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Using Regression based Control Limits and Probability Mixture Models for Monitoring Customer Behavior

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ABSTRACT

In order to achieve the maximum flexibility in adaptation to ever changing customer's expectations in customer relationship management, appropriate measures of customer behavior should be continually monitored. To this end, control charts adjusted for buyer's/visitor's prior intention to repurchase or visit again are suitable means taking into account the heterogeneity across customers. In the case of a subscription-based service provider, this paper discusses three types of adjusted control charts considering grouped data on attribute usage measures are available at each subscription period. With appreciating the characterizing effect of customer's overall satisfaction on his future behavior, regression based models and probability mixture models are used to account for heterogeneity in customers' mean usage rate. Besides adjusted Shewhart and CUSUM control charts for Bernoulli and Poisson distributed usage indicators, the likelihood ratio test based on mixture probability models are investigated in term of detect ability of the shifts in usage behavior through a comparative simulation study.

Keywords: customer usage behavior, attribute control charts, mixture probability model, CUSUM control chart.

1. INTRODUCTION

Customer relationship management (CRM) requires that appropriate measures of customer behavior to be monitored regularly to provide a diagnostic mechanism which helps marketing managers adapt corporate policies to the permanent changes of customer expectations. Control charts as the most widely used tool among statistical process control (SPC) tools provide suitable means for this purpose considering the stochastic nature of the measures of usage behavior. However, there are considerable differences between units inspected in industrial manufacturing processes where the SPC tools are originated from and customers, the subjects of usage monitoring. Unlike in standard industrial applications, usage data may be markedly non-homogenous. Hence, traditional SPC charts which are developed under the assumption of identically distributed data should be modified to be applicable for monitoring customer behavior. Taking customer's underlying pre-usage intention into account allows customers to be treated distinctly depending on their different usage rates when a monitoring scheme is to be devised.

There are several ways to derive predictive models containing customer's behavioral information. For this purpose, it is required to determine features affecting customer usage rate. According to a

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broad consensus in marketing research literature, overall satisfaction can be used as an important factor affecting customer's future behavior. Fornell et al. (1996) employ structural equation modeling to develop a causal model that describes customer's future behavior based on his/her overall satisfaction. There are several examples from telephone service industry and the manufacturing sector, which clearly show that customer retention depends strongly on satisfaction level as it may be concluded explicitly that the high level of satisfaction is required to achieve a high loyalty (Gryna, 1999). Xerox reports that totally satisfied customers are six times more likely to repurchase its products than are its "merely satisfied" customers (Jones and Sasser, 1995). Overall satisfaction will be used in this research as the main explanatory variable to develop models predicting customers' future usage behavior.

Considering the growing rate of online service providers, there are numerous types of services supplied according to a subscription for a given period of time. Internet service providers, online content providers, different kinds of telecommunication services, satellite television providers are among the subscription-based services. Customers of such firms purchase a monthly, quarterly, an annual, or an "annual billed monthly" subscription. Whether a customer renews his/her subscription as well as the number of purchases or visits made during a given time interval are the main criteria characterizing the frequency of usage for this sort of service providers. From a probabilistic point of view, Bernoulli distribution is a well-known model to represent the binary outcome of the former while the Poisson distribution is an appropriate random model for the latter.

In industrial quality control, the quality characteristics following aforementioned discrete distributions are classified as attribute quality characteristics (Montgomery, 2005). More specifically, based on Shewhart's theory, p and u are widely-used control charts of SPC to monitor the variation of the quality measures following binomial and Poisson distributions, respectively. Identically distributed observations resulted from a homogeneous population is an important assumption to employ Shewhart control charts. However, heterogeneity of customers is an inherent feature of usage behavior. Therefore some modifications of conventional control charts are required to meet the purpose of monitoring non-homogenous Bernoulli and Poisson distributed measures of usage behavior. Several types of control charts are proposed and compared in this research to monitor non-homogenous usage behavior. Present study has been inspired by recent researches under the subject of risk-adjusted control charts used for surveillance of surgical outcomes in the field of healthcare quality control. A brief introduction of this idea is presented in the next section.

The remainder of this paper is organized as follows. The next section provides a brief survey of pertinent studies in the area of SPC. Monitoring schemes of usage behavior are proposed and discussed in section three. Section four presents a comparison study of the proposed control charts. Lastly, section five provides concluding remarks and final discussions on the application of the proposed methods.

2. CONTROL CHARTING HETEROGENEOUS ATTRIBUTES

There are different approaches proposed in this paper to deal with heterogeneous usage rate of customers. The first approach involves a modified version of traditional Shewhart charts used for monitoring attribute data. The control charts presented in this paper are developed on the basis of adjustment for individual buyer's/visitor's particular characteristics. To avoid ambiguity, the modified versions of p and u charts are hereafter called case-adjusted p chart and case-adjusted u chart, respectively.

Although Shewhart charts do not possess good efficiency to detect small shifts in process parameters, there are more sensitive control charts like cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) which are capable of detecting small to moderate sustained shifts more efficiently as is of great importance to marketing managers to detect creeping trends in usage rate as quickly as possible. As the second approach to monitoring usage rate, case-adjusted CUSUM charts for Bernoulli and Poisson random variables are used in this research. Although CUSUM charts are often used for individual observations, they are employed here for sample statistics. It should be noticed that CUSUM is designed based on a predetermined size of the parameter shift. This issue will be discussed later in detail.

As the third approach, mixture probability models "i.e.," beta-binomial and gamma-Poisson models are used to incorporate the heterogeneity of customers into the monitoring statistics. A point of note is that usage rate is considered independent of time in this study. Hence, dynamic usage behavior which allows the usage rate to be described as a function of time or of previously recorded usage behavior is not investigated in the proposed methods.

