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Integrative Cell Formation and Layout Design in Cellular Manufacturing Systems

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ABSTRACT

This paper proposes a new integrative view of manufacturing cell formation and both inter-cell and intra-cell layout problems. Cells formation and their popular bi-directional linear layout are determined simultaneously through a Dynamic Programming algorithm (with the objective of minimizing the inter-cell flow cost under a cell size constraint). This Dynamic Programming algorithm is implemented in a Simulated Annealing approach with Genetic operators to reach near optimal solutions. Moreover, within this approach and by using an Ant Colony Optimization technique, we also solve the intra-cell layout problem, i.e., we also determine how to lay out machines within relative cells. In contrast with most of the available approaches in the literature, we consider: (1) An integrated objective function to minimize overall inter-cell and intra-cell flow costs instead of merely minimizing the number of inter-cell movements/costs. (2) The integrative and simultaneous determination of cell formation and their layout instead of using sequential approaches. (3) All three phases of cell formation, inter-cell and intra-cell layout design problems, which are all important for overall performance of the system, and (4) An easy to code and solve integrated procedure through implementing metaheuristic approaches. Our computational results show that by incorporating intra-cell decisions in cell formation and inter-cell design process through implementing our proposed integrated approach, a manufacturer can largely reduce her total material flow cost. Particularly, our computational tests show good quality solutions in comparison with the most similar available approach in the literature with an average improvement of 24.97% in total flow cost for a set of randomly generated test problems.

***Keywords*:** Cell Formation, Intra-cell Layout, Inter-cell Layout, Simulated Annealing, Dynamic Programming, Ant Colony Optimization, Graph Theory.

1. INTRODUCTION

As Irani et al. (1999) describe in the handbook of Cellular Manufacturing Systems, Cellular Manufacturing (CM) is an application of the Group Technology (GT) concepts to factory reconfiguration and shop floor layout design. Group technology is a manufacturing concept that seeks to identify and group similar parts to take advantage of their similarities in manufacturing and design. The paper of R.E.Flanders in 1925 can be considered as the start point of GT. He described

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how in the early 1920 a machine tool manufacturer, Jones and Lamson Machine Company, had standardized its products and then organized manufacturing around them. The alternative he suggested was to arrange facilities by product such that any individual piece stay in a single department until it is completely finished. The concept of GT was originally proposed by Mitrofanov (1966) and Burbidge (1975). Mitrofanov (1966) defined GT as a method of manufacturing piece parts by the classification of these parts into group and subsequently applying to each group similar technological operations. The modern definition of GT is given by Shunk (1987): the realization that many problems are similar, and that by grouping them, a single solution can be found to a set of problems, thus saving time and efforts. However, the most general definition of GT defines it as a manufacturing philosophy which identifies and exploits the underlying proximity of parts and manufacturing processes (Ham et al. (1985)). Wemmerlov and Hyer (2002) define a cell as a group of closely located work stations where multiple sequential operations are performed on one or more families of similar row materials, parts, components, products or information carries. They define a manufacturing cell as a cell whose main purpose is to physically process, transform, transmit and add value to materials whose end state are products or components and an office cell as a cell whose main task is to process, transform, transmit, and add value to information. Moreover, they discuss four perspectives on cells: resource perspective, spatial perspective, transformation perspective and organizational perspective. Ham et al. (1985) define a manufacturing cell as an independent group of functionally dissimilar machines, located together on the floor, dedicated to the manufacturer of a family of similar parts.

A Cellular Manufacturing System (CMS) design is usually partitioned to several phases, including the selection of parts and part families, machines and machine cells, tools and fixtures, material handling facilities and layout (Wemmerlov and Hyer (1987)). Obviously, these phases are not independent and should all be considered through cell design goals. The overall goal for the design process is to achieve performance improvement with respect to lead time, inventory, quality or other measures (Wemmerlov and Hyer (2002)), or in other words, to achieve advantages of a CMS. Irani et al. (1999) presented a complete list of advantages of cellular manufacturing. The most important ones, also reported by Wemmerlov and Hyer (1989), Wemmerlov and Johnson (1997) and Olorunniwo (1997), include reduction in move times, throughput and lead time, WIP and finished goods inventory levels, setup times, as well as improvement in quality, capacity and equipment utilization.

There are many available studies in the field of CMS design. Most of them, however, are focused on the cell formation problem. Moreover, there are not so many papers that concurrently consider all three phases of the design, namely part/machines grouping, intra-cell and inter-cell layout designs within an integrated approach. As Salum (2000) states, one of the major drawbacks of cell formation techniques is that they do not lay out machines in cells (intra-cell layout) and on the shop floor (inter-cell layouts). There are also a few approaches for the layout problem in CM (see Alfa et al. (1992), Irani et al. (1993), Arvindh and Irani (1994), Verma and Ding (1995), Bazargan-Lari and Kaebernick (1996), Wang et al. (1998), Salum (2000), Akturk and Turkcan (2000), Lee and Chiang (2002), and Solimanpour et al. (2004)). Moreover, most of the available studies for the cell formation problem are concerned with creating machine cells with minimal number of inter-cell movements and not with minimal flow cost (see for instance, Askin and Subramanian (1987), Harhalkis et al. (1990), Vakharia and Wemmerlov (1990) and Albadawi et al. (2005)). However, a cell formation with minimal number of inter-cell movements is not always consistent with the one with minimal inter-cell material flow cost, due to lack of layout data in the cell formation process. Additionally, although the cell formation and the inter-cell layout problems have been jointly considered in the literature, most of the available methods are either based on the sequential approaches (in which machine cells are found in first phase and then inter-cell layout is constructed

based on the given cell formation; thus the quality of final solution largely depends on the given cell formation) or are based on difficult mathematical models (in which solving the problem is not easy, and therefore cannot be easily coded and implemented). Moreover, as mentioned before, the literature lacks studies that concurrently consider all three phases of a CMS design within an integrated approach is not rich.

