risks. Then, if the obtained ratio is presented to them, it will be acceptable to them both which shows that the calculated ratio in this paper is reliable in the real case.

5. COMPARISON OF THE MODEL WITH OTHER MODELS

As explained before, based on authors' investigations, there are a few research efforts relating to the way contracting parties arrive at suitable cost sharing ratio that is acceptable to both of them before contract conclusion. Gandhi and Al-Subhi's models are close examples of these efforts, but Gandhi developed a strategy for reducing risk from the contractor's view point and in Al-Subhi model each party - considering the expected utility values of owner's final cost and contractor's final profit -

determines an acceptable sharing ratio from his point of view then contracting parties negotiate with each other for arriving at a suitable ratio.

Table 8 Comparison of Kamal Al Subhi's model and the proposed model

Kamal Al Sobhi's model	Proposed model
-Considering only owner's cost and contractor's profit for calculating their utilities	-Considering different criteria for calculating contracting parties utilities
-Calculating expected utilities of contracting parties only for three values of actual cost (Considers actual cost of project as a discrete random variable)	-Calculating expected utilities of contracting parties for every deviation of actual cost from the target cost (Considers actual cost of project as a continuous random variable)
-Determination of the win-win cost sharing ratio through negotiation	-Determination of the win-win cost sharing ratio through mathematical methods

Indeed the proposed model in this paper is a development of Al-Subhi's model. Table 8 presents advantages of the proposed model in comparison with Al Subhi's model.

Regarding Table 8, the proposed model in this paper is suitable for calculating the equitable cost sharing ratio of risks before contract conclusion.

6. CONCLUSION

This paper presents a mathematical model for calculation of a cost overrun sharing ratio for uncontrollable risks which would be beneficial to both owner and contractor.
Uncontrollable risks are imposed to the project by external factors and the contracting parties can not prevent their occurrence or mitigate their probability of occurrence. Since, allocating all of their costs to one party may cause the party's loss or decrease his motivation to participate in the project, determining a proper ratio for sharing costs of such risks in projects is very essential before project execution.

The proposed model enables us to evaluate the satisfaction of contract parties from participating in project execution considering different criteria with utilizing multi-attribute utility function. Additionally, the model uses actual cost probability density function in order to calculate expected utilities of contract parties, therefore the model can be used for every deviation of actual cost from the target cost; furthermore, the contractor would be able to estimate cost sharing ratios that may make his loss before beginning of project. As an extension, the model can be upgraded to handle more than two parties. Considering contractor's sharing fraction as a continuous variable is another context for upgrading of the model.

REFERENCES

[1] Abednego Martinus P., Ogunlana Stephen O. (2006), Good project governance for proper risk allocation in public-private partnerships in Indonesia; *International Journal of Project Management* 24; 622-634.

[2] Al-Subhi Al-Harbi Kamal M. (1998), Sharing fractions in cost-plus-incentive-fee contracts; *International Journal of Project Management* 16(2); 73-80.

[3] An American National Standard (2004), A Guide to the Project Management Body of Knowledge (PMBOK Guide); an American National Standard ANSI/PMI 99-001.

[4] ANDI (2006), The importance and allocation of risks in Indonesian construction projects; *Construct Manage Econom* 24(1); 69-80.

[5] Baourakis G., Matsatsinis N.F., Siskos Y.(1996), Agricultural product development using multidimensional and multicriteria analyses: The case of wine; *European Journal of Operational Research* 94; 321-334.

[6] Beuthe Michel, Scannella Giuseppe (2001), Comparative analysis of UTA multi-criteria methods; *European Journal of Operational Research* 130; 246-262.

[7] Bolton P., Harris C. (1997), The continuous-time principal-agent problem: first-best risk-sharing contracts and their decentralization; mimeo, Princeton.

[8] Burgert Christian, Ruschendorf Ludger (2006), On the optimal risk allocation problem; *Statistics & decisions* 24; 153–171.

[9] Cooper Dale F., Grey Stephen, Raymond Geoffrey, Walker Phil (2005), Project risk management guidelines; John Wiley & Sons, England.

[10] Chung T.-Y. (1991), Incomplete contracts, specific investments, and risk-sharing; *Review of Economic Studies* 58(5); 1031-42.

[11] Diepenbrock Eileen M., Davison Jay C., Lichtig William A., Rudolph Steven P. (2002), Risk allocation in construction projects; Lorman Risk Allocation Seminar (30 July).

[12] Edwards P.J., Bowen P.A. (1998), Risk and risk management in construction: a review and future directions for research; *Eng Construct Archit Manage* 5(4); 339-349.

[13] El-Sayegh Sameh Monir (2008), Risk assessment and allocation in the UAE construction industry; *International Journal of Project Management* 26; 431-438.

[14] Figueira Jose, Greco Salvatore, Ehrgott Matthias (2005), Multiple criteria decision analysis: state of the art surveys; Springer.

[15] Fisk E.R. (2000), Construction project administration; New Jersey, Prentice Hall.

[16] Gandhi D. (1979), A strategy for risk reduction in incentive contracting; *Decision Sciences* 10(3); 371-386.

[17] Jacquet-Lagreze, E., Siskos, Y. (1978), Une method de construction de fonctions d' utilite additives explicatives d' une preference globale; Technical report Cahiers du LAMSADE 16, Universite de Paris-Dauphine, Paris.

[18] Jacquet-Lagreze, E., Siskos, Y. (1982), Assesing a set of additive utility functions for multicriteria decision making: the UTA method; *European Journal of Operations Research*, 10:151-164.

[19] Kaplan Pervin Özge (2006), A new multiple criteria decision making methodology for environmental decision support; Faculty of North Carolina State University, Unpublished PhD dissertation.

[20] Kartam Nabil A., Kartam Saied A. (2001), Risk and its management in the Kuwaiti construction industry: a contractors' perspective; *International Journal of Project Management* 19; 325-335.

[21] Katz Gerald I. (2001), Contract risk allocation; *Contractual Risk Transfer Seminar*.

[22] Lam K.C., Wang D., Lee Patricia T.K., Tsang Y.T. (2007), Modeling risk allocation decision in construction contracts; *International Journal of Project Management* 25; 485-493.

[23] Medda Francesca (2007), A game theory approach for the allocation of risks in transport public private partnerships; *International Journal of Project Management* 25; 213-218.

[24] Melbourne Victorian Department of Treasury and Finance (2001), Risk Allocation and Contractual Issues a guide; The Department of Treasury and Finance, Melbourne.

[25] Ng A., Loosemore Martin (2007), Risk allocation in the private provision of public infrastructure; *International Journal of Project Management* 25; 66-76.

[26] Siskos Y., Yanacopoulos D. (1985), UTA STAR – an ordinal regression method for building additive value functions; *Investigacao Operational*, 5:39-53.

[27] The American council of engineering companies and the associated general contractors of America (1998), Enlightened risk allocation, February.

[28] Thompson P., Perry J. (1992), Engineering construction risks: a guide to project risk analysis and risk management; Thomas Telford Ltd, London.

[29] Wang M., Chou H. (2003), Risk allocation and risk handling of highway projects in Taiwan; *J Manage Eng* 19(2); 60-68.

[30] Zaghloul Ramzy, Hartman Francis (2003), Construction contracts: the cost of mistrust; *International Journal of Project Management* 21; 419-424.