2-1. Overview of existent approaches

Consider the following equation as the general form of the adjustment model for customer *i*.

$$E(X_i;\boldsymbol{\beta}) = g^{-1}(\mathbf{u}_i' \cdot \boldsymbol{\beta}) \tag{1}$$

where X_i is the response variable representing customer usage behavior. $\mathbf{u}_i$ denotes corresponding vector of regressors indicating factors/covariates characterizing specification of each customer and $\boldsymbol{\beta}$ is the vector of model coefficients. Function $g(\cdot)$ may take the form of *logit* and *log* link functions to predict respectively the mean of the Bernoulli and Poisson distributed response variables. These are logistic regression and Poisson regression models respectively, two special cases of the broad family of the generalized linear models (GLM). Accordingly the left hand side of equation (1) may be formulated as $1/[1+\exp(-\mathbf{u}_i' \cdot \boldsymbol{\beta})]$ when the mean of the Bernoulli distribution is to be predicted and it takes the form $\exp(\mathbf{u}_i' \cdot \boldsymbol{\beta})$ when the mean of the Poisson distribution is to be specified (Myers et al., 2002).

This kind of adjustment, based on the regression models, is also used in the area of public health surveillance for monitoring surgical outcomes by Steiner et al. (1999, 2000) and Sego (2006) where the surgical risk assigned to each patient is estimated pre-operatively conditional on some risk factors. Indeed, the so called risk-adjusted control charts that have been outlined in most recent papers in the area of healthcare quality monitoring are adopted in this research for monitoring customer usage behavior.

Steiner et al. (2000) use a risk-adjusted CUSUM chart for binary surgical outcomes. Since each patient has a different prior surgical risk depending upon his own characteristics, the variation among patients must be taken into account when the probability of a surgical failure is examined. Cook et al. (2003) employ risk-adjusted *p* chart to monitor in-hospital mortality rate for an intensive care unit. An overview of various risk-adjusted control charts applied to outcomes from Bernoulli and Poisson distributions is offered by Grigg and Farewell (2004). Sego (2006) provides an interesting discussion on the risk-adjusted monitoring of clinical outcomes. Following the definition of the risk adjustment procedure (Sego, 2006), the process of incorporating the customer's information that is known prior to purchase, into a control chart will be called the case adjustment.

Woodall (2006) addresses the similarity between risk-adjustment for attribute charts through logistic regression and regression adjustment for variable charts in industrial quality control. For instance, Hawkins (1993) and Wade and Woodall (1993) employ multiple and simple linear regression to account for variation in quality of products as they pass through successive stages of the process under monitoring. This approach is of paramount interest for monitoring cascade processes wherein the variables of downstream process stages are affected by the variables of the upstream stages (Hawkins, 1993, Hauck et al., 1999). Recent applications of regression adjustment in developing SPC charts for multiple stage processes with attribute quality characteristics includes Skinner et al. (2003) and Jearkpaporn et al. (2007). GLM is used in these studies to adjust the mean of the several process output variables for a number of input variables.

2-2. Representing heterogeneity via mixture probability models

Mixture probability models have been widely used in marketing literature for customer-base analysis to represent non-homogenous purchase behaviors of customers. Schmittlein et al. (1987) is among the first studies which describes repeat-buying behavior in a situation where customers buy at a steady rate for a period of time, and then become inactive. Time to 'dropout' is modeled using the Pareto (exponential-gamma mixture) timing model and while the customer is still active, his repeat-buying behavior is modeled using the negative binomial distribution (NBD) counting model (Poisson purchasing at the individual level with gamma heterogeneity). Nadarajah and Kotz (2009) give a detailed presentation on this kind of probability models. Although the Pareto/NBD is counted as a powerful model for customer-base analysis, its empirical application can be challenging, especially because of the intensive computational effort required to estimate model parameters. A simpler model called beta-geometric/NBD (BG/NBD) is proposed by Fader et al. (2005). This model seems to be more useful than its predecessor because it can provide similar results and in addition, it does not possess difficulty to be implemented (Fader and Hardie, 2005). As another instance among the many, a remarkable study by Moe and Fader (2004) models cross-section variation in usage behavior using an exponential-gamma mixture distribution modified in such a way that can capture the non-stationarity of an individual's visit frequency over time.

In addition to the modified Shewhart and CUSUM control charts, mixture probability models are used in this research for adjustment purposes in monitoring customers' usage behavior. To this end, beta and gamma distributions are employed to represent respectively the heterogeneity of the Bernoulli and Poisson distributed response variables. The next section discusses different approaches in detail.

3. CASE ADJUSTED MONITORING SCHEMES

3-1. Case-adjusted Shewhart charts

Two different settings of usage behavior are discussed here. First consider the situation where customers use optionally the proposed services for a given period of time. Generally, subscription-based services like telecommunication service suppliers, online content providers, or insurance services can be sorted in this group. If a customer does not renew his subscription for the next period he may be deemed a churned customer. Considering the inherent variability of usage probability among customers, the goal is to design a control chart which monitors churn rate and provides warning signal in case of a significant change. Mathematically, customer usage from a single service at each period may be modeled as a Bernoulli random variable whose parameter indicates the usage probability that depends on customer characteristics. Case-adjusted p chart which takes into account the heterogeneity across customers is the first proposed method here to monitor customers' usage probability. Control limits of the case-adjusted *p* chart are calculated as follows (Cook et al., 2003):

$$\frac{1}{n}\left(\sum_{i=1}^{n} p_{0i} \pm K\sqrt{\sum_{i=1}^{n} p_{0i}(1-p_{0i})}\right) \tag{2}$$

where p_{0i} shows the hypothesized usage probability for customer i, ($i = 1,...,n$) and can be estimated using logistic regression on customer overall satisfaction score of the last subscription period. It is assumed that the parameters of the regression model can be obtained retrospectively using information on customers' usage behavior from a base time interval. The effect of estimation error is not investigated in this research. n denotes the sample size and K represents the coefficient of control limits that can be determined according to a pre-specified probability of error type I.