In this paper, by improving and combining the studies of Lee and Chiang (2001), Chiang and Lee (2004) and Solimanpur et al. (2004), we propose an integrative and easy to code approach which integrates all three phases of a CMS design: cell formation, its location sequence on the bi-directional linear flow layout and the intra-cell machine layouts. Indeed, the work of Solimanpur et al. (2004) develops a strong Ant Colony Optimization technique for the *inter-cell* layout problem without considering other phases of a CMS design, i.e., cell formation and intra-cell layout problems. We modify their approach to be able to implement it for our *intra-cell* (and not inter-cell) layout decision making as a part of our integrated approach. Moreover, the paper by Chiang and Lee (2004) only considers the joint problem of manufacturing cell formation and its layout assignment. The objective of that study is to minimize the inter-cell flow cost under the cell size constraint. They, however, do not consider the effect of incorporating intra-cell decisions (within inter-cell and cell formation ones) on a more important criterion, i.e., the *total material flow cost*. In this paper, we consider this criterion as the objective function. Moreover, we notice that all these design decisions are *correlated* in the sense that they affect each other. Hence, we propose an *integrative* and *simultaneous* consideration of different design decisions instead of available sequential approaches. Our computational results, as will be discussed, show that by incorporating intra-cell decisions in cell formation and inter-cell design process, and through implementing our proposed integrated approach, a manufacturer can largely reduce her *total material flow cost*.

In contrast with most of the available approaches in the literature, in this paper we will consider: (1) The objective function of minimizing overall inter-cell and intra-cell flow costs instead of minimizing the number of inter-cell movements/costs, which is more precise and applicable. (2) The integrative and simultaneous determination of cell formation and their layout instead of using sequential approaches, which can be critical regarding the objective function. (3) All three phase of cell formation, inter-cell and intra-cell layout design problems, which are all important for overall performance of the system, and (4) An easy to code and solve integrated procedure for addressed problems by implementing metaheuristic approaches.

The remainder of this paper is organized as follows. In Section 2, we discuss the assumptions, techniques and notation. In Section 3 the graphical approach to material flow is discussed. Section 4 describes our integrated solution procedure. Section 5 presents our computational results; and finally Section 6 briefly concludes.

2. ASSUMPTIONS, TECHNIQUES, AND NOTATION

Our objective is to minimize total cost of inter-cell and intra-cell flows subject to a cell size constraint, i.e., the maximum number of machines allowed in each cell. We consider a center-to-center linear distance measure and for simplification, we do not consider any other spatial constraint. However, one may note that such constraints can also be added to the model by some simple modifications to the will-be proposed procedure.

As the problem of partitioning a manufacturing system into several subsystems, with the objective of minimizing inter-cell flow movement cost is NP-complete (Garey and Johnson (1979)), most researchers have focused on developing heuristics or metaheuristics. In this paper, as well, we

propose an enhanced Simulated Annealing (SA) in which the crossover and mutation operators of Genetic Algorithm (GA) are used as generation mechanism to generate neighborhood solution. Kirkpatrick, Gelatt and Vecchi (1983) introduced simulated Annealing (SA) and Creny (1985) considered the analogy between the annealing process of solids and the process of solving combinatorial optimization problems. However, it was originally developed as a simulation model for a physical annealing process of condensed matter (Metropolis et al. (1953)). Laarhoven and Aarts (1987) gave a comprehensive discussion of the theory and review of various applications. Also, they showed that the simulated annealing process converges to the set of global optimal solutions under certain conditions. Koulamas et al. (1994) also applied SA to a large number of optimization problems in a verity of application areas. The main procedure of SA can be described as follows. It starts from an initial solution to the problem, and then generates a new trial solution from the neighborhood at the current solution. If the new solution is better than the current solution it is accepted and used as the new current solution. Otherwise, it may be accepted or rejected depending on an acceptance probability, which is determined by the difference between objective function of the two solutions and by a control parameter called temperature, following the convention in thermodynamics. This process then continues from the new current solution. Initially, the temperature is set at high level, as in annealing, so that almost all moves will be accepted. It is then decreased slowly during the procedure until almost no move will be accepted. In other words, SA procedure can be generally described as following steps.

1. Initialization: set parameters of annealing schedule.

2. Select an iteration mechanism: a simple prescription to generate a transition from current state to another state by a small perturbation.

3. Evaluate the new state and compute $\Delta E = (\text{value of current state - value of new state})$.

4. If the new state is better, make it current state, otherwise probabilistically accept or reject it (with a determined probability function usually called acceptance probability function).

5. Based on stopping rules either stop or continue iterations at step 2.

A simulated annealing algorithm works with a coding of solution configuration. In this study, the configuration of solution, as is common, consists of a string of integer values. Each integer value is the code for one machine and the order of machines in the string is associated with the sequence of machines at the incidence matrix. For instance, the symbolic string π=(8, 1, 3, 5, 4, 2, 7, 6), represents the sequence of eight machines appearing in the row of the incidence matrix. If a cell formation problem involves *m* machines, a string with the defining length of *m* is needed to encode the candidate solution.

This string then will be segmented and each segment represents a machine cell. A Dynamic Programming approach that uses the idea of a graph of material flow is then used to determine the best cell formation, location sequence and inter-cell flow cost of that cell formation regarding the best layout. Consequently, the number of cells, despite of sequential procedures, is obtained through the optimal policy and is not determined before layout design. After such determination and during our integrative procedure, we will deal with intra-cell layout, i.e., to layout machines in each cell. This will be done by modeling the problem as a famous Quadratic Assignment Problem (QAP) model and developing an Ant Colony Optimization (ACO) technique to solve it. We use following notations throughout the paper.

i, j: The index of machines; i, j = 1,…,m;

l,h: The available locations in cells; l,h=1,…C

F: The symmetric flow matrix, F = F_{ij}, where $F_{ij} = f_{ij} + f_{ji}$ and f_{ij} is the amount of material flow from machine i to machine j;

k, v: The index of machine cells (or locations), k; v = 1,…,e; where e is the number of machine cells to be determined;

S: The layout assignment vector of all machine cells, S = *{*s(1),s(2),…,s(e)*}* where s(v) is the location to which cell v is assigned, s(v) = 1, … , e;

π: The permutation of machines in the considered cell for intra-cell layout problem, $\pi = \{\pi(1), \pi(2), ..., \pi(C)\}$ where $\pi(i)$ is the machine placed in the *i*th position of π;

$d_{s(k),s(v)}$: The distance or travel time between cells k and v;

Q: The cell size limit;

$|q_v|$: The number of machines in cell v, which must be equal to or less than Q;

$Tabu_k$: The memory of ant k saving the index of machines already assigned by ant k;

V^k: The memory of ant k saving the moves selected by ant k;

τ_{il}: The pheromone level of move v=(i,l).

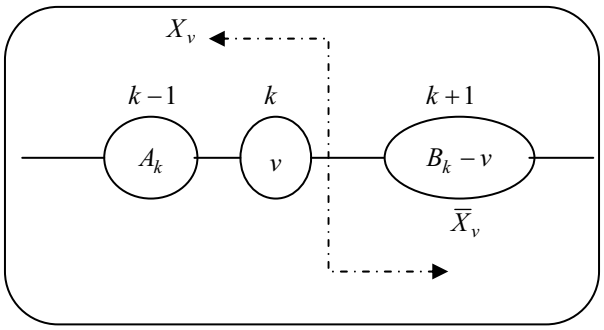

Figure 1 The graphic approach of assigning the cell *v* to the *k*th location

3. GRAPHICAL APPROACH TO MATERIAL FLOW

Here we benefit from the approach first proposed by Lee and Chiang (2001). We know that the material flow F between machines can be described by an undirected graph G(N, A), where N is the set of nodes and each node represents a machine, A is the set of arcs and each arc has the flow F_{ij}

connecting the nodes i and j. In this graphical approach, let A_k denote the set of those (k-1) cells that have already been assigned to the linear sequence of 1,2,…, and k-1, and B_k denote the set of remaining cells that have not been assigned. For any candidate machine cell v in B_k to be assigned to the *k*th site of the linear layout, we partitioned B_k into two distinct sets v and B_k -v and let $X_v = v \cup A_k$, $\overline{X}_v = B_k - v$ (see Figure 1). We define $C_k(X_v, \overline{X}_v)$ as the increased material flow cost of assigning cell v to the *k*th location.

Using Graph Theory, it can be shown that the increased inter-cell flow cost of assigning the partitioned cell v to the *k*th location of linear layout is:

$$C_k(X_v, \overline{X}_v) = \sum_{i \in X_v} \sum_{j \in \overline{X}_v} F_{ij} \ .$$

That is, for the sequence vector $S = \{A_k \cup B_k\}$, the inter-cell flow cost between A_k and B_k is:

$$\sum_{i \in A_k} \sum_{j \in B_k} (d_{s(i),k-1} + d_{k-1,k}) F_{ij} = \sum_{i \in A_k} \sum_{j \in B_k} (\sum_{l=s(i), l \le (k-2)}^{k-2} d_{l,l+1} + d_{k-1,k}) F_{ij} = \sum_{i \in A_k} \sum_{j \in B_k} \sum_{l=s(i), l \le (k-2)}^{k-2} d_{l,l+1} F_{ij} + \sum_{i \in A_k} \sum_{j \in B_k} d_{k-1,k} F_{ij}$$

It can be interpreted as all of the material flow that is transshipped from A_k at the *(k-1)*th sequence to B_k at the *k*th sequence. Thus, for the sequence S,

$$TC(S) = TC(A_k) + \sum_{i \in A_k} \sum_{j \in B_k} \sum_{l=s(i), l \le (k-2)}^{k-2} d_{l,l+1} F_{ij} + \sum_{i \in A_k} \sum_{j \in B_k} d_{k-1,k} F_{ij},$$

where $TC(A_k)$ is the total inter-cell flow between the assigned k -1 machine cells in the first k-1 locations of the linear layout. Once the new cell v in B_k has been partitioned from B_k and assigned to the *k*th layout sequence, the first two items in *TC(S)* become constants. That is,

$$TC(S) = FC + \sum_{i \in A_k} \sum_{j \in B_k} d_{k-1,k} F_{ij},$$

the location vector becomes $S' = \{A_k, v, B_k - v\}$, and its inter-cell flow cost can be estimated by:

$$TC(S') = TC(A_k, v, B_k - v) = FC + \sum_{i \in A_k} \sum_{j \in B_k} d_{k-1,k} F_{ij} + \sum_{i \in v} \sum_{j \in B_k - v} d_{k-1,k} F_{ij} + \sum_{i \in A_k} \sum_{j \in B_k - v} (d_{k-1,k} + d_{k,k+1}) F_{ij}$$

$$= FC + (\sum_{i \in A_k} \sum_{j \in v} d_{k-1,k} F_{ij} + \sum_{i \in A_k} \sum_{j \in B_k - v} d_{k-1,k} F_{ij}) + (\sum_{i \in v} \sum_{j \in B_{k-v}} d_{k,k+1} F_{ij} + \sum_{i \in A_k} \sum_{j \in B_k - v} d_{k,k+1} F_{ij})$$

$$= (FC + \sum_{i \in A_k} \sum_{j \in B_k} d_{k-1,k} F_{ij}) + (\sum_{i \in v} \sum_{j \in B_k - v} d_{k,k+1} F_{ij} + \sum_{i \in A_k} \sum_{j \in B_k - v} d_{k,k+1} F_{ij})$$

$$= TC(S) + (\sum_{i \in v} \sum_{j \in B_k - v} d_{k,k+1} F_{ij} + \sum_{i \in A_k} \sum_{j \in B_k - v} d_{k,k+1} F_{ij}).$$

That is, the increased material flows are those part movements from v to $B_k - v$ and from A_k to $B_k - v$ and the inter-cell flow cost is computed by multiplying the distance $d_{k,k+1}$ between the two locations. Since the cell locations are assumed approximately equally spaced, $d_{k-1,k} = d_{k,k+1} = 1$; and we have:

$$TC(S') = TC(A_k, v, B_k - v)$$

$$= TC(S) + \sum_{i \in v} \sum_{j \in B_k - v} F_{ij} + \sum_{i \in A_k} \sum_{j \in B_k - v} F_{ij}$$

$$= TC(S) + \sum_{i \in A_k \cup v} \sum_{j \in B_k - v} F_{ij}$$

$$= TC(S) + \sum_{i \in X_v} \sum_{j \in \overline{X}_v} F_{ij}$$

$$= TC(S) + C_k(X_v, \overline{X}_v).$$

The increased inter-cell flow cost for the new assignment is equivalent to the minimum cut of the network flow problem in G, where a cut $(X_v, \overline{X}_v)$ is the set of arcs with one end in X_v and the other end in $\overline{X}_v$, and the sum of capacities of all the arcs on this cut is the increased inter-cell flow cost. In other words, the cut value is equivalent to the increased inter-cell flow cost of assigning the cell v to the *k*th location. Starting from the assignment of the first location sequence, the sum of $C_k(X_v, \overline{X}_v)$, k = 1,…,e, is the total inter-cell material flow cost in a linear flow layout, and the formulation to solve the joint problem becomes:

$$\text{Minimize} \quad TC = \sum_{k=1}^{e} C_k(X_v, \overline{X}_v)$$

$$\text{Subject to:} \quad |q_v| \leq Q \qquad v = 1,...,e.$$

Moreover, as discussed previously and despite of most available approaches, we will face the intra-cell layout problem as well. We propose a solution procedure to the related QAP model of this problem as an integrative view of a CMS design. The total intra-cell flow cost of material, which we denote by TC', will be added to TC in order to compute the total material handling cost. Decisions will then be made according to this latter criterion. In next section, we will describe our solution procedure.