Now consider the second situation wherein the number of customer visits or purchases during a given time period is the main indicator of usage behavior. It is assumed that a Poisson random variable can adequately represent customer usage behavior. In contrast to the previous situation, a case-adjusted *u* chart is used here to monitor customers' purchase/visit rate. Through analogy with the case-adjusted *p* chart, control limits of this chart can be computed as follows.

$$\frac{1}{n}\left(\sum_{i=1}^{n} \lambda_{0i} \pm K\sqrt{\sum_{i=1}^{n} \lambda_{0i}}\right) \tag{3}$$

where λ_{0i} which can be predicted by Poisson regression shows the mean usage rate for customer i. As before, customer satisfaction score of the last usage period is considered as the main predictor of usage behavior for the next period. Regression coefficients may be estimated using information of a base period. n and K are defined as done for equation (2).

3-2. Case-adjusted CUSUM charts

CUSUM chart is preferred to the Shewhart chart when detecting small shifts of model parameters is of interest (Montgomery, 2005). Control statistic drawn on the CUSUM chart is computed as follows:

$$C_t = \max(0, C_{t-1} + \omega_t) \quad ,t = 1,2,... \tag{4}$$

and $C_0 = 0$. ω_t indicates the weight assigned to the observation at time *t*. To calculate the weight, both in-control value and presumed out-of-control value of the model parameter are required. Moustakides (1986) proves that the optimal choice for CUSUM weights, ω_t, is the log likelihood ratio of the random variable to be monitored.

It should be noticed that CUSUM charts are employed here for both Bernoulli and Poisson random variables when group observations are periodically collected. For example, consider a cell phone service provider who records the number of customers that renew their subscription at the end of each subscription period. As another example, the number of clients who pay their monthly installments is usually available for a bank. Hence the CUSUM statistic is computed as a composite score for the customers of a sample.

Since heterogeneity of customers constitutes the main assumption of this study, CUSUM control charts proposed here for Bernoulli and Poisson distributed usage indicators are called respectively, case-adjusted Bernoulli CUSUM and case-adjusted Poisson CUSUM.

Consider again the situation of a subscription-based service provider with binary valued observations indicating whether each customer renews his subscription for the next period. In this case the log likelihood ratio statistic, ω_t, for the time period *t* may be calculated as follows.

$$\omega_t = \sum_{i=1}^{n} \log\left(\frac{Q_i^{x_t}(1-Q_i^{x_t})}{p_{0i}^{x_t}(1-p_{0i}^{x_t})} \right) \tag{5}$$

where Q_i denotes the shifted usage probability for customer *i*, ($i = 1,...,n$). It is customary to express the interested shift in the probability of a binary outcome in term of a change in odds ratio (Sego, 2006). The following equation presents a change of size δ in odds ratio for customer *i*.

$$\delta = \frac{Q_i/(1-Q_i)}{p_{0i}/(1-p_{0i})} \tag{6}$$

Or in other words,

$$Q_i = \frac{\delta \cdot p_{0i}}{1 - p_{0i} + \delta \cdot p_{0i}} \tag{7}$$

Thus, equation (5) may be revised as follows.

$$\omega_t = \sum_{i=1}^{n} \log\left(\frac{\delta^{x_t}}{1 - p_{0i} + \delta \cdot p_{0i}} \right) \tag{8}$$

In effect, the CUSUM chart is used to test sequentially the null hypothesis of $H_0 : \delta = 1$ versus the alternative hypothesis $H_A : \delta = \delta^*$ where δ^* indicates shift of interest in customers' odds ratio of usage.

Now consider the second situation where a measure of usage such as the number of visits during a given period is to be monitored. This measure is assumed to follow Poisson distribution. In this case, the weight at time *t* can be computed as shown below.

$$\omega_t = \sum_{i=1}^{n} \log\left(\delta^{x_t}.\exp(\lambda_{0i} - \lambda_{0i}\delta) \right) \tag{9}$$

where $\delta = \lambda_{1i} / \lambda_{0i}$ and λ_{1i} denotes the shifted usage rate. Although detection of decrease in usage rate is of paramount importance in customer churn management, it should be noticed that CUSUM chart could be exploited to detect both increasing and decreasing changes. In this paper, two CUSUM charts, one for detecting decrease and the other for detecting increase of the parameter are employed simultaneously. Let the CUSUM statistic in equation (4) be represented by a superscript of plus sign as X_t^+ which is to be used to detect increase of odds ratio. Another statistic denoted by X_t^- and computed as follows is used to detect decrease of odds ratio.

$$X_t^- = \min\left(0, X_{t-1}^- - \omega_t\right) \quad , \mathrm{t} = 1,2,... \tag{10}$$

As an obvious conclusion, X_t^+ accumulates positive scores and X_t^- accumulates negative scores over consecutive monitoring periods. As soon as one of these two statistics exceed predetermined limits denoted by h^+ and h^-, it is inferred that some assignable causes have already affected usage behavior. In other words the situation wherein either $X_t^+ > h^+$ or $X_t^- < h^-$ occur is an indication of the change in customers' usage rate that requires commensurate root-cause analysis. Usually h^+ and h^- are determined in such a way that a given size of average run length (ARL) is obtained. ARL, a popular measure to assess the performance of a control chart, indicates the average number of points plotted on the chart until an out-of-control signal appears. The CUSUM chart possesses the interesting feature of change point detection which may be very helpful to identify proper root causes of the problem (Montgomery, 2005).