4. INTEGRATED SOLUTION PROCEDURE

As mentioned previously, we implement a Simulated Annealing approach that uses crossover and mutation operators. Additionally, as will be described in details, a Dynamic Programming

procedure and an Ant Colony Optimization (ACO) technique are embedded. The major reasons for implementing this approach include: (a) Our skills and experiences in implementing SA and ACO, and (b) Effective results, achieved by previous researchers.

To describe the procedure in more details, notice that the sequence of machines needs to be partitioned into a number of segments under the cell size constraint. To this end, a Dynamic Programming algorithm is developed to achieve minimum TC, i.e., the minimum inter-cell total flow cost. Each partition then will be treated as a cell. Moreover, TC' (i.e., the intra-cell total flow cost) will be added to TC to build up the total material handling cost. TC' will be determined by an Ant Colony Optimization (ACO) technique, used to solve the QAP model in each stage of our SA procedure. Our main integrated approach is as follows.

Step 1. *Construct a set of initial solutions (parent solution). Each solution is generated randomly by assigning m machines into a sequence.*

Step 2. *Define the initial temperature T and parameters such as the length of equilibrium, the Cooling rate of temperature, and the population size.*

Step 3. *While not yet frozen do:*

Step 3.1. *While not yet equilibrium do*:
Compute the inter-cell and then according to that the intra-cell flow cost (TC & TC' (parent)) of each parent solution in the current set respectively by the Dynamic Programming algorithm and the Ant Colony technique for the QAP model.
Generate a set of neighborhood (child) solution via the crossover and mutation operators in the genetic algorithm.
Compute the inter-cell and then according to that the intra-cell flow cost (TC & TC' (child)) of each child solution set respectively by the Dynamic Programming algorithm and the Ant Colony technique for the QAP model.
For each pair of solutions in the parent set and the child set do:
Calculate the cost improvement Δ = (TC(child) - TC(parent))+(TC' (child)- TC' (parent)).
If $\Delta \le 0$, *the solution in the parent set is replaced by the one in the child set.*
Otherwise, the solution will be replaced by an acceptance probability $\exp(-\Delta / T)$.

Step 3.2. *Reduce the current temperature through a fixed cooling ratio.*

Step 3.3. *End of while*

Step 4. *The algorithm is terminated when either (1) the best solution and the worst solution in the population set are equal, or (2) the temperature has been reduced below the user-defined value. The best solution in the current population set at the frozen temperature is treated as the optimal solution.*

4.1. Embedded Dynamic Programming

As mentioned before, string of machine codes (where machines are sequenced in the row of incident matrix) is used to represent a solution. Machine cells can be obtained by partitioning the string into several segments, and the machines in each segment are clustered as a cell. The partition problem can be stated as follows. In an ordered sequence of m machines, a breaking node is the

machine in the sequence for the partition of one machine cell. This node is at the end of the sequence, which forms the machine cell. The node after the breaking node is the beginning for forming the next machine cell in the linear sequence. The partition problem is to find a set of e breaking nodes, which partition the machine sequence into *e* machine cells with the minimum inter-cell flow cost (minimum *TC*).

Let s_k be the index of breaking node k, $d(s_k)$ denotes the order index of s_k in the sequence of machines, and e_{s_k} be the increased flow cost when s_k is selected as a breaking node. The partition problem can be stated as to find a set of e breaking nodes $s_1, s_2, ..., s_e$ that has the minimal total inter-cell flow cost. It can be formulated as an integer programming model:

$$\text{Minimize} \quad \sum_{k=1}^{e} e_{s_k}$$

$$\text{Subject to:} \quad \sum_{j=d(s_{k-1})+1}^{d(s_k)} g_j \le Q \qquad k = 1,...,e, \quad \text{and} \quad 1 \le d(s_1) \le ... \le d(s_e) = m,$$

where $g_j = 1$, if the *j*th node on the sequence is assigned to machine cell k, it is zero otherwise, and $d(s_0) = 0$. Using dynamic programming, node s_k (or $d(s_k)$) at stage k must be determined, where s_k is the last machine on the sequence to be included in machine cell k; and the clustering of machine cell k + 1 begins at node $d(s_k) + 1$ onward. Let d_k be the number of nodes (machines) included in the machine cell k, then $d(s_{k-1}) = d(s_k) - d_k$. Also, let $f_k(d_k, s_k)$ denote the inter-cell flow cost when node s_k is the last node to be included in the machine cell k (with d_k nodes). At stage k, the partition problem becomes:

$$\text{Minimize} \quad f_k(s_k, d_k) = e_{s_k} + f_{k-1}^*(s_{k-1})$$

$$\text{Subject to:} \quad d_k \ge 1$$

$$\sum_{j=d(s_{k-1})+1}^{d(s_k)} g_j \le Q,$$

where $d(s_0) = 0$, $f_0^*(s_0) = 0$, and $f_k^*(s_k) = \min_{d_k} f_k(s_k, d_k)$. The constraint sets allow only feasible machine cells to be clustered. The dynamic programming algorithm is terminated when the solution at stage k satisfies the condition: $m - d(s_k) \le Q$. That is, the total number of machine cells is e=k +1, since the machine at the end of the sequence is the last breaking node s_e.

4.2. Embedded Ant Colony Optimization Technique

Ant Colony System (ACS), first proposed by Dorigo and Gambardella (1977), is one of the most recent and hopeful metaheuristics for combinatorial optimization problems. Ant Colony Optimization (ACO) as well, has been applied to solve different types of combinatorial optimization problems including TSP (Dorigo and Gambardella (1997)), QAP (Gambardella, Taillard and

Dorigo (1999), Solimanpur et al. (2004)), scheduling problem (Colorni et al. (1994)), vehicle routing (Bullnheimer and Hartt (1999)), graph coloring problem (Costa and Hertz (1997)), Partitioning problem (Kuntz et al. (1994 & 1997)) and telecommunications networks problem (Schoorderwoerd et al. (1997)). Figure 2 illustrates a general ACS algorithm.

As can be seen in our integrated proposed SA procedure, described in Section 4, in order to compute the optimal TC', i.e., the optimal total intra-cell material flow cost, we have implemented an Ant Colony Optimization (ACO) technique to solve the QAP model. That is, first we propos to model the related intra-cell layout problem as QAP and then to solve that with an ACO technique. To do so, we implement the recently developed Ant Colony procedure by Solimanpur, Vrat and Shankar (2004) after making necessary changes. Indeed, they developed a strong ACO algorithm for the QAP model of inter-cell layout problem regardless of other phases of cellular manufacturing design such as cell formation or intra-cell layout problem. Here, we modify their approach to make it suitable for intra-cell layout decisions (and not for the inter-cell layout problem as they have proposed). Then we implement it, in our SA procedure in order to simultaneously compute TC' as a part of our integrated approach.

Suppose that we have $C \le Q$ available locations for placing C machines in particular cell k *(k=1,2,...,e)*. While machines related to each cell are determined in previous stages of our integrated SA procedure, the constraint $C \le Q$ is previously satisfied for each cell and additionally machines to be placed in each cell are known. Thus, the QAP model to minimize intra-cell problem for cell k *(k=1,2,...,e)*, i.e., TC'_k is as follows:

$$Min \qquad TC'_k = \sum_{i=1}^{C}\sum_{j=1}^{C}\sum_{l=1}^{C}\sum_{h=1}^{C} f_{ij} d_{lh} x_{il} x_{jh} \tag{1}$$

subject to:

$$\sum_{l=1}^{C} x_{il} = 1, \qquad i = 1,2,..,C \tag{2}$$

$$\sum_{i=1}^{C} x_{il} = 1 \qquad l = 1,2,...,C \tag{3}$$

$$x_{il} \in \{0,1\}, \qquad i,l \in \{1,2,...,C\} \tag{4}$$

where x_{il} is equal to 1 if machine i is placed in the location l of the cell and is 0 otherwise. f_{ij} refers to the flow between machine i and j, and d_{lh} is the known distance between location l and h in the considered cell. Objective function (1) minimizes the intra-cell flow cost of cell k (TC'_k). Constraints (2) and (3) guarantee both the allocation of each machine to exactly one place and each place exactly to one machine, respectively. After solving the model for all e cells, one can easily obtain the TC' by a simple summation of all resulted TC'_k for *k=1,2,..,e*.

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place exactly to one machine, respectively. After solving the model for all *e* cells, one can easily obtain the TC' by a simple summation of all resulted TC'_k for *k=1,2,..,e*.

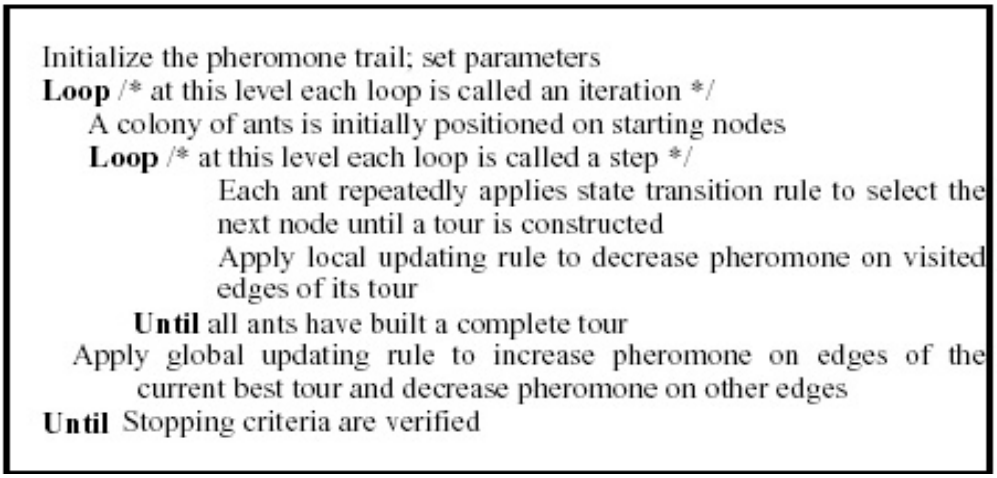

Figure 2 A general ACS Algorithm.

where x_{il} is equal to 1 if machine *i* is placed in the location *l* of the cell and is 0 otherwise. f_{ij} refers to the flow between machine *i* and *j*, and d_{lh} is the known distance between location *l* and *h* in the considered cell. Objective function (1) minimizes the intra-cell flow cost of cell *k* (TC'_k). Constraints (2) and (3) guarantee both the allocation of each machine to exactly one place and each place exactly to one machine, respectively. After solving the model for all *e* cells, one can easily obtain the TC' by a simple summation of all resulted TC'_k for *k=1,2,..,e*.

As is well-known, generally any Ant Colony algorithm must specify the following elements by which we describe our embedded ACO procedure:

- *Construction of solutions*: A feasible and complete solution of the formulated QAP model of intra-cell layout problem is considered as a permutation of machine assignments. Hereafter we call assigning machine *i* to location *l* a move and represent it by *v=(i,l)*.

- *Heuristic information*: Artificial ants can include some heuristic information while assigning machines at different locations in related cells. The heuristic information pertaining to move *v=(i,l)* is denoted by η_{il} which indicates the desirability of assigning machine *i* to location *l* . While calculating the desirability of move *v=(i,l)* the permutation of machines till position l-1 is known, η_{il} can be calculated by the following formula:

$$\eta_{il} = [1 + \sum_{s=1}^{l-1} (f_{\pi(s)i} \times d_{sl} + f_{i\pi(s)} \times d_{ls})]^{-1}$$

in which π denote a permutation of machines under construction and $\pi(s)$ is the machine placed in the *s*th position of permutation π. The reason due to which number 1 in the above formula is added to the summation is to avoid division by 0. Due to this hyperbolic function of desirability, the moves with small augmentation in the objective function would be more desirable for selection.