3-3. Control charts using mixture probability models

To the best of our knowledge, it seems there is no research so far which uses mixture probability models for monitoring purposes along with an adjustment procedure. Indeed, the third approach proposed here incorporates employing mixture probability models to capture heterogeneity of usage behavior. In the case of a subscription-based service provider, suppose that the probability of customer subscription renewal for the next period is denoted by p_{0i}, $(i = 1,...,n)$. Naturally different customers have various degrees of purchase intention. Some customers possess higher interest while the others possess lower tendency to repurchase the product or renew their subscription. To represent this heterogeneity, it is assumed that the parameter p_{0i} follows beta distribution. Therefore a Bernoulli distribution is employed to provide a probabilistic model for the binary outcome of customer action at each period and a beta distribution is used to capture the heterogeneity in the underlying purchase probability. These two distributions are given respectively by the following mass and density functions.

$$\begin{aligned} f(x_i; p_i) &= p_i^{x_i}(1-p_i)^{1-x_i}; \quad x_i = 0,1 \\ g(p_i; \alpha, \beta) &= \frac{p_i^{\alpha-1}(1-p_i)^{\beta-1}}{B(\alpha,\beta)}; \quad 0 < p_i < 1 \end{aligned} \tag{11}$$

where α and β denote parameters of the beta distribution. Integration of these two probability distributions leads to the following mixture model called beta-binomial model.

$$f(x_i;\alpha,\beta)=\int_0^1 f(x_i;p_i)g(p_i;\alpha,\beta)dp_i=\frac{B(\alpha+x_i,\beta+1-x_i)}{B(\alpha,\beta)} \tag{12}$$

Some detailed discussions on such mixture probability models can be found in Johnson et al. (2005). Parameters α and β can be estimated through maximum likelihood estimation (MLE) method. For this purpose, likelihood function to be maximized with respect to the parameters can be obtained in the form of the product of individual likelihood functions shown in equation (12). Eventually, the log likelihood function may be derived as follows.

$$LL(\alpha,\beta)=\sum_{i=1}^{n}\ln\left[L(\alpha,\beta|x_i)\right]=\sum_{i=1}^{n}\ln\left[\frac{B(\alpha+x_i,\beta+1-x_i)}{B(\alpha,\beta)}\right] \tag{13}$$

The log likelihood function should be maximized to obtain estimates of the parameters α and β. Therefore, the original problem of detecting changes in odds ratio may be recast as testing the null hypothesis $H_0:\alpha=\alpha_0,\beta=\beta_0$ versus its general alternative. Considering maximum likelihood estimates of parameters α and β, generalized likelihood ratio (GLR) test method can be used as a suitable approach to provide required monitoring statistic. The derivation of GLR statistic is discussed in this section.

Similar to two previous subsections, the second problem incorporates monitoring the number of purchases/visits made by customers during a given time interval. As stated before, Poisson distribution with parameter λ_i, ($i=1,\ldots,n$), is used to model the number of purchases/visits made by customer *i*. Since customers have different usage rates, a gamma distribution with parameters b_1 and b_2 is considered to represent heterogeneity of λ_i across customers. Functions presented below indicate respectively the Poisson probability mass function and the gamma density function.

$$\begin{aligned} f(x_i;\lambda_i)&=\frac{e^{-\lambda_i}\lambda_i^{x_i}}{x_i!};\quad x_i=0,1,2,\ldots \\ g(\lambda_i;b_1,b_2)&=\frac{1}{b_1^{b_2}\Gamma(b_2)}e^{-\lambda_1/b_1}\lambda_1^{b_2-1};\quad 0<\lambda_i \end{aligned} \tag{14}$$

As the result of integrating functions given in equation (14), the mixture gamma-Poisson model is obtained in the following form.

$$f(x_i;b_1,b_2)=\int_0^\infty f(x_i;\lambda_i)\times g(\lambda_i;b_1,b_2)d\lambda_i=\frac{\Gamma(b_2+x_i)}{\Gamma(b_2)\times x_i!}\times\frac{b_1^{x_i}}{(1+b_1)^{x_i+b_2}} \tag{15}$$

The likelihood function consolidated over all independent observations can be computed as the product of individual likelihood functions resulted in equation (15). Subsequently, the log likelihood function to be maximized to obtain estimates of b_1 and b_2 may be written as follows.

$$LL(b_1, b_2) = \sum_{i=1}^{n} \ln\left[L(b_1, b_2 | x_i)\right] = \sum_{i=1}^{n} \ln\left[\frac{\Gamma(b_2 + x_i)}{\Gamma(b_2) \times x_i!} \times \frac{b_1^{\,x_i}}{(1+b_1)^{x_i + b_2}}\right] \tag{16}$$

In this case the null hypothesis $H_0 : b_1 = b_{10}, b_2 = b_{20}$, will construct the hypothesis on customers' usage rate.

Based on the discussion on how to extract maximum likelihood estimates of the model parameters, the method to derive monitoring statistic is explained. The proposed method called generalized likelihood ratio (GLR) test, is applicable for both situations mentioned in this section. Let $L_{reduced}$ denote likelihood function computed while assuming that the null hypothesis holds true. Also L_{full} will indicate likelihood function computed via maximum likelihood estimates of the model parameters while neglecting the constraints set by the null hypothesis. According to equations (12) and (15) model parameters contain α and β for the first situation while b_1 and b_2 are the parameters of the second model.

It can be shown that when H_0 is true and a large volume of data is provided then the statistic $-2\ln\left[L_{reduced} / L_{full}\right]$ is approximately distributed as χ_2^2, a chi square distribution with two degrees of freedom (Mood et al., 1974). Therefore, considering a specific rate of type I error, it is possible to determine an upper control limit. Once a value of the monitoring statistics exceeds $\chi_{\alpha,2}^2$ which denotes the upper-tail α percentage point of the χ_2^2 distribution, it indicates an unnatural usage behavior contrary to the expected behavior specified by the null hypothesis.