- *Pheromone updating rule* : we use the modification of Maniezzo (1999) mechanism which is also proposed by Solimanpur et al. (2004) for updating Pheromone trail levels and is applied through the following equation:

$$\tau_{il}(t) = \tau_{il}(t-1) + \sum_{k} \Delta\tau_{il}^{k}$$

where

$$\Delta\tau_{il}^{k} = \tau_0 \times (1 - \frac{z^k - LB}{\bar{z} - LB}), \forall v = (i,l) \in V^k .$$

In this formula z_k is the objective function value of the solution obtained by ant k, $\bar{z}$ is the average of the objective function values of the solutions obtained in iteration t and LB is a lower bound to the optimum value of objective function. The control parameter τ_0 is a small real number indicating the maximum increment in the trail levels due to one ant.

- *Selection probability* : Ant *k* chooses machine i to be assigned to location l by the following probability:

$$p_{il}^{k} = \frac{\alpha \times \tau_{il} + (1-\alpha) \times \eta_{il}}{\sum_{i \notin Tabu_k} (\alpha \times \tau_{il} + (1-\alpha) \times \eta_{il})}, \qquad i \notin Tabu_k$$

where p_{il}^{k} is the probability that ant k chooses machine i to be assigned to location l, and α is a control parameter used to map the relative importance of trail level and the desirability of each move.

- *Termination condition:* Finally the algorithm will be stopped when the ants converge to the same solution and the solution cannot be improved. The algorithm will also be terminated if the counter becomes equal or greater than maximum allowed iterations, i.e. the termination condition is: $counter \geq max\_iter$.

By above descriptions our embedded Ant Colony algorithm briefly is as follows:

Step 1. *Set the values of A, max_iter,* α, τ_0. *Set Counter=0.*

Step 2. *Initialize the trail levels of all moves* τ_{il} *(*$\forall i,l \in \{1,2,...,C\}$*).*

Step 3. *Compute the lower bound of the problem.*

Step 4. *For k=1 to A:*
Set $tabu_k$*=0*
for l=1 to C:
compute η_{ij} *and* p_{il}^{k} *for all allowed i (*$\forall i \notin Tabu_k$*).*
Save the solution in the memory of ant k, V^k *and update* $Tabu_k$

Step 5. *For k=1 to A*
Update the trail levels of moves in the memory of ant k, V^k.

Step 6. *If the best solution in the last iteration has been improved in the current iteration record the solution and set counter=0, otherwise counter=counter+1.*

Step 7. *if counter* $\geq$ *max_iter then stop, otherwise go to step 4.*

5. COMPUTATIONA RESULTS

To illustrate the efficiency of our proposed integrated procedure, we have implemented our algorithm to solve 15 randomly generated test problems. We have compared the results of our approach with the previous available one in the literature, i.e., the proposed approach of Chiang and Lee (2004). We use "C&L" (Chiang and Lee) to refer to their approach and "S&A" (Saghafian and Akbari) to refer to our proposed procedure. We consider the criterion of total material flow cost and compare these two approaches. It is noteworthy that in both approaches intra-cell flow costs can be computed based on the objective function of the described QAP model; where this objective is heuristically optimized in "S&A" (using the described embedded Ant Colony Optimization technique as a part of our integrated procedure) but this is not the case for "C&L".

In order to maximize the similarity of our test problems with what "C&L" implemented as their test suites, we have used same data to generate our 15 randomly generated test suites. We also have coded both approaches and implemented them with same input data on a same platform. Table 1 shows the dimension of test problems as well as obtained results. Moreover, Table 1 also presents the resulting solution of each procedure. In this table we denote the cell boundaries by "/". Hence, the sequence of machines in each cell (from left to right) shows the intra-cell layout of that cell according to the QAP model. For instance "1 2/ 3 4" describes that there are two cells in the layout: machine 1 and 2 form the first cell, and machines 3 and 4 form the second one. Machine 1 is placed in the first available place of the first cell, and machine 2 in the second place of this cell. Also, machines 3 and 4 are placed in the first and second available places of cell 2, respectively.

The results presented in Table 1 reveal that our proposed approach achieves solutions with lower total cell flow cost for all implemented test problems. This is mainly due to the fact that our approach is more integrated than the previous available one in the literature. In other words, it considers cell formation, intra-cell material, and inter-cell flows in a jointly manner. The average percent of improvement in total cost is 24.97% with the best improvement of 66.41% for n=6 and the worse of 3.32% for n=10. Figure 3 illustrates the percentage of improvement in total cost for different number of machines. Figure 4 depicts the comparison of our proposed algorithm (S&A) with that of Chiang and Lee (C&L) based on the total flow cost. One can observe that as *n* increases, the difference between resulted total flow costs grows approximately in an exponential way.

Moreover, Figure 5 compares two approaches by considering the resulted inter-cell flow costs. Although our procedure and that of Chiang and Lee (2004) are approximately similar in dealing with the inter-cell layout design, one can observe a slight difference and that is related to the random nature of both algorithms. However, since our proposed procedure modifies a strong Ant Colony Optimization (ACO) technique to also incorporate the intra-cell layout problem, a major difference happens regarding this important component of total flow cost. This difference highlights the importance of considering intra-cell decisions in cell formation and inter-cell layout problems

with respect to total material flow costs. Figure 6 demonstrates this statement by comparing two procedures based on their intra-cell flow costs.

6. CONCLUSION

In this paper, we proposed an integrative view for all three phases of cellular manufacturing systems design, i.e., cell formation, inter and intra-cell layout design. The main algorithm was a Simulated Annealing approach in which a Dynamic Programming (based on the graph representation of material flow) and an Ant Colony algorithm (based on a QAP model) were implemented, respectively for inter-cell and intra-cell layout decisions. The main difference between this paper and other available studies is that in contrast with most of available approaches in literature, in this

Table 1 Computational results with dimensions of random generated test problems.

Problem No.	n	Q	Resulting Solution (S&A)	Resulting Solution (C&L)	Inter-cell Cost (S&A)	Inter- cell Cost (C&L)	Intra-cell Cost (S&A)	Intra-cell Cost (C&L)	Total Cost(S&A)	Total Cost(C&L)	Cost Improvement Percentage
1	5	3	3 2 /5 4 1	2 3/5 1 4	80	80	18	42	98	122	19.67
2	6	3	6 2 3 / 1 4 5	2 6 3/ 4 1 5	100	100	31	290	131	390	66.41
3	7	3	7 3 2 /6/ 4 1 5	6 2 3/ 7/ 4 1 5	208	264	120	396	328	660	50.30
4	8	3	6 2 3 / 5 7 8 / 1 4	6 2 3 / 7 8 / 4 5 1	612	550	60	821	672	1371	50.98
5	9	4	1 5 2 8 / 6 7 3 4 / 9	8 / 6 5 1 2 / 7 4 9 3	988	1074	384	668	1374	1742	21.13
6	10	5	7 / 2 9 5 6 3 / 4 10 1 8	2 7 3 9 5 / 8 10 4 6 1	428	405	416	468	844	873	3.32
7	11	5	7 / 6 / 2 5 3 11 9 / 10 1 8 4	3 / 5 2 11 7 9 / 1 8 10 6 4	950	881	446	801	1396	1682	17.00
8	12	5	13 / 2 3 7 9 11 / 1 5/ 10 6 8 4	11 / 3 / 1 / 9 5 2 7 / 8 4 10 6 12	798	935	162	186	960	1121	14.36
9	13	5	11/ 5 7 1 2 9 / 3/ 12 4 13 / 10 6 8	9/ 2 3 13 7 5 / 6 / 10 8 11 4 12	1050	968	282	597	1332	1565	14.89
10	14	5	5 9 2 7 / 12 11 / 3/ 4 6 14 / 8 1 10 13	2/ 9/ 5 7 11 14 / 8 / 10 3 / 13 1 4 6 12	1953	1850	292	472	2245	2322	3.32
11	15	5	5 / 4 14 / 7 / 8 2 3 15 9 / 10 / 6 11 1 13 12	9 / 8 / 10 / 3 5 15 4 14 / 13 7 2 12 11 / 6 1	1950	2197	525	524	2475	2721	9.04
12	16	6	10/ 4 3 5 14 16 / 9 11 15 /1/ 2 13 6 7 12 8	2 3 4 5 14 16 / 11 9 15 / 10/ 1 6 13 7 8 12	2078	2174	490	1198	2568	3372	23.84
13	17	6	5/ 3 15 16 / 9 11/ 4 / 8 14/ 1 10 2 13 12 7/ 6 17	8 /4 14 16 10 3 5/ 9 11 15 17 6/ 12 1 13 7 2	2540	2857	380	1207	2970	4065	26.94
14	18	6	3 14 4 5 18 16 / 12 13 / 8 7 2 15 / 9 1 6 17 10 11	9 11 /15 /3 5 4 16 18 14 /7 8 /10 2 13 1 12/ 6	2103	2827	640	1190	2743	4017	31.72
15	19	6	17 8 / 11 9 / 12 3 16 13 / 19 14 5 6 / 15 4 7 18 1 2 / 10	5 /3 / 8 4 15 9 17 11 / 16 18 /16 18 / 1 12 14 6 19 3 /7 10 2	4396	4451	404	1670	4800	6121	21.58

paper we considered: (1) The objective function of minimizing overall inter-cell and intra-cell flow costs instead of minimizing the number of inter-cell movements/costs. (2) The integrative and simultaneous determination of cell formation and their layout instead of using sequential approaches. (3) All three phase of cell formation, inter-cell and intra-cell layout design problems, which are all important for overall performance of the system, and finally (4) An easy to code and solve integrated procedure through implementing metaheuristic approaches.

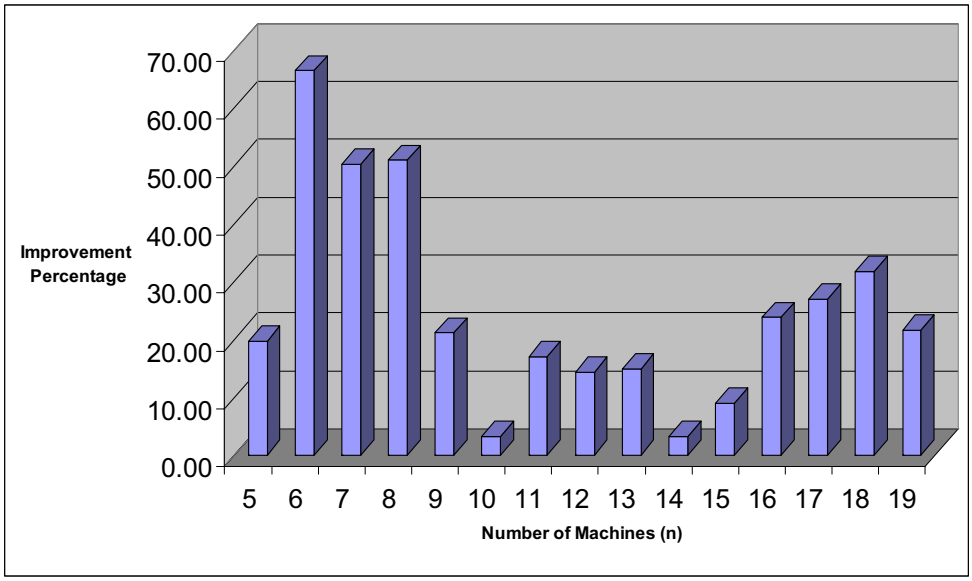

Figure 3 Improvement percentage in total flow costs

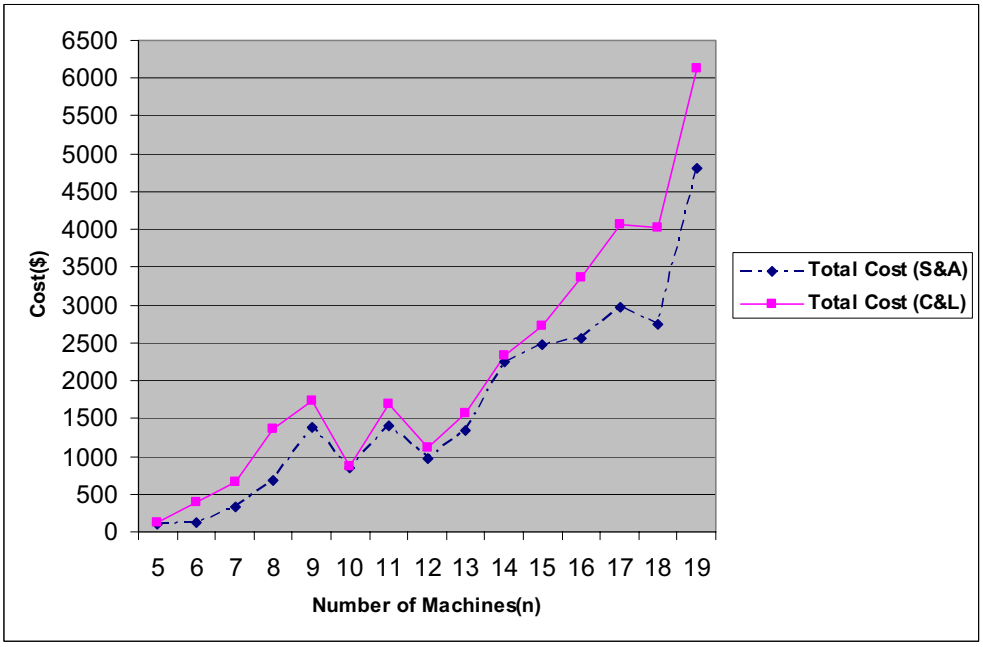

Figure 4 Comparison of proposed procedure (S&A) with that of Chiang and Lee (C&L) regarding resulted total costs.