4. PERFORMANCE COMPARISON

To compare the efficiency of the proposed methods to detect decrease or increase of usage rate, well-known performance criteria, "i.e.," probability of detection and ARL are used. Two different situations are discussed. In the first situation, marketing manager of a subscription based service provider is interested in monitoring periodically the number of customers who repeat their purchase or the number of visitors who renew their subscription. Since there are some intrinsic differences among customers' intention to buy again, it is not reasonable for monitoring purposes to employ the common *p* chart which is used conventionally with the assumption of a homogenous population. Instead, case-adjusted *p* chart, case-adjusted CUSUM chart for the binomial distribution and GLR method based on the beta-binomial model are used in this study. The second situation involves a different measure for monitoring usage behavior. In this situation, the number of purchases made by a customer or the number of site visits made by a visitor over a given time interval are of interest. Traditional SPC charts like *c* and *u* that are used for count attribute characteristics are not applicable in this case because of the underlying heterogeneity of usage rate across customers. Case-adjusted *u* chart, case-adjusted CUSUM chart for the Poisson distribution and GLR method based on the gamma-Poisson model are alternative methods whose abilities in detection of shifts in customers' usage behavior are discussed and compared below.

In the first case, consider that there are 200 units of data sampled periodically to monitor usage probability. To implement the simulation procedure it is assumed that the change occurs at 25th time period after start of monitoring. The information collected during first 25 periods can be used to estimate the model parameters. It should be noticed that a two sided CUSUM chart whose log-likelihood weights are computed based on increasing and decreasing shifts equal to 2 and 0.5 is used here. Moreover, control limits obtained through simulation are set at h^+=3.5 and h^-=-3.5.

Table 1 indicates the probability of detection computed by simulation with 250 replications. According to equation (6), the multiplier δ in Table 1 indicates the magnitude of the shift changing odds ratio and varying in the range of 0.25 to 6 times as much as the in-control value. To provide this table, it is assumed that the usage behavior of all customers is equally affected by an overall change.

For example, consider the change of size 0.5. Case-adjusted *p* chart detects this change at first point with probability 0.480 and, furthermore it can detect this change before the second period with the probability 0.716. The correspondent quantities for case-adjusted CUSUM and GLR charts are equal to 0.552, 0.9 and 0.204, 0.328 respectively. This simple comparison indicates the preeminence of CUSUM chart when compared with other approaches for detecting the change. It should be noticed that in order to compare control charts with respect to probability of detection, it is necessary to equalize their false alarm rate. The false alarm rate is fixed at 1 percentage in this study as shown in the row corresponding to δ=1. Generally, concerning the probability of detection, it is concluded that case-adjusted CUSUM chart possesses the best capability to detect decreasing shifts in odds ratio while for increasing shifts the *p* chart and CUSUM chart equally outperform the GLR chart.

ARL is another useful measure to compare control charts. The ARL is usually defined as the expected number of periods required before a point exceeds the control limits. Tables 4 and 5 present this measure along with the standard deviation of run length (SDRL) for different amounts of shifts in odds ratio. These results indicate that CUSUM chart is preferred over other approaches for detection of decreasing shifts but for increasing shifts CUSUM and *p* chart have similar performance with a negligible difference.

The second situation involves a hypothetical case on monitoring the usage rate. In this situation, the frequency of usage is assumed to follow Poisson distribution. The details of data generation are the same as the previous situation except for the fact that five hundred customers are sampled at each period. It is remarkable that for sample sizes less than 200 the asymptotic chi square distribution of GLR statistic discussed in subsection 3.3 cannot be reached easily.

Tables 6, 7, and 8 contain probability of detection before specified time periods after shift, when the mean of usage rate changes from the in-control value of λ_0 to a new value of λ_1. The results are calculated through simulation. It is mentioned again that the shift size which is specified as δ is considered equally across customers. The comparison between the first columns of these three tables indicates that there is a slight difference between correspondent values of Tables 6 and 7. This implies a rather identical efficiency of case-adjusted *u* chart and case-adjusted CUSUM which are used to detect shifts in the mean of usage rate. However, studying the first column of Table 8 indicates the observable weakness of GLR chart in contrast to the first two approaches.

Average and standard deviation of run lengths are reported respectively in Tables 9 and 10. Examining Table 9 infers the same conclusion about the equivalent ability of the *u* chart and CUSUM chart. Regarding an increasing shift, in contrast to a decreasing shift, GLR chart detects the change in usage rate more slowly. From a computational point of view, the process of estimating the parameters which are required to run the GLR procedure is a time-intensive work.

Table 1 Case-adjusted p chart: Probability of detecting a multiplicative shift of size δ in odds ratio

δ	$\Pr(T\text{-}\tau \le 1)$	$\Pr(T\text{-}\tau \le 2)$	$\Pr(T\text{-}\tau \le 3)$	$\Pr(T\text{-}\tau \le 4)$	$\Pr(T\text{-}\tau \le 5)$	$\Pr(T\text{-}\tau \le 6)$	$\Pr(T\text{-}\tau \le 7)$	$\Pr(T\text{-}\tau \le 8)$	$\Pr(T\text{-}\tau \le 9)$	$\Pr(T\text{-}\tau \le 10)$
0.25	0.992	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	0.480	0.716	0.820	0.888	0.948	0.964	0.976	0.996	1.000	1.000
0.75	0.040	0.096	0.132	0.168	0.220	0.256	0.312	0.356	0.384	0.408
0.9	0.024	0.024	0.036	0.040	0.044	0.068	0.076	0.088	0.100	0.116
1	0.004	0.024	0.036	0.044	0.052	0.060	0.068	0.084	0.096	0.112
1.2	0.060	0.116	0.168	0.216	0.232	0.268	0.288	0.340	0.368	0.388
1.5	0.232	0.380	0.516	0.628	0.708	0.772	0.828	0.864	0.888	0.928
1.75	0.516	0.768	0.888	0.948	0.976	0.984	0.996	0.996	1.000	1.000
2	0.796	0.944	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.5	0.980	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 2 Case-adjusted CUSUM chart: Probability of detecting a multiplicative shift of size $\boldsymbol{\delta}$ in odds ratio

δ	$\Pr(T\text{-}\tau \le 1)$	$\Pr(T\text{-}\tau \le 2)$	$\Pr(T\text{-}\tau \le 3)$	$\Pr(T\text{-}\tau \le 4)$	$\Pr(T\text{-}\tau \le 5)$	$\Pr(T\text{-}\tau \le 6)$	$\Pr(T\text{-}\tau \le 7)$	$\Pr(T\text{-}\tau \le 8)$	$\Pr(T\text{-}\tau \le 9)$	$\Pr(T\text{-}\tau \le 10)$
0.25	0.992	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	0.552	0.900	0.960	0.984	0.996	0.996	1.000	1.000	1.000	1.000
0.75	0.068	0.160	0.244	0.352	0.456	0.516	0.576	0.632	0.672	0.716
0.9	0.032	0.044	0.056	0.076	0.084	0.116	0.144	0.164	0.184	0.204
1	0.010	0.036	0.044	0.048	0.056	0.060	0.068	0.084	0.100	0.124
1.2	0.044	0.096	0.136	0.172	0.200	0.228	0.248	0.280	0.304	0.320
1.5	0.148	0.364	0.536	0.676	0.788	0.848	0.896	0.916	0.932	0.960
1.75	0.424	0.764	0.908	0.984	1.000	1.000	1.000	1.000	1.000	1.000
2	0.748	0.944	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.5	0.976	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 3 GLR chart: Probability of detecting a multiplicative shift of size δ in odds ratio

δ	Pr(T-τ ≤1)	Pr(T-τ ≤2)	Pr(T-τ ≤3)	Pr(T-τ ≤4)	Pr(T-τ ≤5)	Pr(T-τ ≤6)	Pr(T-τ ≤7)	Pr(T-τ ≤8)	Pr(T-τ ≤9)	Pr(T-τ ≤10)
0.25	0.852	0.956	0.992	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	0.204	0.328	0.420	0.512	0.600	0.640	0.684	0.736	0.748	0.788
0.75	0.016	0.056	0.088	0.112	0.148	0.176	0.216	0.240	0.260	0.276
0.9	0.012	0.016	0.024	0.024	0.032	0.060	0.064	0.068	0.076	0.080
1	0.008	0.020	0.020	0.032	0.040	0.048	0.052	0.060	0.072	0.100
1.2	0.036	0.044	0.060	0.084	0.092	0.116	0.116	0.140	0.148	0.152
1.5	0.036	0.088	0.140	0.204	0.268	0.332	0.388	0.412	0.448	0.488
1.75	0.160	0.296	0.400	0.508	0.544	0.568	0.612	0.672	0.720	0.756
2	0.344	0.540	0.700	0.776	0.840	0.860	0.876	0.904	0.924	0.936
2.5	0.640	0.816	0.908	0.936	0.956	0.964	0.980	0.984	0.992	1.000
3	0.788	0.944	0.980	0.996	0.996	1.000	1.000	1.000	1.000	1.000
4	0.956	0.996	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 4 Average run length of control charts: case-adjusted p (CA p) chart, case-adjusted CUSUM (CA CUSUM) chart and GLR chart

Control Chart	δ= 0.25	δ= 0.5	δ= 0.75	δ= 0.9	δ= 1	δ= 1.2	δ= 1.5	δ= 1.75	δ= 2	δ= 2.5	δ= 3	δ= 4	δ= 6
CA u	1.010	2.089	16.756	78.653	103.610	21.382	3.954	1.949	1.321	1.035	1.003	1.000	1.000
CA CUSUM	1.007	1.638	8.306	45.654	101.690	24.949	3.796	1.956	1.379	1.061	1.005	1.000	1.000
GLR	1.045	38.705	81.110	83.690	99.025	29.240	4.855	2.090	1.360	1.020	1.015	1.000	1.000

Table 5 Standard deviation of run length

Control Chart	δ= 0.25	δ= 0.5	δ= 0.75	δ= 0.9	δ= 1	δ= 1.2	δ= 1.5	δ= 1.75	δ= 2	δ= 2.5	δ= 3	δ= 4	δ= 6
CA u	0.100	1.573	16.609	79.295	105.940	20.480	3.518	1.355	0.661	0.184	0.055	0.000	0.000
CA CUSUM	0.083	0.822	7.887	42.351	102.780	23.667	3.014	1.178	0.628	0.239	0.071	0.000	0.000
GLR	0.208	89.212	124.760	113.710	100.219	66.229	12.344	2.462	0.982	0.140	0.122	0.000	0.000

Table 6 Case-adjusted u chart: Probability of detecting a multiplicative shift of size δ in usage rate,

δ	$Pr(T\text{-}\tau \le 1)$	$Pr(T\text{-}\tau \le 2)$	$Pr(T\text{-}\tau \le 3)$	$Pr(T\text{-}\tau \le 4)$	$Pr(T\text{-}\tau \le 5)$	$Pr(T\text{-}\tau \le 6)$	$Pr(T\text{-}\tau \le 7)$	$Pr(T\text{-}\tau \le 8)$	$Pr(T\text{-}\tau \le 9)$	$Pr(T\text{-}\tau \le 10)$
0.9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.925	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.95	0.822	0.969	0.994	0.999	1.000	1.000	1.000	1.000	1.000	1.000
0.975	0.218	0.359	0.484	0.578	0.651	0.721	0.780	0.818	0.852	0.885
1	0.014	0.024	0.031	0.039	0.049	0.059	0.068	0.076	0.087	0.097
1.05	0.817	0.961	0.994	0.999	1.000	1.000	1.000	1.000	1.000	1.000
1.075	0.992	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 7 Case-adjusted CUSUM chart: Probability of detecting a multiplicative shift of size δ in usage rate