Our computational results showed high quality solutions compared to the most similar available approach in the literature. The average improvement percent in total flow cost for a set of randomly generated test problems was 24.97%. This demonstrates the importance of having an integrated view of all phases of Cellular Manufacturing Systems (CMS) design (especially the intra-cell phase which is ignored in many available approaches in the literature). This improvement in average cost also highlights the quality of the proposed procedure. In fact, our computational results showed that by incorporating intra-cell decisions in cell formation and inter-cell design process (through implementing our proposed integrated approach) a manufacturer can largely reduce her total material flow costs. These results, as stated above, reveals the importance of simultaneous consideration of different design aspects of a Cellular Manufacturing System.

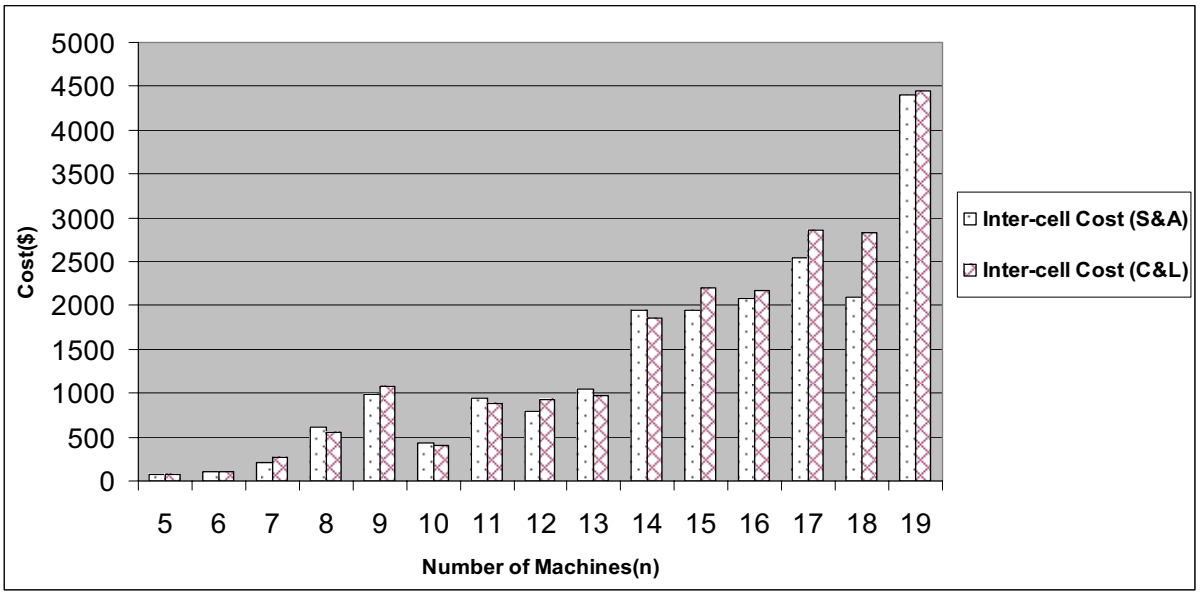

Figure 5 Comparison of two procedures by total inter-cell flow costs.

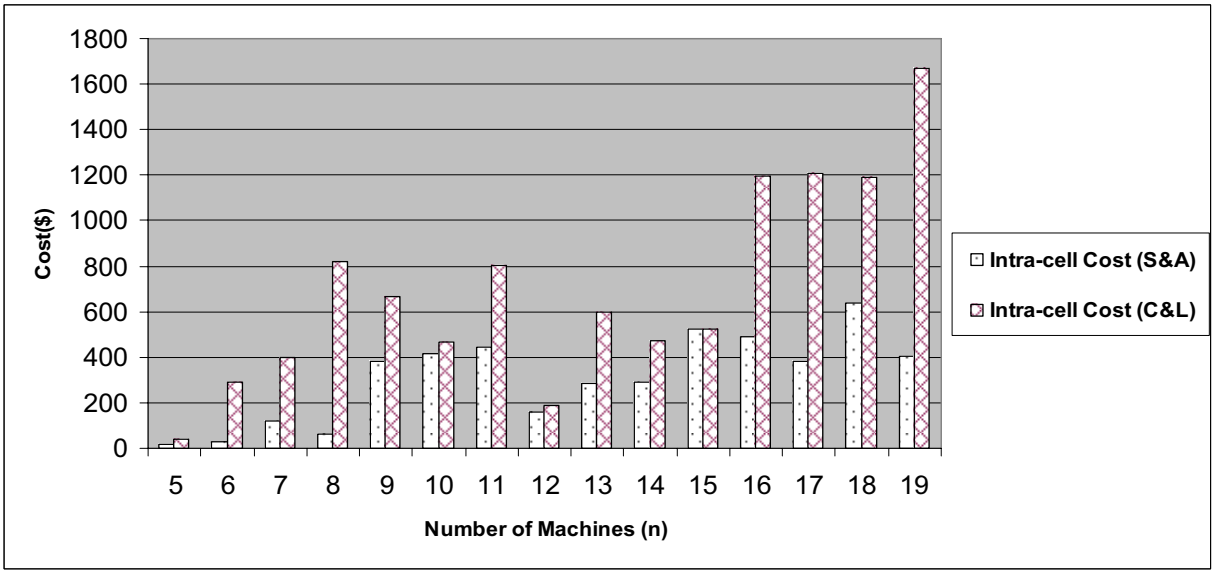

Figure 6 Comparison of two procedures with total intra-cell flow costs.

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