δ	$Pr(T\text{-}\tau \le 1)$	$Pr(T\text{-}\tau \le 2)$	$Pr(T\text{-}\tau \le 3)$	$Pr(T\text{-}\tau \le 4)$	$Pr(T\text{-}\tau \le 5)$	$Pr(T\text{-}\tau \le 6)$	$Pr(T\text{-}\tau \le 7)$	$Pr(T\text{-}\tau \le 8)$	$Pr(T\text{-}\tau \le 9)$	$Pr(T\text{-}\tau \le 10)$
0.9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.925	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.95	0.796	0.979	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.975	0.211	0.379	0.530	0.650	0.718	0.781	0.839	0.874	0.900	0.928
1	0.012	0.022	0.029	0.037	0.045	0.053	0.061	0.067	0.076	0.088
1.05	0.791	0.966	0.996	0.998	1.000	1.000	1.000	1.000	1.000	1.000
1.075	0.992	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 8 GLR chart: Probability of detecting a multiplicative shift of size δ in usage rate

δ	Pr(T-τ ≤1)	Pr(T-τ ≤2)	Pr(T-τ ≤3)	Pr(T-τ ≤4)	Pr(T-τ ≤5)	Pr(T-τ ≤6)	Pr(T-τ ≤7)	Pr(T-τ ≤8)	Pr(T-τ ≤9)	Pr(T-τ ≤10)
0.9	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.925	0.845	0.950	0.990	0.995	0.995	0.995	0.995	0.995	0.995	0.995
0.95	0.460	0.630	0.770	0.840	0.895	0.925	0.945	0.955	0.965	0.965
0.975	0.080	0.145	0.205	0.260	0.300	0.325	0.375	0.410	0.455	0.480
1	0.010	0.030	0.050	0.050	0.050	0.055	0.080	0.085	0.100	0.105
1.05	0.010	0.010	0.030	0.040	0.050	0.050	0.060	0.070	0.070	0.080
1.075	0.120	0.190	0.260	0.290	0.340	0.380	0.430	0.480	0.530	0.560
1.1	0.420	0.610	0.700	0.770	0.800	0.820	0.850	0.890	0.890	0.900
1.15	0.940	0.990	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 9 Average run length of control charts: case-adjusted u (CA u) chart, case-adjusted CUSUM (CA CUSUM) chart and GLR chart

Control Chart	δ= 0.9	δ= 0.925	δ= 0.95	δ= 0.975	δ= 1	δ= 1.08	δ= 1.09	δ= 1.095	δ= 1.1	δ= 1.11	δ= 1.12
CA u	1.000	1.010	1.180	4.740	96.760	1.000	1.000	1.000	1.000	1.000	1.000
CA CUSUM	1.000	1.010	1.170	3.920	107.620	1.000	1.000	1.000	1.000	1.000	1.000
GLR	1.000	1.040	2.510	29.040	104.714	96.510	87.023	38.390	2.460	1.080	1.020

Table 10 Standard deviation of run length of control charts

Control Chart	δ= 0.9	δ= 0.925	δ= 0.95	δ= 0.975	δ= 1	δ= 1.08	δ= 1.09	δ= 1.095	δ= 1.1	δ= 1.11	δ= 1.12
CA u	0.000	0.100	0.435	5.683	84.739	0.000	0.000	0.000	0.000	0.000	0.000
CA CUSUM	0.000	0.100	0.378	4.009	95.441	0.000	0.000	0.000	0.000	0.000	0.000
GLR	0.000	0.197	2.780	102.250	232.309	220.190	210.203	125.570	4.106	0.339	0.141

Hence, it was preferred to run the simulation procedure with fewer repetitions namely 100 iterations, to obtain the less precise results. Moreover, unlike other two charts, GLR chart could hardly detect a small increase in lambda. Therefore the results are reported for changes greater than 1.08. For the shifts of sizes 1.05 and 1.075, the ARLs of the *u* chart are obtained as 1.266 and 1.009. Corresponding values for CUSUM chart are 1.258 and 1.01. Altogether, the results of this illustrative case indicate the superiority of the *u* and CUSUM charts against the GLR chart when customers' usage rate follows non-homogenous Poisson distribution. In case of the second situation it remains to add that with reference to equations (4) and (10), the log-likelihood weights of upper-sided CUSUM and lower-sided CUSUM are chosen as 1.05 and 0.95 here. Upper and lower control limits were obtained as ± 3.2 through a simulation procedure to achieve approximately the 1 percentage false alarm rate.

5. CONCLUSIONS

Monitoring usage behavior helps managers to have an accurate interpretation of customer behavior. This issue is investigated here for the case of a subscription-based service provider. It is assumed that grouped data are available at the end of each period. Two univariate usage measures were examined separately. Implementation of proposed control charts requires information consisting of the degree of customer's overall satisfaction from preceding period in conjunction with the information about his/her usage at the current period. Whether a customer renews his subscription, and the number of purchases or visits made by a customer during each period determine customer usage at the current period. Using this information, three different approaches are employed in this research to design various control charts which are constructed based on the adjustment to the specification of each customer in the sample.

With the assumption that the parameter of consequent Bernoulli and Poisson distributions are predicted precisely using a regression model, case-adjusted *p* and *u* charts constitutes the first approach. Then case-adjusted CUSUM charts are also applied. For this purpose, weights assigned to each customer in the sample are computed through log-likelihood ratio statistic.

The third approach used in this study involved using mixture probability models. In first model a Bernoulli distribution is used to model customer usage with respect to the subscription renewal and a beta distribution is employed to represent the heterogeneity across customers. On the other hand, the second model uses a Poisson distribution to account for the number of visits at a given period while its parameter is required to follow a gamma distribution to capture the non-homogenous usage rate across customers. Using data of customers' usage behavior the parameters of these two mixture probability models may be estimated through MLE method. Therefore, the method of generalized likelihood ratio (GLR) test can be employed to test the null hypothesis on the parameters of the mixture models.

Section 4 provides a comparative study on the proposed control charts. First consider the problem of monitoring non-homogenous Bernoulli random variables. In this situation, case-adjusted *p* chart and case-adjusted CUSUM equivalently detect increases in odds ratio. However, when a decreasing change occurs in odds ratio, CUSUM chart performs slightly better than *p* chart. For the second situation, monitoring non-homogenous Poisson random variables is of interest. Simulation results indicate that case-adjusted *u* chart and case-adjusted CUSUM are both capable to detect overall changes in the hypothesized value of usage rate very quickly. To summarize, in both situations, case-adjusted Shewhart charts and case-adjusted CUSUM charts performed much better than GLR charts developed based on mixture probability models. Generally, from a computational point of view, the implementation of GLR chart was very time consuming and effort-intensive.

A noteworthy remark is that the impact of precision of case adjustment "i.e.," the effect of estimation errors as well as choosing a set of appropriate regressors in establishing the adjustment equation for predicting usage model parameters are not examined in this study. Steiner et al. (2000) provides interesting conclusions on the consequences of estimating procedure on the efficiency of risk-adjusted Bernoulli CUSUM chart when a logistic regression model is employed to perform adjustment. Another improvement of the proposed monitoring schemes may be resulted from finding a suitable way to take into account the importance weight of each customer. Customer life-time value (CLV) provides a good scoring mechanism which differentiates customers depending on their profitability to the organization. An approach such as demerit control chart discussed in detail in SQC textbooks like Montgomery (2005) seems very useful for this purpose.

REFERENCES

[1] Cook D.A., Steiner S.H., Cook R.J., Farewell V.T., Morton A.P. (2003), Monitoring the evolutionary process of quality: risk-adjusted charting to track outcomes in intensive care; *Critical Care in Medicine* 31; 1676–1682.

[2] Fader P.S., Hardie B.G.S., Lee K.L. (2005), Counting your customers' the easy way: an alternative to the Pareto/NBD model; *Marketing Science* 24; 275–284.

[3] Fader P.S., Hardie B.G.S. (2005), The value of simple models in new product forecasting and customer-base analysis; *Applied Stochastic Models in Business and Industry* 21; 461-473.

[4] Fornell C., Johnson M.D., Anderson E.W., Cha J., Bryant B.E. (1996), The American customer satisfaction index: nature, purpose, and findings; *Journal of Marketing* 60(4); 7-18.

[5] Grigg O., Farewell V.T. (2004), An overview of risk-adjusted charts; *Journal of the Royal Statistical Society, Series A* 167; 523–539.

[6] Gryna F.M. (1999), Market research and marketing. In: Juran J., Godfrey A.B., Juran's quality handbook; 5th Edition, McGraw-Hill; New York.

[7] Hauck D.J., Runger G.C., Montgomery D.C. (1999), Multivariate statistical process monitoring and diagnosis with grouped regression-adjusted variables; *Communications in Statistics: Simulation and Computation* 28; 309–328.

[8] Hawkins D.M. (1993), Regression adjustment for variables in multivariate quality control; *Journal of Quality Technology* 25; 170–182.

[9] Jearkpaporn D., Borror C.M., Runger G.C., Montgomery D.C. (2007), Process monitoring for mean shifts for multiple stage processes; *International Journal of Production Research* 45(23); 5547–5570.

[10] Johnson N.L., Kemp A.W., Kotz S. (2005), Univariate discrete distributions; 3rd edition, John Wiley and Sons, Inc., Hoboken; New Jersey.

[11] Jones T.O., Sasser W.E.Jr. (1995), Why satisfied customers defect; *Harvard Business Review* (November–December); 88–99.

[12] Moe W., Fader P.S. (2004), Capturing evolving visit behavior in clickstream data; *Journal of Interactive Marketing* 18; 5-19.

[13] Montgomery D.C. (2005), Introduction to statistical quality control; 5th edition, John Wiley and Sons, Inc.; New York.

[14] Mood A.M., Graybill F.A., Boes D.C. (1974), Introduction to the theory of statistics; 3rd edition, McGraw-Hill; New York.

[15] Moustakides G.V. (1986), Optimal stopping times for detecting changes in distributions; *The Annals of Statistics* 14; 1379–1387.

[16] Myers R.H., Montgomery D.C., Vining G.C. (2002), Generalized linear models, with applications in engineering and the sciences; John Wiley and Sons, Inc.; New York.

[17] Nadaradjah S., Kotz S. (2009), Models for purchase frequency; *European Journal of Operational Research* 192; 1014-1026.

[18] Schmittlein D.C, Morrison D.G., Colombo R. (1987), Counting your customers: who they are and what will they do next?; *Management Science* 33; 1–24.

[19] Sego L.H. (2006), Applications of control charts in medicine and epidemiology; Ph.D. Thesis, Virginia Polytechnic Institute and State University; Blacksburg, VA.

[20] Skinner K.S., Montgomery D.C., Runger G.C. (2003), Process monitoring for multiple count data using generalized linear model-based control charts; *International Journal of Production Research* 41; 1167–1180.

[21] Steiner S.H., Cook R.J., Farewell V.T. (1999), Monitoring paired binary surgical outcomes using cumulative sum charts; *Statistics in Medicine* 18; 69–86.

[22] Steiner S.H., Cook R.J., Farewell V.T., Treasure T. (2000), Monitoring surgical performance using risk-adjusted cumulative sum charts; *Biostatistics* 1; 441–452.

[23] Wade M.R., Woodall W.H. (1993), A review and analysis of cause-selecting control charts; *Journal of Quality Technology*, 25; 161–169.

[24] Woodall W.H. (2006), The use of control charts in health-care and public-health surveillance (with discussion); *Journal of Quality Technology* 38; 89–104.